

Chapter 6 AP Statistics PRACTICE Test

Section I: Multiple Choice Select the best answer for each question.

Questions T6.1 and T6.2 refer to the following setting. A psychologist studied the number of puzzles that subjects were able to solve in a five-minute period while listening to soothing music. Let X be the number of puzzles completed successfully by a subject. The psychologist found that X had the following probability distribution:

Value of X :	1	2	3	4
Probability:	0.2	0.4	0.3	0.1

$x_i p_i$.2 .8 .9 .4

T6.1. What is the probability that a randomly chosen subject completes at least 3 puzzles in the five-minute period while listening to soothing music?

- (a) 0.3
(b) 0.4
(c) 0.6
(d) 0.9
(e) Cannot be determined

$$P(X \geq 3) = .3 + .1 = .4$$

T6.2. Suppose that three randomly selected subjects solve puzzles for five minutes each. The expected value of the total number of puzzles solved by the three subjects is

- (a) 1.8. (b) 2.3. (c) 2.5. (d) 6.9. (e) 7.5.

$$E(X) = \sum x_i p_i = .2 + .8 + .9 + .4 = 2.3$$

$$E(3 \text{ Subjects}) = 2.3 + 2.3 + 2.3 = 6.9$$

T6.3. Suppose a student is randomly selected from your school. Which of the following pairs of random variables are most likely independent?

- (a) X = student's height; Y = student's weight
(b) X = student's IQ; Y = student's GPA
(c) X = student's PSAT Math score; Y = student's PSAT Verbal score
(d) X = average amount of homework the student does per night; Y = student's GPA
(e) X = average amount of homework the student does per night; Y = student's height

NOT IND

NOT IND

NOT IND

NOT IND

INDEPENDENT - ONE DOES NOT INFLUENCE THE OTHER

T6.4. A certain vending machine offers 20-ounce bottles of soda for \$1.50. The number of bottles X bought from the machine on any day is a random variable with mean 50 and standard deviation 15. Let the random variable Y equal the total revenue from this machine on a given day. Assume that the machine works properly and that no sodas are stolen from the machine. What are the mean and standard deviation of Y ?

- (a) $\mu_Y = \$1.50$, $\sigma_Y = \$22.50$
(b) $\mu_Y = \$1.50$, $\sigma_Y = \$33.75$
(c) $\mu_Y = \$75$, $\sigma_Y = \$18.37$
(d) $\mu_Y = \$75$, $\sigma_Y = \$22.50$
(e) $\mu_Y = \$75$, $\sigma_Y = \$33.75$

$$X: \mu_X = 50 \quad \sigma_X = 15$$

$$Y = \text{Total Revenue } (\$1.50 \cdot X)$$

$$\mu_Y = 50 \times 1.5 = \$75$$

$$\sigma_Y = 15 \times 1.5 = \$22.50$$

Questions T6.5 and T6.6 refer to the following setting. The weight of tomatoes chosen at random from a bin at the farmer's market is a random variable with mean $\mu = 10$ ounces and standard deviation $\sigma = 1$ ounce. Suppose we pick four tomatoes at random from the bin and find their total weight T .

Tomato $\mu = 10$ $\sigma = 1$

T6.5. The random variable T has a mean of

- (a) 2.5 ounces. (d) 40 ounces.
(b) 4 ounces. (e) 41 ounces.
(c) 10 ounces.

T6.6. The random variable T has a standard deviation of

- (a) 0.25. (b) 0.50. (c) 0.71. (d) 2. (e) 4.

$SD(4 \text{ Tomatoes}) =$

$\sqrt{1^2 + 1^2 + 1^2 + 1^2} =$

$\sqrt{4} = 2$

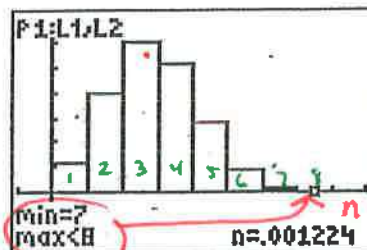
$E(4 \text{ Tomatoes}) = 10 + 10 + 10 + 10 = 40$

T6.7. Which of the following random variables is geometric?

- (a) The number of times I have to roll a die to get two 6s.
(b) The number of cards I deal from a well-shuffled deck of 52 cards until I get a heart.
(c) The number of digits I read in a randomly selected row of the random digits table until I find a 7. *LOOKING FOR THE 1ST occurrence of 7*
(d) The number of 7s in a row of 40 random digits.
(e) The number of 6s I get if I roll a die 10 times.

T6.8. Seventeen people have been exposed to a particular disease. Each one independently has a 40% chance of contracting the disease. A hospital has the capacity to handle 10 cases of the disease. What is the probability that the hospital's capacity will be exceeded?

- (a) 0.011 (b) 0.035 (c) 0.092 (d) 0.965 (e) 0.989



- (a) Binomial with $n = 8, p = 0.1$
(b) Binomial with $n = 8, p = 0.3$ ✓
(c) Binomial with $n = 8, p = 0.8$
(d) Geometric with $p = 0.1$
(e) Geometric with $p = 0.2$

Geometric is typically skewed Right

$n = 17$
 $p = .40$
BINS
 $B(17, .4)$
 $P(X > 10) = 1 - P(X \leq 10)$
 $1 - .965 = .035$
 $\text{binomcdf}(17, .4, 10) = .965$

T6.10. A test for extrasensory perception (ESP) involves asking a person to tell which of 5 shapes—a circle, star, triangle, diamond, or heart—appears on a hidden computer screen. On each trial, the computer is equally likely to select any of the 5 shapes. Suppose researchers are testing a person who does not have ESP and so is just guessing on each trial. What is the probability that the person guesses the first 4 shapes incorrectly but gets the fifth correct?

- (a) $1/5$ (d) $\left(\frac{5}{1}\right) \cdot \left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$
(b) $\left(\frac{4}{5}\right)^4$ (e) $4/5$

(c) $\left(\frac{4}{5}\right)^4 \cdot \left(\frac{1}{5}\right)$

$P(S) = 1/5$

$P(F) = 4/5$

$P(F F F F S)$

T6.11

$Y = \#$ broken eggs in 1 dozen carton

(a) $P(\text{at least 10 eggs unbroken}) = P(Y \leq 2)$
(out of the 12)

Use the probability distribution for Y given:

$$P(Y \leq 2) = P(Y=0) + P(Y=1) + P(Y=2) \\ .78 + .11 + .07 = .96$$

(in context) There is a 96% chance that 2 or fewer eggs are broken. That is there is a 96% chance that at least 10 eggs are unbroken in a randomly selected carton of "store brand" eggs.

(b) $\mu_Y = 0(.78) + 1(.11) + 2(.07) + 3(.03) + 4(.01) = .38$

$\mu_Y = .38$ (in context) We expect, on average, to find .38 broken eggs in a carton of a dozen eggs.

(c) $\sigma_Y^2 = \sum (x_i - \mu_x)^2 \cdot p_i$ $\sigma_Y = \sqrt{(0 - .38)^2(.78) + \dots + (4 - .38)^2(.01)} = .8219$

show either for work

IN CALC> L1 = y_i 's
L2 = p_i 's
1VAR STATS
LIST: L1
FREQ LIST: L2
↓
 $\Sigma x = \mu_x = .38$
 $\sigma_x = .8219$

(in context) Individual cartons will vary from .38 broken eggs by about .82 broken eggs, on average

Cont →

T6.11d

1ST FIND: $P(\text{at least 2 broken eggs}) = P(Y=2) + P(Y=3) + P(Y=4)$
 $P(Y \geq 2) = .07 + .03 + .01 = .11$

2ND - Notice this is a geometric probability because you are looking for the 1ST broken EGG.

Check Geom. Conditions

- B - broken / NOT broken
- I - eggs independent
- T - 1st broken egg
- S - fixed prob success $p = .11$

STATE THE Distribution with appropriate parameters either $G(.11)$ or geometric distribution with $p = .11$

3rd - find the probability for $G(.11)$
1ST Broken egg found in one of first 3 cartons.

$$P(Y \leq 3) = .2950$$

Geometcdf(.11, 3)

4TH

(Context) The probability of finding at least 2 broken eggs in one of the first 3 randomly selected cartons is about 30%.

T6.12

X = the number of owners who greet their dog first

- ④ X is a binomial random variable because it meets the required conditions

B - own greets dog first or does NOT.

I - dog owners are independent

N - fixed trials $n = 12$

S - fixed probability of success $p = .66$

⑥ $P(Y \leq 4) = .0213$

$\text{binomcdf}(12, .66, 4)$ ← NOT Needed to show

← remember to state model

$$B(n, p) = B(12, .66)$$

(Context)

We found the probability of getting a sample of 4 or fewer dog owners greeting their dogs first when they get home is only about 2%.

This is reasonably unlikely to occur, so we would be skeptical that the "Ladies Home Journal's" claim is true.

T6.13

define RV's

E = amount of time ^{for Ed} to complete Hw $\rightarrow N(25, 5)$

A = amount of time for Adelaide
to complete Hw $\rightarrow N(50, 10)$

(a) RV: $D = A - E$

$$E(A - E) = \mu_D = 50 - 25 = 25 \text{ minutes}$$

$$\text{VAR}(A - E) = \sigma_D^2 = 5^2 + 10^2 = 125 \leftarrow \text{Assuming amount of time spent by Ed and Adelaide is independent}$$

$$\text{SD}(A - E) = \sigma_D = \sqrt{125} = 11.18 \text{ mins}$$

(b) Find $P(E > A)$

\downarrow

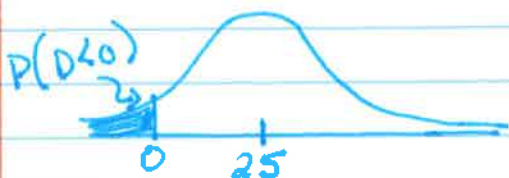
$$P(0 > A - E)$$

$$P(A - E < 0) = P(D < 0)$$

Use your algebra skills to rework this Probability to use the RV " $D = A - E$ " calculated above

STATE model
 $N(25, 11.18)$

Sketch Graph



$$P(D < 0) = .01267$$

normalcdf(-E99, 0, 25, 11.18)

(Context) The probability that Ed will spend more time on home work than Adelaide is very small, about 1.3%.

T6.14

Census Bureau 13% Hispanic adults
Poll - SRS $n = 1,200$ adults

(a) X = the number of hispanic adults

① Binomial Model Conditions

B = Hispanic or Not

I = SRS

N = Fixed trials $n = 1,200$

S = Fixed prob success $p = .13$

② model $B(1200, .13)$

③ $E(X) = \mu_X = np = 1200(.13)$
 $\mu_X = 156$

④ $SD(X) = \sqrt{np(1-p)} = \sqrt{1200(.13)(.87)}$
 $\sigma_X = 11.65$

(b) Suspicious if 15% of the sample is Hispanic
 $15\% = 1200(.15) = 180$ Hispanics

$P(X \geq 180)$

Binomial Model method
remember discrete RV
model $B(1200, .13)$

$$\begin{aligned} P(X \geq 180) &= 1 - P(X \leq 179) \\ &= 1 - .9765 \\ &= \text{binomcdf}(1200, .13, 179) \\ &= .0235 \end{aligned}$$

Normal Approximation model

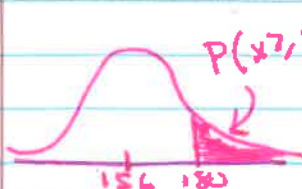
Check Normal condition

$$np = 1200(.13) = 156 \geq 10 \checkmark$$

$$n(1-p) = 1200(.87) = 1,044 \geq 10 \checkmark$$

State model $N(156, 11.56)$
 $E(X) = np = 156$

$$SD(X) = \sqrt{1200(.13)(.87)} = 11.56$$



$$P(X \geq 180) = .0189$$

$$\text{normalcdf}(180, E99, 156, 11.56)$$

(Context) The probability that 15% of the random sample is Hispanic is very small (about 2%).

Therefore we would be suspicious of the opinion poll.