CHAPTER 12 – Solids
Geometry Honors

Formula Reference for Surface area and Volume

NAME OF SOLID	SURFACE AREA	VOLUME
Right Prism	$S=2B+Ph=aP+Ph$ a=apothem of base B=area of base P=Perimeter of base h=height	$V=Bh$ B=area of base h=height
Right Cylinder	$S=2B+Ch=2\pi r^2+2\pi rh$ B=area of base C=Circumference of base r=radius of base h=height	$V=Bh=\pi r^2h$ B=area of base h=height r=radius of base
Regular Pyramid	$S=B+\frac{1}{2}P\ell$ B=area of base P=perimeter of base ℓ=slant height	$V=\frac{1}{3}Bh$ B=area of base h=height
Right Cone	$S=B+\frac{1}{2}C\ell=\pi r^2+\pi r\ell$ B=area of base C=Circumference of base r=radius of base ℓ=slant height	$V=\frac{1}{3}Bh=\frac{1}{3}\pi r^2h$ B=area of base h=height r=radius of base
Sphere	$S=4\pi r^2$ r=radius of sphere	$V=\frac{4}{3}\pi r^3$ r=radius of sphere

12.1 Exploring Solids

Polyhedra	Not Polyhedra
Prism Pyramid	Cylinder Cone Sphere

Enter the number of faces vertices and edges of each solid on the display. used website! www.learner.org 3D shapes - prisms

Solid	Faces	Vertices	Edges
triangular prism	5	6	9
rectangular prism	6	8	12
pentagonal prism	7	10	15
octagonal prism	10	16	24

Use the table above to look for a pattern between the number of faces vertices and edges.

Euler's Theroem

The number of faces (F), vertices (V) and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

* $\text{Edges} = \dfrac{\#\text{faces} \cdot \#\text{sides}}{2}$ *

12.1 Explore Solids

Goal • Identify solids.

Your Notes

VOCABULARY

Polyhedron a solid that is bounded by polygons that enclose a single region of space.

Face The faces of a polyhedron are polygons

Edge An edge of a polyhedron is a line segment formed by the intersection of two faces.

Vertex A vertex of a polyhedron is a point where three or more edges meet

Base A polygon that is used to name the polyhedron.

Regular polyhedron A polyhedron whose faces are all congruent regular polygons.

Convex polyhedron polyhedron such that any two points on its surface can be connected by a line segment that lies entirely inside or on the polyhedron.

Platonic solids One of 5 regular polyhedra:
1. regular tetrahedron (4)
2. cube (6 faces)
3. regular octahedron (8 faces)
4. regular dodecahedron (12 faces)
5. regular icosahedron (20 faces)

Cross section the intersection of a plane and a solid.

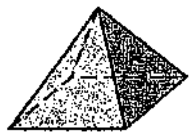
Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.

Your Notes

TYPES OF SOLIDS

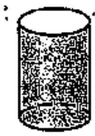
Polyhedra

Prism



Pyramid

Not Polyhedra



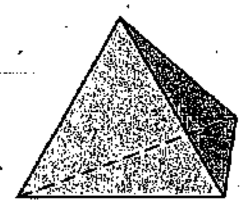
Cylinder

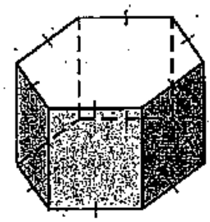
Cone

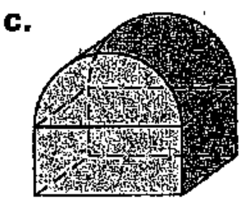
Sphere

Example 1 *Identify and name polyhedra*

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

a. 

b. 

c. 

Solution

a. This is a polyhedron. It has 4 faces so it is a tetrahedron. It has 4 vertices and 6 edges.

b. This is a polyhedron. The two bases are congruent hexagons, so it is a hexagonal prism. It has 8 faces, 12 vertices, and 18 edges.

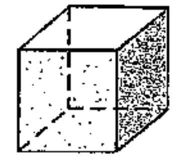
c. This is not a polyhedron. The solid has a curved surface.

Your Notes

THEOREM 12.1: EULER'S THEOREM

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula

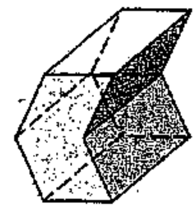
$F + V = E + 2$.



$F = 6, V = 8, E = 12$
$6 + 8 = 12 + 2$

Example 2 *Use Euler's Theorem*

Find the number of faces, vertices, and edges of the polyhedron shown. Check your answers using Euler's Theorem.



The polyhedron has 8 faces, 12 vertices, and 18 edges.

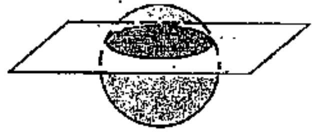
Use Euler's Theorem to check.

$F + V = E + 2$	**Euler's Theorem**
$8 + 12 = 18 + 2$	**Substitute.**
$20 = 20$ ✓	**Check.**

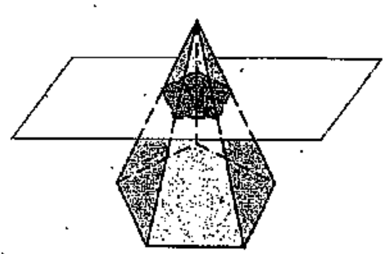
Example 3 *Describe cross sections*

Describe the shape formed by the intersection of the plane and the solid.

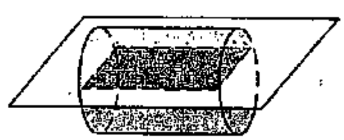
a.



b.



c.



Solution

a. The cross section is a circle.

b. The cross section is a pentagon.

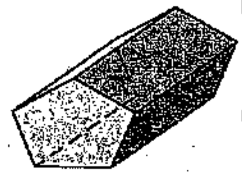
c. The cross section is a rectangle.

Your Notes

Checkpoint **Complete the following exercises.**

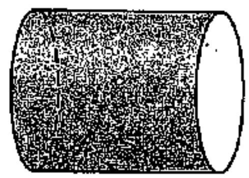
In Exercises 1–3, tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

1.



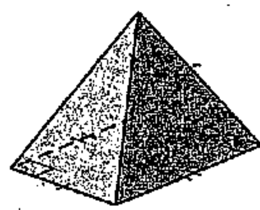
Polyhedron - pentagonal prism.
7 faces 10 vertices
15 edges

2.



Not polyhedron
curved edge

3.



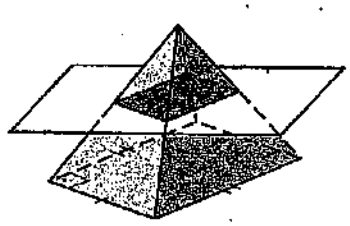
Polyhedron - rectangular pyramid
5 faces
5 vertices
8 edges

4. **Is it possible for a polyhedron to have 16 faces, 34 vertices, and 50 edges?** ***Explain.***

$16+34=50+2$
$50 \neq 52$ NO.

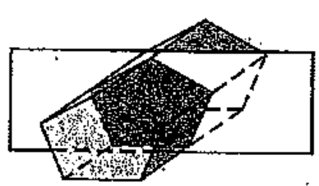
In Exercises 5–7, *describe* the shape formed by the intersection of the plane and the solid.

5.



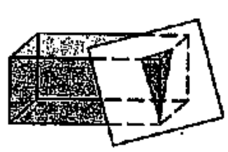
rectangle

6.



pentagon

7.



triangle

Homework

12.2 Surface Area of Prisms and Cylinders

Goal • Find the surface areas of prisms and cylinders.

Your Notes

VOCABULARY

Prism polyhedron with 2 congruent faces, called bases, that lie in parallel lines

Lateral faces The lateral faces of a prism are parallelograms formed by connecting the corresponding vertices of the bases.

Lateral edges The lateral edges of a prism are the segments connecting the corresponding vertices of the bases.

Surface area The sum of the area of its faces

Lateral area The sum of the areas of its lateral faces

Net Two dimensional representation of the faces of a polyhedron.

Right prism In a right prism, each lateral edge is perpendicular to both bases.

Oblique prism Prism with lateral edges that are not perpendicular to the bases.

Cylinder A solid with congruent circular bases that lie in parallel planes

Right cylinder In a right cylinder, the segment joining the centers of the bases is perpendicular to the bases.

Name ______________________ Period __________

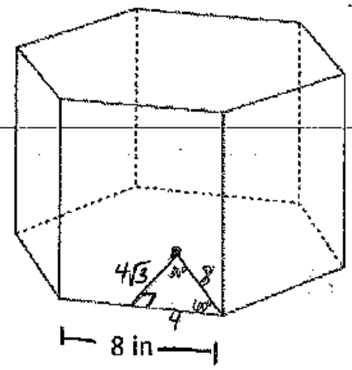
12.2 Surface Area of Prisms and Cylinders

Find the surface area of each right prism below.

1. Regular Hexagonal Prism

$A_{Hex} = \frac{1}{2} a \cdot n \cdot s$
$= \frac{1}{2}(4\sqrt{3})(8)(6)$
$= 96\sqrt{3}$

$A_{rec} = 8(10)$
$= 80$

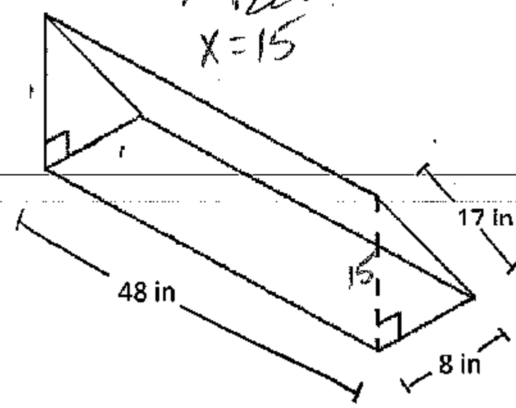


$S = 2(96\sqrt{3}) + 6(80)$
$S = \boxed{(192\sqrt{3} + 480)\ in^2}$
$\approx 812.55\ in^2$

2.

$8^2 + x^2 = 17^2$
$x^2 = 289 - 64$
$x = \sqrt{225}$
$x = 15$

$A_{tr} = \frac{1}{2}(8)(15)$
$= 60$



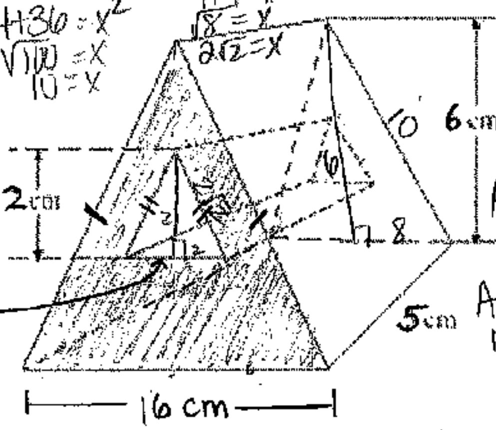
$S = 2(60) + 48(15) + 48(17) + 48(8)$
$= 120 + 720 + 816 + 384$
$= \boxed{2040\ in^2}$

Find the surface area of each hollowed solid below. Assume that all prisms and cylinders are right.

3.

$8^2 + 6^2 = x^2$
$64 + 36 = x^2$
$\sqrt{100} = x$
$10 = x$

$2^2 + 2^2 = x^2$
$4 + 4 = x^2$
$\sqrt{8} = x$
$2\sqrt{2} = x$



$A_{base} = \frac{1}{2}(16)(6) - \frac{1}{2}(4)(2)$
$= 48 - 4$
$= 44$

$A_{out\ lat} = 5(10) + 5(10) + 16(5)$
$= 180$

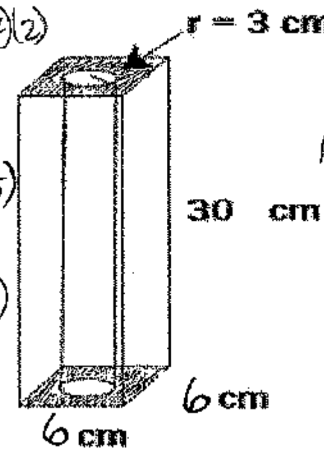
$A_{inn\ lat} = 2\sqrt{2}(5) + 2\sqrt{2}(5) + 4(5)$
$= 10\sqrt{2} + 10\sqrt{2} + 20$
$= 20\sqrt{2} + 20$

S = 2(Base) + Outer Lateral Surface + Inner Lateral Surface.

$= 2(44) + 180 + 20\sqrt{2} + 20$
$= 88 + 200 + 20\sqrt{2}$

$\boxed{288 + 20\sqrt{2}\ cm^2}$
$\approx 316.28\ cm^2$

4.



$A_{base} = 6(6) - \pi(3)^2$
$= 36 - 9\pi$

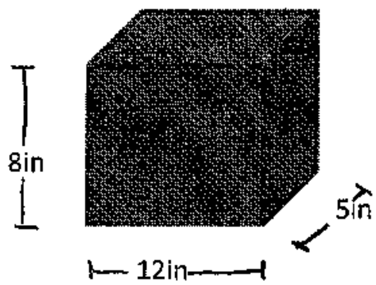
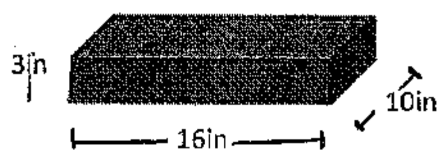
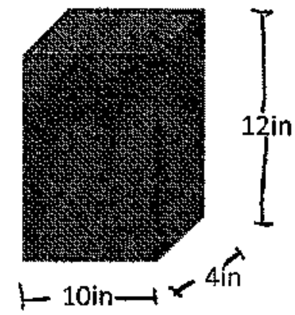
$A_{out\ sur} = 4(6)(30)$
$= 720$

$A_{inn\ sur} = 2\pi(3)(30)$
$= 180\pi$

S = 2(Base) + Outer Surface + Inner Surface
$= 2(36 - 9\pi) + 720 + 180\pi$
$= 72 - 18\pi + 720 + 180\pi$
$= 162\pi + 792$

$\boxed{(162\pi + 792)\ cm^2}$
$\approx 1300.94\ cm^2$

5. You and your partner are designing the packaging box for a brand new snack food. Your team has three different sized options to choose between. Each of the three boxes holds the required amount of snack product, 480 cubic inches. However, your team's job is to ensure that the most cost-effective packaging is recommended to the company. The packaging costs eight cents per square inch.

Box 1	Box 2	Box 3
		

a.) Decide which box is the most cost effective. Explain how you found your answer.

① $S=2(8)(12)+2(5)(8)+2(12)(5)$ ② $2(3)(10)+2(3)(16)+2(16)(10)$ ③ $2(10)(12)+2(4)(12)+2(10)(4)$

$=192+80+120$ $60+96+320$ $240+96+80$

$=392\ in^2$ $476\ in^2$ $=416\ in^2$

$\boxed{\text{Box 1}}$

b.) Determine the cost of materials needed to create 125,000 of the boxes you have recommended.

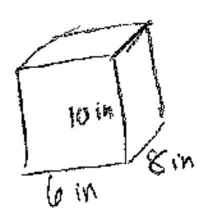
$392(0.08)=31.36$

$\boxed{\$31.36}$

c.) Is there a more cost effective box that your team could design? (It must have the same volume, V=lwh.) If yes, gives its dimensions. If no, explain.

(answers will vary)

Sample



$S=2(6)(10)+2(8)(10)+2(6)(8)$

$=120+160+96$

$=376\ in^2$

d.) If time remains, name and design your new snack box on a separate sheet of paper.

12.3 Surface Area of Pyramids and Cones

Goal • Find the surface areas of pyramids and cones.

Your Notes

Pyramids are classified by the shapes of their bases.

VOCABULARY

Pyramid Polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.

Vertex of a pyramid the common vertex of the lateral faces of a pyramid.

Regular pyramid has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces are congruent isosceles triangles.

Slant height The height of a lateral face of the regular pyramid.

Cone has a circular base and a vertex that is not in the same plane as the base.

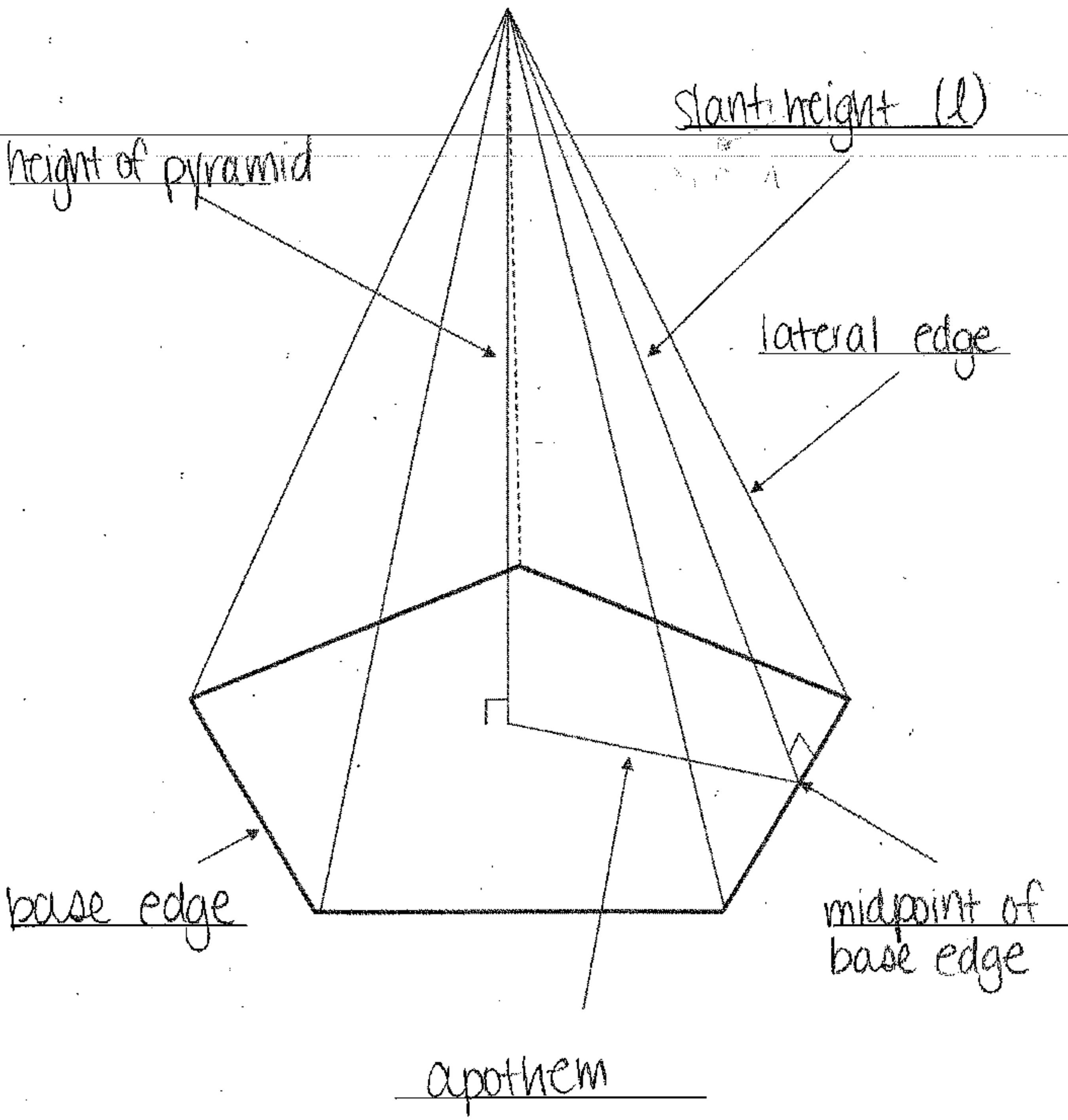
Vertex of a cone The point on the cone that is located at a perpendicular distance from the base, called the height of the cone.

Right cone In a right cone, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

Lateral surface The lateral surface of a cone consists of all segments that connect the vertex with points on the base edge.

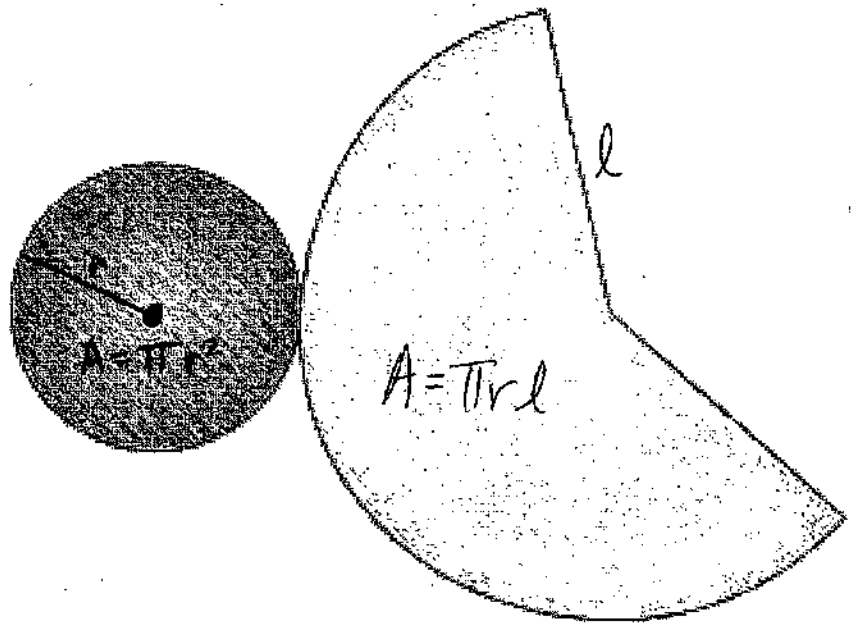
12.3 Surface Area of Pyramids and Cones

Some important reminders for regular pyramids.



To find the slant height, look for right triangles in your pyramid!

Here is a diagram of the net of a cone.



Surface area of a right cone = $\pi r^2 + \pi r l$

Let's show why...

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference}}$$

$$\frac{X}{\pi l^2} = \frac{2\pi r}{2\pi l}$$

$$X = \frac{2\pi r}{2\pi l} \cdot \pi l^2$$

$$X = \frac{\cancel{2}\pi r \cdot \pi l^{\cancel{2}}}{\cancel{2}\cancel{\pi}\cancel{l}}$$

$$X = \pi r l$$

12.4 Volume of Prisms and Cylinders

Goal • Find volumes of prisms and cylinders.

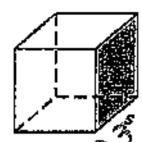
Your Notes

VOCABULARY

Volume volume of a solid is the number of cubic units contained in its interior.

POSTULATE 27: VOLUME OF A CUBE POSTULATE

The volume of a cube is the cube of the length of its side.



$V = s^3$

POSTULATE 28: VOLUME CONGRUENCE POSTULATE

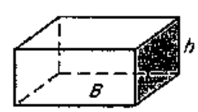
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29: VOLUME ADDITION POSTULATE

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

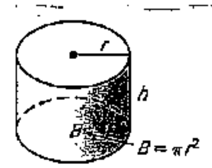
THEOREM 12.6: VOLUME OF A PRISM

The volume V of a prism is $V = Bh$, where B is the area of the base and h is the height.



THEOREM 12.7: VOLUME OF A CYLINDER

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.



Section 12.4

Cavalieri's Principle:

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

12.6 Surface Area and Volume of Spheres

Goal • Find surface areas and volumes of spheres.

Your Notes

VOCABULARY

Sphere the set of all points in space equidistant from a given point.

Center of a sphere the given point from which all points on the sphere are equidistant

Radius of a sphere a segment from the center to a point on the sphere.

Chord of a sphere a segment whose endpoints are on the sphere.

Diameter of a sphere a chord that contains the center of the sphere.

Great circle The intersection of a sphere and a plane that contains the center of the sphere.

Hemisphere One of the congruent halves of a sphere.

THEOREM 12.11: SURFACE AREA OF A SPHERE

The surface area S of a sphere is

$S = 4\pi r^2$,

where r is the radius of the sphere.