

LESSON
3.1**Study Guide**

For use with pages 146–152

GOAL Identify angle pairs formed by three intersecting lines.**Vocabulary**

Two lines are **parallel lines** if they do not intersect and are coplanar.

Two lines are **skew lines** if they do not intersect and are not coplanar.

Two planes that do not intersect are **parallel planes**.

A **transversal** is a line that intersects two or more coplanar lines at different points.

When two lines are cut by a transversal, two angles are **corresponding angles** if they have corresponding positions.

When two lines are cut by a transversal, two angles are **alternate interior angles** if they lie between the two lines and on opposite sides of the transversal.

When two lines are cut by a transversal, two angles are **alternate exterior angles** if they lie outside the two lines and on opposite sides of the transversal.

When two lines are cut by a transversal, two angles are **consecutive interior angles** if they lie between the two lines and on the same side of the transversal.

Postulate 13 Parallel Postulate: If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

Postulate 14 Perpendicular Postulate: If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

LESSON 3.1

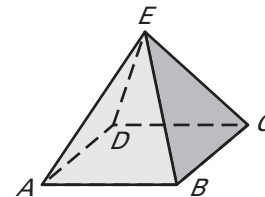
EXAMPLE 1 Identify relationships in space

Think of each segment in the diagram as part of a line. Which line(s) in the diagram appear to fit the description?

- Parallel to \overleftrightarrow{AB}
- Skew to \overleftrightarrow{AB}
- Parallel to \overleftrightarrow{BC}

Solution

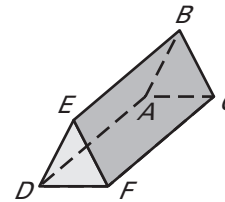
- Only \overleftrightarrow{CD} is parallel to \overleftrightarrow{AB} .
- \overleftrightarrow{ED} and \overleftrightarrow{EC} are skew to \overleftrightarrow{AB} .
- Only \overleftrightarrow{AD} is parallel to \overleftrightarrow{BC} .



LESSON
3.1
Study Guide *continued*
For use with pages 146–152
Exercises for Example 1

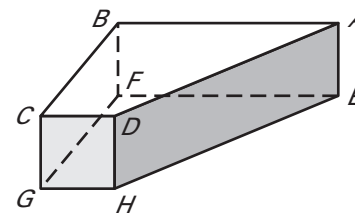
Think of each segment in the diagram as part of a line. Fill in the blank with *parallel*, *skew*, or *perpendicular*.

- \overleftrightarrow{DE} and \overleftrightarrow{CF} are ____.
- \overleftrightarrow{AD} , \overleftrightarrow{BE} , and \overleftrightarrow{CF} are ____.
- Plane ABC and plane DEF are ____.
- \overleftrightarrow{BE} and \overleftrightarrow{AB} are ____.



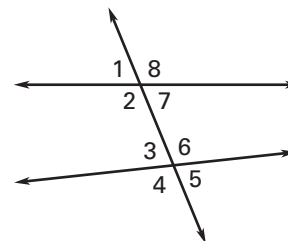
Think of each segment in the diagram as part of a line. There may be more than one right answer.

- Name a line perpendicular to \overleftrightarrow{HD} .
- Name a plane parallel to plane DCH .
- Name a line parallel to \overleftrightarrow{BC} .
- Name a line skew to \overleftrightarrow{FG} .


EXAMPLE 2 **Identify angle relationships**

Identify all pairs of angles of the given type.

- Corresponding
- Alternate interior
- Alternate exterior
- Consecutive interior

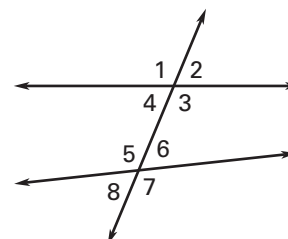

Solution

- | | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| a. $\angle 1$ and $\angle 3$ | b. $\angle 2$ and $\angle 6$ | c. $\angle 1$ and $\angle 5$ | d. $\angle 2$ and $\angle 3$ |
| $\angle 2$ and $\angle 4$ | $\angle 7$ and $\angle 3$ | $\angle 8$ and $\angle 4$ | $\angle 7$ and $\angle 6$ |
| $\angle 8$ and $\angle 6$ | | | |
| $\angle 7$ and $\angle 5$ | | | |

Exercises for Example 2

Complete the statement with *corresponding*, *alternate interior*, *alternate exterior*, or *consecutive interior*.

- $\angle 3$ and $\angle 5$ are ____ angles.
- $\angle 2$ and $\angle 6$ are ____ angles.
- $\angle 1$ and $\angle 7$ are ____ angles.
- $\angle 4$ and $\angle 5$ are ____ angles.



Answer Key

Lesson 3.1

Study Guide

1. skew 2. parallel 3. parallel
4. perpendicular 5. $\overleftrightarrow{AD}, \overleftrightarrow{EH}, \overleftrightarrow{DC}, \overleftrightarrow{HG}$
6. plane ABE 7. \overleftrightarrow{FG} 8. $\overleftrightarrow{AE}, \overleftrightarrow{DH}, \overleftrightarrow{AD}, \overleftrightarrow{DC}$
9. alternate interior 10. corresponding
11. alternate exterior 12. consecutive interior

LESSON
3.2**Study Guide**

For use with pages 153–160

GOAL**Use angles formed by parallel lines and transversals.****Vocabulary**

Postulate 15 Corresponding Angles Postulate: If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

Theorem 3.1 Alternate Interior Angles Theorem: If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Theorem 3.2 Alternate Exterior Angles Theorem: If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Theorem 3.3 Consecutive Interior Angles Theorem: If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

EXAMPLE 1**Identify congruent angles**

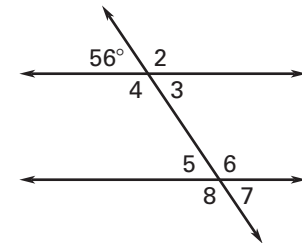
The measure of three of the numbered angles is 56° . Identify the angles. *Explain your reasoning.*

Solution

Using the Vertical Angles Congruence Theorem, $m\angle 3 = 56^\circ$.

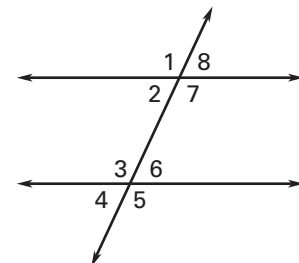
By the Corresponding Angles Postulate, $m\angle 5 = 56^\circ$.

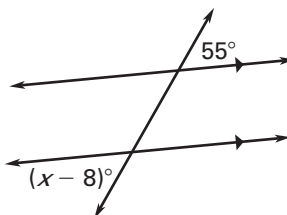
Because $\angle 3$ and $\angle 7$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 7 = 56^\circ$.

**Exercises for Example 1**

Use the diagram at the right.

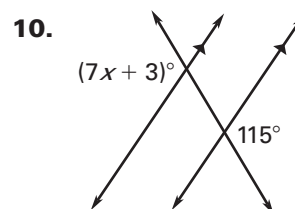
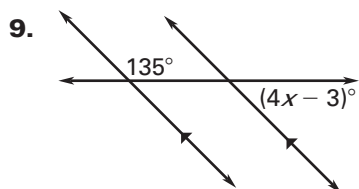
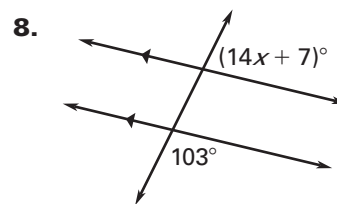
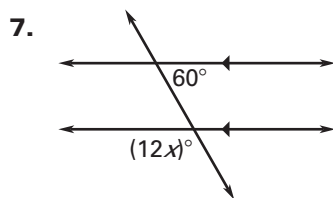
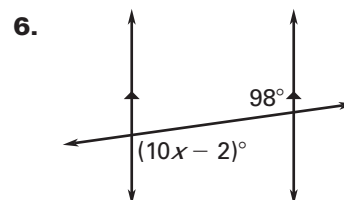
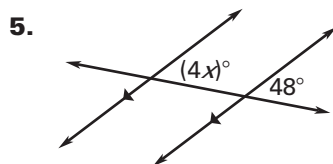
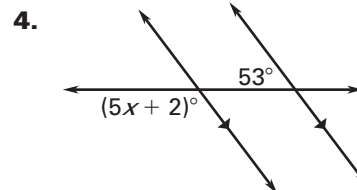
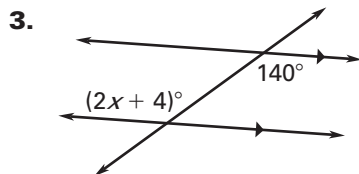
- If $m\angle 2 = 65^\circ$, find three other angles that have a measure of 65° . *Explain your reasoning.*
- If $m\angle 5 = 115^\circ$, find three other angles that have a measure of 115° . *Explain your reasoning.*



LESSON
3.2**Study Guide** *continued*
*For use with pages 153–160***EXAMPLE 2** Use properties of parallel linesFind the value of x .**Solution**

$$x - 8 = 55 \quad \text{Alternate Exterior Angles Theorem}$$

$$x = 63 \quad \text{Add 8 to each side.}$$

Exercises for Example 2Find the value of x .

Answer Key

Lesson 3.2

Study Guide

1. Using the Vertical Angles Congruence Theorem, $m\angle 8 = 65^\circ$. By the Corresponding Angles Postulate, $m\angle 4 = 65^\circ$. Because $\angle 8$ and $\angle 6$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 6 = 65^\circ$.

2. Using the Vertical Angles Congruence Theorem, $m\angle 3 = 115^\circ$. By the Corresponding Angles Postulate, $m\angle 7 = 115^\circ$. Because $\angle 3$ and $\angle 1$ are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 1 = 115^\circ$.

3. 68 **4.** 25 **5.** 12 **6.** 10

7. 10 **8.** 5 **9.** 12 **10.** 16

LESSON
3.3**Study Guide***For use with pages 161–169***GOAL****Use angle relationships to prove that lines are parallel.****Vocabulary**

A proof can be written in paragraph form, called a **paragraph proof**.

Postulate 16 Corresponding Angles Converse: If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

Theorem 3.4 Alternate Interior Angles Converse: If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Theorem 3.5 Alternate Exterior Angles Converse: If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Theorem 3.6 Consecutive Interior Angles Converse: If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

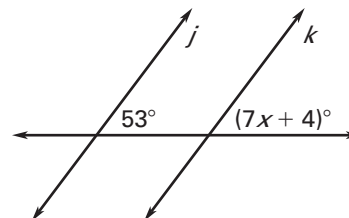
Theorem 3.7 Transitive Property of Parallel Lines: If two lines are parallel to the same line, then they are parallel to each other.

EXAMPLE 1**Apply the Corresponding Angles Converse**

Find the value of x that makes $j \parallel k$.

Solution

Lines j and k are parallel if the marked corresponding angles are congruent.



$$(7x + 4)^\circ = 53^\circ$$

Use Postulate 16 to write an equation.

$$7x = 49$$

Subtract 4 from each side.

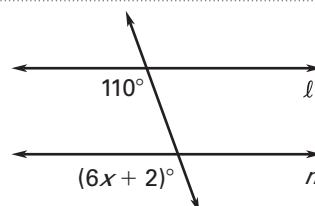
$$x = 7$$

Divide each side by 7.

The lines j and k are parallel when $x = 7$.

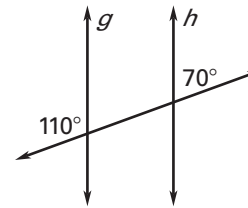
Exercises for Example 1

1. Find the value of x that makes $\ell \parallel m$.



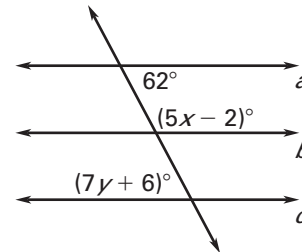
LESSON
3.3**Study Guide** *continued*
For use with pages 161–169

2. Is there enough information in the diagram to conclude that $g \parallel h$? *Explain.*

**EXAMPLE 2** Show lines are parallel

Use the diagram at the right.

- a. Find the value of x that makes $a \parallel b$.
b. Find the value of y that makes $a \parallel c$.

**Solution**

- a. Lines a and b are parallel if the marked consecutive interior angles are supplementary.

$$(5x - 2)^\circ + 62^\circ = 180^\circ$$

Use Theorem 3.6 to write an equation.

$$5x + 60 = 180$$

Combine like terms.

$$5x = 120$$

Subtract 60 from each side.

$$x = 24$$

Divide each side by 5.

The lines a and b are parallel when $x = 24$.

- b. Lines a and c are parallel if the marked alternate interior angles are congruent.

$$(7y + 6)^\circ = 62^\circ$$

Use Theorem 3.4 to write an equation.

$$7y = 56$$

Subtract 6 from each side.

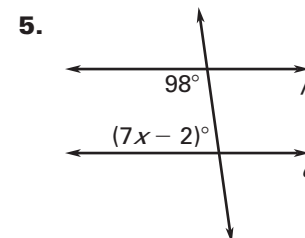
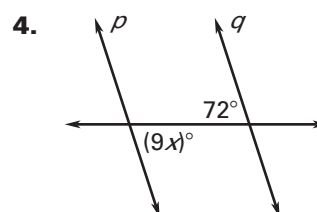
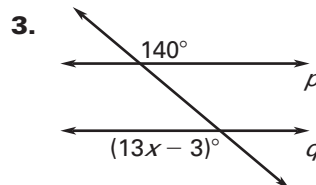
$$y = 8$$

Divide each side by 7.

The lines a and c are parallel when $y = 8$.

Exercises for Example 2

Find the value of x that makes $p \parallel q$.



Answer Key

Lesson 3.3

Study Guide

1. 18

2. Yes; the angle that corresponds with 70° has a measure of 70° because it is a linear pair with the angle that measures 110° .

3. 11 **4.** 8 **5.** 12

LESSON
3.4**Study Guide**

For use with pages 171–179

GOAL Find and compare slopes of lines.**Vocabulary**

The **slope** (m) of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Postulate 17 Slopes of Parallel Lines: In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

Postulate 18 Slopes of Perpendicular Lines: In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1 . Horizontal lines are perpendicular to vertical lines.

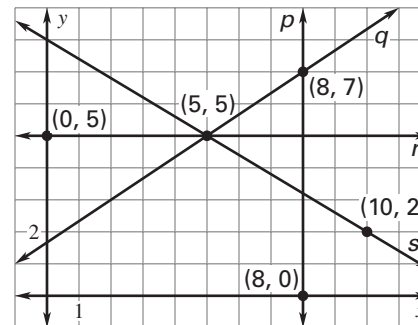
EXAMPLE 1 Find slopes of lines in a coordinate plane

Find the slope of line q and line r .

Solution

$$\text{Slope of line } q: m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{8 - 5} = \frac{2}{3}$$

$$\text{Slope of line } r: m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{5 - 0} = 0$$

**Exercise for Example 1**

1. In Example 1, find the slope of line p and line s .

EXAMPLE 2 Identify parallel lines

Find the slope of each line. Is $a \parallel b$?

Solution

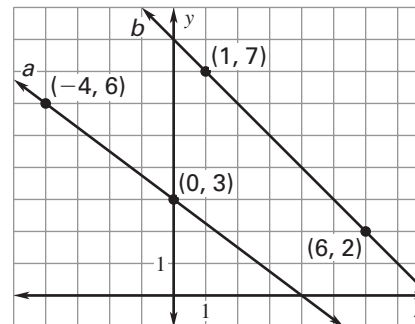
Find the slope of a through $(-4, 6)$ and $(0, 3)$.

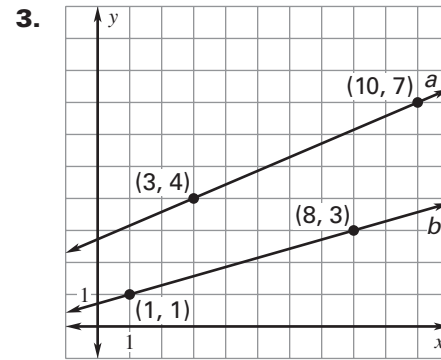
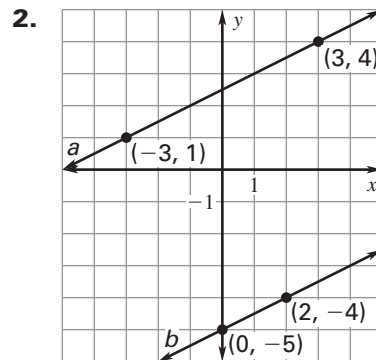
$$m_a = \frac{3 - 6}{0 - (-4)} = -\frac{3}{4}$$

Find the slope of b through $(1, 7)$ and $(6, 2)$.

$$m_b = \frac{2 - 7}{6 - 1} = -1$$

Compare the slopes. Because a and b have different slope, they are not parallel.



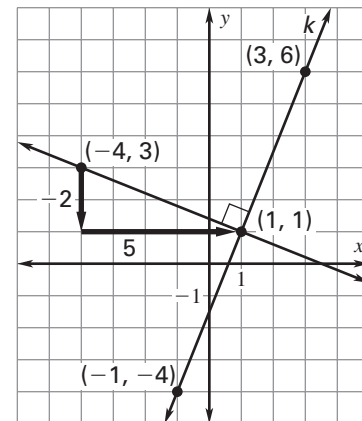
LESSON
3.4**Study Guide** *continued*
For use with pages 171–179**Exercises for Example 2****Find the slope of each line. Is $a \parallel b$?****EXAMPLE 3****Draw a perpendicular line****Line k passes through $(-1, -4)$ and $(3, 6)$. Graph the line perpendicular to k that passes through the point $(-4, 3)$.****Solution****STEP 1** Find the slope m_1 of line k through $(-1, -4)$ and $(3, 6)$.

$$m_1 = \frac{6 - (-4)}{3 - (-1)} = \frac{10}{4} = \frac{5}{2}$$

STEP 2 Find the slope m_2 of a line perpendicular to k . Use the fact that the product of the slopes of two perpendicular lines is -1 .

$$\frac{5}{2} \cdot m_2 = -1$$

$$m_2 = -\frac{2}{5}$$

STEP 3 Use the rise and run to graph the line.**Exercises for Example 3**

4. Line j passes through $(-4, 2)$ and $(6, 0)$. Graph the line perpendicular to j that passes through the point $(0, -4)$.
5. Line n passes through $(-2, 3)$ and $(5, -1)$. Graph the line perpendicular to n that passes through the point $(-6, -1)$.

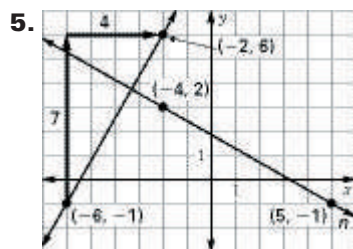
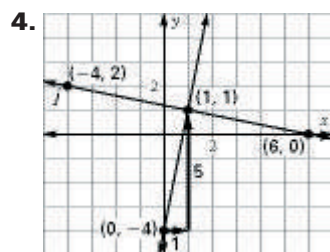
Answer Key

Lesson 3.4

Study Guide

1. undefined, $-\frac{3}{5}$ 2. $m_a = \frac{1}{2}$, $m_b = \frac{1}{2}$; parallel

3. $m_a = \frac{3}{7}$, $m_b = \frac{2}{7}$; not parallel



LESSON
3.5**Study Guide***For use with pages 180–187***GOAL Find equations of lines.****Vocabulary**

The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y -intercept.

A linear equation written as $Ax + By = C$, where A and B are not both zero, is written in **standard form**.

EXAMPLE 1 Write an equation of a parallel line

Write an equation of the line passing through the point (3, 4) that is parallel to the line with the equation $y = -4x + 5$.

STEP 1 Find the slope m . The slope of a line parallel to $y = -4x + 5$ is the same as the given line, so the slope is -4 .

STEP 2 Find the y -intercept b by using $m = -4$ and $(x, y) = (3, 4)$.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$4 = -4(3) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$16 = b \quad \text{Solve for } b.$$

Because $m = -4$ and $b = 16$, an equation of the line is $y = -4x + 16$.

EXAMPLE 2 Write an equation of a perpendicular line

Write an equation of the line p passing through the point (6, -3) that is perpendicular to the line q with the equation $y = 4x - 7$.

STEP 1 Find the slope m of line p . Line q has a slope of 4.

$$4 \cdot m = -1 \quad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$m = -\frac{1}{4} \quad \text{Divide each side by } -2.$$

STEP 2 Find the y -intercept b by using $m = -\frac{1}{4}$ and $(x, y) = (6, -3)$.

$$y = mx + b \quad \text{Use slope-intercept form.}$$

$$-3 = -\frac{1}{4}(6) + b \quad \text{Substitute for } x, y, \text{ and } m.$$

$$-1 = b \quad \text{Solve for } b.$$

Because $m = -\frac{1}{4}$ and $b = -1$, an equation of line p is $y = -\frac{1}{4}x - 1$.

LESSON
3.5**Study Guide** *continued*
*For use with pages 180–187***Exercises for Examples 1 and 2**

Write an equation of the line that passes through point P and is parallel to the line with the given equation.

1. $P(10, 3), y = x - 12$ 2. $P(-5, 2), y = -x - 9$ 3. $P(-1, 2), y = \frac{2}{3}x - 2$

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.

4. $P(8, 7), y = -x + 3$ 5. $P(-4, 5), y = 2x - 6$ 6. $P(2, -3), y = \frac{4}{7}x + 2$

EXAMPLE 3**Graph a line with equation in standard form**

Graph $2x + 3y = 18$.

Solution

STEP 1 Find the intercepts.

To find the x -intercept, let $y = 0$.

$$2x + 3y = 18$$

$$2x + 3(0) = 18$$

$$x = 9$$

To find the y -intercept, let $x = 0$.

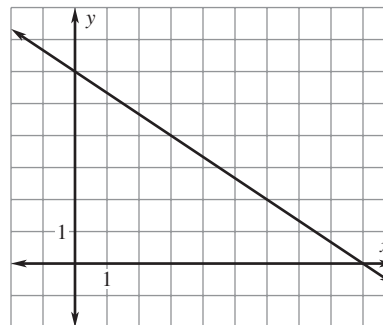
$$2x + 3y = 18$$

$$2(0) + 3y = 18$$

$$y = 6$$

STEP 2 Graph the line.

The line intercepts the axes at $(9, 0)$ and $(0, 6)$. Graph these points, then draw a line through the points.

**Exercises for Example 3**

Graph the equation.

7. $5x + 2y = 20$

8. $x - 6y = 12$

9. $7x + 5y = -14$

Answer Key

Lesson 3.5

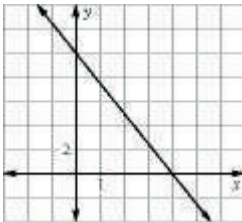
Study Guide

1. $y = x - 7$ 2. $y = -x - 3$ 3. $y = \frac{2}{3}x + \frac{8}{3}$

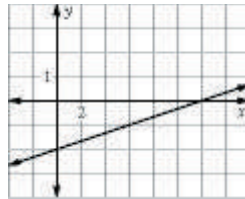
4. $y = x - 1$ 5. $y = -\frac{1}{2}x + 3$

6. $y = -\frac{7}{4}x + \frac{1}{2}$

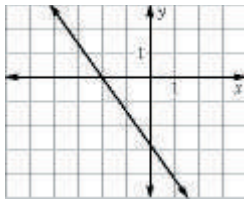
7.



8.



9.



LESSON
3.6**Study Guide**

For use with pages 190–197

GOAL Find the distance between a point and a line.**Vocabulary**

The **distance from a point to a line** is the length of the perpendicular segment from the point to the line.

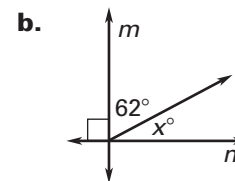
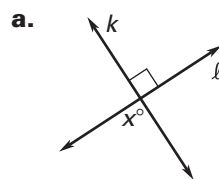
Theorem 3.8: If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

Theorem 3.9: If two lines are perpendicular, then they intersect to form four right angles.

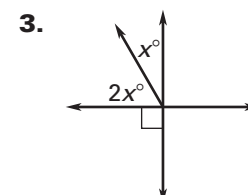
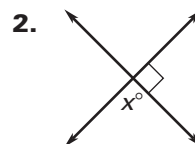
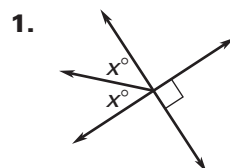
Theorem 3.10: If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

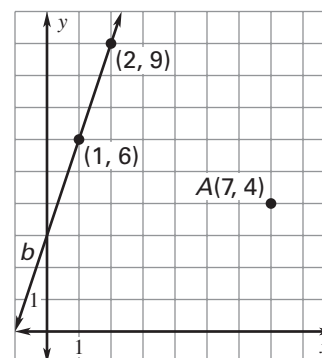
Theorem 3.11 Perpendicular Transversal Theorem: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

Theorem 3.12 Lines Perpendicular to a Transversal Theorem:
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

EXAMPLE 1 Application of the TheoremsFind the value of x .**Solution**

- a. $x = 90$, by Theorem 3.9, because k and l are perpendicular, all four angles formed are right angles. By definition of a right angle, x is 90.
- b. By Theorem 3.9, because m and n are perpendicular, all four angles formed are right angles. By Theorem 3.2, the 62° angle and the x° angle are complementary. Thus $x + 62 = 90$, so $x = 28$.

Exercises for Example 1Find the value of x .

LESSON
3.6**Study Guide** *continued*
*For use with pages 190–197***EXAMPLE 2** Find the distance between a point and a line**What is the distance from point A to line b ?****Solution**

You need to find the slope of line b . Using the points $(1, 6)$ and $(2, 9)$, the slope of the line is

$$m = \frac{9 - 6}{2 - 1} = 3.$$

The distance from point A to line b is the perpendicular segment from the point to the line. The slope of a perpendicular segment from point A to line b is $-\frac{1}{3}$. The segment from $(1, 6)$ to $(7, 4)$ has a slope of $-\frac{1}{3}$.

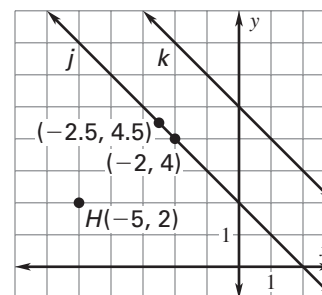
Find the distance between these points.

$$d = \sqrt{(1 - 7)^2 + (6 - 4)^2} \approx 6.3$$

The distance from point A to line b is about 6.3 units.

Exercises for Example 2**Use the graph at the right.**

4. What is the distance from point H to line j ?
5. What is the distance from line j to line k ?



Answer Key

Lesson 3.6

Study Guide

- 1.** 45 **2.** 90 **3.** 30 **4.** about 3.5 units
5. about 2.1 units