

CHAPTER
6

Probability: What are the Chances?

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3	Idea of probability/Myths	D
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13	Simulation to estimate probability	E
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MULTIPLE
CHOICE
PRACTICE
PROBLEMS

TO RECEIVE CREDIT

① CROSS OUT AT
LEAST 2 ANSWERS
THAT ARE
OBVIOUSLY
INCORRECT

② SHOW WORK
WHEN NECESSARY

49	Conditional probability formula	A
50	Venn diagrams	C
51	Venn diagrams	A
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54	Probabilities from tree diagram	A
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58	Conditional probability from 2-way table	A
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60	Tree diagram from probabilities	A
61	Venn diagrams	D
62	Venn diagrams	D
63	Conditional probability from 2-way table	D
64	Conditional probability from 2-way table	D
65	Conditional probability from 2-way table	E
66	Venn diagrams	E
67	Venn diagrams	D
68	Venn diagrams	B
69	Conditional probability from 2-way table	E
70	Conditional probability from 2-way table	D
71	Conditional probability from 2-way table	C

1. I toss a penny and observe whether it lands heads up or tails up. Suppose the penny is fair, i.e., the probability of heads is $1/2$ and the probability of tails is $1/2$. This means that

- ~~A) every occurrence of a head must be balanced by a tail in one of the next two or three tosses.~~
- ~~B) if I flip the coin 10 times, it would be almost impossible to obtain 7 heads and 3 tails.~~
- C) if I flip the coin many, many times the proportion of heads will be approximately $1/2$, and this proportion will tend to get closer and closer to $1/2$ as the number of tosses increases.**
- ~~D) regardless of the number of flips, half will be heads and half tails.~~
- ~~E) all of the above.~~

ANS: C

TOP: Idea of probability

2. If the individual outcomes of a phenomenon are uncertain, but there is nonetheless a regular distribution of outcomes in a large number of repetitions, we say the phenomenon is

- A) random.**
- B) predictable.
- C) uniform.
- D) probable.
- E) normal.

ANS: A

TOP: Idea of randomness

3. When two coins are tossed, the probability of getting two heads is 0.25. This means that

- ~~A) of every 100 tosses, exactly 25 will have two heads.~~
- ~~B) the odds against two heads are 4 to 1.~~
- ~~C) in the long run, the average number of heads is 0.25.~~
- D) in the long run two heads will occur on 25% of all tosses.**
- ~~E) if you get two heads on each of the first five tosses of the coins, you are unlikely to get heads the fourth time.~~

ANS: D

TOP: Idea of probability/Myths

4. If I toss a fair coin 5000 times

- ~~A) and I get anything other than 2500 heads, then something is wrong with the way I flip coins.~~
- B) the proportion of heads will be close to 0.5**
- ~~C) a run of 10 heads in a row will increase the probability of getting a run of 10 tails in a row.~~
- ~~D) the proportion of heads in these tosses is a parameter~~
- ~~E) the proportion of heads will be close to 50.~~

ANS: B

TOP: Idea of probability/Myths

5. You read in a book on poker that the probability of being dealt three of a kind in a five-card poker hand is $1/50$. What does this mean?

- A) If you deal thousands of poker hands, the fraction of them that contain three of a kind will be very close to $1/50$.**
- ~~B) If you deal 50 poker hands, then one of them will contain three of a kind.~~
- ~~C) If you deal 10,000 poker hands, then 200 of them will contain three of a kind.~~
- ~~D) A probability of 0.02 is somebody's best guess for a probability of being dealt three of a kind.~~
- ~~E) It doesn't mean anything, because $1/50$ is just a number.~~

ANS: A

TOP: Idea of probability/Myths

6. A basketball player makes 160 out of 200 free throws. We would estimate the probability that the player makes his next free throw to be

- A) 0.16.
- B) 50-50; either he makes it or he doesn't.
- C) 0.80.
- D) 1.2.
- E) 80.

$$160/200 = .8$$

ANS: C

TOP: Idea of probability/Myths

7. In probability and statistics, a *random* phenomenon is

- A) ~~something~~ that is ~~completely unexpected~~ or surprising
- B) ~~something~~ that has a limited set of outcomes, but when each outcome occurs is completely unpredictable.
- C) something that appears unpredictable, but each individual outcome ~~can be accurately predicted~~ with appropriate mathematical or computer modeling.
- D) something that is unpredictable from one occurrence to the next, but over the course of many occurrences follows a predictable pattern
- E) something whose outcome ~~defies description~~.

ANS: D

TOP: Idea of randomness

8. You are playing a board game with some friends that involves rolling two six-sided dice. For eight consecutive rolls, the sum on the dice is 6. Which of the following statements is true?

- A) Each time you roll another 6, the probability of getting yet another 6 on the next roll goes down.
- B) Each time you roll another 6, the probability of getting yet another 6 on the next roll goes up.
- C) You should find another set of dice: eight consecutive 6's is impossible with fair dice.
- D) The probability of rolling a 6 on the ninth roll is the same as it was on the first roll.
- E) None of these statements is true.

ANS: D

TOP: Probability Myths

9. A poker player is dealt poor hands for several hours. He decides to bet heavily on the last hand of the evening on the grounds that after many bad hands he is due for a winner.

- A) He's right, because the winnings have to average out.
- B) He's wrong, because successive deals are independent of each other.
- C) He's right, because successive deals are independent of each other.
- D) He's wrong, because he's clearly on a "cold streak."
- E) Whether he's right or wrong depends on how many bad hands he's been dealt so far.

ANS: B

TOP: Probability Myths

10. You want to use simulation to estimate the probability of getting exactly one head and one tail in two tosses of a fair coin. You assign the digits 0, 1, 2, 3, 4 to heads and 5, 6, 7, 8, 9 to tails. Using the following random digits to execute as many simulations as possible, what is your estimate of the probability?

19226 95034 05756 07118

$$H: 0-4 = 5/10 = 1/2$$

$$T: 5-9 = 5/10 = 1/2$$

- A) 1/20
- B) 1/10
- C) 5/10
- D) 6/10
- E) 2/3

$$6/10$$

expanded to show simulation

ANS: D

TOP: Simulation to estimate probability

SIMULATE GAME

- We flip 2 coins
- We roll 2 dice
- So pair the random #'s

19 HT	22 HH	69 TT	50 TH	34 HH	05 HT	75 TT	60 TH	71 TH	18 HT
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88

- ASSIGN H(0-4), T(5-9)
- COUNT OUTCOMES OF TH or HT - This happens 6 out of 10 times

11. A box has 10 tickets in it, two of which are winning tickets. You draw a ticket at random. If it's a winning ticket, you win. If not, you get another chance, as follows: your losing ticket is replaced in the box by a winning ticket (so now there are 10 tickets, as before, but 3 of them are winning tickets). You get to draw again, at random. Which of the following are legitimate methods for using simulation to estimate the probability of winning?

I. Choose, at random, a two-digit number. If the first digit is 0 or 1, you win on the first draw; If the first digit is 2 through 9, but the second digit is 0, 1, or 2, you win on the second draw. Any other two-digit number means you lose.

II. Choose, at random, a one-digit number. If it is 0 or 1, you win. If it is 2 through 9, pick a second number. If the second number is 8, 9, or 0, you win. Otherwise, you lose.

III. Choose, at random, a one-digit number. If it is 0 or 1, you win on the first draw. If it is 2, 3, or 4, you win on the second draw; If it is 5 through 9, you lose.

A) I only

B) II only

C) III only

D) I and II

E) I, II, and III

ANS: D

TOP: Simulation to estimate probability

12. A basketball player makes $\frac{2}{3}$ of his free throws. To simulate a single free throw, which of the following assignments of digits to making a free throw are appropriate?

I. 0 and 1 correspond to making the free throw and 2 corresponds to missing the free throw.

II. 01, 02, 03, 04, 05, 06, 07, and 08 correspond to making the free throw and 09, 10, 11, and 12 correspond to missing the free throw.

III. Use a die and let 1, 2, 3, and 4 correspond to making a free throw while 5 and 6 correspond to missing a free throw.

A) I only

B) II only

C) III only

D) I and III

E) I, II, and III

ANS: E

TOP: Simulation to estimate probability

13. A basketball player makes 75% of his free throws. We want to estimate the probability that he makes 4 or more free throws out of 5 attempts (we assume the shots are independent). To do this, we use the digits 1, 2, and 3 to correspond to making the free throw and the digit 4 to correspond to missing the free throw. If the table of random digits begins with the digits below, how many free throw does he hit in our first simulation of five shots?

192289563458301

MAKES 5

A) 1

B) 2

C) 3

D) 4

E) 5

ANS: E

TOP: Simulation to estimate probability

1 9 2 2 3 9 4 6 3 MAKE
MAKE MAKE MAKE MAKE

4 shots { 1-3 MAKE FREE THROW
4 MISSES
0, 5-9 IGNORE

Use the following to answer questions 14 – 15:

Scenario 5-1

To simulate a toss of a coin we let the digits 0, 1, 2, 3, and 4 correspond to a head and the digits 5, 6, 7, 8, and 9 correspond to a tail. Consider the following game: We are going to toss the coin until we either get a head or we get two tails in a row, whichever comes first. If it takes us one toss to get the head we win \$2, if it takes us two tosses we win \$1, and if we get two tails in a row we win nothing. Use the following sequence of random digits to simulate this game as many times as possible:

12975 13258 45144

H \$2
TH \$1
TT \$0

14. Scenario 5-1. Based on your simulation, the estimated probability of winning \$2 in this game is

- A) 1/4.
- B) 5/15.
- C) 7/15.
- D) 9/15.
- E) 7/11.

ANS: E

TOP: Simulation to estimate probability

WIN \$2
7 Times
played 11 times

\$2	\$2	\$0	\$1	\$2	\$2	\$0	\$2	\$1	\$2	\$2
1	2	97	51	3	2	58	4	51	4	4
H	H	TT	TH	H	H	TT	H	TH	TH	H

15. Scenario 5-1. Based on your simulation, the estimated probability of winning nothing is

- A) 1/2.
- B) 2/11.
- C) 2/15.
- D) 6/15.
- E) 7/11.

ANS: B

TOP: Simulation to estimate probability

LOST TWICE
OUT OF 11

16. The collection of all possible outcomes of a random phenomenon is called

- A) a census.
- B) the probability.
- C) a chance experiment
- D) the sample space.
- E) the distribution.

ANS: D

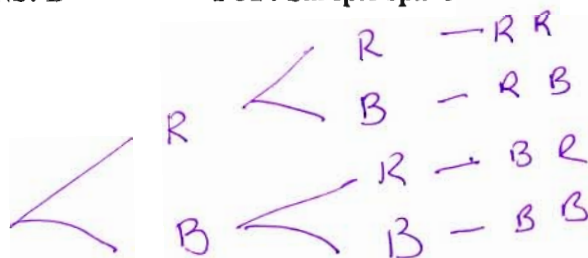
TOP: Sample space

17. I select two cards from a deck of 52 cards and observe the color of each (26 cards in the deck are red and 26 are black). Which of the following is an appropriate sample space S for the possible outcomes?

- A) $S = \{\text{red, black}\}$
- B) $S = \{(\text{red, red}), (\text{red, black}), (\text{black, red}), (\text{black, black})\}$, where, for example, (red, red) stands for the event "the first card is red and the second card is red."
- C) $S = \{(\text{red, red}), (\text{red, black}), (\text{black, black})\}$, where, for example, (red, red) stands for the event "the first card is red and the second card is red."
- D) $S = \{0, 1, 2\}$.
- E) All of the above.

ANS: B

TOP: Sample space



18. A basketball player shoots 8 free throws during a game. The sample space for counting the number she makes is

A) $S =$ any number between 0 and 1.

B) $S =$ whole numbers 0 to 8.

C) $S =$ whole numbers ~~X~~ to 8.

D) $S =$ all sequences of 8 hits or misses, like HMMHHHMH.

E) $S = \{ \text{HMMMMMMM, MHMMMMMM, MMHMMMMM, MMMHMMMM, MMMMHMMM, MMMMMHMM, MMMMMMHM, MMMMMMMH} \}$

ANS: B

TOP: Sample space

Count
made

0 miss all shots
& makes all shots

19. A game consists of drawing three cards at random from a deck of playing cards. You win \$3 for each red card that is drawn. It costs \$2 to play. For one play of this game, the sample space S for the net amount you win (after deducting the cost of play) is

A) $S = \{ \$0, \$1, \$2, \$3 \}$

B) $S = \{ -\$6, -\$3, \$0, \$6 \}$

C) $S = \{ -\$2, \$1, \$4, \$7 \}$

D) $S = \{ -\$2, \$3, \$6, \$9 \}$

E) $S = \{ \$0, \$3, \$6, \$9 \}$

ANS: C

TOP: Sample space

0 Red = $0 - 2 = -\$2$
1 Red = $3 - 2 = \$1$
2 Red = $6 - 2 = \$4$
3 Red = $9 - 2 = \$7$

20. Suppose there are three cards in a deck, one marked with a 1, one marked with a 2, and one marked with a 5. You draw two cards at random and without replacement from the deck of three cards. The sample space $S = \{ (1, 2), (1, 5), (2, 5) \}$ consists of these three equally likely outcomes. Let X be the sum of the numbers on the two cards drawn. Which of the following is the correct set of probabilities for X ?

A)

X	P(X)
1	1/3
2	1/3
5	1/3

B)

X	P(X)
3	1/3
6	1/3
7	1/3

C)

X	P(X)
3	3/16
6	6/16
7	7/16

D)

X	P(X)
3	1/4
6	1/2
7	1/2

E)

X	P(X)
1	1/4
2	1/2
5	1/2

ANS: B

TOP: Sample space

21. An assignment of probabilities must obey which of the following?

✓ A) The probability of any event must be a number between 0 and 1, inclusive.

✓ B) The sum of all the probabilities of all outcomes in the sample space must be exactly 1.

✓ C) The probability of an event is the sum of the probabilities of outcomes in the sample space in which the event occurs.

✓ D) All three of the above.

E) A and B only.

ANS: D

TOP: Basic Probability Rules

22. Event A has probability 0.4. Event B has probability 0.5. If A and B are disjoint, then the probability that both events occur is

A) 0.0.

B) 0.1.

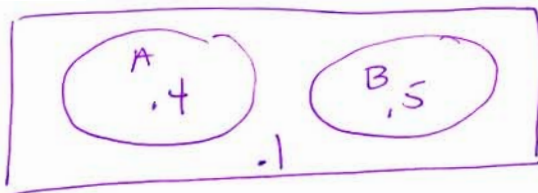
C) 0.2.

D) 0.7.

E) 0.9.

ANS: A

TOP: Addition of disjoint events



23. Event A has probability 0.4. Event B has probability 0.5. If A and B are independent, then the probability that both events occur is

A) 0.0.

B) 0.1.

C) 0.2.

D) 0.7.

E) 0.9.

ANS: C

$$P(A \cap B) = (.4)(.5)$$

TOP: Multiplication Rule, Independent events

Use the following to answer questions 24 – 26:

Scenario 5-2

If you draw an M&M candy at random from a bag of the candies, the candy you draw will have one of six colors. The probability of drawing each color depends on the proportion of each color among all candies made. The table below gives the probability that a randomly chosen M&M had each color before blue M & M's replaced tan in 1995.

Color	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	0.1

= 1

24. Use Scenario 5-2. The probability of drawing a yellow candy is

A) 0.

B) .1.

C) .2.

D) .3.

E) impossible to determine from the information given.

ANS: C

TOP: Basic Probability Rules

The sum must equal 1

25. Use Scenario 5-2. The probability that you do not draw a red candy is

A) .2.

B) .3.

C) .7.

D) .8.

E) impossible to determine from the information given.

ANS: D

TOP: Complement rule

$$1 - .2 = .8$$

26. Use Scenario 5-2. The probability that you draw either a brown or a green candy is

A) .1.

B) .3.

C) .4.

D) .6.

E) .7.

ANS: C

TOP: Addition of disjoint events

$$.3 + .1 = .4$$

27. Here is an assignment of probabilities to the face that comes up when rolling a die once:

Outcome	1	2	3	4	5	6
Probability	1/7	2/7	0	3/7	0	1/7

$$= 7/7 = 1$$

Which of the following is true?

- Dumb Problem*
- A) This isn't a legitimate assignment of probability, because every face of a die must have probability 1/6.
 - B) This isn't a legitimate assignment of probability, because it gives probability zero to rolling a 3 or a 5. *So why have*
 - C) This isn't a legitimate assignment of probability, because the probabilities do not add to exactly 1. *ADD TO 1*
 - D) This isn't a legitimate assignment of probability, because we must actually roll the die many times to learn the true probabilities.
 - E) This is a legitimate assignment of probability.

ANS: E

TOP: Basic Probability Rules

28. Students at University X must have one of four class ranks—freshman, sophomore, junior, or senior. At University X, 35% of the students are freshmen and 30% are sophomores. If a University X student is selected at random, the probability that he or she is either a junior or a senior is

- A) 30%.
- B) 35%.
- C) 65%.
- D) 70%.
- E) 89.5%.

$$1 - .35 - .3 = .35$$

ANS: B

TOP: Addition of disjoint events

29. If the knowledge that an event A has occurred implies that a second event B cannot occur, the events A and B are said to be

- A) independent.
- B) disjoint. *Ha! Ha!*
- C) mutually exhaustive.
- D) the sample space.
- E) complementary.

ANS: B

TOP: Mutually exclusive events

Use the following for questions 30 – 32:

Scenario 5-3

Ignoring twins and other multiple births, assume that babies born at a hospital are independent random events with the probability that a baby is a boy and the probability that a baby is a girl both equal to 0.5.

30. Use Scenario 5-3. The probability that the next five babies are girls is

- A) 1.0.
- B) 0.5.
- C) 0.1.
- D) 0.0625.
- E) 0.03125.

$$(.5)^5$$

ANS: E

TOP: Multiplication Rule, Independent events

31. Use Scenario 5-3. The probability that at least one of the next three babies is a boy is

- A) 0.125.
- B) 0.333.
- C) 0.667.
- D) 0.750.
- E) 0.875.

$$P(B) = 1/2$$

$$1 - (1/2)^3 = .875$$

ANS: E

TOP: Complement rule

32. Use Scenario 5-3. The events A = the next two babies are boys, and B = the next two babies are girls are

- A) disjoint.
- B) conditional.
- C) independent.
- D) complementary.
- E) one of the above.

ANS: A

TOP: Mutually exclusive events

33. Event A occurs with probability 0.3. If event A and B are disjoint, then

- A) $P(B) \leq 0.3$.
- B) $P(B) \geq 0.3$.
- C) $P(B) \leq 0.7$.
- D) $P(B) \geq 0.7$.
- E) $P(B) = 0.21$.

$$P(A) = .3$$

$$P(B) = 1 - .3 = .7 \text{ since disjoint } B \text{ must be less than } .7.$$

ANS: C

TOP: Mutually exclusive events

34. A stack of four cards contains two red cards and two black cards. I select two cards, one at a time, and do not replace the first card selected before selecting the second card. Consider the events

A = the first card selected is red
 B = the second card selected is red

4 cards $\begin{cases} 2 \text{ RED} \\ 2 \text{ BLACK} \end{cases}$

The events A and B are

- A) independent and disjoint.
- B) not independent, but disjoint.
- C) independent, not disjoint
- D) not independent, not disjoint.
- E) independent, but we can't tell it's disjoint without further information.

NOT INDEPENDENT: B/C DID NOT REPLACE

NOT DISJOINT - COULD GET A RED ON BOTH 1ST + 2ND DRAWS

ANS: D

TOP: Independent and mutually exclusive events

35. Which of the following statements is not true?

- A) If two events are mutually exclusive, they are not independent. T
- B) If two events are mutually exclusive, then $P(A \cap B) = 0$ T
- C) If two events are independent, then they must be mutually exclusive. F
- D) If two events, A and B , are independent, then $P(A) = P(A|B)$ T

~~E) All four statements above are true.~~

ANS: C

TOP: Independent and mutually exclusive events



BADLY
WORDED

36. In a certain town, 60% of the households have broadband internet access, 30% have at least one high-definition television, and 20% have both. The proportion of households that have neither broadband internet or high-definition television is:

- A) 0%.
 - B) 10%.
 - C) 30%.**
 - D) 80%.
 - E) 90%.
- ANS: C

$$1 - P(A \cup B) = 1 - (.60 + .30 - .20) = .30$$

TOP: General addition rule

37. Suppose that A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.4$.

- $P(A \cup B)$ is: $= P(A) + P(B) - P(A \cap B)$
 $= .2 + .4 - (.2)(.4)$
- A) 0.08.
 - B) 0.12.
 - C) 0.44.
 - D) 0.52.**
 - E) 0.60.
- ANS: D

TOP: General addition rule (and multiplication of indep. events)

38. Suppose that A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.4$.

- $P(A \cap B^c)$ is $(.2)(.8) = .12$
- A) 0.08.
 - B) 0.12.**
 - C) 0.40.
 - D) 0.52.
 - E) 0.60.
- ANS: B

TOP: Multiplication Rule, Independent events; Complement

Use the following to answer questions 39 – 40:

Scenario 5-4

In a particular game, a fair die is tossed. If the number of spots showing is either four or five, you win \$1. If the number of spots showing is six, you win \$4. And if the number of spots showing is one, two, or three, you win nothing. You are going to play the game twice.

39. Use Scenario 5-4. The probability that you win \$4 both times is

- A) 1/36.**
- B) 1/12
- C) 1/6.
- D) 1/4.
- E) 1/3.

ANS: A

TOP: Multiplication Rule, Independent events

$$P(6) = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Dice	
1	+0
2	+0
3	+0
4	+\$1
5	+\$1
6	+\$4

40. Use Scenario 5-4. The probability that you win at least \$1 both times is

- A) $1/36$.
- B) $4/36$.
- C) $1/4$.
- D) $1/2$.
- E) $3/4$.

$$P(4, 5, 6) = \left(\frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)$$

ANS: C

TOP: Multiplication Rule, Independent events; Complement

Use the following to answer questions 41 – 43:

Scenario 5-5

Suppose we roll two six-sided dice--one red and one green. Let A be the event that the number of spots showing on the red die is three or less and B be the event that the number of spots showing on the green die is three or more.

EVENTS : $A = \text{RED DIE } 1-3$

$B = \text{GREEN DIE } 3-6$

41. Use Scenario 5-5. The events A and B are

- A) disjoint. RED + GREEN HAVE 3 IN COMMON
- B) conditional.
- C) independent.
- D) reciprocals.
- E) complementary.

ANS: C

TOP: Independent and mutually exclusive events

42. Use Scenario 5-5. $P(A \cap B) =$

- A) $1/6$.
- B) $1/4$.
- C) $1/3$. ($2/6$)
- D) $5/6$.
- E) none of these.

ANS: C

TOP: Multiplication Rule, Independent events

$$\frac{1}{2} \cdot \frac{4}{6} = \frac{4}{12} = \frac{1}{3}$$

	GREEN		
	1-2	3-6	
RED	1-3	$A \cap B = \frac{1}{2} \cdot \frac{4}{6}$	$\frac{1}{2}$
	4-6		$\frac{1}{2}$
	$\frac{2}{6}$	$\frac{4}{6}$	

43. Use Scenario 5-5. $P(A \cup B) =$

- A) $1/6$.
- B) $1/4$.
- C) $2/3$.
- D) $5/6$.
- E) 1.

ANS: D

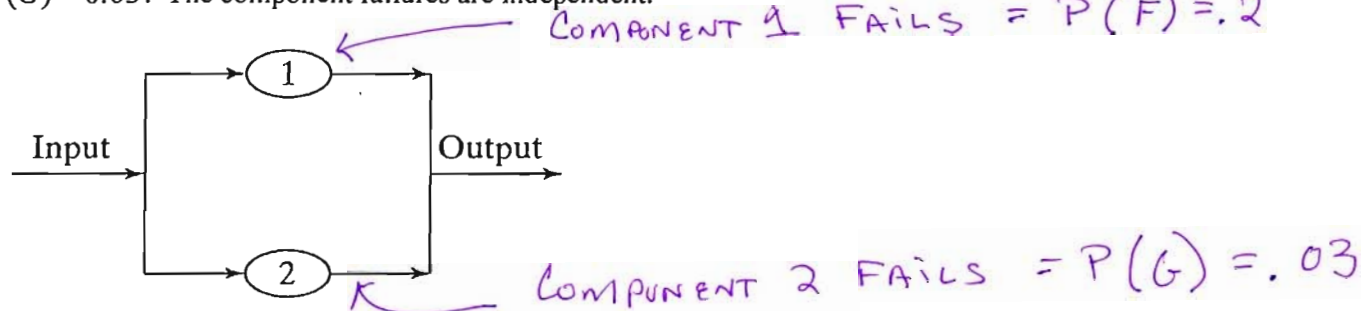
TOP: General addition rule (and multiplication of indep. events)

$$P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$$

Use the following to answer questions 44 – 46:

Scenario 5-6

A system has two components that operate in parallel, as shown in the diagram below. Because the components operate in parallel, at least one of the components must function properly if the system is to function properly. Let F denote the event that component 1 fails during one period of operation and G denote the event that component 2 fails during one period of operation. Suppose $P(F) = 0.20$ and $P(G) = 0.03$. The component failures are independent.



44. Use Scenario 5-6. The event corresponding to the system failing during one period of operation is
- A) F and G . ← THE SYSTEM FAILS WHEN BOTH COMPONENTS 1 + 2 FAIL.
 - B) F or G . X ← THE SYSTEM CAN FUNCTION WITH EITHER COMPONENT 1 OR COMPONENT 2.
 - C) not F or not G .
 - D) not F and not G .
 - E) not F or G .

ANS: A

TOP: Intersection of events

45. Use Scenario 5-6. The event corresponding to the system functioning properly during one period of operation is

- A) F and G .
- B) F or G .
- C) not F or not G .
- D) not F and not G .
- E) not F or G .

$P(F) + P(G)$ denote failures

ANS: C

TOP: Union of events

46. Use Scenario 5-6. The probability that the system functions properly during one period of operation is closest to

- A) 0.5.
- B) 0.776.
- C) 0.940.
- D) 0.970.
- E) 0.994.

AT LEAST 1 COMPONENT MUST WORK

$$P(\text{AT LEAST}) = 1 - (.2)(.03) = .994$$

ANS: E

TOP: Multiplication Rule, Independent events

47. Event A occurs with probability 0.8. The conditional probability that event B occurs, given that A occurs, is 0.5. The probability that both A and B occur

A) is 0.3.

B) is 0.4.

C) is 0.625.

D) is 0.8.

E) cannot be determined from the information given.

ANS: B

TOP: Conditional probability formula

$$P(A) = .8$$

$$P(B|A) = .5$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$(.5)(.8) = P(A \cap B) = .4$$

FROM FORMULA SHEET

48. Event A occurs with probability 0.3, and event B occurs with probability 0.4. If A and B are independent, we may conclude that

A) $P(A \text{ and } B) = 0.12$. $(.3)(.4) \checkmark$

B) $P(A|B) = 0.3 = P(A) \checkmark$

C) $P(B|A) = 0.4 = P(B) \checkmark$

D) all of the above.

E) none of the above.

ANS: D

TOP: Conditional probability formula

$$P(A) = .3$$

$$P(B) = .4$$

INDEPENDENT

FACE CARDS

49. The card game Euchre uses a deck with 32 cards: Ace, King, Queen, Jack, 10, 9, 8, 7 of each suit. Suppose you choose one card at random from a well-shuffled Euchre deck. What is the probability that the card is a Jack, given that you know it's a face card?

A) $1/3$

B) $1/4$

C) $1/8$

D) $1/9$

E) $1/12$

ANS: A

TOP: Conditional probability formula

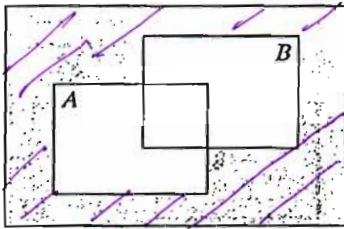
$$P(\text{JACK} | \text{FACE CARD}) = \frac{P(\text{JACK} \cap \text{FACE CARD})}{P(\text{FACE CARD})}$$

$$= \frac{4}{4+4+4} = \frac{4}{12}$$

J
K Q J

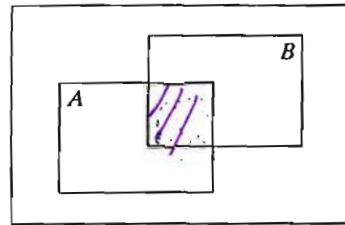
50. A plumbing contractor puts in bids on two large jobs. Let A = the event that the contractor wins the first contract and let B = the event that the contractor wins the second contract. Which of the following Venn diagrams has correctly shaded the event that the contractor wins exactly one of the contracts?

A)



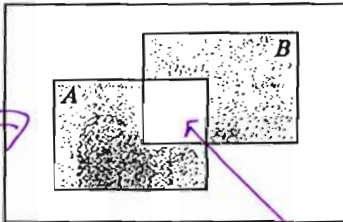
NEITHER
 $A \text{ or } B$

B)



$A \cap B$

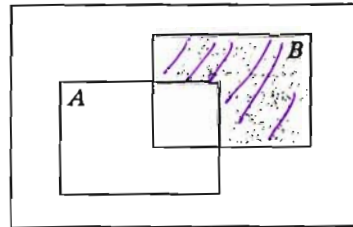
C)



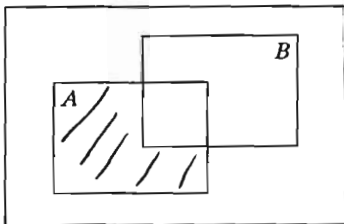
EXACTLY
ONE

NOT INCLUDED

D)



E)

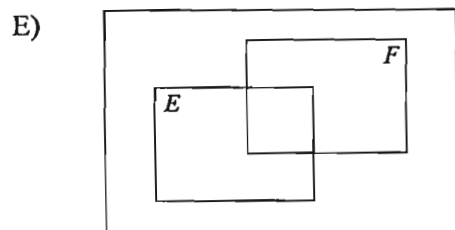
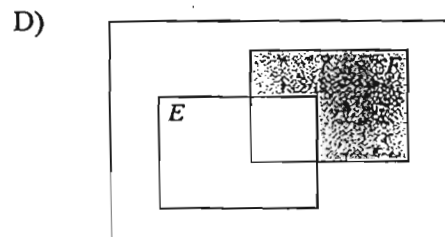
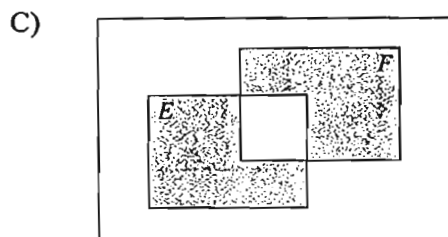
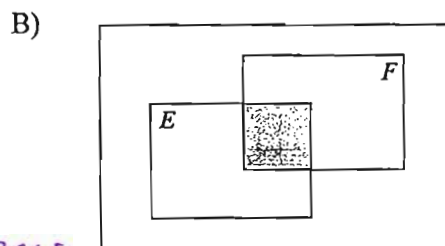
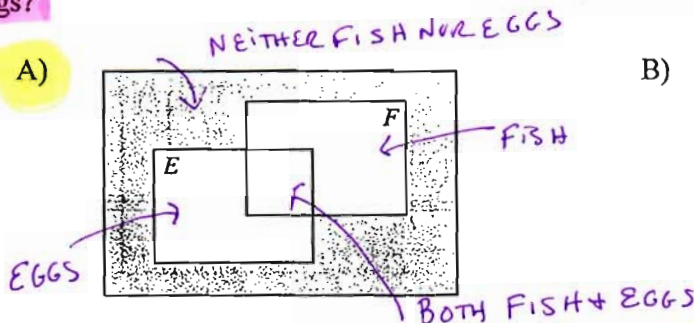


BECAUSE THIS WOULD
BE BOTH AND WE WANT
EXACTLY 1. THIS IS CORRECT.

ANS: C

TOP: Venn diagrams

51. Among the students at a large university who describe themselves as vegetarians, some eat fish, some eat eggs, some eat both fish and eggs, and some eat neither fish nor eggs. Choose a vegetarian student at random. Let E = the event that the student eats eggs, and let F = the event that the student eats fish. Which of the following Venn diagrams has **correctly shaded the event that the student eats neither fish nor eggs?**



ANS: A

TOP: Venn diagrams

Use the following for questions 52 – 53:

Scenario 5-7

The probability of a randomly selected adult having a rare disease for which a diagnostic test has been developed is 0.001. The diagnostic test is not perfect. The probability the test will be positive (indicating that the person has the disease) is 0.99 for a person with the disease and 0.02 for a person without the disease.

52. Use Scenario 5-7. The proportion of adults for which the test would be positive is

A) 0.00002.

B) 0.00099.

C) 0.01998.

D) 0.02097.

E) 0.02100.

$$.00099 + .01998$$

ANS: D

TOP: Multiplication rule, dependent events

53. Use Scenario 5-7. If a randomly selected person is tested and the result is positive, the probability the individual has the disease is

A) 0.001.

B) 0.019.

C) 0.020.

D) 0.021.

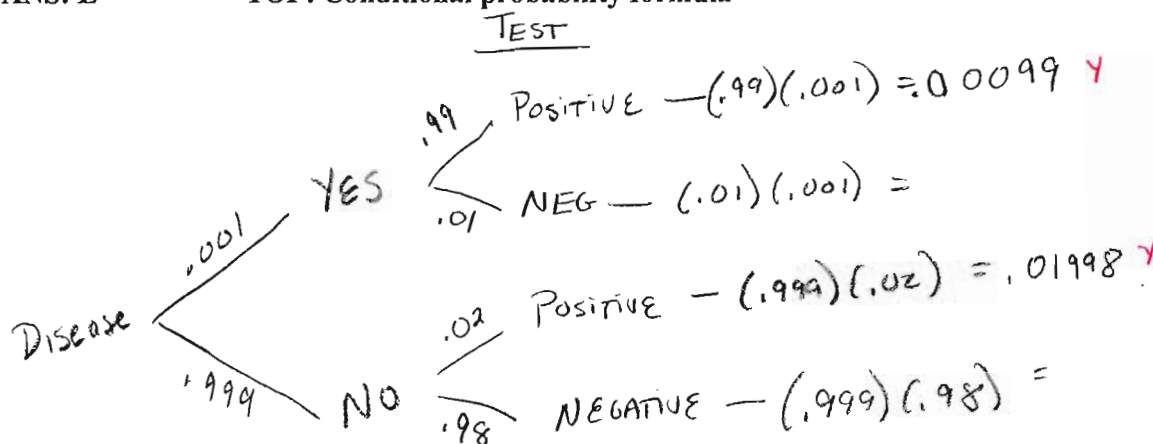
E) 0.047.

$$P(\text{DISEASE} | \text{TESTED POSITIVE}) = \frac{P(\text{DISEASE} \cap \text{POSITIVE})}{P(\text{POSITIVE})}$$

$$= \frac{.00099}{.02097} = .0472$$

ANS: E

TOP: Conditional probability formula



Use the following for questions 54 – 57:

Scenario 5-8

A student is chosen at random from the River City High School student body, and the following events are recorded:

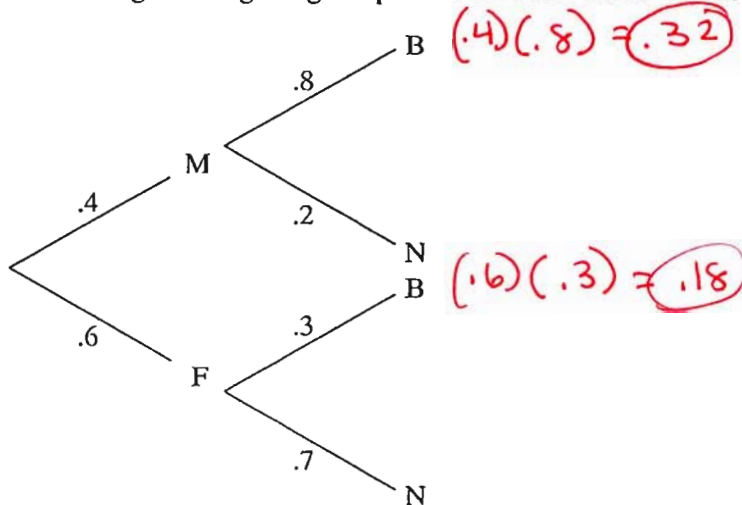
M = The student is male

F = The student is female

B = The student ate breakfast that morning.

N = The student did not eat breakfast that morning.

The following tree diagram gives probabilities associated with these events.



54. Use Scenario 5-8. What is the probability that the selected student is a male and ate breakfast?

A) 0.32

B) 0.40

C) 0.50

D) 0.64

E) 0.80

ANS: A

TOP: Probabilities from tree diagram

55. Use Scenario 5-8. What is the probability that the student had breakfast?

A) 0.32

B) 0.40

C) 0.50

D) 0.64

E) 0.80

ANS: C

TOP: Probabilities from tree diagram

56. Use Scenario 5-8. Given that a student who ate breakfast is selected, what is the probability that he is male?

A) 0.32

B) 0.40

C) 0.50

D) 0.64

E) 0.80

ANS: D

TOP: Probabilities from tree diagram

$$P(\text{male} | \text{Breakfast}) = \frac{P(\text{male} \cap \text{Breakfast})}{P(\text{Breakfast})}$$

$$= \frac{.32}{.50} = .64$$

57. Use Scenario 5-8. Find $P(B|F)$ and write in words what this expression represents.

- A) 0.18; The probability the student ate breakfast and is female.
- B) 0.18; The probability the student ate breakfast, given she is female.
- C) 0.18; The probability the student is female, given she ate breakfast.
- D) 0.30; The probability the student ate breakfast, given she is female.**
- E) 0.30; The probability the student is female, given she ate breakfast.

ANS: D

TOP: Probabilities from tree diagram

Use the following for questions 58-59:

Scenario 5-9

You ask a sample of 370 people, "Should clinical trials on issues such as heart attacks that affect both sexes use subjects of just one sex?" The responses are in the table below.

Suppose you choose one of these people at random

	Yes	No	
Male	34	105	139
Female	46	185	231
	80	290	370

58. Use Scenario 5-9. What is the probability that the person said "Yes," given that she is a woman?

- A) 0.20** $46/231 = .199$
- B) 0.22
- C) 0.25
- D) 0.50
- E) 0.575

ANS: A

TOP: Conditional probability from 2-way table

59. Use Scenario 5-9. What is the probability that the person is a woman, given that she said "Yes?"

- A) 0.20 $46/80 = .575$
- B) 0.22
- C) 0.25
- D) 0.50
- E) 0.575**

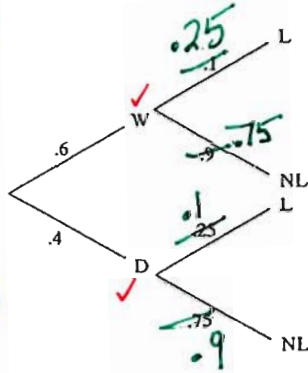
ANS: E

TOP: Conditional probability from 2-way table

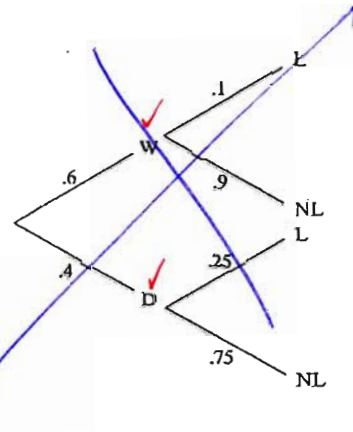
ERROR

60. Each day, Mr. Bayona chooses a one-digit number from a random number table to decide if he will walk to work or drive that day. The numbers 0 through 3 indicate he will drive, 4 through 9 mean he will walk. If he drives, he has a probability of 0.1 of being late. If he walks, his probability of being late rises to 0.25. Let W = Walk, D = Drive, L = Late, and NL = Not Late. Which of the following tree diagrams summarizes these probabilities?

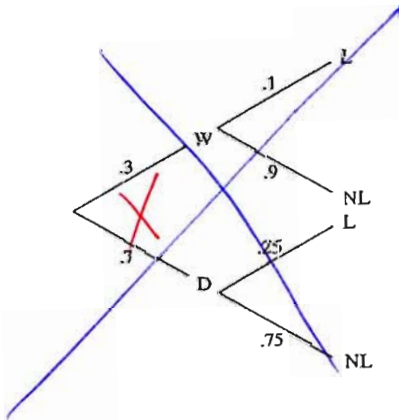
A)



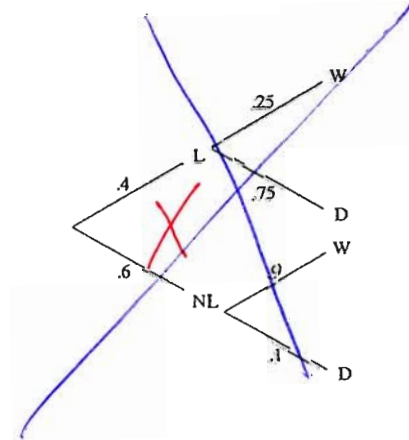
B)



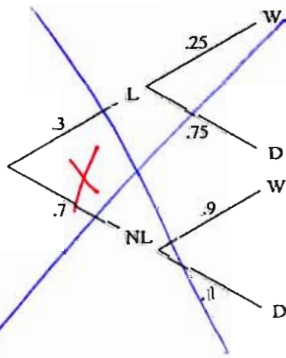
C)



D)



E)



ANS: A

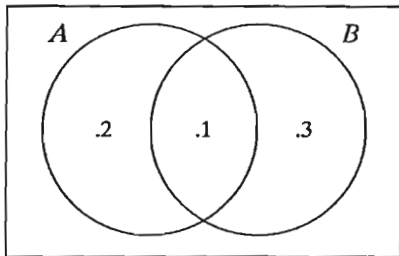
TOP: Tree diagram from probabilities

Use the following for questions 61 – 62.

Scenario 5-10

The Venn diagram below describes the proportion of students who take chemistry and Spanish at Jefferson High School, Where A = Student takes chemistry and B = Students takes Spanish.

Suppose one student is chosen at random.



$$.2 + .1 + .3 = .6$$

61. Use Scenario 5-10. Find the value of $P(A \cup B)$ and describe it in words.

- A) 0.1; The probability that the student takes both chemistry and Spanish.
- B) 0.1; The probability that the student takes either chemistry or Spanish, but not both.
- C) 0.5; The probability that the student takes either chemistry or Spanish, but not both.
- D) 0.6; The probability that the student takes either chemistry or Spanish, or both.
- E) 0.6; The probability that the student takes both chemistry and Spanish.

ANS: D

TOP: Venn diagrams

62. Use Scenario 5-10. The probability that the student takes *neither* Chemistry nor Spanish is

- A) 0.1
- B) 0.2
- C) 0.3
- D) 0.4
- E) 0.6

ANS: D

TOP: Venn diagrams

$$1 - (.2 + .1 + .3) = .4$$

Use the following for questions 63 – 65:

Scenario 5-11

The following table compares the hand dominance of 200 Canadian high-school students and what methods they prefer using to communicate with their friends.

	Cell phone/Text	In person	Online	Total
Left-handed	12	13	9	34 LEFT
Right-handed	43	72	51	166
Total	55	85 IN PERSON	60	200

Suppose one student is chosen randomly from this group of 200.

63. Use Scenario 5-11. What is the probability that the student chosen is left-handed or prefers to communicate with friends in person?

- A) 0.065
- B) 0.17
- C) 0.425
- D) 0.53
- E) 0.595

ANS: D

TOP: Conditional probability from 2-way table

$$\downarrow \quad \downarrow$$

$$\frac{34}{200} + \frac{85}{200} - \frac{13}{200}$$

$$= \frac{106}{200} = 0.53$$

64. Use Scenario 5-11. If you know the person that has been randomly selected is left-handed, what is the probability that they prefer to communicate with friends in person?

- A) 0.065
- B) 0.153
- C) 0.17
- D) 0.382
- E) 0.53

ANS: D

TOP: Conditional probability from 2-way table

$$P(\text{IN PERSON} | \text{LEFT}) = \frac{P(\text{IN PERSON} \cap \text{LEFT})}{P(\text{LEFT})} = \frac{13/200}{34/200}$$

$$= \frac{13}{34} = 0.3823$$

65. Use Scenario 5-11. Which of the following statements supports the conclusion that the event "Right-handed" and the event "Online" are not independent?

- A) $\frac{51}{200} \neq \frac{34}{60}$
- B) $\frac{9}{34} \neq \frac{166}{200}$
- C) $\frac{166}{200} \neq \frac{60}{200}$
- D) $\frac{60}{166} \neq \frac{166}{200}$
- E) $\frac{51}{60} \neq \frac{166}{200}$

ANS: E

TOP: Conditional probability from 2-way table

$$P(\text{RIGHT}) \neq P(\text{RIGHT/ONLINE})$$

$$\frac{166}{200} \neq \frac{51}{60}$$

Double checked ↴

$$P(\text{ONLINE}) \neq P(\text{ONLINE/RIGHT})$$

$$\frac{60}{200} \neq \frac{51}{166}$$

INDEPENDENCE

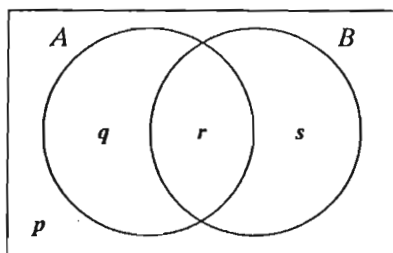
$$P(A) = P(A|B)$$

$$P(B) \text{ OR } P(B|A)$$

Use the follow for questions 66 – 68:

Scenario 5-12

The letters p , q , r , and s represent probabilities for the four distinct regions in the Venn diagram below. For each question, indicate which expression describes the probability of the event indicated.



66. Use Scenario 5-12. $P(A \cup B)$ UNION

- A) p
- B) r
- C) $q + s$
- D) $q + s - r$
- E) $q + s + r$

ANS: E

TOP: Venn diagrams

67. Use Scenario 5-12. $P(B|A)$

- A) s
- B) $s - r$
- C) $\frac{s}{r}$
- D) $\frac{r}{q+r}$
- E) $\frac{r+s}{q+r+s}$

$$\frac{P(A \cap B)}{P(A)} = \frac{r}{q+r}$$

ANS: D

TOP: Venn diagrams

68. Use Scenario 5-12. The probability associated with the intersection of A and B .

- A) p
- B) r
- C) $q + s$
- D) $q + s - r$
- E) $q + s + r$

ANS: B

TOP: Venn diagrams

$$P(A \cap B)$$

Use the following for questions 69 – 71:

Scenario 5-13

One hundred high school students were asked if they had a dog, a cat, or both at home. Here are the results.

Cat?	Dog?		Total
	No	Yes	
No	74	4	78
Yes	10	12	22
Total	84	16	100

69. Use Scenario 5-13. If a single student is selected at random and you know she has a dog, what is the probability she also has a cat?

- A) 0.04
- B) 0.12
- C) 0.22
- D) 0.25
- E) 0.75

$$P(\text{CAT} | \text{DOG}) = 12/16 = 75\%$$

ANS: E

TOP: Conditional probability from 2-way table

70. Use Scenario 5-13. If a single student is selected at random, what is the probability associated with the union of the events “has a dog” and “does not have a cat?”

- A) 0.04
- B) 0.16
- C) 0.78
- D) 0.9
- E) 0.94

$$P(\text{DOG} \cup \text{NOT CAT}) = .78 + .16 - .12 = .90$$

ANS: D

TOP: Conditional probability from 2-way table

71. Use Scenario 5-13. If two students are selected at random, what is the probability that neither of them has a dog or a cat?

- A) 0.37
- B) 0.540
- C) 0.548
- D) 0.655
- E) 0.74

$$P(\text{NEITHER})^2 = (.74)^2$$

ANS: C

TOP: Conditional probability from 2-way table