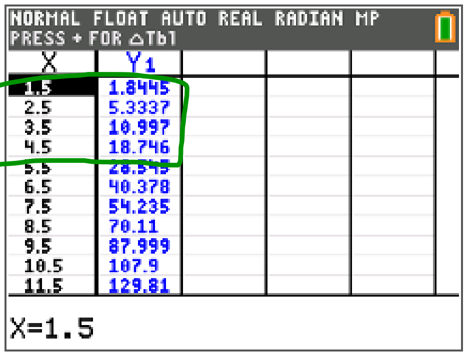
1. Consider the region enclosed between the graph of $f(x) = x^2 - \ln x$ and the x-axis for $1 \le x \le 5$.

(a) Find MRAM_4, the area estimate obtained using 4 midpoint rectangles.

3 decimal places

$[1,2]\,[2,3]\,[3,4]\,[4,5]$

$\frac{3}{2} \quad \frac{5}{2} \quad \frac{7}{2} \quad \frac{9}{2}$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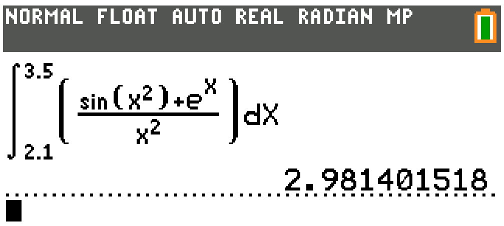
X	Y_1
1.5	1.8445
2.5	5.3337
3.5	10.997
4.5	18.746
5.5	28.545
6.5	40.378
7.5	54.235
8.5	70.11
9.5	87.999
10.5	107.9
11.5	129.81

X=1.5

$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - \ln \frac{3}{2}$

$1(1.8445 + 5.3337 +$
$10.997 + 18.746)$

4. Use NINT to evaluate $\int_{2.1}^{3.5} \frac{\sin(x^2) + e^x}{x^2}\,dx$. $= 2.981$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$\int_{2.1}^{3.5} \left(\frac{\sin(x^2)+e^X}{x^2} \right) dX$

2.981401518

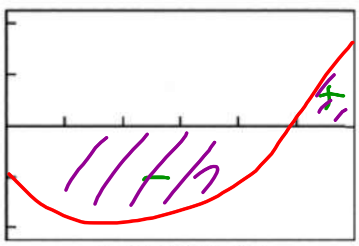
radian mode

8. **(a)** Graph the function $y = 0.2x^2 - 0.8x - 1$ over the interval $[0, 6]$.

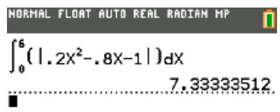
(b) Integrate $y = 0.2x^2 - 0.8x - 1$ over $[0, 6]$.

(c) Find the area of the region between the graph in part (a) and the x-axis.

8. **(a)**



$[0, 6]$ by $[-2.25, 2.25]$



b) $\int_0^6 (0.2x^2 - 0.8x - 1)\,dx = -6$

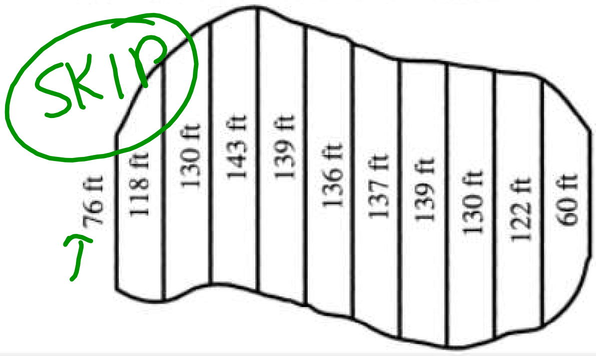
$\left|\int_0^5 f(x)\,dx\right| + \int_5^6 f(x)\,dx$

OR

$\int_0^6 (|0.2x^2 - 0.8x - 1|)\,dx =$

7.333

12. A meadow has the shape shown, where the measurements shown were taken at 30-foot intervals. Use Simpson's Rule to estimate the area of the meadow.



$h = \frac{b-a}{n}$

h = length of each subinterval

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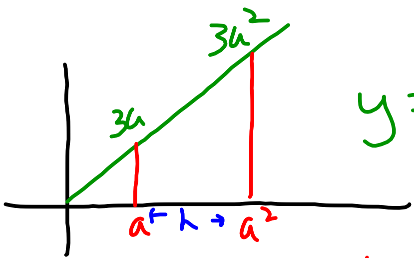
$T = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots y_n)$

$\frac{30}{2}(76 + 2(118) + 2(130) \cdots\cdots 60)$

(3) $\int_a^{a^2} 3x\,dx$



$y = 3x$

trapezoid

$h = a^2 - a$ $\quad \frac{3a^2 + 3a}{2}$

$\int_a^{a^2} 3x\,dx$

$A = (a^2 - a)\left(\frac{3a^2 + 3a}{2}\right)$

$\frac{3}{2}x^2 \Big|_a^{a^2}$

$(a^2 - a)\,\frac{3}{2}(a^2 + a)$

$\frac{3}{2}(a^4 - a^2)$

$\boxed{\frac{3}{2}(a^4 - a^2)}$

5. Suppose that f and g are continuous functions and that

$\int_3^5 f(x)\,dx = 7$, $\int_3^5 g(x)\,dx = 2$, and $\int_0^5 g(x)\,dx = 4$.

Which of the following must be true?

I. $\int_0^3 g(x)\,dx = 2$

II. $\int_3^5 [f(x)g(x)]\,dx = 14$

III. $\int_3^5 [f(x) - g(x)]\,dx = 5$

(A) III only (B) I and II (C) I and III

(D) II and III (E) I, II, and III

$$\underbrace{\int_0^3 g(x)\,dx}_{2} + \underbrace{\int_3^5 g(x)\,dx}_{2} = \underbrace{\int_0^5 g(x)\,dx}_{4}$$

$7 - 2$

6. Evaluate $\int_2^7 (4x - 10)\,dx$.

FTC $F(7) - F(2)$

$$2x^2 - 10x \Big|_2^7$$

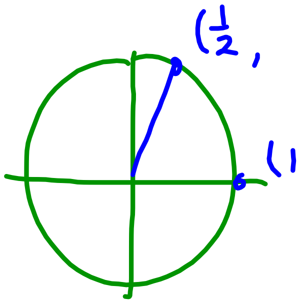
$$2(49) - 70 - (8 - 20)$$

$$98 - 70 - (-12)$$

$$28 + 12 = 40$$

7. Evaluate $\int_0^{\pi/3} \sec x \tan x \, dx$ using Part 2 of the Fundamental Theorem of Calculus.

$$\sec x \Big|_0^{\frac{\pi}{3}}$$



$$\sec \frac{\pi}{3} - \sec 0$$

$$2 - 1 = \boxed{1}$$

9. (a) Let $f(x) = 4x - 9$. Find K so that $\int_{-1}^{x} f(t)\,dt + K = \int_{3}^{x} f(t)\,dt$.

(b) Find $\frac{d}{dx}\int_{-1}^{x} f(t)\,dt$.

$$\int_{-1}^{x} f(t)dt + \int_{3}^{-1} f(t)dt = \int_{3}^{x} f(t)dt$$

$$K = \int_{3}^{-1} f(t)dt$$

$$2t^2 - 9t \Big|_3^{-1}$$

$$(2+9) - (-9)$$

$$11 + 9 = 20$$

9. **(a)** Let $f(x) = 4x - 9$. Find K so that

$$\int_{-1}^{x} f(t)\,dt + K = \int_{3}^{x} f(t)\,dt.$$

$\boxed{\textbf{(b)}\ \text{Find } \dfrac{d}{dx}\displaystyle\int_{-1}^{x} f(t)\,dt.}$ b) $4x-9$ FTC

$$\int_{-1}^{x} f(t)dt + K = \int_{3}^{x} f(t)dt$$

$$\int_{-1}^{x} (4t-9)dt + K = \int_{3}^{x} (4t-9)dt$$

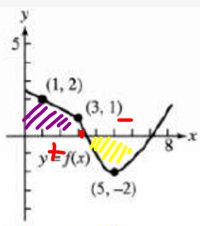
$$2t^2-9t\Big|_{-1}^{x} + K = 2t^2-9t\Big|_{3}^{x}$$

$$\cancel{2x^2} - \cancel{9x} - (2+9) + K = \cancel{2x^2} - \cancel{9x} - (18-27)$$

$$-11 + K = 9$$

$$K = 20$$

10. A particle moves along a coordinate axis. Its position at time t (sec) $\boxed{\text{is } s(t) = \int_0^t f(x)\,dx \text{ cm,}}$ where f is the function whose graph is shown.



f(x) is velocity

(a) What is the particle's position at $t = 0$?
(b) What is the particle's position at $t = 3$?
(c) What is the particle's velocity at $t = 5$?
(d) Approximately when is the acceleration zero?
(e) At what time during the first 7 sec does s have its largest value?

d) at 5 sec

e) at 3.5 sec

a) $s(0) = \int_0^0 f(x)dx = 0$

b) $s(3) =$ trapezoid $A = h\left(\frac{b_1+b_2}{2}\right)$

$= 3\left(\frac{1+2.5}{2}\right)$

$= \frac{3}{2}\left(\frac{7}{2}\right) = \boxed{\frac{21}{4}}$ cm

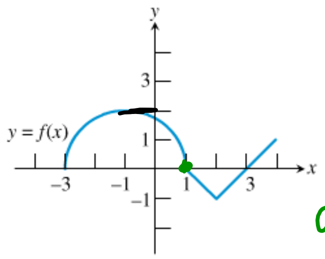
c) velocity at $t=5$

-2 cm/sec

54. The graph of a function f consists of a semicircle and two line segments as shown below.

$c)\ g(-1) = \frac{1}{4}(\pi)(2^2)$

$= -\pi$



$a)\ g(1) = \int_1^1 f(t)\,dt = 0$

Let $g(x) = \int_1^x f(t)\,dt$.

(a) Find $g(1)$.

(b) Find $g(3)$.

(c) Find $g(-1)$.

(d) Find all values of x on the open interval $(-3, 4)$ at which g has a relative maximum.

$x=1$

(e) Write an equation for the line tangent to the graph of g at $x = -1$.

$b\ \ g(3) = \int_1^3 f(t)\,dt$

$= -1$

Point + slope

$(-1, -\pi)$

$m = 2$

$\boxed{y + \pi = 2(x+1)}$

Approximations with Unequal Subintervals

When dealing with real-world data, the subdivisions, or subintervals, are often of unequal length. On recent AP Calculus Exams, problems such as these have been particularly difficult for students. This may be because students view Riemann and trapezoidal sums as rules rather than concepts, which is what these sums actually represent. Consider the following example in which the values of a function are given at unequal subintervals.

x	0	2	3	7	9
$f(x)$	3	6	7	6	8

[0,2] [2,3] [3,7] [7,9]

$2(4.5) + 1(6.5) + 4(6.5) + 2(7)$

$9 + 6.5 + 26 + 14 \qquad 49 + 6.5 = 55.5$

11. Use the Trapezoidal Rule with $n = 4$ to approximate the value of $\int_0^2 (x^2 - 4x + 4)\,dx$.

$y = x^2 - 4x + 4$

table st 0

$\Delta .5$

$[0, \tfrac{1}{2}]\ [\tfrac{1}{2}, 1]\ [1, \tfrac{3}{2}]\ [\tfrac{3}{2}, 2]$

$\frac{1}{2}\left(\frac{4+2.25}{2}\right) + \frac{1}{2}\left(\frac{2.25+1}{2}\right)$

$+\frac{1}{2}\left(\frac{1+.25}{2}\right) + \frac{1}{2}\left(\frac{.25+0}{2}\right)$

$\frac{1}{4}(4+2.25+2.25+1+1+.5)$

$\frac{6+4.5+.5}{4} = \boxed{\frac{11}{4}}$

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X	Y_1
0	4
.5	2.25
1	1
1.5	.25
2	0
2.5	.25
3	1
3.5	2.25
4	4
4.5	6.25
5	9

X=0