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Date

LESSON	Study	Guide
Ζ.Ι	For use with pa	ges 72–78



Describe patterns and use inductive reasoning.

Vocabulary

A **conjecture** is an unproven statement that is based on observations.

You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

A counterexample is a specific case for which a conjecture is false.

EXAMPLE 1 Describe a visual pattern

Sketch the next figure in the pattern.



Solution

Each figure looks like the one before it except that it has rotated 90° . The next figure in the pattern is shown at the right.



Exercise for Example 1

1. Sketch the next figure in the pattern.



EXAMPLE2 Describe a number pattern

Describe the pattern in the numbers 2, 8, 32, 128, . . . and write the next three numbers in the pattern.

Solution

Notice that each number in the pattern is four times the previous number.



Continue the pattern. The next three numbers are 512, 2048, and 8192.

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Exercises for Example 2

Describe the pattern in the numbers. Give the next number in the pattern.

2. 1, 5, 9, 13, . . .

3. 1, 3, 9, 27, 81, . . .

EXAMPLE 3 Make and test a conjecture

Make and test a conjecture about the product of any two odd integers.

Solution

STEP 1 Find a pattern using a few groups of small numbers.

$3 \times 13 = 39$	$5 \times 9 = 45$
$7 \times 21 = 147$	$11 \times 15 = 165$

Conjecture The product of any two odd integers is odd.

STEP 2 Test your conjecture using other numbers. For example, test that it works with the pairs 17, 19 and 23, 31.

 $17 \times 19 = 323 \checkmark \qquad \qquad 23 \times 31 = 713 \checkmark$

Exercises for Example 3

- 4. Make and test a conjecture about the product of any two even integers.
- **5.** Make and test a conjecture about the product of an even integer and an odd integer.

EXAMPLE 4 Find a counterexample

Find a counterexample to show that the conjecture is false.

Conjecture All odd numbers are prime.

Solution

To find a counterexample, you need to find an odd number that is a composite number.

The number 9 is odd but is a composite number, not a prime number.

Exercise for Example 4

6. Find a counterexample to show that the conjecture is false.

Conjecture The difference of two positive numbers is always positive.

Lesson 2.1





- **2.** Each number is 4 more than the previous number; 17
- **3.** Each number is 3 times the previous number; 243
- **4.** The product of any two even integers is even.
- **5.** The product of an even integer and an odd integer is even.
- **6.** 2 5 = -3, which is not positive

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Study Guide For use with pages 79–85

GOAL Write definitions as conditional statements.

Vocabulary

A **conditional statement** is a logical statement that has two parts, a hypothesis and a conclusion.

When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

The **negation** of a statement is the opposite of the original statement.

To write the **converse** of a conditional statement, switch the hypothesis and conclusion.

To write the **inverse** of a conditional statement, negate both the hypothesis and conclusion.

To write the **contrapositive** of a conditional statement, first write the converse and then negate both the hypothesis and the conclusion.

When two statements are both true or are both false, they are called **equivalent statements.**

If two lines intersect to form a right angle, then they are **perpendicular lines.**

A **biconditional statement** is a statement that contains the phrase "if and only if."

EXAMPLE1 Rewrite four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Basketball players are athletes." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a basketball player, then you are an athlete.

True, basketball players are athletes.

Converse If you are an athlete, then you are a basketball player.

False, not all athletes play basketball.

Inverse If you are not a basketball player, then you are not an athlete.

False, even if you don't play basketball, you can still be an athlete.

Contrapositive If you are not an athlete, then you are not a basketball player.

True, a person who is not an athlete cannot be a basketball player.

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Study Guide continued For use with pages 79–85

Exercises for Example 1

Write the if-then form, the converse, the inverse, and the contrapositive of the statement. Decide whether each statement is *true* or *false*.

1. All 180° angles are straight angles. **2.** All cats are mammals.

EXAMPLE2 Use definitions

Decide whether each statement about

the diagram is true. Explain your answer using the definitions you have learned.

a. $\overrightarrow{AC} \perp \overrightarrow{CD}$

b. $\angle ACD$ and $\angle BDC$ are complementary.

Solution

- **a.** This statement is true. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So the lines are perpendicular.
- **b.** This statement is false. Both angles are right angles, so the sum of their measures is not 90°.

Exercises for Example 2

Use the diagram in Example 2. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

3. $\angle ACD$ and $\angle BDC$ are supplementary. **4.** $\overrightarrow{AC} \perp \overrightarrow{BD}$

EXAMPLE3 Write a biconditional

Write the definition of supplementary angles as a biconditional.

Solution

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Definition If two angles are supplementary angles, then the sum of their measures is 180° .

Converse If the sum of the measures of two angles is 180°, then they are supplementary angles.

Biconditional Two angles are supplementary angles if and only if the sum of their measures is 180°.

Exercises for Example 3

Rewrite the definition as a biconditional.

- **5.** If two angles are complementary angles, then the sum of their measures is 90° .
- **6.** If a polygon is equilateral, then all of its sides are congruent.



Lesson 2.2

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1. If-then: If the measure of an angle is 180°, then the angle is a straight angle. True.

Converse: If an angle is a straight angle, then the measure of the angle is 180°. True. **Inverse:** If the measure of an angle is not 180°, then the angle is not a straight angle. True. **Contrapositive:** If an angle is not a straight angle, then the measure of the angle is not 180°. True.

2. If-then: If an animal is a cat, then it is a mammal. True. Converse: If an animal is a mammal, then it is a cat. False. There are mammals that are not cats. Inverse: If an animal is not a cat, then it is not a mammal. False. There are mammals that are not cats. Contrapositive: If an animal is not a mammal, then it is not a cat. True.

- **3.** True; Both angles are right angles, so the sum of their measures is 180°.
- **4.** False; The two lines do not intersect.
- **5.** Two angles are complementary angles if and only if the sum of their measures is 90°.
- 6. A polygon is equilateral if and only if all of its sides are congruent.

ESSON 2.3

b. Notice that the conclusion of the second statement is the hypothesis of the first statement.

If x > 5, then 2x > 7.

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Use the Law of Detachment EXAMPLE 1

Use the Law of Detachment to make a valid conclusion in the true situation.

If two angles have the same measure, then they are congruent. You know that $m \angle A = m \angle B$.

Solution

First, identify the hypothesis and the conclusion of the first statement. The hypothesis is "If two angles have the same measure." The conclusion is "then they are congruent."

Because $m \angle A = m \angle B$ satisfies the hypothesis of a true conditional statement, the conclusion is also true. So, $\angle A \cong \angle B$.

EXAMPLE 2 Use the Law of Syllogism

If possible, use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

- **a.** If the electric power is off, then the refrigerator does not run. If the refrigerator does not run, then the food will spoil.
- **b.** If 2x > 10, then 2x > 7.

If x > 5, then 2x > 10.

Solution

- **a.** The conclusion of the first statement is the hypothesis of the second statement, so you can write the following statement. If the electric power is off, then the food will spoil.

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GOAL

Study Guide

For use with pages 86–93

Use deductive reasoning to form a logical argument. Vocabulary Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. Law of Detachment If the hypothesis of a true conditional statement is true, then the conclusion is also true. Law of Syllogism If these statements are true, If hypothesis p, then conclusion q.

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If hypothesis q, then conclusion r.

then the following statement is true.

If hypothesis *p*, then conclusion *r*.

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Study Guide continued

For use with pages 86–93

- **Exercises for Examples 1 and 2 1** If $(A = a = b = a)^{2} < m < A < 00^{\circ}$ Angle *P* is an equilation
- **1.** If $\angle A$ is acute, then $0^{\circ} < m \angle A < 90^{\circ}$. Angle *B* is an acute angle. Using the Law of Detachment, what conclusion can you make?
- **2.** If *B* is between *A* and *C*, then AB + BC = AC. *E* is between *D* and *F*. Using the Law of Detachment, what conclusion can you make?
- **3.** If you study hard, you will pass all of your classes. If you pass all of your classes, you will graduate. Using the Law of Syllogism, what statement can you make?
- **4.** If $x^2 > 9$, then $x^2 > 8$. If x > 4, then $x^2 > 9$. Using the Law of Syllogism, what statement can you make?

EXAMPLE 3 Use inductive and deductive reasoning

What conclusion can you make about the sum of two even integers?

Solution

STEP 1 Look for a pattern in several examples. Use inductive reasoning to make a conjecture.

-2 + 4 = 2, -4 + 10 = 6, 6 + 8 = 14, 12 + 6 = 18,

-20 + 14 = -6, -12 + 2 = -10, -6 + 2 = -4, -2 + (-6) = -8

Conjecture: Even integer + Even integer = Even integer

STEP 2 Let *n* and *m* be any integer. Use deductive reasoning to show the conjecture is true.

2n and 2m are even integers because any integer multiplied by 2 is even.

2n + 2m represents the sum of two even integers.

2n + 2m can be written as 2(n + m).

The sum of two integers (n + m) is an integer and any integer multiplied by 2 is even.

The sum of two even integers is an even integer.

Exercise for Example 3

5. What conclusion can you make about the sum of two odd integers? (*Hint:* An odd integer can be written as 2n + 1, where *n* is any integer.)

Lesson 2.3

Study Guide

- **1.** $0^{\circ} < m \angle B < 90^{\circ}$
- **2.** DE + EF = DF
- **3.** If you study hard, you will graduate.
- **4.** If x > 4, then $x^2 > 8$.
- **5.** The sum of two odd integers is an even integer.

Study Guide

For use with pages 96–102

GOAL Use postulates involving points, lines, and planes.

Vocabulary

A line is a **line perpendicular to a plane** if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point.

Postulate 5 Through any two points there exists exactly one line.

Postulate 6 A line contains at least two points.

Postulate 7 If two lines intersect, then their intersection is exactly one point.

Postulate 8 Through any three noncollinear points there exists exactly one plane.

Postulate 9 A plane contains at least three noncollinear points.

Postulate 10 If two points lie in a plane, then the line containing them lies in the plane.

Postulate 11 If two planes intersect, then their intersection is a line.

EXAMPLE 1 Identify a postulate illustrated by a diagram

State the postulate illustrated by the diagram. a. If then If then Solution **a.** Postulate 6 A line contains at least two points. **b.** Postulate 8 Through any three noncollinear points there exists exactly one plane. **Exercises for Example 1** State the postulate illustrated by the diagram. 1. If then If then

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EXAMPLE2 Use given information to sketch a diagram

Sketch a diagram showing \overrightarrow{AB} intersecting \overrightarrow{CD} at point *E*, so that $\overrightarrow{AB} \perp \overrightarrow{CD}$.

Solution

- **STEP 1** Draw \overrightarrow{AB} and label points A and B.
- **STEP 2** Draw point E on \overrightarrow{AB} .
- **STEP 3** Draw \overrightarrow{CD} through *E* perpendicular to \overrightarrow{AB} . Mark a right angle.

Exercise for Example 2

3. Redraw the diagram in Example 2 if the given information also states that $\overline{AE} \cong \overline{EB}$.

EXAMPLE3 Interpret a diagram in three dimensions

Which of the following statements *cannot* be assumed from the diagram?

All points shown are coplanar.

 $\overleftarrow{FG} \perp \overleftarrow{CD}$ or $m \angle CEF = 90^\circ$.

C, E, and D are collinear.

Solution

When you interpret a diagram, you can only assume information about size or measure if it is marked.

With no right angle marked, you cannot assume $\overrightarrow{FG} \perp \overrightarrow{CD}$ or $m \angle CEF = 90^\circ$.

Exercises for Example 3

Use the diagram in Example 3 to determine if the statement is *true* or *false*.

- **4.** $\angle CEF$ and $\angle FED$ are a linear pair.
- **5.** $\angle CEF \cong \angle FED$
- $6. \quad CD = BD$
- **7.** \overrightarrow{FG} and \overrightarrow{CD} intersect at *E*.
- **8.** $\angle CEF$ and $\angle GED$ are vertical angles.
- **9.** \overrightarrow{FG} and \overrightarrow{BD} do not intersect.



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Lesson 2.4

Study Guide 1. Postulate 5

2. Postulate 9



4. true

5. false

6. false

7. true

8. true

9. false

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lse algebrai	c properties in logical arguments.
Vocabula	ry
Algebraic Pr	operties of Equality
Let a, b , and	c be real numbers.
Addition Pro	pperty If $a = b$, then $a + c = b + c$.
Subtraction	Property If $a = b$, then $a - c = b - c$.
Multiplication	pn Property If $a = b$, then $ac = bc$.
Division Pro	perty If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution any equation	Property If $a = b$, then a can be substituted for expression.
Distributive real numbers	Property $a(b + c) = ab + ac$, where a, b , and .
Reflexive Pro	operty of Equality
Real Numbe	rs For any real number $a, a = a$.
Segment Lei	agth For any segment \overline{AB} , $AB = AB$.
Angle Measu	ire For any angle $\angle A$, $m \angle A = m \angle A$.
Symmetric I	Property of Equality
Real Numbe then $b = a$.	rs For any real numbers a and b , if $a = b$,
Segment Let then $CD = A$	ngth For any segments \overline{AB} and \overline{CD} , if $AB = CI$ B.
Angle Measu then $m \angle B =$	tre For any angles $\angle A$ and $\angle B$, if $m \angle A = m \angle m \angle A$.
Transitive P	roperty of Equality
Real Numbe $b = c$, then a	rs For any real numbers a, b , and c , if $a = b$ ar $= c$.
Segment Let if $AB = CD$ a	ngth For any segments \overline{AB} , \overline{CD} , and \overline{EF} , and $CD = EF$, then $AB = EF$.
Angle Measure and $m \neq B =$	tre For any angles $\angle A$, $\angle B$, and $\angle C$, if $m \angle A$ $m \angle C$ then $m \angle A = m \angle C$



EXAMPLE 1 Write reasons for each step

Solve 3(x - 2) = x + 4. Write a reason for each step.

Equation	Explanation	Reason
3(x-2) = x+4	Write original equation.	Given
3x - 6 = x + 4	Multiply.	Distributive Property
3x - 6 - x = x + 4 - x	Subtract <i>x</i> from each side.	Subtraction Property of Equality
2x-6=4	Combine like terms.	Simplify.
2x = 10	Add 6 to each side.	Addition Property of Equality
x = 5	Divide each side by 2.	Division Property of Equality

The value of x is 5.

Exercises for Example 1

Solve the equation. Write a reason for each step.

1. 2x + 10 = 7x **2.** 4 - (3x + 5) = 11 - x

EXAMPLE2 Use properties of equality

In the	diagra	m, <i>WY</i>	= XZ .
Show	that <i>V</i>	VX = Y2	7

Equation	Explanation	Reason
WY = WX + XY	Add lengths of adjacent segments.	Segment Addition Postulate
XZ = XY + YZ	Add lengths of adjacent segments.	Segment Addition Postulate
WY = XZ	Use given information.	Given
WX + XY = XY + YZ	Substitute $WX + XY$ for WY and $XY + YZ$ for YZ .	Substitution Property of Equality
WX = YZ	Subtract <i>XY</i> from each side.	Subtraction Property of Equality

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XY

-2

Exercises for Example 2

Name the property of equality the statement illustrates.

- **3.** If WX = YZ, then YZ = WX.
- **4.** If $m \angle D = m \angle E$ and $m \angle E = 45^{\circ}$, then $m \angle D = 45^{\circ}$.

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- **1.**2
- **2.** -6
- **3.** Symmetric Property of Equality
- **4.** Transitive Property of Equality

LESSON 2.6

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Study Guide For use with pages 112–119

GOAL Write proofs using geometric theorems.

Vocabulary

A **proof** is a logical argument that shows a statement is true.

A two-column proof has numbered statements and corresponding reasons that show an argument in a logical order.

A **theorem** is a statement that can be proven.

Theorem 2.1 Congruence of Segments: Segment congruence is reflexive, symmetric, and transitive.

Theorem 2.2 Congruence of Angles: Angle congruence is reflexive, symmetric, and transitive.

Write a two-column proof EXAMPLE 1

Write a two-column proof for the following situation.



GIVEN: $AD = 8, BC = 8, \overline{BC} \cong \overline{CD}$

PROVE: $\overline{AD} \cong \overline{CD}$

Statements	Reasons	
1. <i>AD</i> = 8 <i>BC</i> = 8	1. Given	
2. $AD = BC$	2. Transitive Property of Equality	
3. $\overline{AD} \cong \overline{BC}$	3. Definition of congruent segments	
4. $\overline{BC} \cong \overline{CD}$	4. Given	
5. $\overline{AD} \cong \overline{CD}$	5. Transitive Property of Equality	

Exercise for Example 1

1. Write a two-column proof for the following situation.

GIVEN: $AD = 12, AB = 12, \overline{BC} \cong \overline{CD}, \overline{AD} \cong \overline{CD}$

PROVE: $\overline{BC} \cong \overline{BA}$



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EXAMPLE2 Name the property shown

Name the property illustrated by the statement.

continued

a. If
$$\overline{XY} \cong \overline{WZ}$$
 and $\overline{WZ} \cong \overline{PQ}$, then $\overline{XY} \cong \overline{PQ}$

b. If
$$\angle M \cong \angle N$$
, then $\angle N \cong \angle M$.

Solution

- a. Transitive Property of Segment Congruence
- **b.** Symmetric Property of Angle Congruence

Exercises for Example 2

Name the property illustrated by the statement.

2. $\angle R \cong \angle R$

4. $\overline{XY} \cong \overline{XY}$

- **3.** If $\overline{XY} \cong \overline{PQ}$, then $\overline{PQ} \cong \overline{XY}$.
- 5. If $\angle X \cong \angle Y$ and $\angle Y \cong \angle Z$, then $\angle X \cong \angle Z$.

EXAMPLE3 Transitive Property of Congruence

Prove the Transitive Property of Angle Congruence.

PROVE: $\angle A \cong \angle C$

GIVEN: $\angle A \cong \angle B, \angle B \cong \angle C$



Exercise for Example 3

6. Prove the Transitive Property of Segment Congruence.

GIVEN: $\overline{AB} \cong \overline{BC}, \overline{BC} \cong \overline{CD}$ **PROVE:** $\overline{AB} \cong \overline{CD}$

Lesson 2.6

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1. AD = 12, AB = 12 (Given); $\overline{AD} \cong \overline{AB}$ (Definition of congruent segments); $\overline{BC} \cong \overline{CD}$,

 $\overline{AD} \cong \overline{CD}$ (Given); $\overline{BC} \cong \overline{BA}$ (Transitive Property of Segment Congruence)

2. Reflexive Property of Angle Congruence

3. Symmetric Property of Segment Congruence

4. Reflexive Property of Segment Congruence

5. Transitive Property of Angle Congruence

6. $\overline{AB} \cong \overline{BC}, \overline{BC} \cong \overline{CD}$, (Given); AB = BC (Definition of congruent segments); BC = CD (Definition of congruent segments); AB = CD (Transitive Property of Equality); $\overline{AB} \cong \overline{CD}$ (Definition of congruent segments)

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Study Guide For use with pages 122–131

GOAL Use properties of special pairs of angles.

Vocabulary

Theorem 2.3 Right Angles Congruence Theorem: All right angles are congruent.

Theorem 2.4 Congruent Supplements Theorem: If two angles are supplementary to the same angle (or to congruent angles), then they are congruent.

Theorem 2.5 Congruent Complements Theorem: If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

Postulate 12 Linear Pair Postulate: If two angles form a linear pair, then they are supplementary.

Theorem 2.6 Vertical Angles Congruence Theorem: Vertical angles are congruent.

EXAMPLE 1 Find angle measures

Complete the statement given that $m \angle AGF = 90^{\circ}$.

- **a.** $m \angle CGD = \underline{?}$
- **b.** If $m \angle BGF = 113^\circ$, then $m \angle DGE = \underline{?}$.

Solution

- **a.** Because $\angle CGD$ and $\angle AGF$ are vertical angles, $\angle CGD \cong \angle AGF$. By the definition of congruent angles, $m \angle CGD = m \angle AGF$. So, $m \angle CGD = 90^{\circ}$.
- **b.** By the Angle Addition Postulate, $m \angle BGF = m \angle AGF + m \angle AGB$. Substitute to get $113^\circ = 90^\circ + m \angle AGB$. By the Subtraction Property of Equality, $m \angle AGB = 23^\circ$. Because $\angle DGE$ and $\angle AGB$ are vertical angles, $\angle DGE \cong \angle AGB$. By the definition of congruent angles, $m \angle DGE = m \angle AGB$. So, $m \angle DGE = 23^\circ$.

Exercises for Example 1

Complete the statement given that $m \angle BHD = m \angle CHE = 90^\circ$.

- **1.** $m \angle AHG = \underline{?}$
- **2.** $m \angle CHA = _?$
- **3.** If $m \angle CHD = 31^\circ$, then $m \angle EHF = \underline{?}$.
- **4.** If $m \angle BHG = 125^\circ$, then $m \angle CHF = \underline{?}$.
- **5.** If $m \angle EHF = 38^\circ$, then $m \angle BHC = \underline{?}$.





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EXAMPLE 2 Find angle measures

If $m \angle BGD = 90^\circ$ and $m \angle CGD = 26^\circ$, find $m \angle 1$, $m \angle 2$, and $m \angle 3$.

Solution

 $\angle BGC$ and $\angle CGD$ are complementary. So, $m \angle 1 = 90^\circ - 26^\circ = 64^\circ$.

 $\angle AGB$ and $\angle BGD$ are supplementary. So, $m \angle 2 = 180^\circ - 90^\circ = 90^\circ$.

 $\angle AGF$ and $\angle CGD$ are vertical angles. So, $m \angle 3 = 26^{\circ}$.

Exercises for Example 2

In Exercises 6 and 7, refer to Example 2.

6. Find $m \angle FGE$.

7. Find $m \angle DGE$.

EXAMPLE3 Use algebra

Solve for *x* in the diagram.

Solution



Because $\angle AEB$ and $\angle BEC$ form a linear pair, the sum of their measures is 180°. So, you can solve for *x* as follows:

(2x + 3) + 25 = 180Definition of supplementary angles.2x + 28 = 180Combine like terms.2x = 152Subtract 28 from both sides.x = 76Divide each side by 2.

Exercises for Example 3

Solve for *x* in the diagram.





Lesson 2.7

Study Guide 1. 90° **2.** 90° **3.** 31° **4.** 125° **5.** 52° **6.** 64° **7.** 90° **8.** 147 **9.** 44