Chapter 2

Negative Numbers (Integers)

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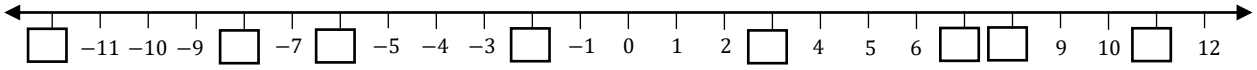
2A Introduction to Negative Numbers (Integers) https://youtu.be/way9i8y2UnY

Give explanations for the following number groups:

Natural Numbers ($\mathbb{N}$)	Integers ($\mathbb{Z}$)

Question 1

Fill in the blanks on the number line below.



Question 2

List the integers from -3 to 5.

Example 1 https://youtu.be/nKFeYWXWp_U

Insert the symbol < or > in the empty boxes below to make each statement true.

(a) $-1 \square 3$ (b) $-5 \square -2$ (c) $-4 \square -9$

Question 3

Insert the symbol < or > in the empty boxes below to make each statement true.

(a) $-2 \square 6$ (b) $7 \square -1$ (c) $-3 \square -8$

(d) $-6 \square 0$ (e) $-5 \square -1$ (f) $0 \square -4$

(g) $-28 \square 13$ (h) $-26 \square -29$ (i) $-56 \square -57$

Question 4

Put the following numbers in **ascending** order.

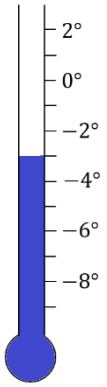
(a) $-4, -12, 13, 0, -1, 5, 6, -3$ (b) $11, -17, -304, 58, 2, -1002, 13$

Question 5

The thermometer at right was placed in a freezer.

a) What is the temperature inside the freezer?

b) If the temperature increases by 4 degrees, what will the new temperature be?



Question 6

Jeremy has a bank account with –$540 in it.

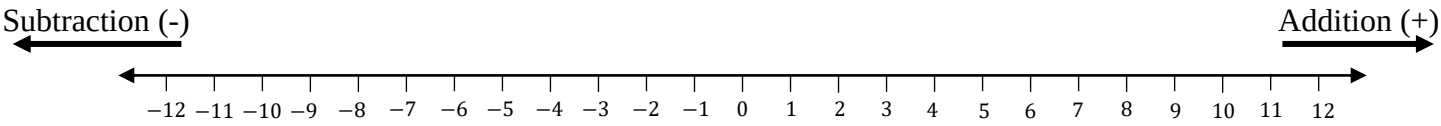
a) Why would a bank use negatives when talking about amounts of money?

b) If he deposited $200 in his bank account, what would be his new balance?

2B Addition and Subtraction of Integers https://youtu.be/akuVsJDRm7U

Answer the questions below:

How can you tell the difference between the subtraction and negative symbol?	How can you tell the difference between the addition and positive symbol?



Example 1 https://youtu.be/NsnUp3PAEy4

Solve the following:

(a) $-7+2$ (b) $3-6$ (c) $-3+5$

(d) $-1-4$ (e) $0-6$ (f) $-5+18$

Question 1

Solve the following:

(a) $-3+4$ (b) $2-8$ (c) $-1+6$

(d) $-3-2$ (e) $0-4$ (f) $-7+3$

(g) $-6+0$ (h) $-5+10$ (i) $-12+7$

(j) $9-17$ (k) $-5-6$ (l) $0-13$

(m) $-12+15$ (n) $-10+19$ (o) $8-16$

Question 2

Find the missing number.

(a) $-5+\square=2$ (b) $-7+\square=-3$ (c) $4-\square=-5$

(d) $-2-\square=-8$ (e) $0-\square=-10$ (f) $-11+\square=-11$

(g) $\square+2=-6$ (h) $\square-5=-10$ (i) $\square+8=5$

Adding and Subtracting Large Integers

When trying to solve problems involving large integers we cannot always use the number line to come up with a solution. There are basically two scenarios that we face when solving these problems.

Scenario 1 https://youtu.be/fftCvYUXkIs

- Both numbers have a plus sign in front of them
- Both numbers have a minus sign in front of them

Example

Solve the following:

(a) $2+5$ (b) $-2-5$

(c) $24+31$ (d) $-24-31$

Question 3
Solve the following:

(a) $11 + 28$ (b) $-23 - 41$ (c) $-16 - 23$

(d) $-48 - 32$ (e) $17 + 26$ (f) $-78 - 19$

(g) $-125 - 506$ (h) $-381 - 429$ (i) $-1268 - 780$

Scenario 2 https://youtu.be/d2wAKUEnMng

- One number has a plus sign in front of it while the other has a minus sign in front of it.

Example
Solve the following:

(a) $5 - 2$ (b) $-5 + 2$ (c) $2 - 5$ (d) $-2 + 5$

(e) $45 - 22$ (f) $-45 + 22$ (g) $22 - 45$ (h) $-22 + 45$

Question 4
Solve the following:

(a) $38 - 14$ (b) $22 - 45$ (c) $-15 + 29$

(d) $-68 + 17$ (e) $72 - 65$ (f) $-19 + 45$

(g) $45 - 73$ (h) $-83 + 33$ (i) $125 - 36$

(j) $-56 + 132$ (k) $-230 + 102$ (l) $761 - 890$

(m) $-1002 + 785$ (n) $6700 - 7209$ (o) $-1205 + 2450$

Question 5
Solve the following:

(a) $-28 - 18$ (b) $32 - 58$ (c) $-78 + 36$

(d) $-29 + 56$ (e) $-101 - 59$ (f) $-121 + 78$

(g) $156 - 212$ (h) $-253 + 368$ (i) $-178 - 234$

(j) $-246 - 520$ (k) $-406 + 212$ (l) $458 - 716$

(m) $-2305 + 3290$ (n) $5409 - 7376$ (o) $-1348 - 3754$

Question 6
Solve the following by working from left to right:

(a) $-2 + 5 - 8$ (b) $6 - 10 + 4$ (c) $-1 - 7 + 13$

(d) $-5 + 14 - 12$ (e) $5 - 22 + 11$ (f) $-16 - 28 + 52$

Question 7
James owes $15 600 on his credit card. He pays back $2350 and then a month later borrows another $1305 for a TV. How much does he now owe on his credit card?

2C Further Addition and Subtraction of Integers https://youtu.be/IO6wDjv8HxU

Quite often in the English language we use what is called a double negative. Here are some examples:

- There ain't no way I am doing that
- I don't got no time
- There aren't no people behind there

Double negatives in the English language work in a similar way for Mathematics.

What to do when two signs are next to each other?

Subtracting a negative (the double negative)	Adding a negative

Example 1 https://youtu.be/5ClyiCYtjpE

Solve the following:

(a) $5 + (-3)$ (b) $4 - (-6)$ (c) $-2 + (-7)$

(d) $-9 - (-5)$ (e) $6 + (-10)$ (f) $-8 - (-11)$

Question 1

Solve the following:

(a) $4 + (-3)$ (b) $7 - (-3)$ (c) $6 + (-6)$

(d) $5 - (-1)$ (e) $3 + (-8)$ (f) $9 + (-11)$

(g) $-1 - (-6)$ (h) $-2 + (-5)$ (i) $-8 - (-3)$

(j) $7 + (-6)$ (k) $15 - (-3)$ (l) $-7 - (-4)$

(m) $12 + (-5)$ (n) $-5 + (-4)$ (o) $-8 - (-16)$

Question 2

Solve the following:

(a) $25 + (-17)$ (b) $45 - (-13)$ (c) $17 + (-37)$

(d) $17 - (-43)$ (e) $9 + (-37)$ (f) $28 + (-56)$

(g) $-26 - (-63)$ (h) $-19 + (-65)$ (i) $-28 - (-130)$

Question 3
Find the missing number.

(a) $5 + \square = -1$ (b) $-3 + \square = -8$ (c) $4 - \square = 10$

(d) $7 + \square = 2$ (e) $-9 - \square = 0$ (f) $-8 + \square = -12$

(g) $-10 - \square = -4$ (h) $12 - \square = 25$ (i) $-36 + \square = -52$

(j) $\square - (-8) = 2$ (k) $\square + (-5) = -12$ (l) $\square - (-11) = 23$

(m) $\square + (-17) = -58$ (n) $\square - (-34) = -13$ (o) $-76 - \square = -34$

Question 4
Solve the following by working from left to right:

(a) $5 + (-5) - (-7)$ (b) $4 + (-10) + (-3)$ (c) $-2 - (-6) - (-1)$

(d) $-7 + (-8) - (-28)$ (e) $5 + (-32) + (-21)$ (f) $-17 - (-29) + (-46)$

Question 5
Gerald has a debt of $560. How much money will he owe if $220 of debt is removed?

Question 6
Jane starts with $56 in her bank account. $135 of debt is added to her account. Later the bank removes $13 of debt due to an error. How much does she now owe the bank?

Question 7
Rewrite the following sentences using mathematical statements then solve it. The first one has been completed for you.

(a) The sum of 2 and -3.
Answer: $2 + (-3) = -1$

(b) The sum of -3 and -6.

(c) The difference between 6 and -5.

(d) The difference between -1 and -12

(e) The difference between -14 and 6

(f) The difference between 4 and the sum of 3 and -8

Question 8
Solve the following:

(a) $5 - (-(-2)$ (b) $12 - (-(-(-6)$ (c) $17 - (-(-(-(-(-23)$

Question 9
If x is a positive number and y is a negative number, state whether each statement below is true or false.

(a) $x - y$ is positive (b) $y - x$ is positive

2D Multiplication and Division of Integers https://youtu.be/7dT_vyMM6B8

There are two possible situations that can occur when multiplying and dividing negative numbers. Watch the video and write notes in the boxes below:

One number is negative and the other is positive	Both numbers are negative

Example 1 https://youtu.be/plHzytUYJhQ

Calculate the following:

(a) $4 \times (-3)$ (b) $-2 \times (-8)$ (c) -6×9

Question 1

Calculate the following:

(a) $3 \times (-7)$ (b) $-2 \times (-9)$ (c) -5×6

(d) $-8 \times (-6)$ (e) -11×5 (f) -10×1

(g) $9 \times (-12)$ (h) $-1 \times (-15)$ (i) -5×6

(j) $-15 \times (-5)$ (k) $45 \times (-2)$ (l) -16×3

Example 2 https://youtu.be/59peomDmsfg

Calculate the following:

(a) $-15 \div 3$ (b) $-18 \div (-6)$ (c) $45 \div (-5)$

Question 2

Calculate the following:

(a) $-8 \div 2$ (b) $-16 \div (-4)$ (c) $50 \div (-10)$

(d) $-19 \div (-19)$ (e) $-28 \div 7$ (f) $23 \div (-1)$

(g) $22 \div (-11)$ (h) $-56 \div (-8)$ (i) $-90 \div 9$

(j) $-78 \div 6$ (k) $-63 \div (-3)$ (l) $91 \div (-7)$

Question 3

Calculate the following by working from left to right:

(a) $3 \times (-2) \times 4$ (b) $10 \times (-3) \div (-5)$

(c) $-6 \div 2 \times (-10) \div 3$ (d) $42 \div (-7) \times 10 \div 5$

(e) $-64 \div (-8) \times 3 \times (-2)$ (f) $12 \times (-4) \div (-2) \div 6 \times (-1)$

(g) $-56 \div (-7) \times 9 \div (-36)$ (h) $58 \div (-2) \times 4 \times (-3) \div 6$

Question 4

Find the missing number.

(a) $2 \times \square = -10$ (b) $-6 \times \square = -18$ (c) $-4 \times \square = 20$

(d) $-16 \div \square = -2$ (e) $-13 \div \square = 1$ (f) $32 \div \square = -4$

(g) $\square \times (-6) = -42$ (h) $\square \div 11 = -9$ (i) $\square \div (-7) = 12$

(j) $-9 \times \square = 108$ (k) $145 \div \square = -5$ (l) $15 \times \square = -75$

Question 5

Place a multiplication or division sign in each box below to make each statement true.

(a) $4 \square (-3) \square 2 = -6$ (b) $-36 \square 12 \square 5 = -15$

(c) $-15 \square 3 \square (-5) = 9$ (d) $-64 \square (-4) \square 3 = 48$

Question 6

When we multiply two negative numbers we get a positive number, therefore $(-4)^2 = -4 \times (-4) = 16$. Notice that we got a positive solution when we square a negative number.

a) Calculate $(-7)^2$.

b) Can you think of a number that will give a negative solution when squared?

c) If $2^3 = 2 \times 2 \times 2 = 8$, Calculate $(-4)^3$.

Question 7

Two numbers have a product of -24 and a difference of 11. What are the two numbers?

Question 8

Two numbers have a difference of 42 and a quotient of -6. What are the two numbers?

2E Order of Operations with Integers https://youtu.be/erT-dXAS-b0

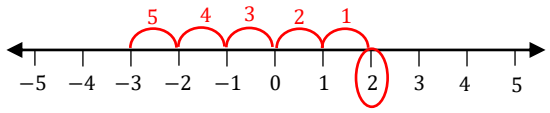
B	I	D	M	A	S
()	3^2 or $\sqrt{5}$	÷	×	+	–

What have we learned so far about performing operations with integers?

Using the number line to add or subtract integers.

- Start at the first number
- Addition – move to the right
- Subtraction – move to the left

$$2 - 5 = -3$$



Multiplying/Dividing Integers

- Ignore the negative at first and multiply/divide
- If the question has one negative then your solution will be negative
- If the question has two negatives (double negative) then your solution will be positive

$$3 \times (-2) = -6 \text{ or } -8 \div (-2) = 4$$

Adding a negative number

When a plus sign and the minus sign are next to each other, we can change them into a minus sign

$$3 + (-2) = 3 - 2$$

Subtracting a negative number (double negative)

When two minus signs are next to each other, we can change them into a plus sign

$$3 - (-2) = 3 + 2$$

Example 1 https://youtu.be/vKvQHCPXIZ8

Evaluate the following using the order of operations:

(a) $4 - 5 \times 2$

(b) $-3 + 2 \times (-4)$

(c) $-5 \times (-2) - 6 \div 3$

(d) $16 \div (-4) - 4 \times (-5)$

Question 1

Evaluate the following using the order of operations:

(a) $-5 + 6 \times 2$

(b) $7 - 4 \times 3$

(c) $-6 + 16 \div 4$

(d) $5 + 3 \times (-3)$

(e) $-12 - 6 \times (-2)$

(f) $17 + 20 \div (-4)$

(g) $3 \times (-2) - 14 \div 7$

(h) $-48 \div 12 + 7 \times (-2)$

(i) $7 \times (-3) - 8 \times (-3)$

Example 2 https://youtu.be/3fbBZbAAVPQ

Evaluate the following using the order of operations:

(a) $-8 \times (-10 + 4)$ (b) $24 \div (4 - 8) \times (-7)$ (c) $-9 + (-2 + 7) \times (-1)$

Question 2

Evaluate the following using the order of operations:

(a) $(3 - 7) \times 5$ (b) $28 \div (2 - 9)$ (c) $(-3 - 7) \times (-15)$

(d) $7 \times (-2 - 3) \div (-35)$ (e) $(7 - 14) \times 2 \div 7$ (f) $-54 \div (3 - 12) \times (-1)$

(g) $(3 - 15) \div (-15 \div 5)$ (h) $(-3 \times 6) - 22 \div (-2)$ (i) $(5 - 12) + (-16 \div 2)$

Example 3 https://youtu.be/WCWJTNQ8ftU

Evaluate the following using the order of operations:

(a) $5 + \dfrac{3 - 7}{2}$ (b) $-\dfrac{3 - 15}{3} + 6$

Question 3

Evaluate the following using the order of operations:

(a) $3 - \dfrac{12 + 8}{4}$ (b) $1 + \dfrac{4 - 12}{2}$ (c) $7 - \dfrac{3 - 21}{6}$

(d) $\dfrac{-11 + 5}{3} + 16$ (e) $-\dfrac{3 - 24}{7}$ (f) $9 - \dfrac{4 \times (-5)}{1 - (-4)}$

Question 4

Insert brackets in order to make each statement true.

(a) $-5 + 3 \times (-9) = 18$ (b) $16 \div 1 - 9 = -2$ (c) $11 \div 1 - 12 + 5 = 4$

(d) $8 + 2 \times 3 - 12 = -10$ (e) $-1 + 6 \div 2 - 7 = -1$ (f) $-2 - 7 - 3 - 12 = 0$

(g) $-13 - 5 \times 2 - 8 = 108$ (h) $-24 \div 3 - 9 + 2 = 6$ (i) $-99 \div 2 - 13 \times 3 = 3$

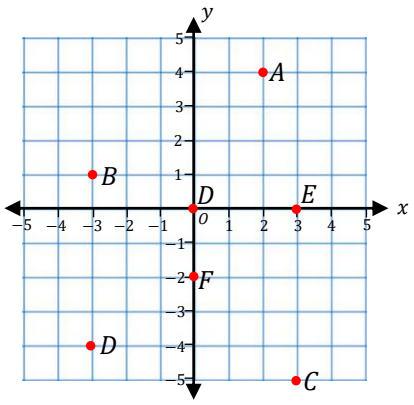
2F The Cartesian Plane https://youtu.be/UXIQapPh3tw

- The cartesian plane has two _____ known as the *x*-axis and the _____.
- A point on the _____ is written in the form _____.
- The two axes meet at a point called the _____, which is usually labelled with an *O*. The coordinates of the origin are _____.

Fill in the blanks above with one of the following:					
(x,y)	axes	$(0,0)$	plane	origin	*y*-axis

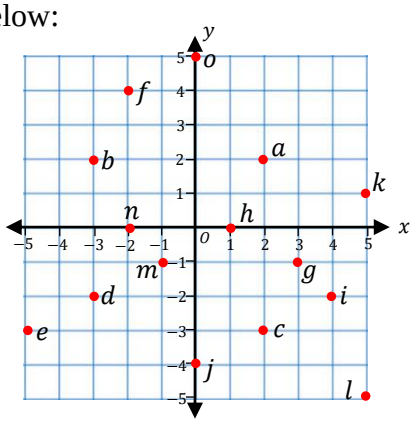
Example 1 https://youtu.be/UCPZoJASof4

Write down the coordinates of each point labelled on the Cartesian plane below:



Question 1

Write down the coordinates of each point labelled on the Cartesian plane below:



(a) (b) (c)

(d) (e) (f)

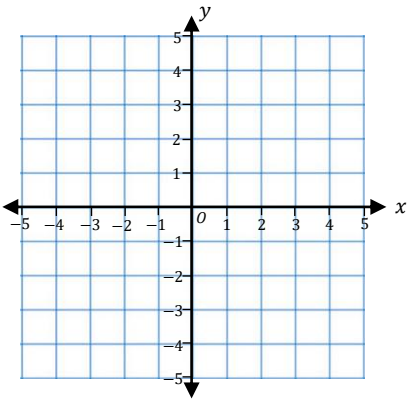
(g) (h) (i)

(j) (k) (l)

(m) (n) (o)

Question 2

Label each point below on the cartesian plane at right:



(a) $(2,1)$ (b) $(5,1)$ (c) $(-3,4)$

(d) $(-1,-4)$ (e) $(3,-1)$ (f) $(-1,3)$

(g) $(-5,0)$ (h) $(2,-4)$ (i) $(0,4)$

(j) $(0,-3)$ (k) $(-3,-2)$ (l) $(1,0)$

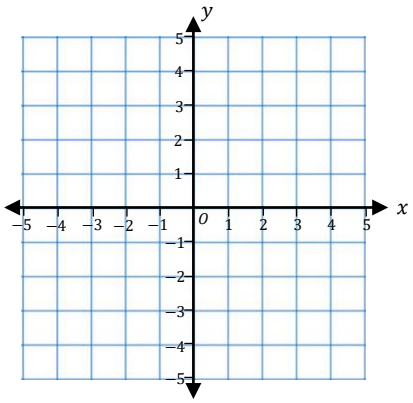
Question 3

Which one of the following points is closest to the origin? $A(1,5)$, $B(-2,4)$ or $C(-3,-2)$?

Example 2 https://youtu.be/uKMM4ODXHa0

Sometimes you will get a set of Cartesian coordinates in a table like the one below. Plot the points on the Cartesian plane at right and connect them to make a straight line.

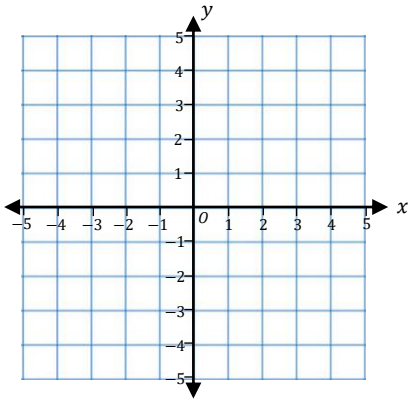
x	-2	-1	0	1	2
y	-4	-2	0	2	4



Question 4

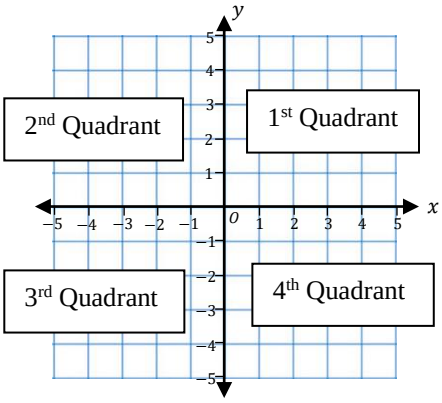
By looking at the set of points on the table below, plot the points on the Cartesian plane at right and connect them to make a straight line.

x	-2	-1	0	1	2
y	3	2	1	0	-1



Question 5

The Cartesian plane is split into four quadrants (as shown to the right). State whether the following is true or false.



a) The point $(2,5)$ lies in the 1st quadrant.

b) The point $(-3,-1)$ lies in the 4th quadrant.

c) The points $(1,-4)$ and $(2,-6)$ lie in the same quadrant.

d) The point $(-7,5)$ lies in the 3rd quadrant.

e) The point $(7,-2)$ is the same as the point $(-2,7)$.

f) A point that lies on the *x*-axis must have a *y* coordinate of 0.

g) A point that lies on the *y*-axis must have an *x* coordinate of 2.

h) A point with a positive *y*-coordinate cannot lie in either Quadrant 3 or 4.

i) A point with a negative *x*-coordinate cannot lie in either Quadrant 1 or 4.

j) The point $(-4,0)$ lies in the 2nd and 3rd quadrant.

Solutions

Chapter 2A
Q1 – $-12, -8, -6, -2, 3, 7, 8, 11$
Q2 – $-3, -2, -1, 0, 1, 2, 3, 4, 5$
Q3 – (a) < **(b)** > **(c)** > **(d)** < **(e)** < **(f)** > **(g)** < **(h)** > **(i)** >
Q4 – (a) $-12, -4, -3, -1, 0, 5, 6, 13$ **(b)** $-1002, -304, -17, 2, 11, 13, 58$
Q5 – (a) $-3°$ **(b)** $1°$
Q6 – (a) a negative amount means that the person owes money.
(b) –$340

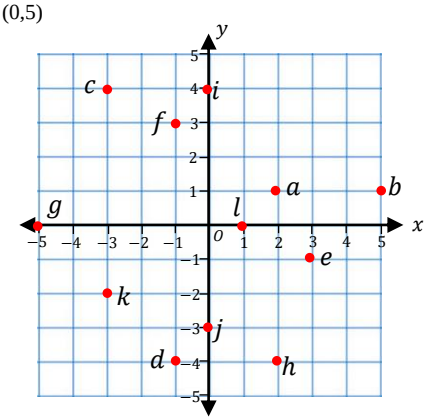
Chapter 2B
Q1 – (a) 1 **(b)** -6 **(c)** 5 **(d)** -5 **(e)** -4 **(f)** -4 **(g)** -6 **(h)** 5 **(i)** -5 **(j)** -8 **(k)** -11 **(l)** -13 **(m)** 3 **(n)** 9 **(o)** -8
Q2 – (a) 7 **(b)** 4 **(c)** 9 **(d)** 6 **(e)** 10 **(f)** 0 **(g)** -8 **(h)** -5 **(i)** -3
Q3 – (a) 39 **(b)** -64 **(c)** -39 **(d)** -80 **(e)** 43 **(f)** -97 **(g)** -631 **(h)** -810 **(i)** -2048
Q4 – (a) 24 **(b)** -23 **(c)** 14 **(d)** -51 **(e)** 7 **(f)** 26 **(g)** -28 **(h)** -50 **(i)** 89 **(j)** 76 **(k)** -128 **(l)** -219 **(m)** -217 **(n)** -509 **(o)** 1245
Q5 – (a) -46 **(b)** -26 **(c)** -42 **(d)** 27 **(e)** -160 **(f)** -43 **(g)** -56 **(h)** 115 **(i)** -412 **(j)** -766 **(k)** -194 **(l)** -258 **(m)** 985 **(n)** -1967 **(o)** -5102
Q6 – (a) -5 **(b)** 0 **(c)** 5 **(d)** -3 **(e)** -6 **(f)** 8
Q7 – $14 555

Chapter 2C
Q1 – (a) 1 **(b)** 10 **(c)** 0 **(d)** 6 **(e)** -5 **(f)** -2 **(g)** 5 **(h)** -7 **(i)** -5 **(j)** 1 **(k)** 18 **(l)** -3 **(m)** 7 **(n)** -9 **(o)** 8
Q2 – (a) 8 **(b)** 58 **(c)** -20 **(d)** 60 **(e)** -28 **(f)** -28 **(g)** 37 **(h)** -84 **(i)** 102
Q3 – (a) -6 **(b)** -5 **(c)** -6 **(d)** -5 **(e)** -9 **(f)** -4 **(g)** -6 **(h)** -13 **(i)** -16 **(j)** -6 **(k)** -7 **(l)** 12 **(m)** -41 **(n)** -47 **(o)** -42
Q4 – (a) 7 **(b)** -9 **(c)** 5 **(d)** 13 **(e)** -48 **(f)** -34
Q5 – $340
Q6 – $66
Q7 – (a) $\mathbf{2 + (-3) = -1}$ **(b)** $-3 + (-6) = -9$ **(c)** $6 - (-5) = 11$
(d) $-1 - (-12) = 11$ **(e)** $6 - (-14) = 20$ **(f)** $4 - \big(3 + (-8)\big) = 9$
Note: part 7(e) should have a positive solution. The difference between 6 and 8 is the same as the difference between 8 and 6.
Q8 – (a) 3 **(b)** 18 **(c)** 40
Q9 – (a) T **(b)** F

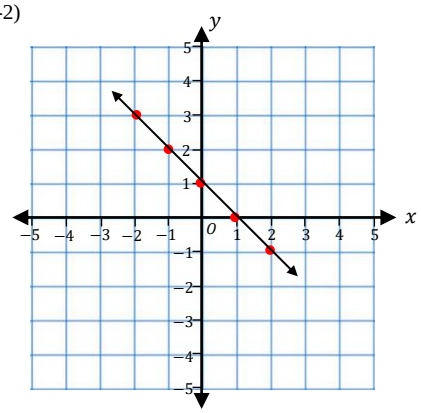
Chapter 2D
Q1 – (a) -21 **(b)** 18 **(c)** -30 **(d)** 48 **(e)** -55 **(f)** -10 **(g)** -108 **(h)** 15 **(i)** -30 **(j)** 75 **(k)** -90 **(l)** -48
Q2 – (a) -4 **(b)** 4 **(c)** -5 **(d)** 1 **(e)** -4 **(f)** -23 **(g)** -2 **(h)** 7 **(i)** -10 **(j)** -13 **(k)** 21 **(l)** -13
Q3 – (a) -24 **(b)** 150 **(c)** 10 **(d)** -12 **(e)** -12 **(f)** -4 **(g)** -2 **(h)** 58
Q4 – (a) -5 **(b)** 3 **(c)** -5 **(d)** 8 **(e)** -13 **(f)** -8 **(g)** 7 **(h)** -99 **(i)** -84 **(j)** -12 **(k)** -29 **(l)** -5
Q5 – (a) ×,÷ **(b)** ÷,× **(c)** ×,÷ **(d)** ÷,×
Q6 – (a) 49 **(b)** None exist **(c)** -64
Q7 – -8 & 3 or -3 & 8
Q8 – -6 & 36 or -36 & 6

Chapter 2E
Q1 – (a) 7 **(b)** -5 **(c)** -2 **(d)** -4 **(e)** 0 **(f)** 12 **(g)** -8 **(h)** -18 **(i)** 3
Q2 – (a) -20 **(b)** -4 **(c)** 150 **(d)** 1 **(e)** -2 **(f)** -6 **(g)** 4 **(h)** -7 **(i)** -15
Q3 – (a) -2 **(b)** -3 **(c)** 10 **(d)** 14 **(e)** 3 **(f)** 13
Q4 – (a) $(-5 + 3) \times (-9) = 18$ **(b)** $16 \div (1 - 9) = -2$
(c) $11 \div (1 - 12) + 5 = 4$ **(d)** $8 + 2 \times (3 - 12) = -10$
(e) $(-1 + 6) \div (2 - 7) = -1$ **(f)** $-2 - 7 - (3 - 12) = 0$
(g) $(-13 - 5) \times (2 - 8) = 108$ **(h)** $-24 \div (3 - 9 + 2) = 6$
(i) $-99 \div ((2 - 13) \times 3) = 3$

Chapter 2F
Filling in the blanks: axes, *y*-axis, plane, (*x*,*y*), origin, (0,0).
Q1 – (a) (2,2) **(b)** (-3,2) **(c)** (2,-3) **(d)** (-3,-2) **(e)** (-5,-3) **(f)** (-2,4) **(g)** (3,-1) **(h)** (1,0) **(i)** (4,-2) **(j)** (0,-4) **(k)** (5,1) **(l)** (5,-5) **(m)** (-1,-1) **(n)** (-2,0) **(o)** (0,5)
Q2 –



Q3 – (-3,-2)
Q4 –



Q5 – (a) T **(b)** F **(c)** T **(d)** F **(e)** F **(f)** T **(g)** F **(h)** T **(i)** T **(j)** False, it does not lie in any quadrant