Chapter 2: The Next Step… Conditional Probability

STATISTICS

Vocabulary

Common Vocabulary

- Event: something that occurs or happens with one or more possible outcomes
- Experiment: process of taking a measurement or making an observation
- Simple Event: simplest outcome of an experiment
- Sample Space: the set of all possible events or outcomes of an experiment

Common Vocabulary

- Union: when two event occur in a single performance of an experiment; an ‘or’ relationship
- Intersection: when two events occur in a single performance of an experiment; an ‘and’ relationship
- Complement: all the outcomes in the sample space that are not in a given event

Vocabulary

- Given the probability experiment we did on Friday, define the given terms in the context of the experiment:
  - Event:
  - Experiment:
  - Simple Event:
  - Sample Space:
  - Union:
  - Intersection:
  - Complement:

Complement of an Event

- Denoted as P(A'), complement of A
  - Complement is anything that is not A
- P(A) + P(A') = 1
- P(A') = 1 – P(A)

Finding the Complement

- I have Starbursts in a bag. 12 are orange, 3 are yellow, 9 are pink, and 7 are red.
  - What is the probability that I don't pick a red Starburst?
  - What is the probability that I don't pick an orange or yellow Starburst?

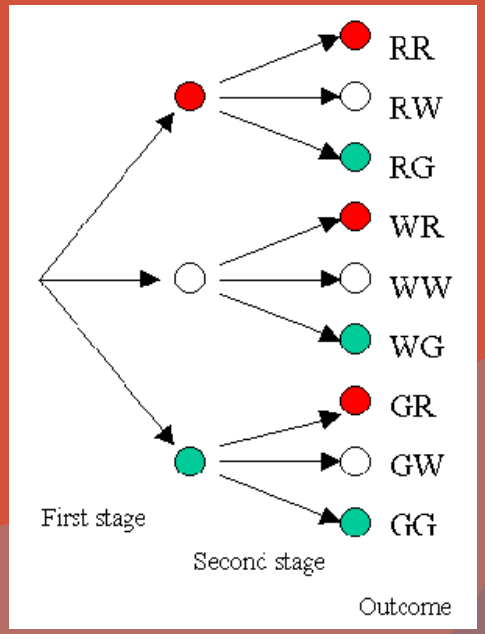
Probabilities

- From your results of the experiment, determine the following:
  - The probability that neither die rolled a 1.
  - The probability that both die rolled the same number.
  - The probability that both die rolled different numbers.
  - The probability that the sum of the two dice was less than 9.
  - The probability that the sum of the two dice was greater than 5.

What are Tree Diagrams?

What is a tree diagram?

- A tree diagram is another way to show the outcomes of simple probability events.
  - Each outcome is represented as a branch on a tree.



Tree Diagrams

- Where have you seen a tree diagram before?
- What was its purpose?
- Was it helpful to what you were doing?

Tree Diagrams

- Let's create a tree diagram for flipping a FAIR coin three times.

  - What is the probability that all the outcomes will be heads?
  - What is the probability that at least two of the outcomes will be heads? (order doesn't matter)

Example

- Irvin opens up his sock drawer to get a pair of socks to wear to school. He looks in the sock drawer and sees 4 red socks, 8 white socks, and 6 brown socks. Irvin reaches in the drawer and pulls out a red sock. He is wearing blue shorts so he replaces it. He then draws out a white sock. What is the probability that Irvin pulls out a red sock, replaces it, and then pulls out a white sock?

Tree Diagrams

- What would a tree diagram look like for the experiment we did on Friday?

  - Would you want to create that diagram?

Importance

- What are some advantages of creating a tree diagram for a probability event?

- Why would creating a tree be useful?
- What are some disadvantages of creating a tree diagram for a probability event?

Order and Probability

Review Worksheet

- Look at the worksheet you were given previously.
- How many different ways did you find for question 1?
  - What method did you use to find these possibilities?
- How many different ways did you find for question 2?
  - What method did you use to find all these possibilities?

Permutation vs. Combination

- Permutation is the number of possible arrangements in an ordered set of objects.
  - Order matters
    - The combination to the safe was 472. We care about the order because 247 wouldn't work.
- Combinations are arrangements of objects without regard to order and without repetition
  - Order doesn't matter
    - My fruit salad is a combination of apples, grapes, and bananas. It doesn't matter the order we say it, we could also say grapes, bananas, and apples.
- Can you think of other examples?

Permutation or Combination

- Selecting three students to attend a conference in Washington D.C.
  - Combination
- Selecting a lead and an understudy for a school play.
  - Permutation
- Assigning students to their seats on the first day of school.
  - Permutation

Fundamental Counting Principle

- The Fundamental Counting Principle states that if an event can be chosen in *p* different ways and another independent event can be chosen in *q* different ways, the number of different ways the 2 events can occur is *p* x *q*.
  - Example:
    - You have 2 pairs of pants and 4 shirts. How many different outfits can you make?

Permutations

Permutations (order matters)

- Permutations without repetition
  - Places in a race; you can't be first and second
- Permutations with repetition
  - Choosing a combination for a lock or phone number; you can use the same number each time

With or Without Repetition

- How many different arrangements for the letters in the word “MATHEMATICS”?
- How many different arrangements for the letters in the word “STATISTICS”?
- How many different arrangements for the letters in the word “LUNCH”?

Notation for Permutations Without Repetition

- N is the total number of objects
- R is the number of objects chosen
- When n = r then, nPr = n!
  - ! means to multiply a series of descending natural numbers
    - Ex: 5! = 5*4*3*2*1 = 120
    - 0! = 1

$$ {}_{n}P_{r} = \frac{n!}{(n-r)!} $$

Example

- How many ways can you arrange the letters in the word “MATH”?
  - With repetition or without repetition?

More Examples

- Solve
  - 4 P 4
  - 6 P 3
  - What is the total number of possible 4-letter arrangements of the letters ‘s’, ‘n’, ‘o’, and ‘w’ if each letter is used only once?
  - A committee is to be formed with a president, vice president, and a treasurer. If there are ten people to select from how many committees are possible?

Notation for Permutations With Repetition

- This is used when there are double objects or repetitions in a permutation problem.
- N is the number of objects
- R is the number of objects chosen
- X is the number of times a letter is repeated

$$\frac{nPr}{x_1!x_2!}$$

Example

- How many ways can we arrange the letters in the word “TOOTH?”

More Examples

- How many different 5- letter arrangements can be formed from the word “APPLE?”

- How many different 6-digit numerals can be written using the following 7 digits? Assume the repeated digits are all used.
  - 3,3,4,4,4,5,6

More Examples

- How many ways can you arrange the letters of the word “loose”?
  - 60 ways
- How many ways can you arrange the letters in the word “appearing”?
  - 90,720 ways
- How many ways can you arrange the letters in the word “Mississippi”?
  - 34,650 ways

Quick Review

- Which of these are with and without repetition?
  - Arranging the letters of the word “ALGORITHM”?
  - Arranging the letters of the word “TOOTHFAIRIES”
- Calculate the number of ways to arrange the letters in both questions above.
  - 362,880 ways
  - 59,875,200 ways

Combinations

Combinations (order doesn't matter)

- Combinations without repetition
  - No repetition allowed
    - Example: numbers in a lottery ticket
- Combinations with repetition
  - Repetition allowed
    - Example: type of coins in your pocket

Notation for Combinations Without Repetition

- N is the number of objects taken
- R is the number of objects chosen

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$

Example

- In how many ways can 3 desserts be chosen in any order from a menu of 10?

  - 120 ways

More Examples

- There are 12 boys and 14 girls in Ms. Peacock's math class. Find the number of ways that Ms. Peacock can select a team of 3 students from the class to work on a group project. The team must consist of 2 boys and 1 girl.
  - 1,092 ways

More Examples

- From a list of 8 books, how many groups of 3 books can be selected?
  - 56 ways

More Examples

- If there are 20 rock songs and 20 rap songs to choose from, in how many different ways can you select 12 rock songs and 7 rap songs for a mixed CD?

  - 9.765 x 10 ^ 9 ways

More Examples

- From a group of 10 men and 12 women, how many committees of 3 men and 3 women can be formed?
  - 26,400 ways

Conditional Probability

What is it?

- Conditional Probability is a probability calculation where the probability of a second event is affected by the probability of the first.

- Can you think of some examples?

Notation

The probability of event A occurring, given that event B has occurred.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = P(A \cap B) / P(A)$$

$$P(A|B) = P(A \cap B) / P(B)$$

$$P(A \cap B) = P(B|A)\,P(A) = P(A|B)\,P(B)$$

$$\frac{P(\textit{first and second event})}{P(\textit{first event})}$$

Example

- In a recent election, 35% of the voters were democrats and 65% were not. Of the democrats, 75% voted for candidate Z and of the non-Democrats, 15% voted for candidate Z. Assume A = voter is Democrat; B = voted for candidate Z. Answer the following:
  - Find P(B/A), P(B/Ac)
  - Find P(A n B), explain its representation
  - Find P(Ac n B), explain its representation
  - Find P(B)

Example

- Find P(B/A), P(B/Ac)
  - .75
- Find P(A n B), explain its representation
  - P(B/A) * P(A) = .75(.35) = .26; that is the probability of being a democrat and voting for candidate Z
- Find P(Ac n B), explain its representation
  - P(B/Ac) * P(Ac) = .15(.65) = .0975; that is the probability of not being a democrat and voting for candidate Z.
- Find P(B)
  - P(A n B) + P(Ac n B) = .26 + .0975 = . 36

Example

- IF P(A) = .3, P(B) = .7, and P(A n B) = .15
  - Find P(A/B).
  - Find P(B/A).

More Examples

- At Dunkerton High School, 90% of the students take Chemistry and 35% of the students take both physics and chemistry. What is the probability that a student from Dunkerton who is taking Chemistry is also taking physics?

More Examples

- Assume two fair coins are tossed.
  - List all the possible outcomes in the sample space.
  - Suppose the two events are defined as follows:
    - A: At least one head appears
    - B: Only one head appears
- Find the probabilities:
  - P(A), P(B), P (A n B), P(A/B), P(B/A)

More Examples

Create your own experiment

- You are going to create your own experiment of conditional probability:
  - EXAMPLE:
    - Would you rather have ice cream or a candy bar? If you would rather have ice cream, would you want white or chocolate? IF you would rather have a candy bar, would you rather have a Snickers or a Twix?