SURFACE AREA AND VOLUME

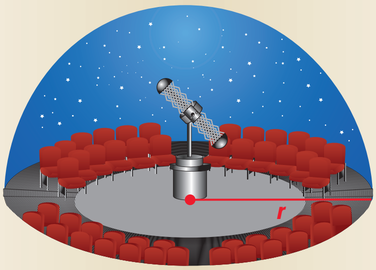
How are geometric solids used in planetarium design?

CHAPTER 12

APPLICATION: Spherical Buildings

Planetariums create space shows by projecting images of the moon, stars, and planets onto the interior surface of a dome, or *hemisphere*. The planetarium in these architectural drawings is shaped like a ball, or *sphere*.

When constructing the dome of a planetarium, the builders need to estimate the amount of material needed to cover the interior surface of the dome. The diagram below shows the radius *r* of the dome of a planetarium.



Think & Discuss

1. Use the formula $2\pi r^2$ to find the amount of material needed to cover the interior of a hemisphere with a radius of 40 feet.
2. Describe any other buildings that are shaped like spheres or hemispheres.

Learn More About It

You will investigate spherical buildings in Exercises 38–40 on p. 764.

INTERNET APPLICATION LINK Visit www.mcdougallittell.com for more information about spheres in architecture.

CHAPTER 12

Study Guide

PREVIEW

What's the chapter about?

Chapter 12 is about **surface area and volume of solids**. Surface area and volume are the measurements used to describe three-dimensional geometric figures. In Chapter 12, you'll learn

- how to calculate the surface area and volume of various solids.
- how to use surface area and volume in real-life situations, such as finding the amount of wax needed to make a candle.

KEY VOCABULARY

▶ **Review**
- equilateral triangle, p. 194
- polygon, p. 322
- convex, p. 323
- nonconvex, p. 323
- ratio, p. 457
- scale factor, p. 474
- locus, p. 642

▶ **New**
- polyhedron, p. 719
- Platonic solids, p. 721
- prism, p. 728
- cylinder, p. 730
- pyramid, p. 735
- circular cone, p. 737
- sphere, p. 759
- similar solids, p. 766

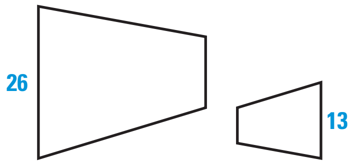
PREPARE

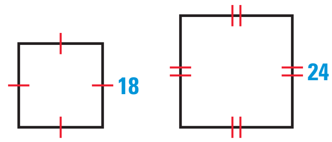
Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you'll apply in this chapter. See the given **reference page** if there is something you don't understand.

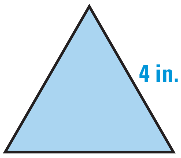
> **STUDENT HELP**
>
> **Study Tip**
> "Student Help" boxes throughout the chapter give you study tips and tell you where to look for extra help in this book and on the Internet.

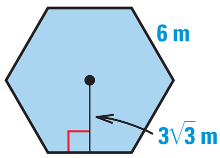
Find the scale factor of the similar polygons. (Review p. 474)

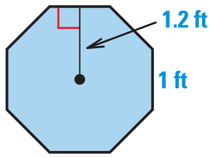
1. 

2. 

Calculate the area of the regular polygon. (Review pp. 669–671)

3. 

4. 

5. 

STUDY STRATEGY

Here's a study strategy!

Generalizing Formulas

When faced with having to remember many formulas, try to find an underlying concept that links some or all of the formulas together. Then, you only have to remember the concept instead of all the formulas.

12.1 Exploring Solids

What *you should learn*

GOAL 1 Use properties of polyhedra.

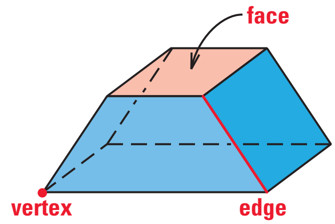
GOAL 2 Use Euler's Theorem in **real-life** situations, such as analyzing the molecular structure of salt in **Example 5**.

Why *you should learn it*

▼ You can use properties of polyhedra to classify various crystals, as in **Exs. 39–41**.

GOAL 1 USING PROPERTIES OF POLYHEDRA

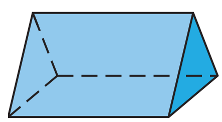
A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra*, or polyhedrons.



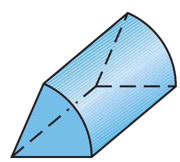
EXAMPLE 1 *Identifying Polyhedra*

Decide whether the solid is a polyhedron. If so, count the number of faces, vertices, and edges of the polyhedron.

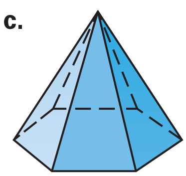
a.



b.



c.



SOLUTION

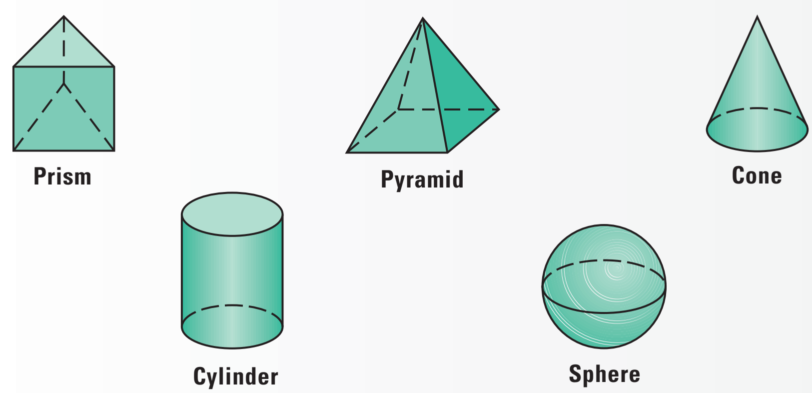
a. This is a polyhedron. It has 5 faces, 6 vertices, and 9 edges.

b. This is not a polyhedron. Some of its faces are not polygons.

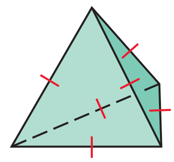
c. This is a polyhedron. It has 7 faces, 7 vertices, and 12 edges.

CONCEPT SUMMARY — TYPES OF SOLIDS

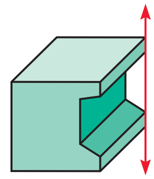
Of the five solids below, the prism and pyramid are polyhedra. The cone, cylinder, and sphere are not polyhedra.



A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is *nonconvex*, or *concave*.



regular, convex

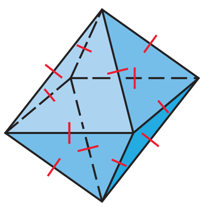


nonregular, nonconvex

EXAMPLE 2 *Classifying Polyhedra*

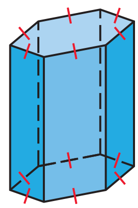
Is the octahedron convex? Is it regular?

a.



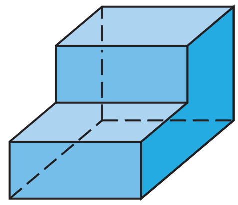
convex, regular

b.



convex, nonregular

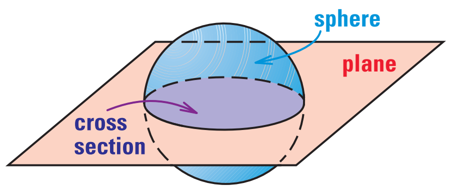
c.



nonconvex, nonregular

. . . .

Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For instance, the diagram shows that the intersection of a plane and a sphere is a circle.



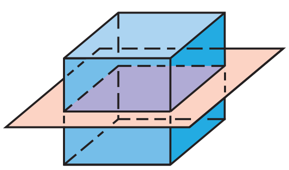
EXAMPLE 3 *Describing Cross Sections*

STUDENT HELP

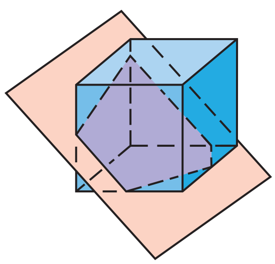
Study Tip
When sketching a cross section of a polyhedron, first sketch the solid. Then, locate the vertices of the cross section and draw the sides of the polygon.

Describe the shape formed by the intersection of the plane and the cube.

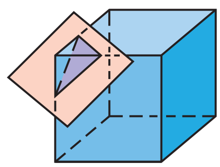
a.



b.



c.



SOLUTION

a. This cross section is a square.

b. This cross section is a pentagon.

c. This cross section is a triangle.

. . . .

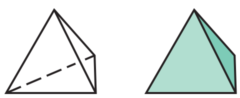
The square, pentagon, and triangle cross sections of a cube are described in Example 3. Some other cross sections are the rectangle, trapezoid, and hexagon.

GOAL 2 USING EULER'S THEOREM

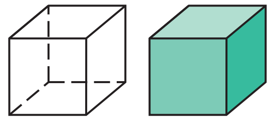
STUDENT HELP

Study Tip
Notice that four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." A cube is sometimes called a *hexahedron*.

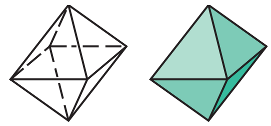
There are five regular polyhedra, called *Platonic solids*, after the Greek mathematician and philosopher Plato. The **Platonic solids** are a regular **tetrahedron** (4 faces), a cube (6 faces), a regular **octahedron** (8 faces), a regular **dodecahedron** (12 faces), and a regular **icosahedron** (20 faces).



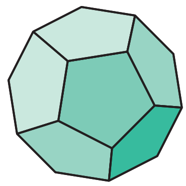
Regular tetrahedron
4 faces, 4 vertices, 6 edges



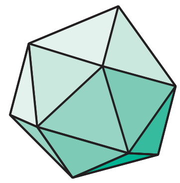
Cube
6 faces, 8 vertices, 12 edges



Regular octahedron
8 faces, 6 vertices, 12 edges



Regular dodecahedron
12 faces, 20 vertices, 30 edges



Regular icosahedron
20 faces, 12 vertices, 30 edges

Notice that the sum of the number of faces and vertices is two more than the number of edges in the solids above. This result was proved by the Swiss mathematician Leonhard Euler (1707–1783).

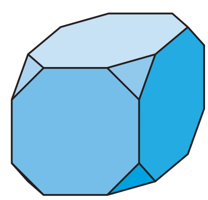
THEOREM

THEOREM 12.1 ***Euler's Theorem***

The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

EXAMPLE 4 *Using Euler's Theorem*

The solid has 14 faces; 8 triangles and 6 octagons. How many vertices does the solid have?



SOLUTION

On their own, 8 triangles and 6 octagons have $8(3) + 6(8)$, or 72 edges. In the solid, each side is shared by exactly two polygons. So, the number of edges is one half of 72, or 36. Use Euler's Theorem to find the number of vertices.

$F + V = E + 2$ Write Euler's Theorem.

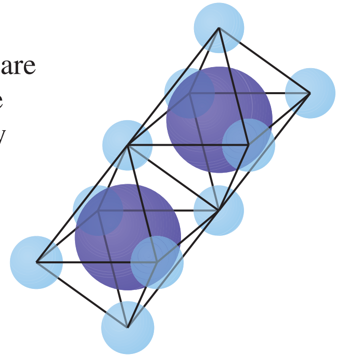
$14 + V = 36 + 2$ Substitute.

$V = 24$ Solve for V.

▶ The solid has 24 vertices.

EXAMPLE 5 *Finding the Number of Edges*

CHEMISTRY In molecules of sodium chloride, commonly known as table salt, chloride atoms are arranged like the vertices of regular octahedrons. In the crystal structure, the molecules share edges. How many sodium chloride molecules share the edges of one sodium chloride molecule?



SOLUTION

To find the number of molecules that share edges with a given molecule, you need to know the number of edges of the molecule.

You know that the molecules are shaped like regular octahedrons. So, they each have 8 faces and 6 vertices. You can use Euler's Theorem to find the number of edges, as shown below.

$F + V = E + 2$ **Write Euler's Theorem.**

$8 + 6 = E + 2$ **Substitute.**

$12 = E$ **Simplify.**

▶ So, 12 other molecules share the edges of the given molecule.

FOCUS ON APPLICATIONS

GEODESIC DOME The dome has the same underlying structure as a soccer ball, but the faces are subdivided into triangles.

APPLICATION LINK
www.mcdougallittell.com

EXAMPLE 6 *Finding the Number of Vertices*

SPORTS A soccer ball resembles a polyhedron with 32 faces; 20 are regular hexagons and 12 are regular pentagons. How many vertices does this polyhedron have?

SOLUTION

Each of the 20 hexagons has 6 sides and each of the 12 pentagons has 5 sides. Each edge of the soccer ball is shared by two polygons. Thus, the total number of edges is as follows:

$E = \frac{1}{2}(6 \cdot 20 + 5 \cdot 12)$ **Expression for number of edges**

$= \frac{1}{2}(180)$ **Simplify inside parentheses.**

$= 90$ **Multiply.**

Knowing the number of edges, 90, and the number of faces, 32, you can apply Euler's Theorem to determine the number of vertices.

$F + V = E + 2$ **Write Euler's Theorem.**

$32 + V = 90 + 2$ **Substitute.**

$V = 60$ **Simplify.**

▶ So, the polyhedron has 60 vertices.

Guided Practice

Vocabulary Check ✓

1. Define *polyhedron* in your own words.

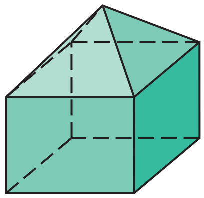
Concept Check ✓

2. Is a regular octahedron convex? Are all the Platonic solids convex? Explain.

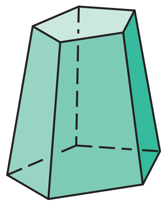
Skill Check ✓

Decide whether the solid is a polyhedron. Explain.

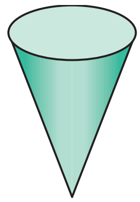
3.



4.



5.



Use Euler's Theorem to find the unknown number.

6. Faces: _?_
Vertices: 6
Edges: 12

7. Faces: 5
Vertices: _?_
Edges: 9

8. Faces: _?_
Vertices: 10
Edges: 15

9. Faces: 20
Vertices: 12
Edges: _?_

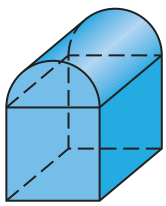
Practice and Applications

Student Help

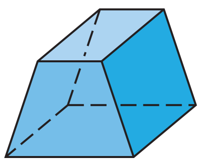
Extra Practice to help you master skills is on p. 825.

IDENTIFYING POLYHEDRA **Tell whether the solid is a polyhedron. Explain your reasoning.**

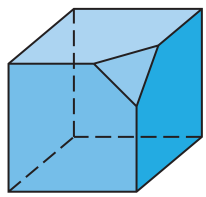
10.



11.

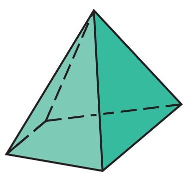


12.

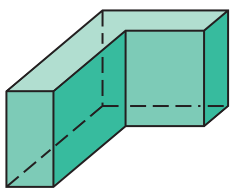


ANALYZING SOLIDS **Count the number of faces, vertices, and edges of the polyhedron.**

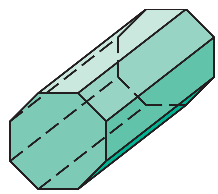
13.



14.



15.

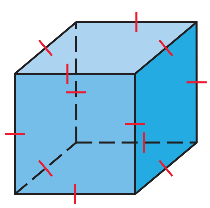


Student Help

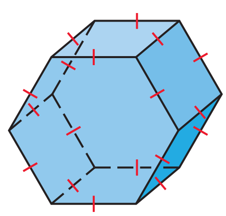
HOMEWORK HELP
Example 1: Exs. 10–15
Example 2: Exs. 16–24
Example 3: Exs. 25–35
Example 4: Exs. 36–52
Example 5: Ex. 53
Example 6: Exs. 47–52

ANALYZING POLYHEDRA **Decide whether the polyhedron is regular and/or convex. Explain.**

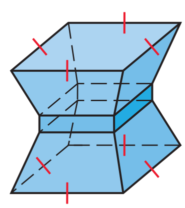
16.



17.



18.



LOGICAL REASONING **Determine whether the statement is *true* or *false*. Explain your reasoning.**

19. Every convex polyhedron is regular.

20. A polyhedron can have exactly 3 faces.

21. A cube is a regular polyhedron.

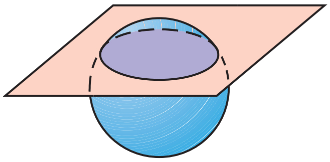
22. A polyhedron can have exactly 4 faces.

23. A cone is a regular polyhedron.

24. A polyhedron can have exactly 5 faces.

CROSS SECTIONS **Describe the cross section.**

25.



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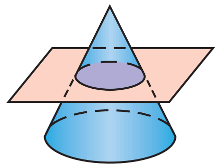
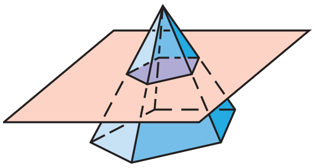
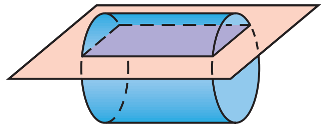


27.

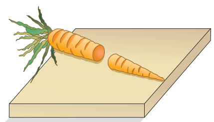


28.

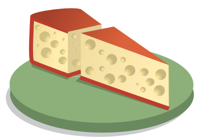


COOKING **Describe the shape that is formed by the cut made in the food shown.**

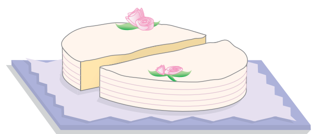
29. Carrot



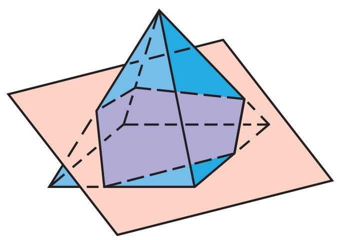
30. Cheese



31. Cake



CRITICAL THINKING **In the diagram, the bottom face of the pyramid is a square.**



32. Name the cross section shown.

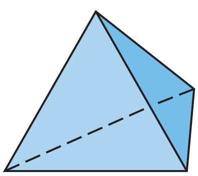
33. Can a plane intersect the pyramid at a point? If so, sketch the intersection.

34. Describe the cross section when the pyramid is sliced by a plane parallel to its bottom face.

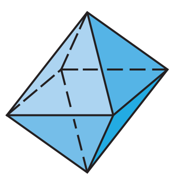
35. Is it possible to have an isosceles trapezoid as a cross section of this pyramid? If so, draw the cross section.

POLYHEDRONS **Name the regular polyhedron.**

36.



37.



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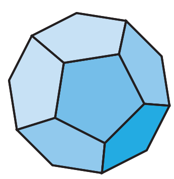


FOCUS ON CAREERS

REAL LIFE **MINERALOGY** By studying the arrangement of atoms in a crystal, mineralogists are able to determine the chemical and physical properties of the crystal.

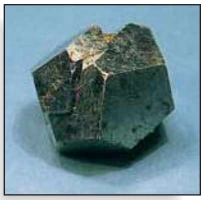
INTERNET **CAREER LINK** www.mcdougallittell.com

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 47–52.

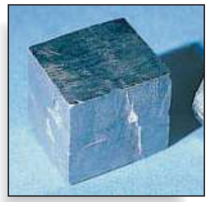
CRYSTALS **In Exercises 39–41, name the Platonic solid that the crystal resembles.**

39. Cobaltite



40. Fluorite

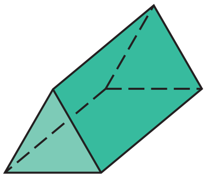
41. Pyrite



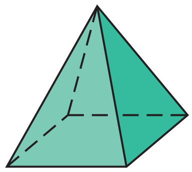
42. VISUAL THINKING Sketch a cube and describe the figure that results from connecting the centers of adjoining faces.

EULER'S THEOREM **In Exercises 43–45, find the number of faces, edges, and vertices of the polyhedron and use them to verify Euler's Theorem.**

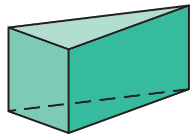
43.



44.



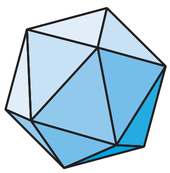
45.



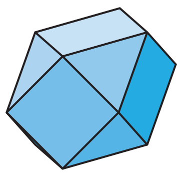
46. MAKING A TABLE Make a table of the number of faces, vertices, and edges for the Platonic solids. Use it to show Euler's Theorem is true for each solid.

USING EULER'S THEOREM **In Exercises 47–52, calculate the number of vertices of the solid using the given information.**

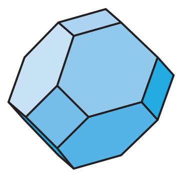
47. 20 faces; all triangles



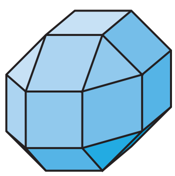
48. 14 faces; 8 triangles and 6 squares



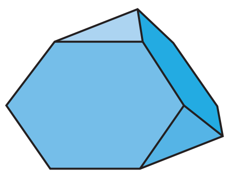
49. 14 faces; 8 hexagons and 6 squares



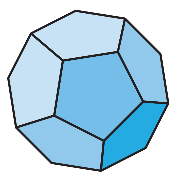
50. 26 faces; 18 squares and 8 triangles



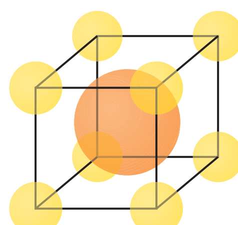
51. 8 faces; 4 hexagons and 4 triangles



52. 12 faces; all pentagons



53. **SCIENCE CONNECTION** In molecules of cesium chloride, chloride atoms are arranged like the vertices of cubes. In its crystal structure, the molecules share faces to form an array of cubes. How many cesium chloride molecules share the faces of a given cesium chloride molecule?

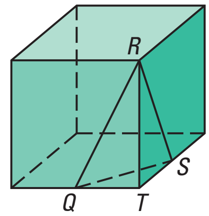


Test Preparation

54. **MULTIPLE CHOICE** A polyhedron has 18 edges and 12 vertices. How many faces does it have?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

55. **MULTIPLE CHOICE** In the diagram, Q and S are the midpoints of two edges of the cube. What is the length of $\overline{QS}$, if each edge of the cube has length h?



(A) $\frac{h}{2}$ (B) $\frac{h}{\sqrt{2}}$ (C) $\frac{2h}{\sqrt{2}}$

(D) $\sqrt{2}h$ (E) $2h$

★ Challenge

SKETCHING CROSS SECTIONS **Sketch the intersection of a cube and a plane so that the given shape is formed.**

56. An equilateral triangle

57. A regular hexagon

58. An isosceles trapezoid

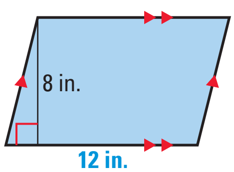
59. A rectangle

EXTRA CHALLENGE
www.mcdougallittell.com

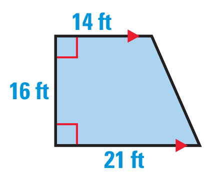
MIXED REVIEW

FINDING AREA OF QUADRILATERALS **Find the area of the figure.** (Review 6.7 for 12.2)

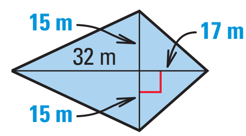
60.



61.



62.



FINDING AREA OF REGULAR POLYGONS **Find the area of the regular polygon described. Round your answer to two decimal places.** (Review 11.2 for 12.2)

63. An equilateral triangle with a perimeter of 48 meters and an apothem of 4.6 meters.

64. A regular octagon with a perimeter of 28 feet and an apothem of 4.22 feet.

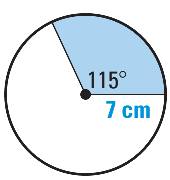
65. An equilateral triangle whose sides measure 8 centimeters.

66. A regular hexagon whose sides measure 4 feet.

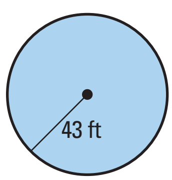
67. A regular dodecagon whose sides measure 16 inches.

FINDING AREA **Find the area of the shaded region. Round your answer to two decimal places.** (Review 11.5)

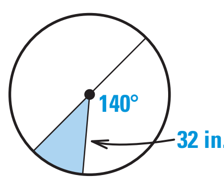
68.



69.



70.



12.2 Surface Area of Prisms and Cylinders

What you should learn

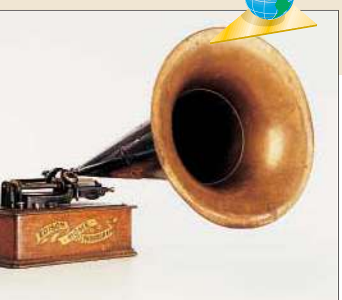
GOAL 1 Find the surface area of a prism.

GOAL 2 Find the surface area of a cylinder.

Why you should learn it

▼ You can find the surface area of **real-life** objects, such as the cylinder records used on phonographs during the late 1800s. See **Ex. 43**.

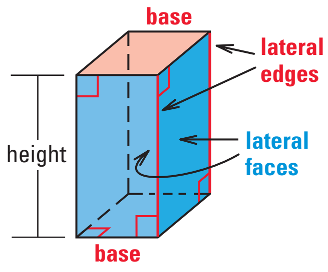
REAL LIFE



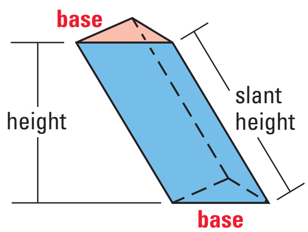
GOAL 1 FINDING THE SURFACE AREA OF A PRISM

A **prism** is a polyhedron with two congruent faces, called **bases**, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are *lateral edges.*

The *altitude* or *height* of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. Prisms that have lateral edges that are not perpendicular to the bases are **oblique prisms**. The length of the oblique lateral edges is the *slant height* of the prism.



Right rectangular prism



Oblique triangular prism

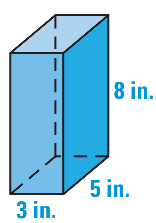
Prisms are classified by the shapes of their bases. For example, the figures above show one rectangular prism and one triangular prism. The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

EXAMPLE 1 *Finding the Surface Area of a Prism*

Find the surface area of a right rectangular prism with a height of 8 inches, a length of 3 inches, and a width of 5 inches.

SOLUTION

Begin by sketching the prism, as shown. The prism has 6 faces, two of each of the following:



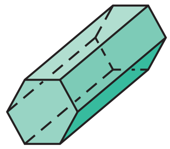
Faces	Dimensions	Area of faces
Left and right	8 in. by 5 in.	40 in.^{2}
Front and back	8 in. by 3 in.	24 in.^{2}
Top and bottom	3 in. by 5 in.	15 in.^{2}

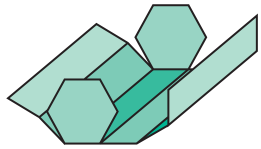
▶ The surface area of the prism is $S = 2(40) + 2(24) + 2(15) = 158$ in.^{2}

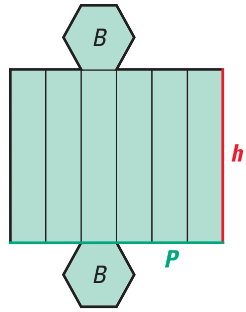
STUDENT HELP

Study Tip
When sketching prisms, first draw the two bases. Then connect the corresponding vertices of the bases.

Imagine that you cut some edges of a right hexagonal prism and unfolded it. The two-dimensional representation of all of the faces is called a **net**.







In the net of the prism, notice that the lateral area (the sum of the areas of the lateral faces) is equal to the perimeter of the base multiplied by the height.

THEOREM

THEOREM 12.2 ***Surface Area of a Right Prism***

The surface area S of a right prism can be found using the formula $S = 2B + Ph$, where B is the area of a base, P is the perimeter of a base, and h is the height.

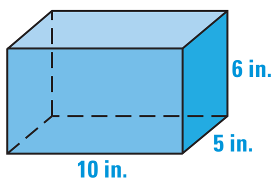
EXAMPLE 2 *Using Theorem 12.2*

STUDENT HELP

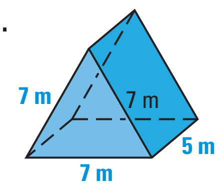
Study Tip
The prism in part (a) has three pairs of parallel, congruent faces. Any pair can be called bases, whereas the prism in part (b) has only one pair of parallel, congruent faces that can be bases.

Find the surface area of the right prism.

a.



b.



SOLUTION

a. Each base measures 5 inches by 10 inches with an area of

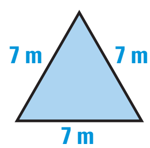
$$B = 5(10) = 50 \text{ in.}^2$$

The perimeter of the base is $P = 30$ in. and the height is $h = 6$ in.

▶ So, the surface area is

$$S = 2B + Ph = 2(50) + 30(6) = 280 \text{ in.}^2$$

b. Each base is an equilateral triangle with a side length, s, of 7 meters. Using the formula for the area of an equilateral triangle, the area of each base is



$$B = \frac{1}{4}\sqrt{3}\left(s^2\right) = \frac{1}{4}\sqrt{3}\left(7^2\right) = \frac{49}{4}\sqrt{3} \text{ m}^2.$$

The perimeter of each base is $P = 21$ m and the height is $h = 5$ m.

▶ So, the surface area is

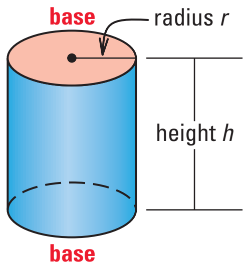
$$S = 2B + Ph = 2\left(\frac{49}{4}\sqrt{3}\right) + 21(5) \approx 147 \text{ m}^2.$$

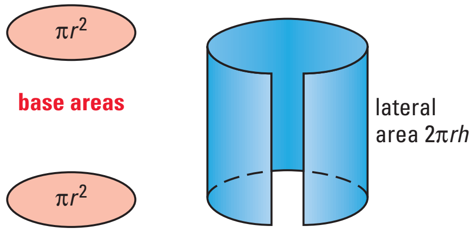
STUDENT HELP

Look Back
For help with finding the area of an equilateral triangle, see p. 669.

GOAL 2 FINDING THE SURFACE AREA OF A CYLINDER

A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The *altitude*, or *height*, of a cylinder is the perpendicular distance between its bases. The radius of the base is also called the *radius* of the cylinder. A cylinder is called a **right cylinder** if the segment joining the centers of the bases is perpendicular to the bases.





The **lateral area of a cylinder** is the area of its curved surface. The lateral area is equal to the product of the circumference and the height, which is $2\pi rh$. The entire **surface area of a cylinder** is equal to the sum of the lateral area and the areas of the two bases.

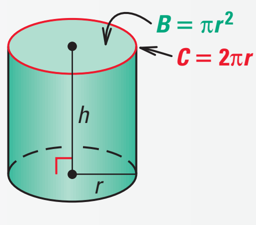
THEOREM

THEOREM 12.3 ***Surface Area of a Right Cylinder***

The surface area S of a right cylinder is

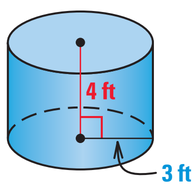
$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



EXAMPLE 3 *Finding the Surface Area of a Cylinder*

Find the surface area of the right cylinder.



SOLUTION

Each base has a radius of 3 feet, and the cylinder has a height of 4 feet.

$S = 2\pi r^2 + 2\pi rh$	Formula for surface area of cylinder
$= 2\pi(3^2) + 2\pi(3)(4)$	Substitute.
$= 18\pi + 24\pi$	Simplify.
$= 42\pi$	Add.
≈ 131.95	Use a calculator.

▶ The surface area is about 132 square feet.

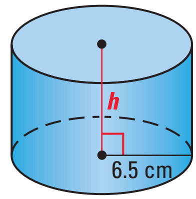
STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

Using Algebra

EXAMPLE 4 *Finding the Height of a Cylinder*

Find the height of a cylinder which has a radius of 6.5 centimeters and a surface area of 592.19 square centimeters.



SOLUTION

Use the formula for the surface area of a cylinder and solve for the height h.

$S = 2\pi r^2 + 2\pi rh$	Formula for surface area
$592.19 = 2\pi(6.5)^2 + 2\pi(6.5)h$	Substitute 6.5 for *r*.
$592.19 = 84.5\pi + 13\pi h$	Simplify.
$592.19 - 84.5\pi = 13\pi h$	Subtract 84.5π from each side.
$326.73 \approx 13\pi h$	Simplify.
$8 \approx h$	Divide each side by 13π.

▶ The height is about 8 centimeters.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the differences between a prism and a cylinder. Describe their similarities.

Concept Check ✓

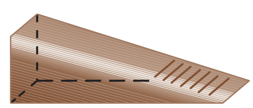
2. Sketch a triangular prism. Then sketch a net of the triangular prism. Describe how to find its lateral area and surface area.

Skill Check ✓

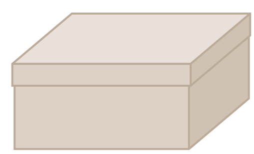
Give the mathematical name of the solid.

3. Soup can

4. Door stop



5. Shoe box



Use the diagram to find the measurement of the right rectangular prism.

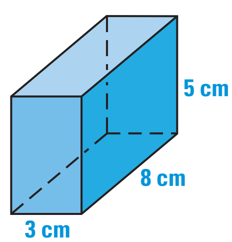
6. Perimeter of a base

7. Length of a lateral edge

8. Lateral area of the prism

9. Area of a base

10. Surface area of the prism



Make a sketch of the described solid.

11. Right rectangular prism with a 3.4 foot square base and a height of 5.9 feet

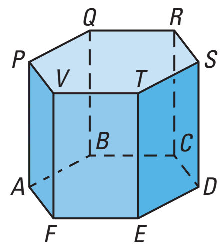
12. Right cylinder with a diameter of 14 meters and a height of 22 meters

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 825.

STUDYING PRISMS **Use the diagram at the right.**



13. Give the mathematical name of the solid.

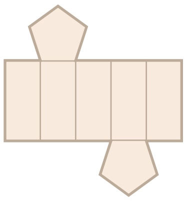
14. How many lateral faces does the solid have?

15. What kind of figure is each lateral face?

16. Name four lateral edges.

ANALYZING NETS **Name the solid that can be folded from the net.**

17.



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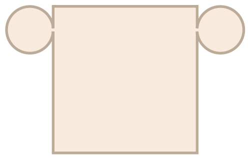

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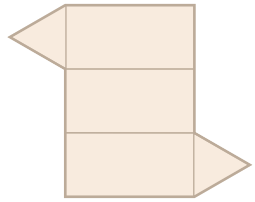
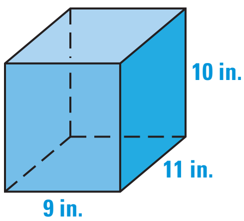


SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your result to two decimal places.**

20.



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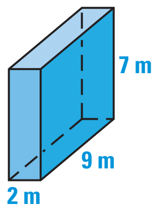

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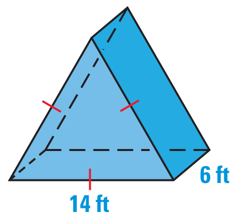

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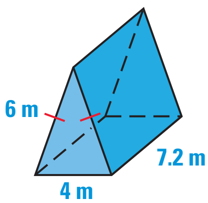

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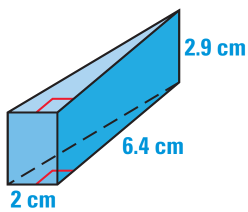
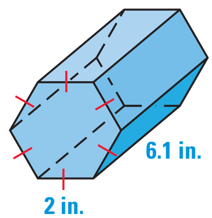
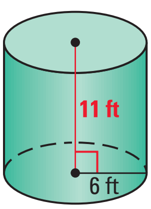


25.



SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder. Round the result to two decimal places.**

26.



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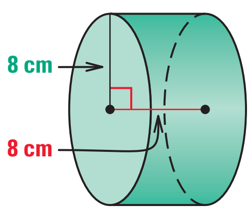

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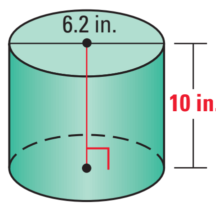


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–16, 20–25
Example 2: Exs. 20–25, 29–31, 35–37
Example 3: Exs. 26–28
Example 4: Exs. 32–34

VISUAL THINKING **Sketch the described solid and find its surface area.**

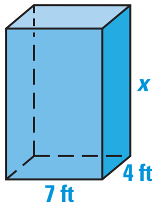
29. Right rectangular prism with a height of 10 feet, length of 3 feet, and width of 6 feet

30. Right regular hexagonal prism with all edges measuring 12 millimeters

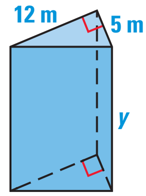
31. Right cylinder with a diameter of 2.4 inches and a height of 6.1 inches

USING ALGEBRA **Solve for the variable given the surface area S of the right prism or right cylinder. Round the result to two decimal places.**

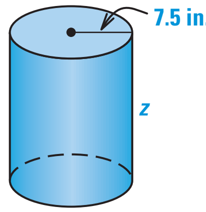
32. $S = 298\ \text{ft}^2$



33. $S = 870\ \text{m}^2$

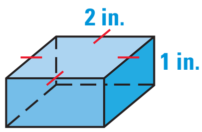


34. $S = 1202\ \text{in.}^2$



LOGICAL REASONING **Find the surface area of the right prism when the height is 1 inch, and then when the height is 2 inches. When the height doubles, does the surface area double?**

35.



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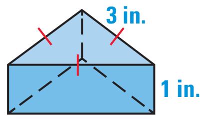

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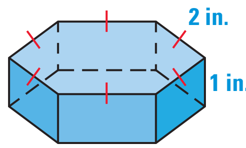
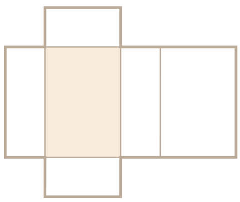


PACKAGING **In Exercises 38–40, sketch the box that results after the net has been folded. Use the shaded face as a base.**

38.



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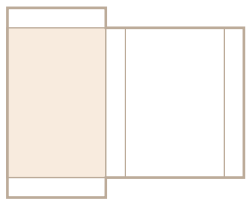

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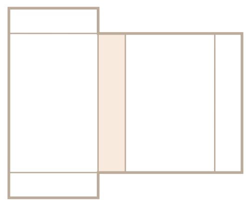
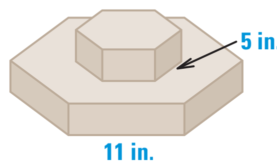


41. CRITICAL THINKING If you were to unfold a cardboard box, the cardboard would not match the net of the original solid. What sort of differences would there be? Why do these differences exist?

42. ARCHITECTURE A skyscraper is a rectangular prism with a height of 414 meters. The bases are squares with sides that are 64 meters. What is the surface area of the skyscraper (including both bases)?

43. WAX CYLINDER RECORDS The first versions of phonograph records were hollow wax cylinders. Grooves were cut into the lateral surface of the cylinder, and the cylinder was rotated on a phonograph to reproduce the sound. In the late 1800's, a standard sized cylinder was about 2 inches in diameter and 4 inches long. Find the exterior lateral area of the cylinder described.

44. CAKE DESIGN Two layers of a cake are right regular hexagonal prisms as shown in the diagram. Each layer is 3 inches high. Calculate the area of the cake that will be frosted. If one can of frosting will cover 130 square inches of cake, how many cans do you need? (*Hint:* The bottom of each layer will not be frosted and the entire top of the bottom layer will be frosted.)



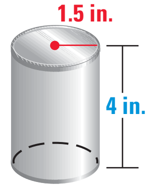
FOCUS ON CAREERS

REAL LIFE

ARCHITECTS use the surface area of a building to help them calculate the amount of building materials needed to cover the outside of a building.

Test Preparation

MULTI-STEP PROBLEM Use the following information.
A canned goods company manufactures cylindrical cans resembling the one at the right.



45. Find the surface area of the can.

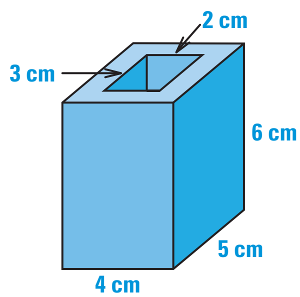
46. Find the surface area of a can whose radius and height are twice that of the can shown.

47. *Writing* Use the formula for the surface area of a right cylinder to explain why the answer in Exercise 46 is not twice the answer in Exercise 45.

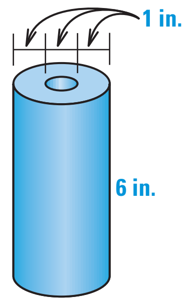
★ Challenge

FINDING SURFACE AREA Find the surface area of the solid. Remember to include both lateral areas. Round the result to two decimal places.

48.



49.

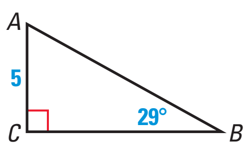


EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

EVALUATING TRIANGLES Solve the right triangle. Round your answers to two decimal places. (Review 9.6)

50.



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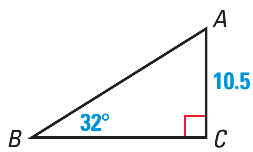

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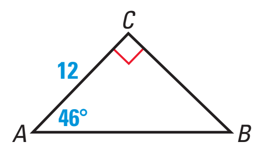
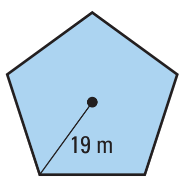
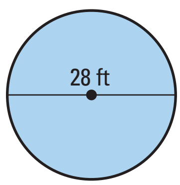


FINDING AREA Find the area of the regular polygon or circle. Round the result to two decimal places. (Review 11.2, 11.5 for 12.3)

53.



54.



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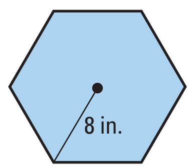
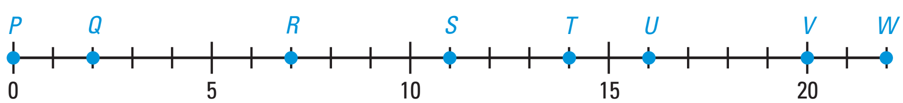


FINDING PROBABILITY Find the probability that a point chosen at random on $\overline{PW}$ is on the given segment. (Review 11.6)



56. $\overline{QS}$ **57.** $\overline{PU}$ **58.** $\overline{QU}$ **59.** $\overline{TW}$ **60.** $\overline{PV}$

12.3 Surface Area of Pyramids and Cones

What *you should learn*

GOAL 1 Find the surface area of a pyramid.

GOAL 2 Find the surface area of a cone.

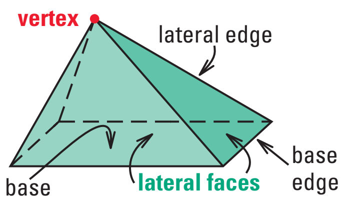
Why *you should learn it*

▼ To find the surface area of solids in **real life**, such as the Pyramid Arena in Memphis, Tennessee, shown below and in **Example 1**.

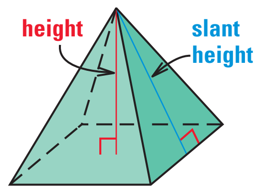
REAL LIFE

GOAL 1 FINDING THE SURFACE AREA OF A PYRAMID

A **pyramid** is a polyhedron in which the *base* is a polygon and the *lateral faces* are triangles with a common *vertex*. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The *altitude*, or *height*, of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid



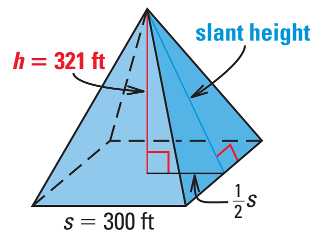
Regular pyramid

A **regular pyramid** has a regular polygon for a base and its height meets the base at its center. The *slant height* of a regular pyramid is the altitude of any lateral face. A nonregular pyramid does not have a slant height.

EXAMPLE 1 *Finding the Area of a Lateral Face*

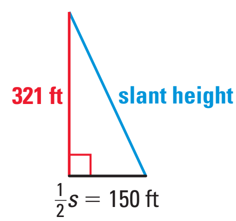
REAL LIFE

ARCHITECTURE The lateral faces of the Pyramid Arena in Memphis, Tennessee, are covered with steel panels. Use the diagram of the arena at the right to find the area of each lateral face of this regular pyramid.



SOLUTION

To find the slant height of the pyramid, use the Pythagorean Theorem.



$(\text{Slant height})^2 = h^2 + \left(\frac{1}{2}s\right)^2$ **Write formula.**

$(\text{Slant height})^2 = 321^2 + 150^2$ **Substitute.**

$(\text{Slant height})^2 = 125{,}541$ **Simplify.**

$\text{Slant height} = \sqrt{125{,}541}$ **Take the positive square root.**

$\text{Slant height} \approx 354.32$ **Use a calculator.**

▶ So, the area of each lateral face is $\frac{1}{2}$(base of lateral face)(slant height), or about $\frac{1}{2}(300)(354.32)$, which is about 53,148 square feet.

STUDENT HELP

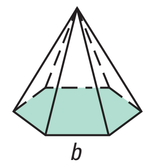
Study Tip

A *regular pyramid* is considered a regular polyhedron only if *all* its faces, including the base, are congruent. So, the only pyramid that is a regular polyhedron is the regular triangular pyramid, or *tetrahedron*. See page 721.

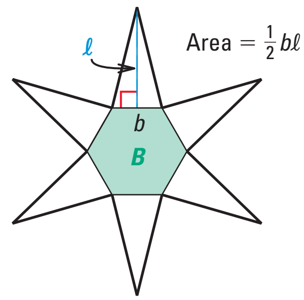
STUDENT HELP

Study Tip
When sketching the net of a pyramid, first sketch the base. Then sketch the lateral faces.

A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.



The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area is as follows:



S = (Area of base) + 6(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

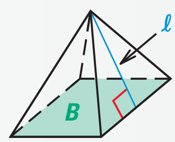
$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

THEOREM

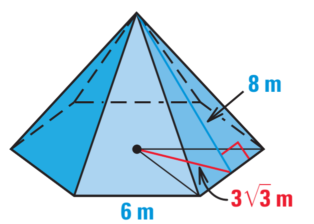
THEOREM 12.4 ***Surface Area of a Regular Pyramid***

The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



EXAMPLE 2 *Finding the Surface Area of a Pyramid*

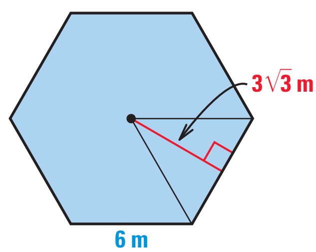
To find the surface area of the regular pyramid shown, start by finding the area of the base.



STUDENT HELP

Look Back
For help with finding the area of regular polygons see pp. 669–671.

Use the formula for the area of a regular polygon, $\frac{1}{2}$(apothem)(perimeter). A diagram of the base is shown at the right. After substituting, the area of the base is $\frac{1}{2}(3\sqrt{3})(6 \cdot 6)$, or $54\sqrt{3}$ square meters.



Now you can find the surface area, using $54\sqrt{3}$ for the area of the base, B.

$S = B + \frac{1}{2}P\ell$ **Write formula.**

$= 54\sqrt{3} + \frac{1}{2}(36)(8)$ **Substitute.**

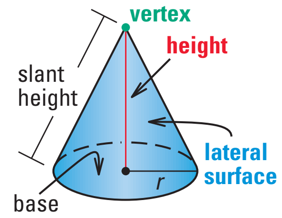
$= 54\sqrt{3} + 144$ **Simplify.**

≈ 237.5 **Use a calculator.**

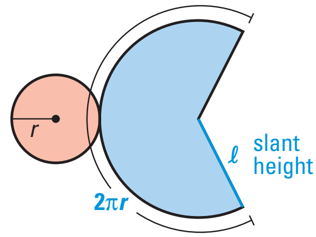
▶ So, the surface area is about 237.5 square meters.

GOAL 2 FINDING THE SURFACE AREA OF A CONE

A **circular cone**, or **cone**, has a circular *base* and a *vertex* that is not in the same plane as the base. The *altitude*, or *height*, is the perpendicular distance between the vertex and the base. In a **right cone**, the height meets the base at its center and the *slant height* is the distance between the vertex and a point on the base edge.



The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge. When you cut along the slant height and lie the cone flat, you get the net shown at the right. In the net, the circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can find the area of this sector by using a proportion, as shown below.



$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ **Substitute.**

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ **Multiply each side by $\pi \ell^2$.**

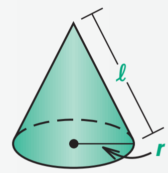
$$\text{Area of sector} = \pi r \ell$$ **Simplify.**

The surface area of a cone is the sum of the base area and the lateral area, $\pi r \ell$.

THEOREM

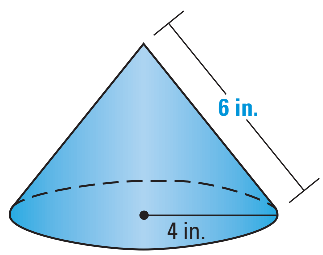
THEOREM 12.5 ***Surface Area of a Right Cone***

The surface area S of a right cone is $S = \pi r^2 + \pi r \ell$, where r is the radius of the base and ℓ is the slant height.



EXAMPLE 3 *Finding the Surface Area of a Right Cone*

To find the surface area of the right cone shown, use the formula for the surface area.



$S = \pi r^2 + \pi r \ell$ **Write formula.**

$= \pi 4^2 + \pi(4)(6)$ **Substitute.**

$= 16\pi + 24\pi$ **Simplify.**

$= 40\pi$ **Simplify.**

▶ The surface area is 40π square inches, or about 125.7 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the differences between pyramids and cones. Describe their similarities.

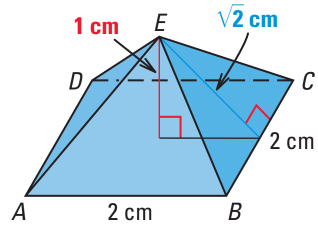
Concept Check ✓

2. Can a pyramid have rectangles for lateral faces? Explain.

Skill Check ✓

Match the expression with the correct measurement.

3. Area of base	**A.** $4\sqrt{2}$ cm^2
4. Height	**B.** $\sqrt{2}$ cm
5. Slant height	**C.** 4 cm^2
6. Lateral area	**D.** $(4 + 4\sqrt{2})$ cm^2
7. Surface area	**E.** 1 cm



In Exercises 8–11, sketch a right cone with $r = 3$ ft and $h = 7$ ft.

8. Find the area of the base.

9. Find the slant height.

10. Find the lateral area.

11. Find the surface area.

Find the surface area of the regular pyramid described.

12. The base area is 9 square meters, the perimeter of the base is 12 meters, and the slant height is 2.5 meters.

13. The base area is $25\sqrt{3}$ square inches, the perimeter of the base is 30 inches, and the slant height is 12 inches.

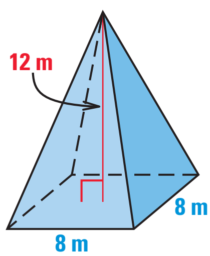
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 825.

AREA OF A LATERAL FACE **Find the area of a lateral face of the regular pyramid. Round the result to one decimal place.**

14.



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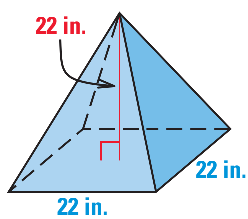

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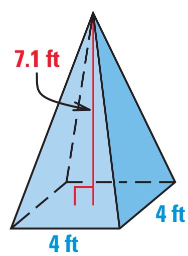
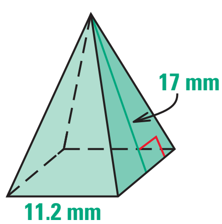


SURFACE AREA OF A PYRAMID **Find the surface area of the regular pyramid.**

17.



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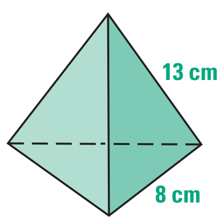

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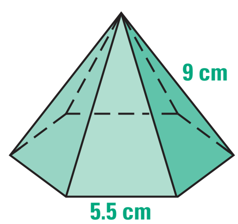
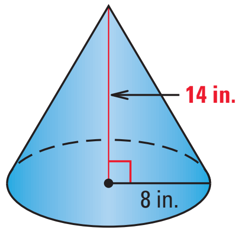


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 14–16
Example 2: Exs. 17–19
Example 3: Exs. 20–25

FINDING SLANT HEIGHT Find the slant height of the right cone.

20.



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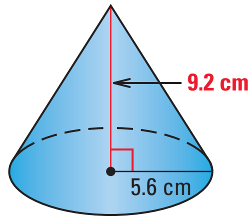
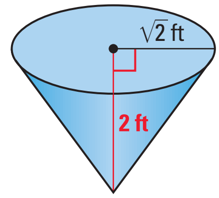
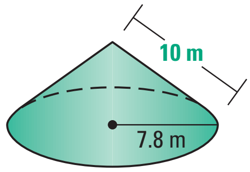


22.

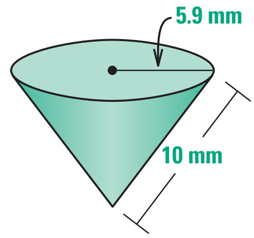


SURFACE AREA OF A CONE Find the surface area of the right cone. Leave your answers in terms of π.

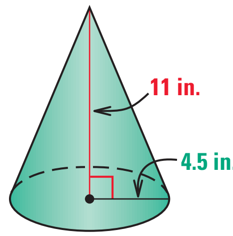
23.



24.

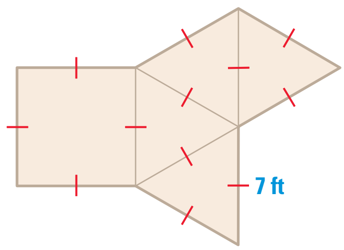


25.

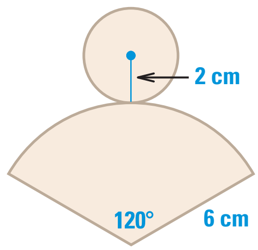


USING NETS Name the figure that is represented by the net. Then find its surface area. Round the result to one decimal place.

26.



27.



VISUAL THINKING Sketch the described solid and find its surface area. Round the result to one decimal place.

28. A regular pyramid has a triangular base with a base edge of 8 centimeters, a height of 12 centimeters, and a slant height of 12.2 centimeters.

29. A regular pyramid has a hexagonal base with a base edge of 3 meters, a height of 5.8 meters, and a slant height of 6.2 meters.

30. A right cone has a diameter of 11 feet and a slant height of 7.2 feet.

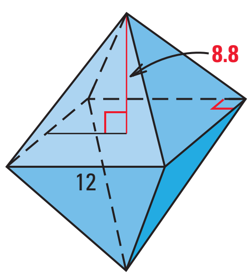
31. A right cone has a radius of 9 inches and a height of 12 inches.

STUDENT HELP

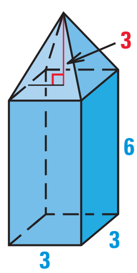
INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 32–34.

COMPOSITE SOLIDS Find the surface area of the solid. The pyramids are regular and the prisms, cylinders, and cones are right. Round the result to one decimal place.

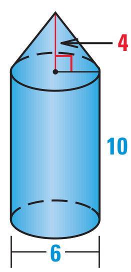
32.



33.

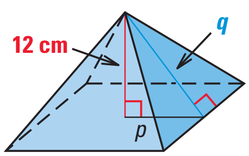


34.

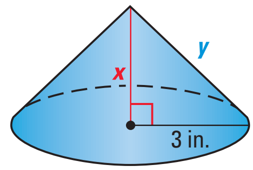


USING ALGEBRA **In Exercises 35–37, find the missing measurements of the solid. The pyramids are regular and the cones are right.**

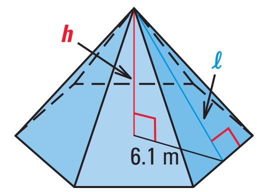
35. $P = 72$ cm



36. $S = 75.4$ in.^{2}



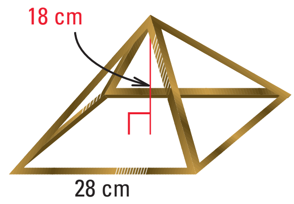
37. $S = 333$ m^2, $P = 42$ m



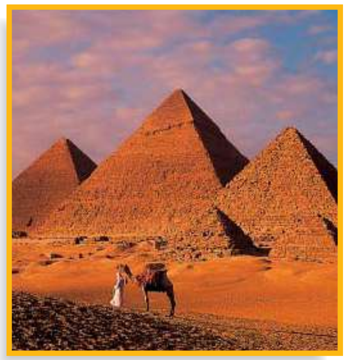
FOCUS ON APPLICATIONS

LAMPSHADES Many stained-glass lampshades are shaped like cones or pyramids. These shapes help direct the light down.

38. **LAMPSHADES** Some stained-glass lampshades are made out of decorative pieces of glass. Estimate the amount of glass needed to make the lampshade shown at the right by calculating the lateral area of the pyramid formed by the framing. The pyramid has a square base.



39. **PYRAMIDS** The three pyramids of Giza, Egypt, were built as regular square pyramids. The pyramid in the middle of the photo is Chephren's Pyramid and when it was built its base edge was $707\frac{3}{4}$ feet, and it had a height of 471 feet. Find the surface area of Chephren's Pyramid, including its base, when it was built.

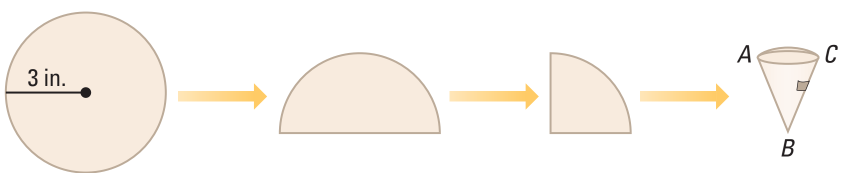


40. **DATA COLLECTION** Find the dimensions of Chephren's Pyramid today and calculate its surface area. Compare this surface area with the surface area you found in Exercise 39.

41. **SQUIRREL BARRIER** Some bird feeders have a metal cone that prevents squirrels from reaching the bird seed, as shown. You are planning to manufacture this metal cone. The slant height of the cone is 12 inches and the radius is 8 inches. Approximate the amount of sheet metal you need.

42. **CRITICAL THINKING** A regular hexagonal pyramid with a base edge of 9 feet and a height of 12 feet is inscribed in a right cone. Find the lateral area of the cone.

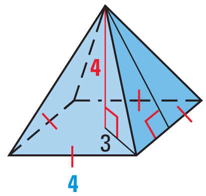
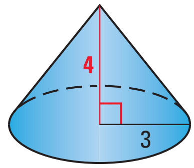
43. **PAPER CUP** To make a paper drinking cup, start with a circular piece of paper that has a 3 inch radius, then follow the steps below. How does the surface area of the cup compare to the original paper circle? Find $m\angle ABC$.



Test Preparation

QUANTITATIVE COMPARISON **Choose the statement that is true about the given quantities.**

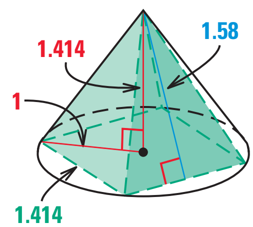
- (A) The quantity in column A is greater.
- (B) The quantity in column B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the given information.

	Column A	Column B
		
44.	Area of base	Area of base
45.	Lateral edge length	Slant height
46.	Lateral area	Lateral area

★ Challenge

INSCRIBED PYRAMIDS **Each of three regular pyramids are inscribed in a right cone whose radius is 1 unit and height is $\sqrt{2}$ units. The dimensions of each pyramid are listed in the table and the square pyramid is shown.**

Base	Base edge	Slant height
Square	1.414	1.58
Hexagon	1	1.65
Octagon	0.765	1.68



47. Find the surface area of the cone.

48. Find the surface area of each of the three pyramids.

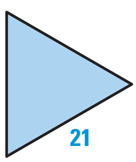
49. What happens to the surface area as the number of sides of the base increases? If the number of sides continues to increase, what number will the surface area approach?

EXTRA CHALLENGE
www.mcdougallittell.com

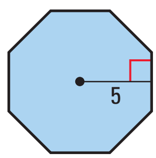
MIXED REVIEW

FINDING AREA **In Exercises 50–52, find the area of the regular polygon. Round your result to two decimal places.** **(Review 11.2 for 12.4)**

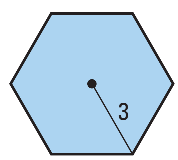
50.



51.



52.



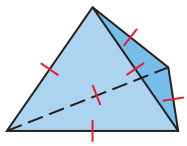
53. **AREA OF A SEMICIRCLE** A semicircle has an area of 190 square inches. Find the approximate length of the radius. **(Review 11.5 for 12.4)**

QUIZ 1

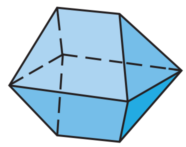
Self-Test for Lessons 12.1–12.3

State whether the polyhedron is regular and/or convex. Then calculate the number of vertices of the solid using the given information. (Lesson 12.1)

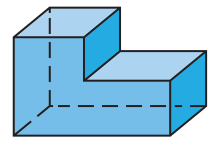
1. 4 faces; all triangles



2. 8 faces; 4 triangles and 4 trapezoids

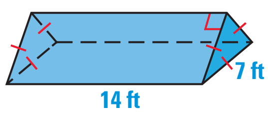


3. 8 faces; 2 hexagons and 6 rectangles

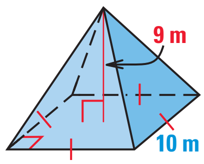


Find the surface area of the solid. Round your result to two decimal places. (Lesson 12.2 and 12.3)

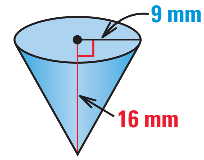
4.



5.



6.



MATH & History

History of Containers

APPLICATION LINK www.mcdougallittell.com

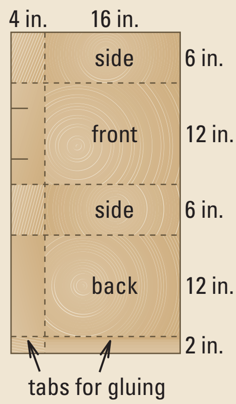
THEN

THROUGHOUT HISTORY, people have created containers for items that were important to store, such as liquids and grains. In ancient civilizations, large jars called *amphorae* were used to store water and other liquids.

NOW

TODAY, containers are no longer used just for the bare necessities. People use containers of many shapes and sizes to store a variety of objects.

1. How much paper is required to construct a paper grocery bag using the pattern at the right?
2. The sections on the left side of the pattern are folded to become the rectangular base of the bag. Find the dimensions of the base. Then find the surface area of the completed bag.



c. 525 B.C.

Amphorae are used in Ancient Greece to store water and oils.

1810

Tin containers are first used to package food.

1870

Margaret Knight patents machine to make paper bags.

1990s

Water bottles come in all shapes and sizes.

12.4 Volume of Prisms and Cylinders

What *you should learn*

GOAL 1 Use volume postulates.

GOAL 2 Find the volume of prisms and cylinders in **real life**, such as the concrete block in **Example 4**.

Why *you should learn it*

▼ Learning to find the volumes of prisms and cylinders is important in **real life**, such as in finding the volume of a fish tank in **Exs. 7–9, and 46**.

REAL LIFE

GOAL 1 EXPLORING VOLUME

The **volume of a solid** is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic meters (m^3).

VOLUME POSTULATES

POSTULATE 27 ***Volume of a Cube***

The volume of a cube is the cube of the length of its side, or $V = s^3$.

POSTULATE 28 ***Volume Congruence Postulate***

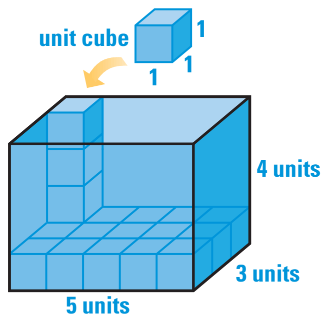
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 ***Volume Addition Postulate***

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

EXAMPLE 1 *Finding the Volume of a Rectangular Prism*

The box shown is 5 units long, 3 units wide, and 4 units high. How many unit cubes will fit in the box? What is the volume of the box?



SOLUTION

The base of the box is 5 units by 3 units. This means $5 \cdot 3$, or 15 unit cubes, will cover the base.

Three more layers of 15 cubes each can be placed on top of the lower layer to fill the box. Because the box contains 4 layers with 15 cubes in each layer, the box contains a total of $4 \cdot 15$, or 60 unit cubes.

▶ Because the box is completely filled by the 60 cubes and each cube has a volume of 1 cubic unit, it follows that the volume of the box is $60 \cdot 1$, or 60 cubic units.

In Example 1, the area of the base, 15 square units, multiplied by the height, 4 units, yields the volume of the box, 60 cubic units. So, the volume of the prism can be found by multiplying the area of the base by the height. This method can also be used to find the volume of a cylinder.

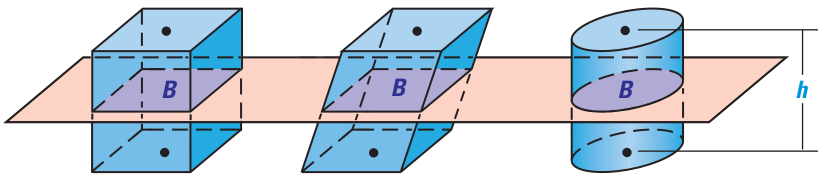
GOAL 2 FINDING VOLUMES OF PRISMS AND CYLINDERS

THEOREM

THEOREM 12.6 ***Cavalieri's Principle***

If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Theorem 12.6 is named after mathematician Bonaventura Cavalieri (1598–1647). To see how it can be applied, consider the solids below. All three have cross sections with equal areas, B, and all three have equal heights, h. By Cavalieri's Principle, it follows that each solid has the same volume.



VOLUME THEOREMS

THEOREM 12.7 ***Volume of a Prism***

The volume V of a prism is $V = Bh$, where B is the area of a base and h is the height.

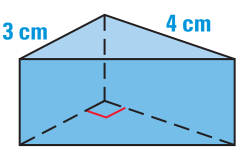
THEOREM 12.8 ***Volume of a Cylinder***

The volume V of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

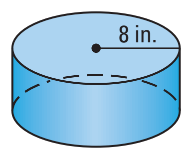
EXAMPLE 2 *Finding Volumes*

Find the volume of the right prism and the right cylinder.

a.



b.



SOLUTION

a. The area B of the base is $\frac{1}{2}(3)(4)$, or 6 cm^2. Use $h = 2$ to find the volume.

$$V = Bh = 6(2) = 12 \text{ cm}^3$$

b. The area B of the base is $\pi \cdot 8^2$, or 64π in.^{2} Use $h = 6$ to find the volume.

$$V = Bh = 64\pi(6) = 384\pi \approx 1206.37 \text{ in.}^3$$

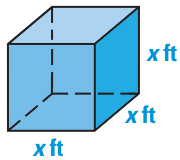
Using Algebra

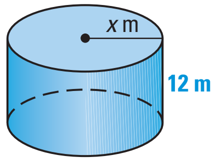
EXAMPLE 3 *Using Volumes*

Use the measurements given to solve for x.

a. Cube, $V = 100 \text{ ft}^3$

b. Right cylinder, $V = 4561 \text{ m}^3$





STUDENT HELP

KEYSTROKE HELP

If your calculator does not have a cube root key, you can raise a number to the $\frac{1}{3}$ to find its cube root. For example, the cube root of 8 can be found as follows:

8 ^ (1 ÷ 3)

SOLUTION

a. A side length of the cube is x feet.

$V = s^3$ Formula for volume of cube

$100 = x^3$ Substitute.

$4.64 \approx x$ Take the cube root.

▶ So, the height, width, and length of the cube are about 4.64 feet.

b. The area of the base is πx^2 square meters.

$V = Bh$ Formula for volume of cylinder

$4561 = \pi x^2(12)$ Substitute.

$4561 = 12\pi x^2$ Rewrite.

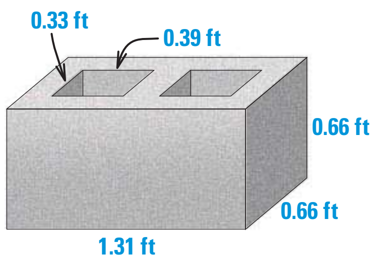
$\frac{4561}{12\pi} = x^2$ Divide each side by 12π.

$11 \approx x$ Find the positive square root.

▶ So, the radius of the cylinder is about 11 meters.

EXAMPLE 4 *Using Volumes in Real Life*

CONSTRUCTION Concrete weighs 145 pounds per cubic foot. To find the weight of the concrete block shown, you need to find its volume. The area of the base can be found as follows:



$$B = \boxed{\text{Area of large rectangle}} - 2 \cdot \boxed{\text{Area of small rectangle}}$$

$$= (1.31)(0.66) - 2(0.33)(0.39)$$

$$\approx 0.61 \text{ ft}^2$$

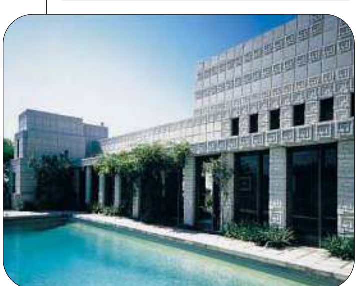
Using the formula for the volume of a prism, the volume is

$$V = Bh \approx 0.61(0.66) \approx 0.40 \text{ ft}^3.$$

▶ To find the weight of the block, multiply the pounds per cubic foot, 145 lb/ft^3, by the number of cubic feet, 0.40 ft^3.

$$\text{Weight} = \frac{145 \text{ lb}}{1 \text{ ft}^3} \cdot 0.4 \text{ ft}^3 \approx 58 \text{ lb}$$

FOCUS ON APPLICATIONS



CONSTRUCTION The Ennis-Brown House, shown above, was designed by Frank Lloyd Wright. It was built using concrete blocks.

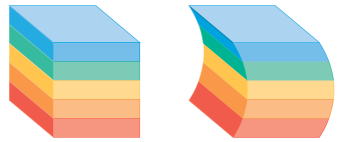
Guided Practice

Vocabulary Check ✓

1. Surface area is measured in __?__ and volume is measured in __?__.

Concept Check ✓

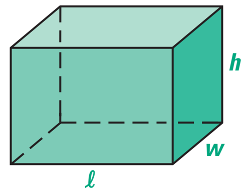
2. Each stack of memo papers shown contains 500 sheets of paper. Explain why the stacks have the same volume. Then calculate the volume, given that each sheet of paper is 3 inches by 3 inches by 0.01 inches.



Skill Check ✓

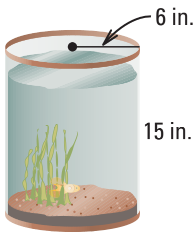
Use the diagram to complete the table.

	ℓ	w	h	Volume
3.	17	3	5	?
4.	?	8	10	160
5.	4.8	6.1	?	161.04
6.	$6t$	?	$3t$	$54t^3$

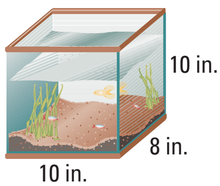


FISH TANKS **Find the volume of the tank.**

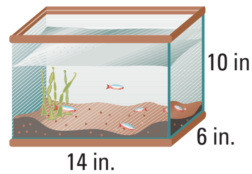
7.



8.



9.



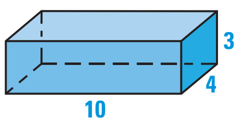
Practice and Applications

STUDENT HELP

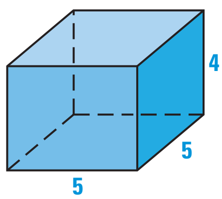
Extra Practice to help you master skills is on p. 826.

USING UNIT CUBES **Find the number of unit cubes that will fit in the box. Explain your reasoning.**

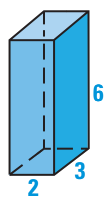
10.



11.



12.

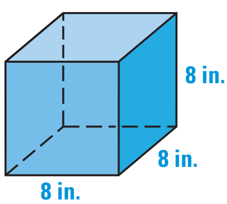


STUDENT HELP

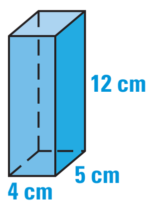
HOMEWORK HELP
Example 1: Exs. 10–12
Example 2: Exs. 13–27
Example 3: Exs. 28–33
Example 4: Exs. 34–37

VOLUME OF A PRISM **Find the volume of the right prism.**

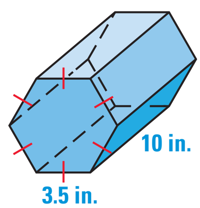
13.



14.

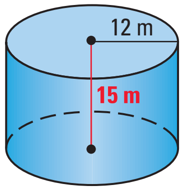


15.

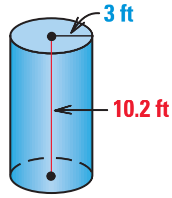


VOLUME OF A CYLINDER **Find the volume of the right cylinder. Round the result to two decimal places.**

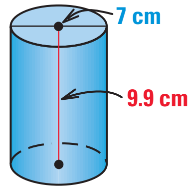
16.



17.



18.



VISUAL THINKING **Make a sketch of the solid and find its volume. Round the result to two decimal places.**

19. A prism has a square base with 4 meter sides and a height of 15 meters.

20. A pentagonal prism has a base area of 24 square feet and a height of 3 feet.

21. A prism has a height of 11.2 centimeters and an equilateral triangle for a base, where each base edge measures 8 centimeters.

22. A cylinder has a radius of 4 meters and a height of 8 meters.

23. A cylinder has a radius of 21.4 feet and a height of 33.7 feet.

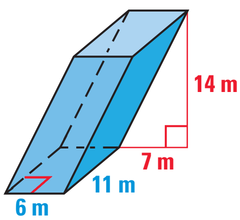
24. A cylinder has a diameter of 15 inches and a height of 26 inches.

STUDENT HELP

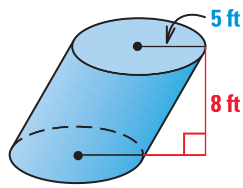
INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with Exs. 25–27.

VOLUMES OF OBLIQUE SOLIDS **Use Cavalieri's Principle to find the volume of the oblique prism or cylinder.**

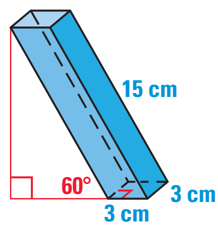
25.



26.

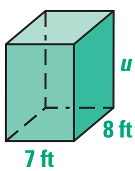


27.

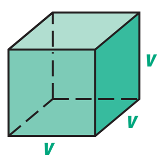


USING ALGEBRA **Solve for the variable using the given measurements. The prisms and the cylinders are right.**

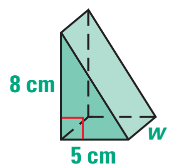
28. Volume = 560 ft^3



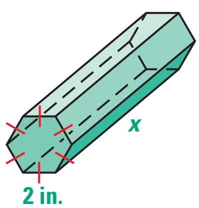
29. Volume = 2700 yd^3



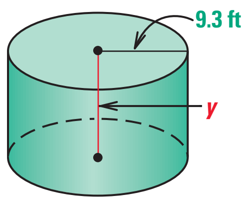
30. Volume = 80 cm^3



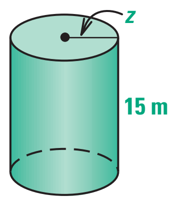
31. Volume = 72.66 $in.^3$



32. Volume = 3000 ft^3



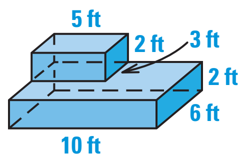
33. Volume = 1696.5 m^3



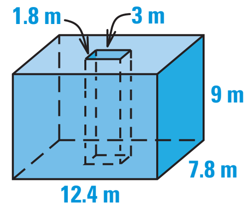
34. **CONCRETE BLOCK** In Example 4 on page 745, find the volume of the entire block and subtract the volume of the two rectangular prisms. How does your answer compare with the volume found in Example 4?

FINDING VOLUME **Find the volume of the entire solid. The prisms and cylinders are right.**

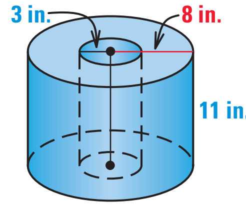
35.



36.



37.



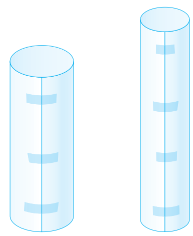
CONCRETE **In Exercises 38–40, determine the number of yards of concrete you need for the given project. To builders, a "yard" of concrete means a cubic yard. (A cubic yard is equal to $(36 \text{ in.})^3$, or $46{,}656 \text{ in.}^3$.)**

38. A driveway that is 30 feet long, 18 feet wide, and 4 inches thick

39. A tennis court that is 100 feet long, 50 feet wide, and 6 inches thick

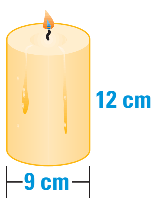
40. A circular patio that has a radius of 24 feet and is 8 inches thick

41. **LOGICAL REASONING** Take two sheets of paper that measure $8\frac{1}{2}$ inches by 11 inches and form two cylinders; one with the height as $8\frac{1}{2}$ inches and one with the height as 11 inches. Do the cylinders have the same volume? Explain.

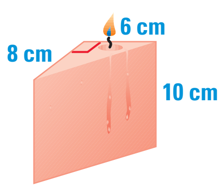


CANDLES **In Exercises 42–44, you are melting a block of wax to make candles. How many candles of the given shape can be made using a block that measures 10 cm by 9 cm by 20 cm? The prisms and cylinder are right.**

42.



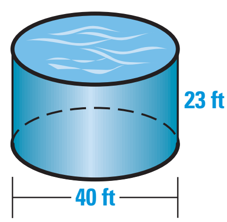
43.



44.

45. **CANNED GOODS** Find the volume and surface area of a prism with a height of 4 inches and a 3 inch by 3 inch square base. Compare the results with the volume and surface area of a cylinder with a height of 5.1 inches and a diameter of 3 inches. Use your results to explain why canned goods are usually packed in cylindrical containers.

AQUARIUM TANK **The Caribbean Coral Reef Tank at the New England Aquarium is a cylindrical tank that is 23 feet deep and 40 feet in diameter, as shown.**

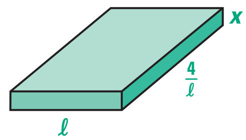


46. How many gallons of water are needed to fill the tank? (One gallon of water equals 0.1337 cubic foot.)

47. Determine the weight of the water in the tank. (One gallon of salt water weighs about 8.56 pounds.)

Test Preparation

48. MULTIPLE CHOICE If the volume of the rectangular prism at the right is 1, what does x equal?



Ⓐ $\frac{1}{4}$ Ⓑ $\frac{\ell}{4}$ Ⓒ ℓ

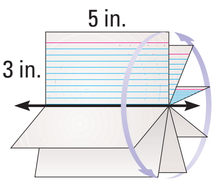
Ⓓ 4 Ⓔ 4ℓ

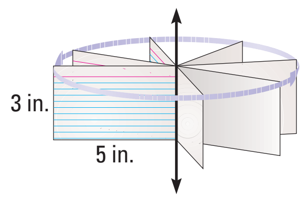
49. MULTIPLE CHOICE What is the volume of a cylinder with a radius of 6 and a height of 10?

Ⓐ 60π Ⓑ 90π Ⓒ 120π Ⓓ 180π Ⓔ 360π

★ **Challenge**

50. Suppose that a 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? Explain your reasoning.





EXTRA CHALLENGE

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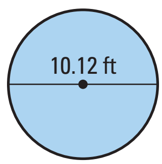
MIXED REVIEW

USING RATIOS **Find the measures of the angles in the triangle whose angles are in the given extended ratio.** (Review 8.1)

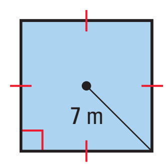
51. 2:5:5 **52.** 1:2:3 **53.** 3:4:5

FINDING AREA **In Exercises 54–56, find the area of the figure. Round your result to two decimal places.** (Review 11.2, 11.5 for 12.5)

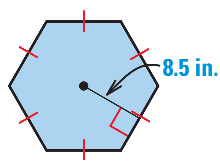
54.



55.



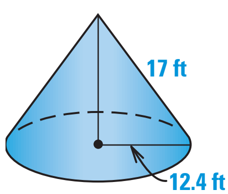
56.



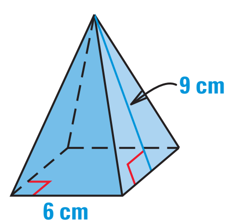
57. SURFACE AREA OF A PRISM A right rectangular prism has a height of 13 inches, a length of 1 foot, and a width of 3 inches. Sketch the prism and find its surface area. (Review 12.2)

SURFACE AREA **Find the surface area of the solid. The cone is right and the pyramids are regular.** (Review 12.3)

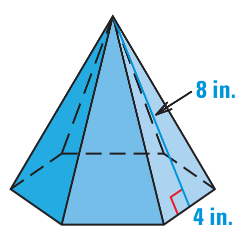
58.



59.



60.



12.5 Volume of Pyramids and Cones

***What** you should learn*

GOAL 1 Find the volume of pyramids and cones.

GOAL 2 Find the volume of pyramids and cones in **real life**, such as the nautical prism in **Example 4**.

***Why** you should learn it*

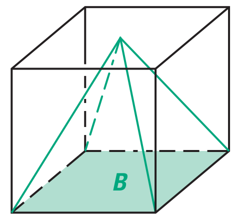
▼ Learning to find volumes of pyramids and cones is important in **real life**, such as in finding the volume of a volcano shown below and in **Ex. 34**.

REAL LIFE

Mount St. Helens

GOAL 1 FINDING VOLUMES OF PYRAMIDS AND CONES

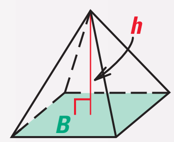
In Lesson 12.4, you learned that the volume of a prism is equal to Bh, where B is the area of the base and h is the height. From the figure at the right, it is clear that the volume of the pyramid with the same base area B and the same height h must be less than the volume of the prism. The volume of the pyramid is one third the volume of the prism.



THEOREMS

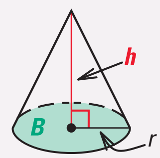
THEOREM 12.9 ***Volume of a Pyramid***

The volume V of a pyramid is $V = \frac{1}{3}Bh$, where B is the area of the base and h is the height.



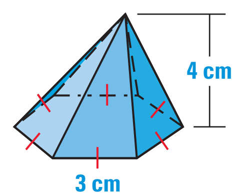
THEOREM 12.10 ***Volume of a Cone***

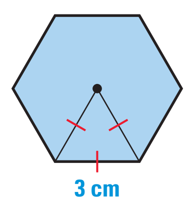
The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$, where B is the area of the base, h is the height, and r is the radius of the base.



EXAMPLE 1 *Finding the Volume of a Pyramid*

Find the volume of the pyramid with the regular base.





SOLUTION

The base can be divided into six equilateral triangles. Using the formula for the area of an equilateral triangle, $\frac{1}{4}\sqrt{3} \cdot s^2$, the area of the base B can be found as follows:

$$6 \cdot \frac{1}{4}\sqrt{3} \cdot s^2 = 6 \cdot \frac{1}{4}\sqrt{3} \cdot 3^2 = \frac{27}{2}\sqrt{3}\text{ cm}^2.$$

Use Theorem 12.9 to find the volume of the pyramid.

$V = \frac{1}{3}Bh$ — **Formula for volume of pyramid**

$= \frac{1}{3}\left(\frac{27}{2}\sqrt{3}\right)(4)$ — **Substitute.**

$= 18\sqrt{3}$ — **Simplify.**

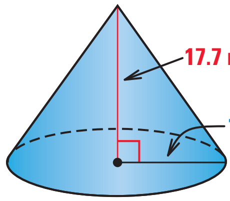
▶ So, the volume of the pyramid is $18\sqrt{3}$, or about 31.2 cubic centimeters.

> **STUDENT HELP**
>
> **Study Tip**
> The formulas given in Theorems 12.9 and 12.10 apply to all pyramids and cones, whether right or oblique. This follows from Cavalieri's Principle, stated in Lesson 12.4.

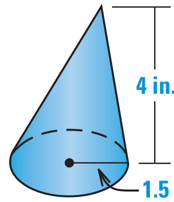
EXAMPLE 2 *Finding the Volume of a Cone*

Find the volume of each cone.

a. Right circular cone



b. Oblique circular cone



SOLUTION

a. Use the formula for the volume of a cone.

$V = \frac{1}{3}Bh$ **Formula for volume of cone**

$= \frac{1}{3}(\pi r^2)h$ **Base area equals πr^2.**

$= \frac{1}{3}(\pi 12.4^2)(17.7)$ **Substitute.**

$\approx 907.18\pi$ **Simplify.**

▶ So, the volume of the cone is about 907.18π, or 2850 cubic millimeters.

b. Use the formula for the volume of a cone.

$V = \frac{1}{3}Bh$ **Formula for volume of cone**

$= \frac{1}{3}(\pi r^2)h$ **Base area equals πr^2.**

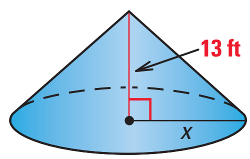
$= \frac{1}{3}(\pi 1.5^2)(4)$ **Substitute.**

$= 3\pi$ **Simplify.**

▶ So, the volume of the cone is 3π, or about 9.42 cubic inches.

EXAMPLE 3 *Using the Volume of a Cone*

Use the given measurements to solve for x.



Volume = 2614 ft^3

SOLUTION

$V = \frac{1}{3}\pi r^2 h$ **Formula for volume**

$2614 = \frac{1}{3}(\pi x^2)(13)$ **Substitute.**

$7842 = 13\pi x^2$ **Multiply each side by 3.**

$192 \approx x^2$ **Divide each side by 13π.**

$13.86 \approx x$ **Find positive square root.**

▶ So, the radius of the cone is about 13.86 feet.

> **STUDENT HELP**
>
> **Study Tip**
> To eliminate the fraction in an equation, you can multiply each side by the reciprocal of the fraction. This was done in Example 3.

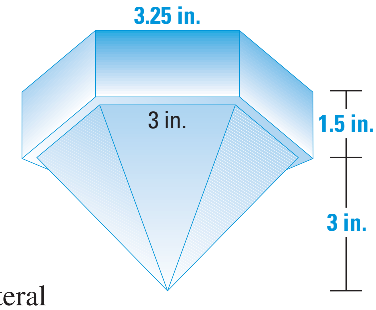
FOCUS ON APPLICATIONS

NAUTICAL PRISMS Before electricity, nautical prisms were placed in the decks of sailing ships. By placing the hexagonal face flush with the deck, the prisms would draw light to the lower regions of the ship.

GOAL 2 USING VOLUME IN REAL-LIFE PROBLEMS

EXAMPLE 4 *Finding the Volume of a Solid*

NAUTICAL PRISMS A nautical prism is a solid piece of glass, as shown. Find its volume.



SOLUTION

To find the volume of the entire solid, add the volumes of the prism and the pyramid. The bases of the prism and the pyramid are regular hexagons made up of six equilateral triangles. To find the area of each base, B, multiply the area of one of the equilateral triangles by 6, or $6\left(\frac{\sqrt{3}}{4}s^2\right)$, where s is the base edge.

Volume of prism $= 6\left(\frac{\sqrt{3}}{4}s^2\right)h$ — Formula for volume of prism

$= 6\left(\frac{\sqrt{3}}{4}(3.25)^2\right)(1.5)$ — Substitute.

≈ 41.16 — Use a calculator.

Volume of pyramid $= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4}s^2\right)h$ — Formula for volume of pyramid

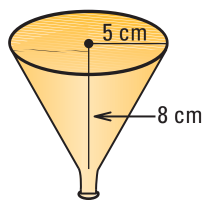
$= \frac{1}{3} \cdot 6\left(\frac{\sqrt{3}}{4} \cdot 3^2\right)(3)$ — Substitute.

≈ 23.38 — Use a calculator.

▶ The volume of the nautical prism is **41.16** + **23.38** or 64.54 cubic inches.

EXAMPLE 5 *Using the Volume of a Cone*

AUTOMOBILES If oil is being poured into the funnel at a rate of 147 milliliters per second and flows out of the funnel at a rate of 42 milliliters per second, estimate the time it will take for the funnel to overflow. $(1 \text{ mL} = 1 \text{ cm}^3)$



SOLUTION

First, find the approximate volume of the funnel.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5^2)(8) \approx 209 \text{ cm}^3 = 209 \text{ mL}$$

The rate of accumulation of oil in the funnel is $147 - 42 = 105$ mL/s. To find the time it will take for the oil to fill the funnel, divide the volume of the funnel by the rate of accumulation of oil in the funnel as follows:

$$209 \text{ mL} \div \frac{105 \text{ mL}}{1 \text{ s}} = 209 \text{ mL} \times \frac{1 \text{ s}}{105 \text{ mL}} \approx 2 \text{ s}$$

▶ The funnel will overflow after about 2 seconds.

Guided Practice

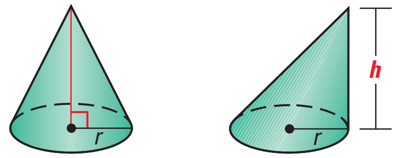
Vocabulary Check ✓

1. The volume of a cone with radius r and height h is $\frac{1}{3}$ the volume of a __?__ with radius r and height h.

Concept Check ✓

Do the two solids have the same volume? Explain your answer.

2.



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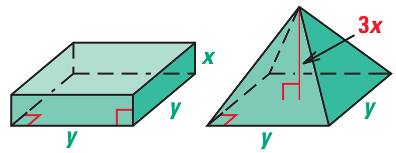
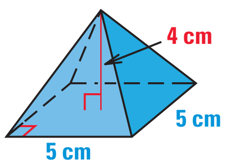


Skill Check ✓

In Exercises 4–6, find (a) the area of the base of the solid and (b) the volume of the solid.

4.



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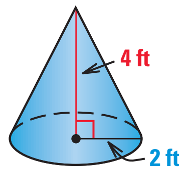
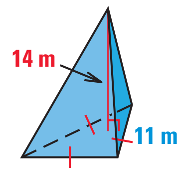


6.



7. **Critical Thinking** You are given the radius and the slant height of a right cone. Explain how you can find the height of the cone.

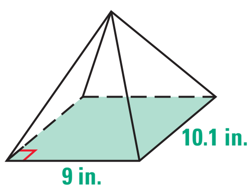
Practice and Applications

Student Help

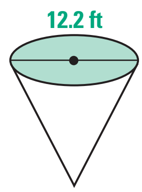
Extra Practice to help you master skills is on p. 826.

Finding Base Areas **Find the area of the base of the solid.**

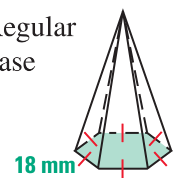
8.



9.

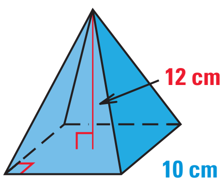


10. Regular base



Volume of a Pyramid **Find the volume of the pyramid. Each pyramid has a regular polygon for a base.**

11.



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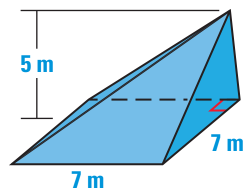

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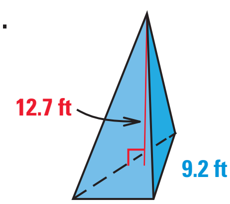

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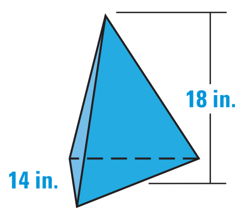

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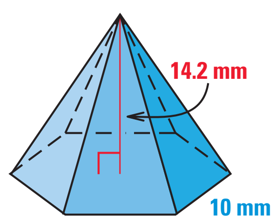

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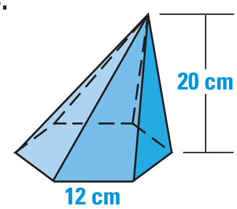
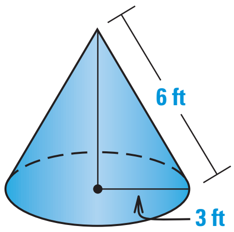


Student Help

Homework Help
Example 1: Exs. 11–16
Example 2: Exs. 17–19
Example 3: Exs. 20–22
Example 4: Exs. 23–28
Example 5: Ex. 29

VOLUME OF A CONE **Find the volume of the cone. Round your result to two decimal places.**

17.



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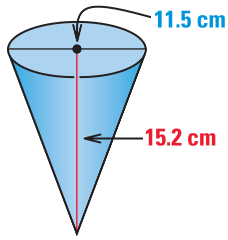
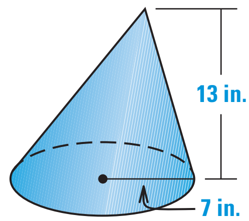
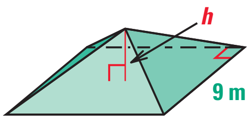


19.

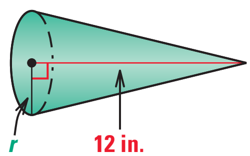


USING ALGEBRA **Solve for the variable using the given information.**

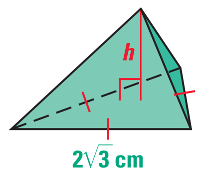
20. Volume = 270 m^3



21. Volume = $100\pi \text{ in.}^3$



22. Volume = $5\sqrt{3} \text{ cm}^3$

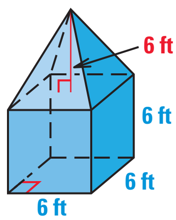


STUDENT HELP

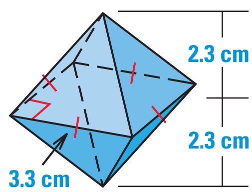
HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 23–25.

COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round the result to two decimal places.**

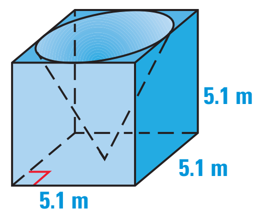
23.



24.



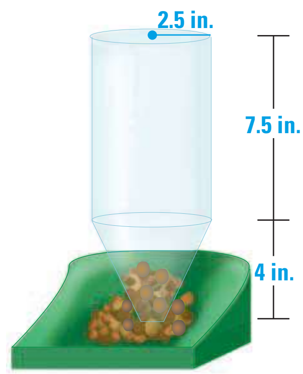
25.



AUTOMATIC FEEDER **In Exercises 26 and 27, use the diagram of the automatic pet feeder.** $\left(\textbf{1 cup} = \textbf{14.4 in.}^3\right)$

26. Calculate the amount of food that can be placed in the feeder.

27. If a cat eats half of a cup of food, twice per day, will the feeder hold enough food for three days?



28. **ANCIENT CONSTRUCTION** Early civilizations in the Andes Mountains in Peru used cone-shaped adobe bricks to build homes. Find the volume of an adobe brick with a diameter of 8.3 centimeters and a slant height of 10.1 centimeters. Then calculate the amount of space 27 of these bricks would occupy in a mud mortar wall.

29. **SCIENCE CONNECTION** During a chemistry lab, you use a funnel to pour a solvent into a flask. The radius of the funnel is 5 centimeters and its height is 10 centimeters. If the solvent is being poured into the funnel at a rate of 80 milliliters per second and the solvent flows out of the funnel at a rate of 65 milliliters per second, how long will it be before the funnel overflows? $\left(1 \text{ mL} = 1 \text{ cm}^3\right)$

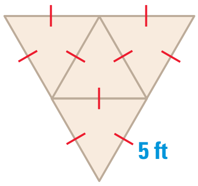
Focus on Careers

VOLCANOLOGY Volcanologists collect and interpret data about volcanoes to help them predict when a volcano will erupt.

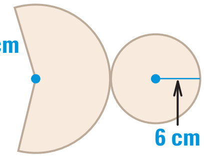
CAREER LINK www.mcdougallittell.com

USING NETS In Exercises 30–32, use the net to sketch the solid. Then find the volume of the solid. Round the result to two decimal places.

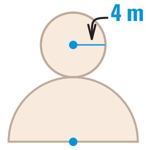
30.



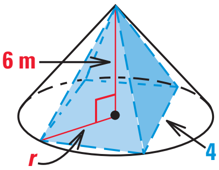
31.



32.



33. FINDING VOLUME In the diagram at the right, a regular square pyramid with a base edge of 4 meters is inscribed in a cone with a height of 6 meters. Use the dimensions of the pyramid to find the volume of the cone.



34. VOLCANOES Before 1980, Mount St. Helens was cone shaped with a height of about 1.83 miles and a base radius of about 3 miles. In 1980, Mount St. Helens erupted. The tip of the cone was destroyed, as shown, reducing the volume by 0.043 cubic mile. The cone-shaped tip that was destroyed had a radius of about 0.4 mile. How tall is the volcano today? (*Hint:* Find the height of the destroyed cone-shaped tip.)

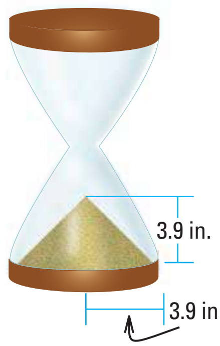
Test Preparation

MULTI-STEP PROBLEM Use the diagram of the hourglass below.

35. Find the volume of the cone-shaped pile of sand.

36. The sand falls through the opening at a rate of one cubic inch per minute. Is the hourglass a true "hour"-glass? Explain. (1 hr = 60 min)

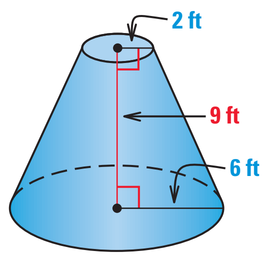
37. *Writing* The sand in the hourglass falls into a conical shape with a one-to-one ratio between the radius and the height. Without doing the calculations, explain how to find the radius and height of the pile of sand that has accumulated after 30 minutes.



★ Challenge

FRUSTUMS A *frustum* of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information below to complete Exercises 38 and 39.

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.



38. Use the measurements in the diagram to calculate the volume of the frustum.

39. Write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h.

STUDENT HELP

Look Back
For help with finding geometric means, see p. 466.

MIXED REVIEW

FINDING ANGLE MEASURES **Find the measure of each interior and exterior angle of a regular polygon with the given number of sides.** **(Review 11.1)**

40. 9 **41.** 10 **42.** 19

43. 22 **44.** 25 **45.** 30

FINDING THE AREA OF A CIRCLE **Find the area of the described circle.** **(Review 11.5 for 12.6)**

46. The diameter of the circle is 25 inches.

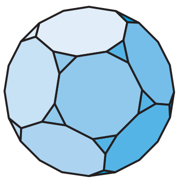
47. The radius of the circle is 16.3 centimeters.

48. The circumference of the circle is 48π feet.

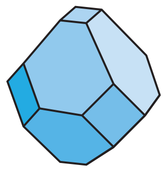
49. The length of a $36°$ arc of the circle is 2π meters.

USING EULER'S THEOREM **Calculate the number of vertices of the solid using the given information.** **(Review 12.1)**

50. 32 faces; 12 octagons and 20 triangles



51. 14 faces; 6 squares and 8 hexagons

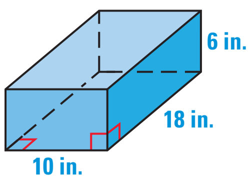


QUIZ 2

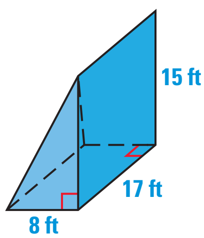
Self-Test for Lessons 12.4 and 12.5

In Exercises 1–6, find the volume of the solid. **(Lessons 12.4 and 12.5)**

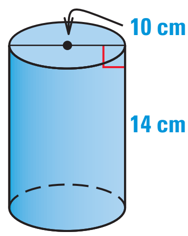
1.



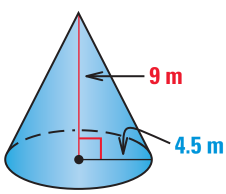
2.



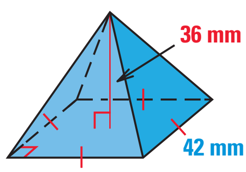
3.



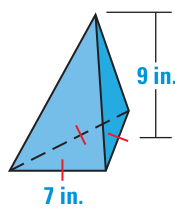
4.



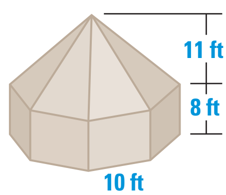
5.



6.



7. **STORAGE BUILDING** A road-salt storage building is composed of a regular octagonal pyramid and a regular octagonal prism as shown. Find the volume of salt that the building can hold. **(Lesson 12.5)**



12.6 Surface Area and Volume of Spheres

What you should learn

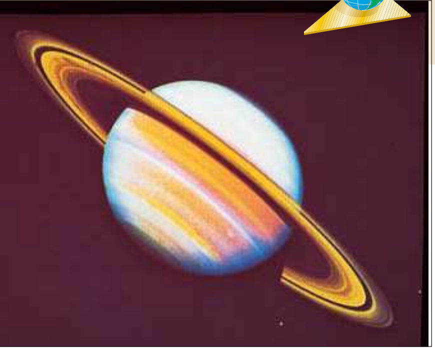
GOAL 1 Find the surface area of a sphere.

GOAL 2 Find the volume of a sphere in **real life**, such as the ball bearing in **Example 4**.

Why you should learn it

▼ You can find the surface area and volume of **real-life** spherical objects, such as the planets and moons in **Exs. 18 and 19**.

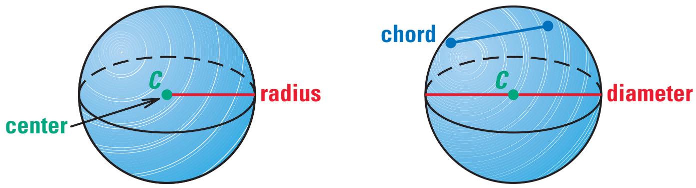
REAL LIFE



Saturn

GOAL 1 FINDING THE SURFACE AREA OF A SPHERE

In Lesson 10.7, a circle was described as the locus of points in a plane that are a given distance from a point. A **sphere** is the locus of points in *space* that are a given distance from a point. The point is called the **center of the sphere**. A **radius of a sphere** is a segment from the center to a point on the sphere.



A **chord of a sphere** is a segment whose endpoints are on the sphere. A **diameter** is a chord that contains the center. As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM

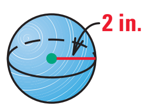
THEOREM 12.11 ***Surface Area of a Sphere***

The surface area S of a sphere with radius r is $S = 4\pi r^2$.

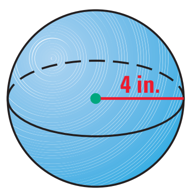
EXAMPLE 1 ***Finding the Surface Area of a Sphere***

Find the surface area. When the radius doubles, does the surface area double?

a.



b.



SOLUTION

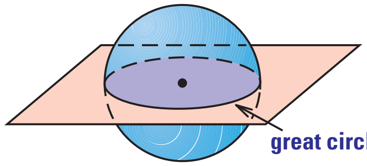
a. $S = 4\pi r^2 = 4\pi(2)^2 = 16\pi \text{ in.}^2$

b. $S = 4\pi r^2 = 4\pi(4)^2 = 64\pi \text{ in.}^2$

The surface area of the sphere in part (b) is four times greater than the surface area of the sphere in part (a) because $16\pi \cdot 4 = 64\pi$.

▶ So, when the radius of a sphere doubles, the surface area does *not* double.

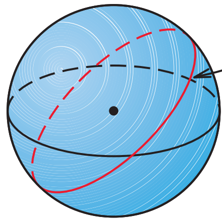
If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called **hemispheres**.



EXAMPLE 2 *Using a Great Circle*

STUDENT HELP
INTERNET
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

The circumference of a great circle of a sphere is 13.8π feet. What is the surface area of the sphere?



SOLUTION

Begin by finding the radius of the sphere.

$C = 2\pi r$ — Formula for circumference of circle

$13.8\pi = 2\pi r$ — Substitute 13.8π for C.

$6.9 = r$ — Divide each side by 2π.

Using a radius of 6.9 feet, the surface area is

$$S = 4\pi r^2 = 4\pi(6.9)^2 = 190.44\pi \text{ ft}^2.$$

▶ So, the surface area of the sphere is 190.44π, or about 598 ft^2.

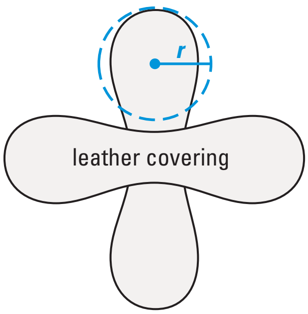
EXAMPLE 3 *Finding the Surface Area of a Sphere*

REAL LIFE

BASEBALL A baseball and its leather covering are shown. The baseball has a radius of about 1.45 inches.

a. Estimate the amount of leather used to cover the baseball.

b. The surface of a baseball is sewn from two congruent shapes, each of which resembles two joined circles. How does this relate to the formula for the surface area of a sphere?



SOLUTION

a. Because the radius r is about 1.45 inches, the surface area is as follows:

$S = 4\pi r^2$ — Formula for surface area of sphere

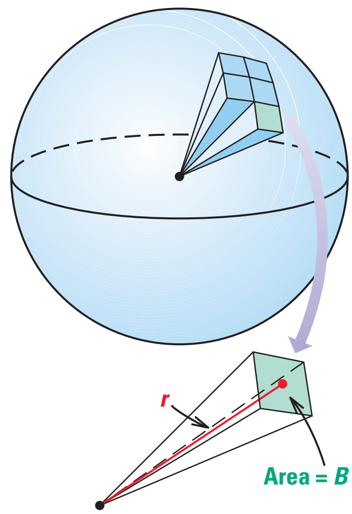
$\approx 4\pi(1.45)^2$ — Substitute 1.45 for r.

$\approx 26.4 \text{ in.}^2$ — Use a calculator.

b. Because the covering has two pieces, each resembling two joined circles, then the entire covering consists of four circles with radius r. The area of a circle of radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the same as the formula for the surface area of a sphere.

GOAL 2 FINDING THE VOLUME OF A SPHERE

Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r, as shown. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\frac{1}{3}Br$	**Each pyramid has a volume of $\frac{1}{3}Br$.**
$= \frac{1}{3}(nB)r$	**Regroup factors.**
$\approx \frac{1}{3}(4\pi r^2)r$	**Substitute $4\pi r^2$ for nB.**
$= \frac{4}{3}\pi r^3$	**Simplify.**

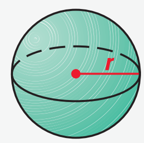
STUDENT HELP

Study Tip
If you understand how a formula is derived, then it will be easier for you to remember the formula.

THEOREM

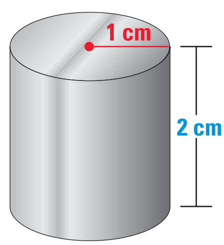
THEOREM 12.12 ***Volume of a Sphere***

The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.

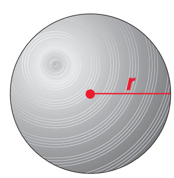


EXAMPLE 4 *Finding the Volume of a Sphere*

BALL BEARINGS To make a steel ball bearing, a cylindrical *slug* is heated and pressed into a spherical shape with the same volume. Find the radius of the ball bearing below.



slug



ball bearing

SOLUTION

To find the volume of the slug, use the formula for the volume of a cylinder.

$$V = \pi r^2 h = \pi(1^2)(2) = 2\pi \text{ cm}^3$$

To find the radius of the ball bearing, use the formula for the volume of a sphere and solve for r.

$V = \frac{4}{3}\pi r^3$	**Formula for volume of sphere**
$2\pi = \frac{4}{3}\pi r^3$	**Substitute 2π for V.**
$6\pi = 4\pi r^3$	**Multiply each side by 3.**
$1.5 = r^3$	**Divide each side by 4π.**
$1.14 \approx r$	**Use a calculator to take the cube root.**

▶ So, the radius of the ball bearing is about 1.14 centimeters.

FOCUS ON PEOPLE

IN-LINE SKATING Ball bearings help the wheels of an in-line skate turn smoothly. The two brothers above, Scott and Brennan Olson, pioneered the design of today's in-line skates.

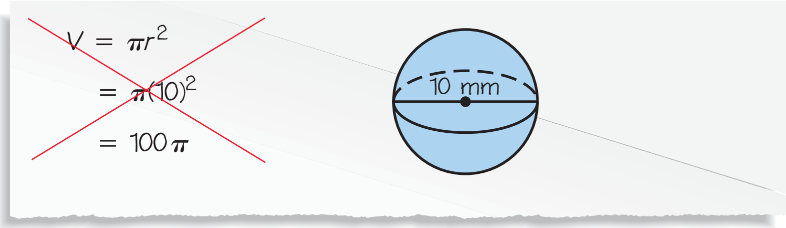
GUIDED PRACTICE

Vocabulary Check ✓

1. The locus of points in space that are ___?___ from a ___?___ is called a sphere.

Concept Check ✓

2. **ERROR ANALYSIS** Melanie is asked to find the volume of a sphere with a diameter of 10 millimeters. Explain her error(s).



Skill Check ✓

In Exercises 3–8, use the diagram of the sphere, whose center is *P*.

3. Name a chord of the sphere.

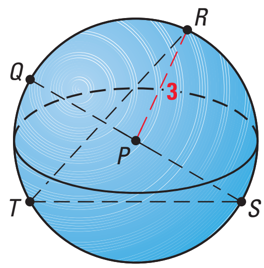
4. Name a segment that is a radius of the sphere.

5. Name a segment that is a diameter of the sphere.

6. Find the circumference of the great circle that contains Q and S.

7. Find the surface area of the sphere.

8. Find the volume of the sphere.



9. **CHEMISTRY** A helium atom is approximately spherical with a radius of about 0.5×10^{-8} centimeter. What is the volume of a helium atom?

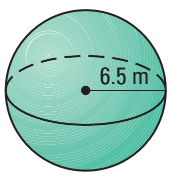
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 826.

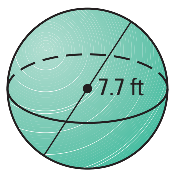
FINDING SURFACE AREA **Find the surface area of the sphere. Round your result to two decimal places.**

10.



11.

12.



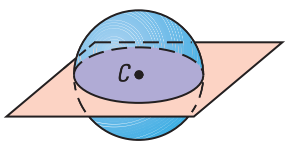
USING A GREAT CIRCLE **In Exercises 13–16, use the sphere below. The center of the sphere is *C* and its circumference is 7.4π inches.**

13. What is half of the sphere called?

14. Find the radius of the sphere.

15. Find the diameter of the sphere.

16. Find the surface area of half of the sphere.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 10–12
Example 2: Exs. 13–16
Example 3: Ex. 17
Example 4: Exs. 20–22, 41–43

17. **SPORTS** The diameter of a softball is 3.8 inches. Estimate the amount of leather used to cover the softball.

FOCUS ON APPLICATIONS

REAL LIFE **PLANETS** Jupiter is the largest planet in our solar system. It has a diameter of 88,730 miles, or 142,800 kilometers.

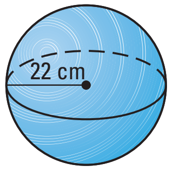
INTERNET APPLICATION LINK
www.mcdougallittell.com

18. **PLANETS** The circumference of Earth at the equator (great circle of Earth) is 24,903 miles. The diameter of the moon is 2155 miles. Find the surface area of Earth and of the moon to the nearest hundred. How does the surface area of the moon compare to the surface area of Earth?

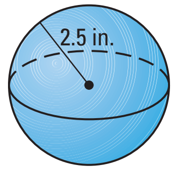
19. **DATA COLLECTION** Research to find the diameters of Neptune and its two moons, Triton and Nereid. Use the diameters to find the surface area of each.

FINDING VOLUME **Find the volume of the sphere. Round your result to two decimal places.**

20.



21.



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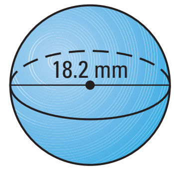
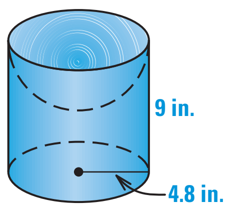


USING A TABLE **Copy and complete the table below. Leave your answers in terms of π.**

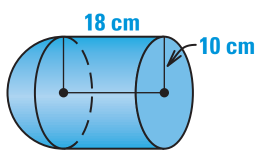
	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
23.	7 mm	?	?	?
24.	?	?	144π in.^{2}	?
25.	?	10π cm	?	?
26.	?	?	?	$\frac{4000\pi}{3}$ m^3

COMPOSITE SOLIDS **Find (a) the surface area of the solid and (b) the volume of the solid. The cylinders and cones are right. Round your results to two decimal places.**

27.



28.



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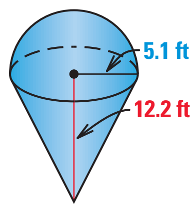


TECHNOLOGY **In Exercises 30–33, consider five spheres whose radii are 1 meter, 2 meters, 3 meters, 4 meters, and 5 meters.**

30. Find the volume and surface area of each sphere. Leave your results in terms of π.

31. Use your answers to Exercise 30 to find the ratio of the volume to the surface area, $\frac{V}{S}$, for each sphere.

32. Use a graphing calculator to plot the graph of $\frac{V}{S}$ as a function of the radius. What do you notice?

33. *Writing* If the radius of a sphere triples, does its surface area triple? Explain your reasoning.

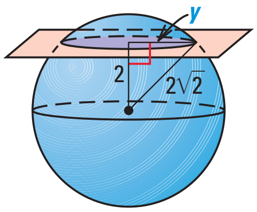
34. **VISUAL THINKING** A sphere with radius r is inscribed in a cylinder with height $2r$. Make a sketch and find the volume of the cylinder in terms of r.

STUDENT HELP

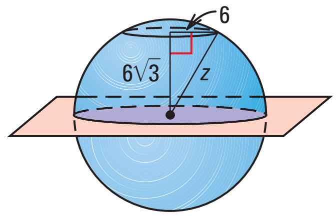
INTERNET **HOMEWORK HELP**
Visit our Web site www.mcdougallittell.com for help with Exs. 35 and 36.

USING ALGEBRA **In Exercises 35 and 36, solve for the variable. Then find the area of the intersection of the sphere and the plane.**

35.

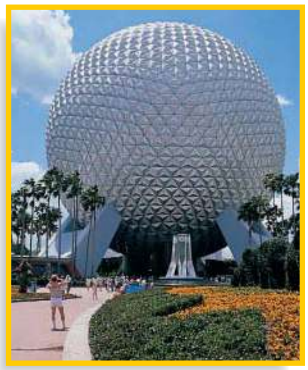


36.



37. **CRITICAL THINKING** Sketch the intersection of a sphere and a plane that does not pass through the center of the sphere. If you know the circumference of the circle formed by the intersection, can you find the surface area of the sphere? Explain.

SPHERES IN ARCHITECTURE **The spherical building below has a diameter of 165 feet.**



38. Find the surface area of a sphere with a diameter of 165 feet. Looking at the surface of the building, do you think its surface area is the same? Explain.

39. The surface of the building consists of 1000 (nonregular) triangular pyramids. If the lateral area of each pyramid is about 267.3 square feet, estimate the actual surface area of the building.

40. Estimate the volume of the building using the formula for the volume of a sphere.

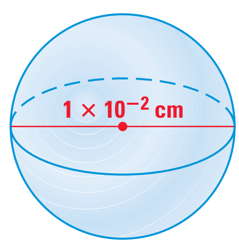
BALL BEARINGS **In Exercises 41–43, refer to the description of how ball bearings are made in Example 4 on page 761.**

41. Find the radius of a steel ball bearing made from a cylindrical slug with a radius of 3 centimeters and a height of 6 centimeters.

42. Find the radius of a steel ball bearing made from a cylindrical slug with a radius of 2.57 centimeters and a height of 4.8 centimeters.

43. If a steel ball bearing has a radius of 5 centimeters, and the radius of the cylindrical slug it was made from was 4 centimeters, then what was the height of the cylindrical slug?

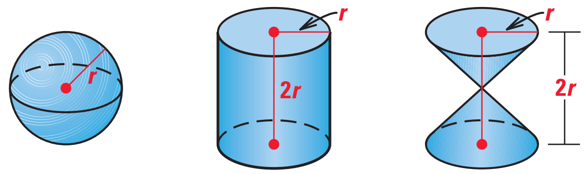
44. **COMPOSITION OF ICE CREAM** In making ice cream, a mix of solids, sugar, and water is frozen. Air bubbles are whipped into the mix as it freezes. The air bubbles are about 1×10^{-2} centimeter in diameter. If one quart, 946.34 cubic centimeters, of ice cream has about 1.446×10^{9} air bubbles, what percent of the ice cream is air? (*Hint:* Start by finding the volume of one air bubble.)



Air bubble

Test Preparation

MULTI-STEP PROBLEM **Use the solids below.**



45. Write an expression for the volume of the sphere in terms of r.

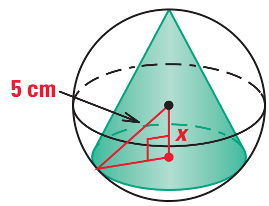
46. Write an expression for the volume of the cylinder in terms of r.

47. Write an expression for the volume of the solid composed of two cones in terms of r.

48. Compare the volumes of the cylinder and the cones to the volume of the sphere. What do you notice?

★ **Challenge**

49. A cone is inscribed in a sphere with a radius of 5 centimeters, as shown. The distance from the center of the sphere to the center of the base of the cone is x. Write an expression for the volume of the cone in terms of x. (*Hint:* Use the radius of the sphere as part of the height of the cone.)



EXTRA CHALLENGE
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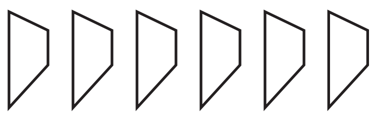
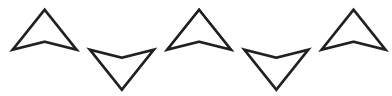
MIXED REVIEW

CLASSIFYING PATTERNS **Name the isometries that map the frieze pattern onto itself.** (Review 7.6)

50.



51.



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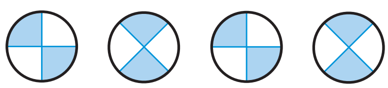

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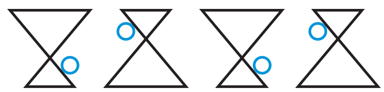
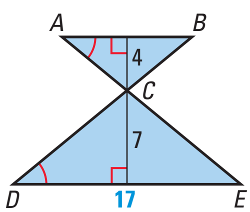
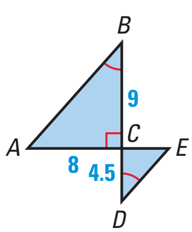


FINDING AREA **In Exercises 54–56, determine whether $\triangle ABC$ is similar to $\triangle EDC$. If so, then find the area of $\triangle ABC$.** (Review 8.4, 11.3 for 12.7)

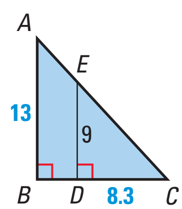
54.



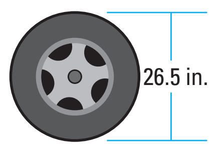
55.



56.



57. MEASURING CIRCLES The tire at the right has an outside diameter of 26.5 inches. How many revolutions does the tire make when traveling 100 feet? (Review 11.4)



12.7 Similar Solids

What you should learn

GOAL 1 Find and use the scale factor of similar solids.

GOAL 2 Use similar solids to solve **real-life** problems, such as finding the lift power of the weather balloon in **Example 4**.

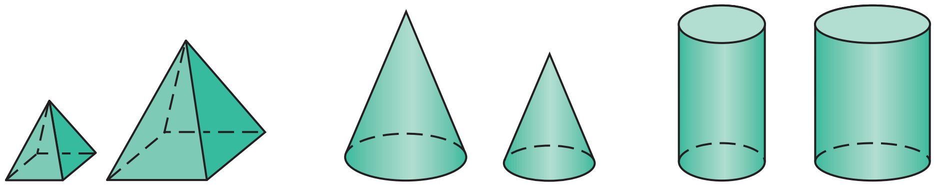
Why you should learn it

▼ You can use similar solids when building a model, such as the model planes below and the model car in **Exs. 25–27**.

REAL LIFE

GOAL 1 COMPARING SIMILAR SOLIDS

Two solids with equal ratios of corresponding *linear* measures, such as heights or radii, are called **similar solids**. This common ratio is called the *scale factor* of one solid to the other solid. Any two cubes are similar; so are any two spheres. Here are other examples of similar and nonsimilar solids.

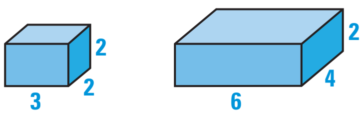


Similar pyramids **Similar cones** **Nonsimilar cylinders**

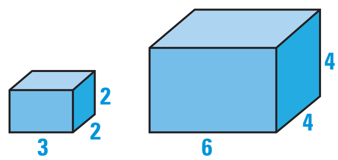
EXAMPLE 1 *Identifying Similar Solids*

Decide whether the two solids are similar. If so, compare the surface areas and volumes of the solids.

a.



b.



SOLUTION

a. The solids are not similar because the ratios of corresponding linear measures are not equal, as shown.

lengths: $\frac{3}{6} = \frac{1}{2}$ **widths:** $\frac{2}{4} = \frac{1}{2}$ **heights:** $\frac{2}{2} = \frac{1}{1}$

b. The solids are similar because the ratios of corresponding linear measures are equal, as shown. The solids have a scale factor of 1 : 2.

lengths: $\frac{3}{6} = \frac{1}{2}$ **widths:** $\frac{2}{4} = \frac{1}{2}$ **heights:** $\frac{2}{4} = \frac{1}{2}$

The surface area and volume of the solids are as follows:

Prism	Surface area	Volume
Smaller	$S = 2B + Ph = 2(6) + 10(2) = \mathbf{32}$	$V = Bh = 6(2) = \mathbf{12}$
Larger	$S = 2B + Ph = 2(24) + 20(4) = \mathbf{128}$	$V = Bh = 24(4) = \mathbf{96}$

▶ The ratio of side lengths is 1 : 2, the ratio of surface areas is **32** : **128**, or 1 : 4, and the ratio of volumes is **12** : **96**, or 1 : 8.

THEOREM

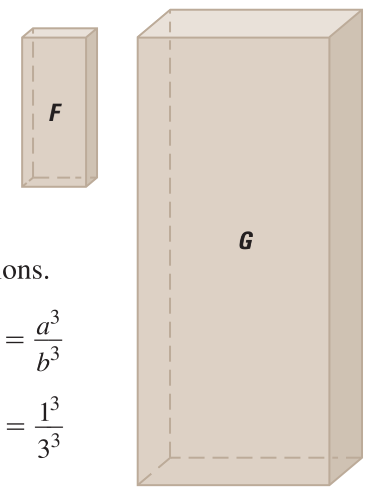
THEOREM 12.13 ***Similar Solids Theorem***

If two similar solids have a scale factor of $a:b$, then corresponding areas have a ratio of $a^2:b^2$, and corresponding volumes have a ratio of $a^3:b^3$.

The term *areas* in the theorem above can refer to any pair of corresponding areas in the similar solids, such as lateral areas, base areas, and surface areas.

EXAMPLE 2 *Using the Scale Factor of Similar Solids*

The prisms are similar with a scale factor of 1:3. Find the surface area and volume of prism G given that the surface area of prism F is 24 square feet and the volume of prism F is 7 cubic feet.



SOLUTION

Begin by using Theorem 12.13 to set up two proportions.

$$\frac{\text{Surface area of } F}{\text{Surface area of } G} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of } F}{\text{Volume of } G} = \frac{a^3}{b^3}$$

$$\frac{24}{\text{Surface area of } G} = \frac{1^2}{3^2} \qquad \frac{7}{\text{Volume of } G} = \frac{1^3}{3^3}$$

$$\text{Surface area of } G = 216 \qquad \text{Volume of } G = 189$$

▶ So, the surface area of prism G is 216 square feet and the volume of prism G is 189 cubic feet.

✓ **CHECK** Check your answers by substituting back into the original proportions.

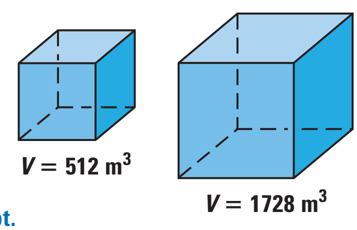
$$\frac{\text{Surface area of } F}{\text{Surface area of } G} = \frac{24}{216} = \frac{1}{9} \qquad \frac{\text{Volume of } F}{\text{Volume of } G} = \frac{7}{189} = \frac{1}{27}$$

STUDENT HELP

Look Back
For help with solving a proportion with an unknown, see p. 459.

EXAMPLE 3 *Finding the Scale Factor of Similar Solids*

To find the scale factor of the two cubes, find the ratio of the two volumes.



$$\frac{a^3}{b^3} = \frac{512}{1728}$$ **Write ratio of volumes.**

$$\frac{a}{b} = \frac{8}{12}$$ **Use a calculator to take the cube root.**

$$= \frac{2}{3}$$ **Simplify.**

▶ So, the two cubes have a scale factor of 2:3.

GOAL 2 USING SIMILARITY IN REAL LIFE

FOCUS ON CAREERS

METEOROLOGY Meteorologists rely on data collected from weather balloons and radar to make predictions about the weather.

CAREER LINK www.mcdougallittell.com

EXAMPLE 4 *Using Volumes of Similar Solids*

METEOROLOGY The lift power of a weather balloon is the amount of weight the balloon can lift. Find the missing measures in the table below, given that the ratio of the lift powers is equal to the ratio of the volumes of the balloons.

Diameter	Volume	Lift power
8 ft	? ft^3	17 lb
16 ft	? ft^3	? lb

SOLUTION

Find the volume of the smaller balloon, whose radius is 4 feet.

smaller balloon $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4)^3 \approx 85.3\pi \text{ ft}^3$

The scale factor of the two balloons is $\frac{8}{16}$, or 1:2. So, the ratio of the volumes is $1^3:2^3$, or 1:8. To find the volume of the larger balloon, multiply the volume of the smaller balloon by 8.

larger balloon $V \approx 8(85.3\pi) \approx 682.4\pi \text{ ft}^3$

The ratio of the lift powers is 1:8. To find the lift power of the larger balloon, multiply the lift power of the smaller balloon by 8, as follows: 8(17) = 136 lb.

Diameter	Volume	Lift power
8 ft	85.3π ft^3	17 lb
16 ft	682.4π ft^3	136 lb

EXAMPLE 5 *Comparing Similar Solids*

SWIMMING POOLS Two swimming pools are similar with a scale factor of 3:4. The amount of a chlorine mixture to be added is proportional to the volume of water in the pool. If two cups of the chlorine mixture are needed for the smaller pool, how much of the chlorine mixture is needed for the larger pool?

SOLUTION

Using the scale factor, the ratio of the volume of the smaller pool to the volume of the larger pool is as follows:

$$\frac{a^3}{b^3} = \frac{3^3}{4^3} = \frac{27}{64} \approx \frac{1}{2.4}$$

The ratio of the volumes of the mixtures is 1:2.4. The amount of the chlorine mixture for the larger pool can be found by multiplying the amount of the chlorine mixture for the smaller pool by 2.4 as follows: 2(2.4) = 4.8 c.

► So, the larger pool needs 4.8 cups of the chlorine mixture.

STUDENT HELP

Study Tip
To rewrite a fraction so that it has a 1 in the numerator, divide both the numerator and the denominator by the numerator. For example,

$\frac{27}{64} = \frac{27 \div 27}{64 \div 27} \approx \frac{1}{2.4}$.

GUIDED PRACTICE

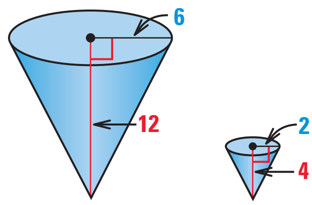
Vocabulary Check

1. If two solids are similar with a scale factor of $p:q$, then corresponding areas have a ratio of __?__, and corresponding volumes have a ratio of __?__.

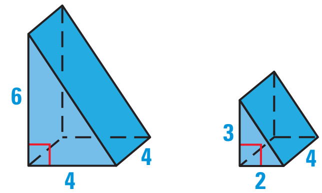
Concept Check

Determine whether the pair of solids are similar. Explain your reasoning.

2.



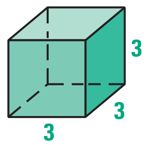
3.



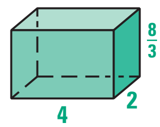
Skill Check

In Exercises 4–6, match the right prism with a similar right prism.

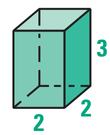
A.



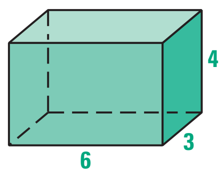
B.



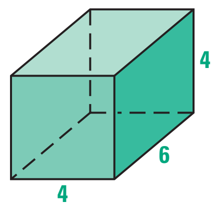
C.



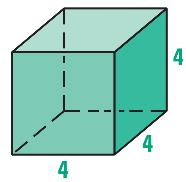
4.



5.



6.



7. Two cubes have volumes of 216 cubic inches and 1331 cubic inches. Find their scale factor.

8. Two spheres have a scale factor of 1 : 3. The smaller sphere has a surface area of 36π square meters. Find the surface area of the larger sphere.

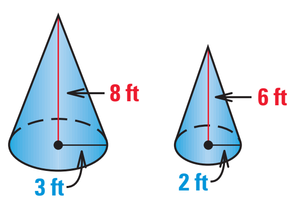
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 826.

IDENTIFYING SIMILAR SOLIDS **Decide whether the solids are similar.**

9.



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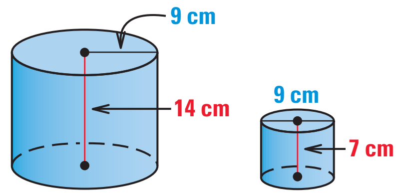

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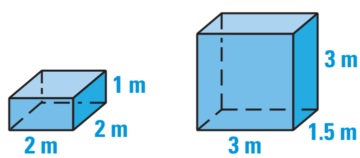

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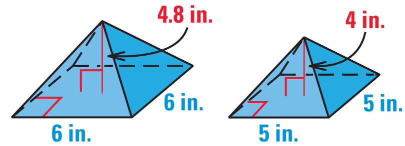


STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–16
Example 2: Exs. 17–20
Example 3: Exs. 21–24
Example 5: Ex. 34

LOGICAL REASONING **Complete the statement using *always, sometimes,* or *never*.**

13. Two cubes are __?__ similar.

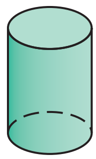
14. Two cylinders are __?__ similar.

15. A solid is __?__ similar to itself.

16. A pyramid is __?__ similar to a cone.

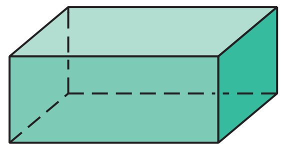
USING SCALE FACTOR **The solid is similar to a larger solid with the given scale factor. Find the surface area S and volume V of the larger solid.**

17. Scale factor 1 : 2



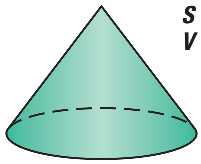
$S = 28\pi$ cm^2
$V = 20\pi$ cm^3

18. Scale factor 1 : 3



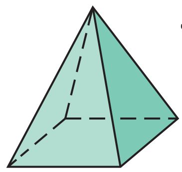
$S = 125.5$ m^2
$V = 87$ m^3

19. Scale factor 1 : 4



$S = 24\pi$ ft^2
$V = 12\pi$ ft^3

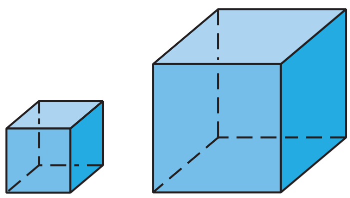
20. Scale factor 2 : 5



$S = 360$ in.^{2}
$V = 400$ in.^{3}

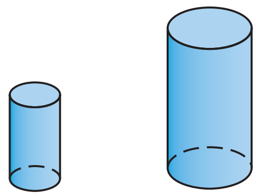
FINDING SCALE FACTOR **Use the given information about the two similar solids to find their scale factor.**

21.



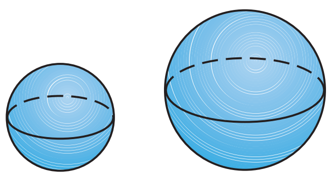
$V = 27$ ft^3 $V = 216$ ft^3

22.



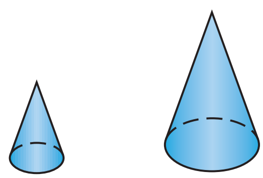
$V = 27\pi$ cm^3 $V = 125\pi$ cm^3

23.



$V = 36\pi$ m^3 $V = 121.5\pi$ m^3

24.



$S = 24\pi$ in.^{2} $S = 384\pi$ in.^{2}

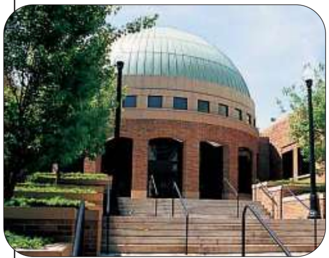
MODEL CAR **The scale factor of the model car at the right to the actual car is 1 : 16. Use the scale factor to complete the exercises.**

25. The model has a height of 5.5 inches. What is the height of the actual car?

26. Each tire of the model has a surface area of 12.9 square inches. What is the surface area of each tire of the actual car?

27. The model's engine has a volume of 2 cubic inches. Find the volume of the actual car's engine.

FOCUS ON APPLICATIONS



ARCHITECTURE The Civil Rights Institute, in Birmingham, Alabama, was completed in 1992. Its roof is in the shape of a hemisphere.

CRITICAL THINKING Decide whether the statement is true. Explain your reasoning.

28. If sphere A has a radius of x and sphere B has a radius of y, then the corresponding volumes have a ratio of $x^3 : y^3$.

29. If cube A has an edge length of x and cube B has an edge length of y, then the corresponding surface areas have a ratio of $x^2 : y^2$.

ARCHITECTURE In Exercises 30–33, you are building a scale model of the Civil Rights Institute shown at the left.

30. You decide that 0.125 inch in your model will correspond to 12 inches of the actual building. What is your scale factor?

31. The dome of the building is a hemisphere with a diameter of $50\frac{2}{3}$ feet. Find the surface area of the hemisphere.

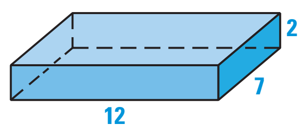
32. Use your results from Exercises 30 and 31 to find the surface area of the dome of your model. $\left(1 \text{ ft}^2 = 144 \text{ in.}^2\right)$

33. Use your results from Exercises 30 and 31 to find the volume of the actual dome. What is the volume of your model's dome?

34. **MAKING JUICE** Two similar cylindrical juice containers have a scale factor of $2:3$. To make juice in the smaller container, you use $\frac{1}{2}$ cup of concentrated juice and fill the rest with water. Find the amount of concentrated juice needed to make juice in the larger container. (*Hint:* Start by finding the ratio of the volumes of the containers.)

Test Preparation

35. **MULTIPLE CHOICE** The dimensions of the right rectangular prism shown are doubled. How many times larger is the volume of the new prism?

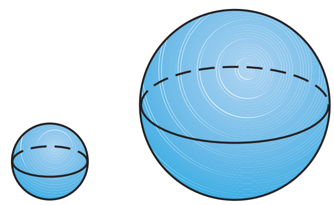


Ⓐ $\frac{1}{4}$ Ⓑ $\frac{1}{2}$ Ⓒ 2 Ⓓ 4 Ⓔ 8

36. **MULTIPLE CHOICE** What is the ratio of the surface areas of the spheres shown?

Ⓐ $\frac{\sqrt{2}}{\sqrt{5}}$ Ⓑ $\frac{2}{5}$ Ⓒ $\frac{\sqrt{8}}{\sqrt{125}}$

Ⓓ $\frac{4}{25}$ Ⓔ $\frac{8}{125}$



Volume $= 8\pi$ Volume $= 125\pi$

★ Challenge

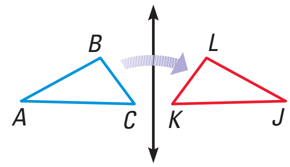
37. **SPORTS** Twelve basketballs, each with a diameter of 9.55 inches, fill a crate. Estimate the number of volleyballs it would take to fill the crate. The diameter of a volleyball is 8.27 inches. Explain why your answer is an estimate and not an exact number.

38. **CRITICAL THINKING** Two similar cylinders have surface areas of 96π square feet and 150π square feet. The height of each cylinder is equal to its diameter. Find the dimensions of one cylinder and use their scale factor to find the dimensions of the other cylinder.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

TRANSFORMATIONS **Use the diagram of the isometry to complete the statement.** (Review 7.1)



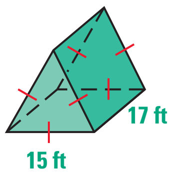
39. $\overline{BC} \rightarrow$ _?_ **40.** $\overline{AB} \rightarrow$ _?_

41. _?_ $\rightarrow \overline{KJ}$ **42.** $\angle BCA \rightarrow$ _?_

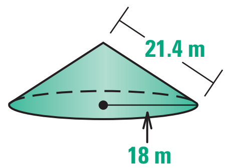
43. _?_ $\rightarrow \angle LJK$ **44.** $\triangle ABC \rightarrow$ _?_

FINDING SURFACE AREA **In Exercises 45–47, find the surface area of the solid.** (Review 12.2, 12.3, and 12.6)

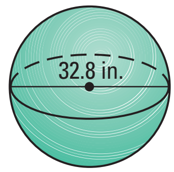
45.



46.



47.



48. The volume of a cylinder is about 14,476.46 cubic meters. If the cylinder has a height of 32 meters, what is its diameter? (Review 12.4)

49. The volume of a cone is about 40,447.07 cubic inches. If the cone has a radius of 22.8 inches, what is its height? (Review 12.5)

QUIZ 3

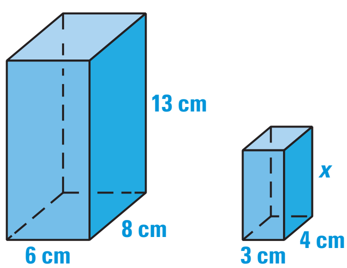
Self-Test for Lessons 12.6 and 12.7

You are given the diameter *d* of a sphere. Find the surface area and volume of the sphere. Round your result to two decimal places. (Lesson 12.6)

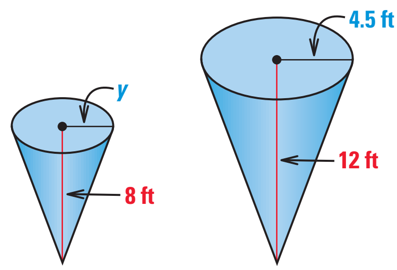
1. $d = 20$ cm **2.** $d = 3.76$ in. **3.** $d = 10.8$ ft **4.** $d = 30\sqrt{5}$ m

In Exercises 5 and 6, you are given two similar solids. Find the missing measurement. Then calculate the surface area and volume of each solid. (Lesson 12.7)

5.



6.



7. **WORLD'S FAIR** The Trylon and Perisphere were the symbols of the New York World's Fair in 1939–40. The Perisphere, shown at the left, was a spherical structure with a diameter of 200 feet. Find the surface area and volume of the Perisphere. (Lesson 12.6)

8. **SCALE MODEL** The scale factor of a model of the Perisphere to the actual Perisphere is 1 : 20. Use the information in Exercise 7 and the scale factor to find the radius, surface area, and volume of the model. (Lesson 12.7)

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Use properties of polyhedra. **(12.1)**	Classify crystals by their shape. **(p. 725)**
Find the surface area of prisms and cylinders. **(12.2)**	Determine the surface area of a wax cylinder record. **(p. 733)**
Find the surface area of pyramids and cones. **(12.3)**	Find the area of each lateral face of a pyramid, such as the Pyramid Arena in Tennessee. **(p. 735)**
Find the volume of prisms and cylinders. **(12.4)**	Find the volume of a fish tank, such as the tank at the New England Aquarium. **(p. 748)**
Find the volume of pyramids and cones. **(12.5)**	Find the volume of a volcano, such as Mount St. Helens. **(p. 757)**
Find the surface area and volume of a sphere. **(12.6)**	Find the surface area of a planet, such as Earth. **(p. 763)**
Find the surface area and volume of similar solids. **(12.7)**	Use the scale factor of a model car to determine dimensions on the actual car. **(p. 770)**

How does Chapter 12 fit into the BIGGER PICTURE of geometry?

Solids can be assigned three types of measure. For instance, the height and radius of a cylinder are one-dimensional measures. The surface area of a cylinder is a two-dimensional measure, and the volume of a cylinder is a three-dimensional measure. Assigning measures to plane regions and to solids is one of the primary goals of geometry. In fact, the word geometry means "Earth measure."

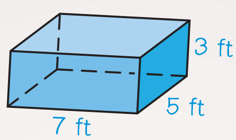
STUDY STRATEGY

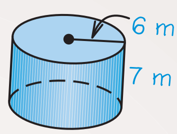
How did generalizing formulas help you?

The list of similar concepts you made, following the **Study Strategy** on p. 718, may resemble this one.

Generalizing Formulas

The same concept is used to find the surface area of a prism and the surface area of a cylinder. For example, the surface areas can be found by adding twice the area of the base, $2B$, to the lateral area L.





$$S = 2B + L$$
$$= 2(l \cdot w) + Ph$$
$$= 2(7 \cdot 5) + 24 \cdot 3$$
$$= 142 \text{ ft}^2$$

$$S = 2B + L$$
$$= 2(\pi r^2) + Ch$$
$$= 2(\pi(6)^2) + (\pi \cdot 12)7$$
$$= 156\pi \text{ m}^2$$

CHAPTER 12

Chapter Review

VOCABULARY

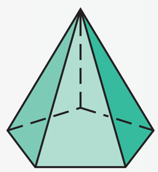
- polyhedron, p. 719
- face, p. 719
- edge, p. 719
- vertex, p. 719
- regular polyhedron, p. 720
- convex, p. 720
- cross section, p. 720
- Platonic solids, p. 721
- tetrahedron, p. 721
- octahedron, p. 721
- dodecahedron, p. 721
- icosahedron, p. 721
- prism, p. 728
- bases, p. 728
- lateral faces, p. 728
- right prism, p. 728
- oblique prism, p. 728
- surface area of a polyhedron, p. 728
- lateral area of a polyhedron, p. 728
- net, p. 729
- cylinder, p. 730
- right cylinder, p. 730
- lateral area of a cylinder, p. 730
- surface area of a cylinder, p. 730
- pyramid, p. 735
- regular pyramid, p. 735
- circular cone, p. 737
- lateral surface of a cone, p. 737
- right cone, p. 737
- volume of a solid, p. 743
- sphere, p. 759
- center of a sphere, p. 759
- radius of a sphere, p. 759
- chord of a sphere, p. 759
- diameter of a sphere, p. 759
- great circle, p. 760
- hemisphere, p. 760
- similar solids, p. 766

12.1 EXPLORING SOLIDS

Examples on pp. 719–722

EXAMPLE The solid at the right has 6 faces and 10 edges. The number of vertices can be found using Euler's Theorem.

$$F + V = E + 2$$
$$6 + V = 10 + 2$$
$$V = 6$$



Use Euler's Theorem to find the unknown number.

1. Faces: 32
Vertices: ___?___
Edges: 90

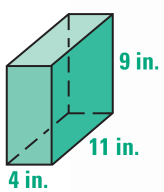
2. Faces: ___?___
Vertices: 6
Edges: 10

3. Faces: 5
Vertices: 5
Edges: ___?___

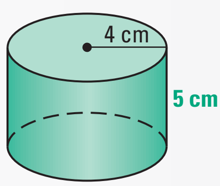
12.2 SURFACE AREA OF PRISMS AND CYLINDERS

Examples on pp. 728–731

EXAMPLES The surface area of a right prism and a right cylinder are shown.



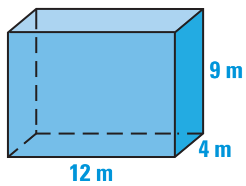
$$S = 2B + Ph$$
$$= 2(44) + 30(9)$$
$$= 358 \text{ in.}^2$$



$$S = 2\pi r^2 + 2\pi rh$$
$$= 2\pi(4^2) + 2\pi(4)(5)$$
$$\approx 226.2 \text{ cm}^2$$

Find the surface area of the right prism or right cylinder. Round your result to two decimal places.

4.



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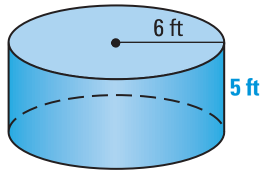

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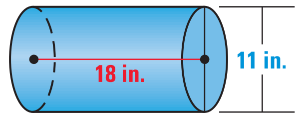
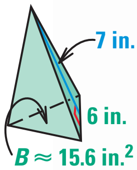


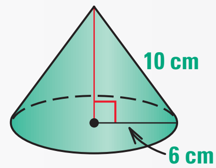
12.3 SURFACE AREA OF PYRAMIDS AND CONES

Examples on pp. 735–737

EXAMPLES The surface area of a regular pyramid and a right cone are shown.



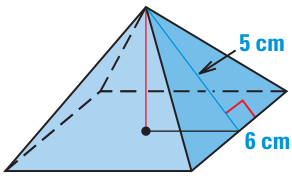
$$S = B + \frac{1}{2}P\ell$$
$$\approx 15.6 + \frac{1}{2}(18)(7)$$
$$\approx 78.6 \text{ in.}^2$$



$$S = \pi r^2 + \pi r\ell$$
$$= \pi(6)^2 + \pi(6)(10)$$
$$\approx 301.6 \text{ cm}^2$$

Find the surface area of the regular pyramid or right cone. Round your result to two decimal places.

7.



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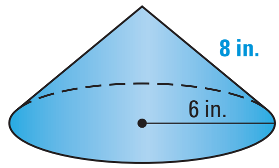

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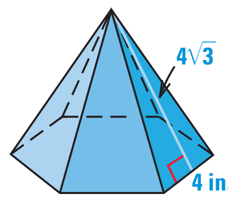
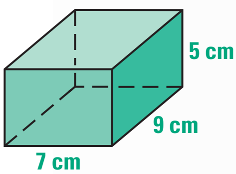
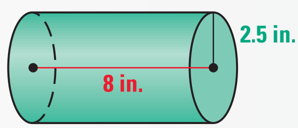


12.4 VOLUME OF PRISMS AND CYLINDERS

Examples on pp. 743–745

EXAMPLES The volume of a rectangular prism and a right cylinder are shown.





$V = Bh = (7 \cdot 9)(5) = 315 \text{ cm}^3$

$V = \pi r^2 h = \pi(2.5^2)(8) \approx 157.1 \text{ in.}^3$

Find the volume of the described solid.

10. A side of a cube measures 8 centimeters.

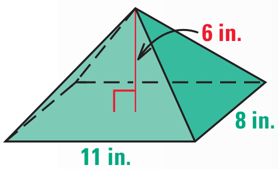
11. A right prism has a height of 37.2 meters and regular hexagonal bases, each with a base edge of 21 meters.

12. A right cylinder has a radius of 3.5 inches and a height of 8 inches.

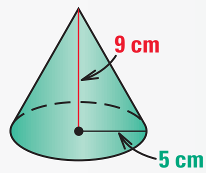
12.5 VOLUME OF PYRAMIDS AND CONES

Examples on pp. 752–754

EXAMPLES The volume of a right pyramid and a right cone are shown.



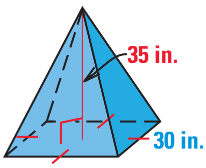
$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(11 \cdot 8)(6)$$
$$= 176 \text{ in.}^3$$



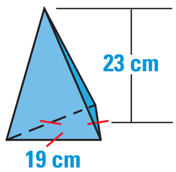
$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(5^2)(9)$$
$$\approx 235.6 \text{ cm}^3$$

Find the volume of the pyramid or cone.

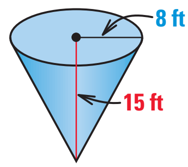
13.



14.



15.



12.6 SURFACE AREA AND VOLUME OF SPHERES

Examples on pp. 759–761

EXAMPLES The surface area and volume of the sphere are shown.

$$S = 4\pi r^2 = 4\pi(7^2) \approx 615.8 \text{ in.}^2$$
$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7^3) \approx 1436.8 \text{ in.}^3$$

16. Find the surface area and volume of a sphere with a radius of 14 meters.

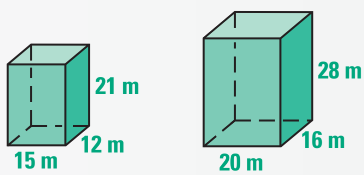
17. Find the surface area and volume of a sphere with a radius of 0.5 inch.

12.7 SIMILAR SOLIDS

Examples on pp. 766–768

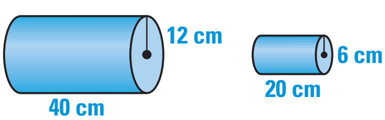
EXAMPLES The ratios of the corresponding linear measurements of the two right prisms are equal, so the solids are similar with a scale factor of 3 : 4.

lengths: $\frac{15}{20} = \frac{3}{4}$ **widths:** $\frac{12}{16} = \frac{3}{4}$ **heights:** $\frac{21}{28} = \frac{3}{4}$

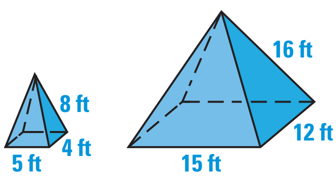


Decide whether the solids are similar. If so, find their scale factor.

18.



19.

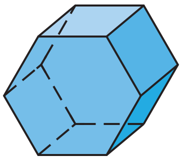


CHAPTER 12

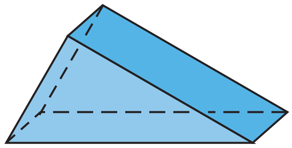
Chapter Test

Determine the number of faces, vertices, and edges of the solids.

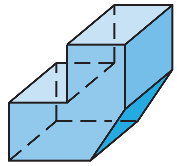
1.



2.



3.



USING ALGEBRA Sketch the solid described and find its missing measurement. (*B* is the base area, *P* is the base perimeter, *h* is the height, *S* is the surface area, *r* is the radius, and ℓ is the slant height.)

4. Right rectangular prism: $B = 44 \text{ m}^2$, $P = 30$ m, $h = 7$ m, $S =$ __?__

5. Right cylinder: $r = 8.6$ in., $h =$ __?__, $S = 784\pi \text{ in.}^2$

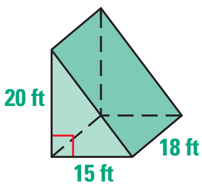
6. Regular pyramid: $B = 100 \text{ ft}^2$, $P = 40$ ft, $\ell =$ __?__, $S = 340 \text{ ft}^2$

7. Right cone: $r = 12$ yd, $\ell = 17$ yd, $S =$ __?__

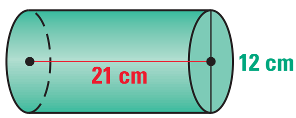
8. Sphere: $r = 34$ cm, $S =$ __?__

In Exercises 9–11, find the volume of the right solid.

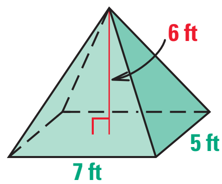
9.



10.



11.

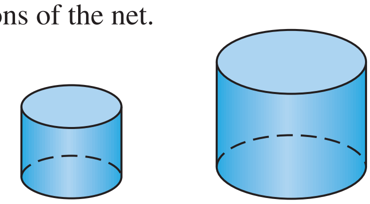


12. Draw a net for each solid in Exercises 9–11. Label the dimensions of the net.

13. The scale factor of two spheres is 1 : 5. The radius of the smaller sphere is 3 centimeters. What is the volume of the larger sphere?

14. Describe the possible intersections of a plane and a sphere.

15. What is the scale factor of the two cylinders at the right?



$V = 8\pi \text{ m}^3$ $\quad$ $V = 27\pi \text{ m}^3$

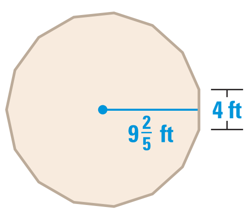
16. **CANNED GOODS** Find the volume and surface area of a prism with a height of 6 inches and a 4 inch by 4 inch square base. Compare the results with the volume and surface area of a cylinder with a height of 7.64 inches and a diameter of 4 inches.

SILOS Suppose you are building a silo. The shape of your silo is a right prism with a regular 15-gon for a base, as shown. The height of your silo is 59 feet.

17. What is the area of the floor of your silo?

18. Find the lateral area and volume of your silo.

19. What are the lateral area and volume of a larger silo that is in a 1 : 1.25 ratio with yours?



Cumulative Practice

for Chapters 1–12

1. Two lines intersect to form vertical angles with measures of $(4x - 2)°$ and $6(x - 3)°$. Find the measures of the four angles formed at the intersection of the two lines. **(2.6)**

2. Sketch two parallel lines intersected by a transversal. Then sketch the bisectors of two consecutive interior angles. What kind of triangle is formed by the transversal and the two bisectors? Explain your answer. **(3.3, 4.1)**

3. Write a coordinate proof to show that in a right triangle, the length of the median to the hypotenuse is half the length of the hypotenuse. Use $\triangle RST$, which is right with vertices $R(0, 0)$, $S(2h, 0)$, and $T(0, 2k)$. **(4.7, 5.3)**

Decide whether the triangle is *acute, right,* or *obtuse*. Name the largest and the smallest angles of the triangle. **(5.5, 9.3)**

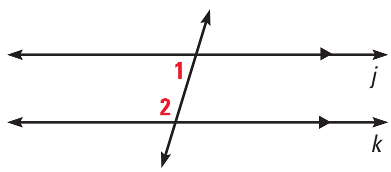
4. $\triangle ABC$, $AB = 12$, $BC = 8$, and $AC = 15$

5. $\triangle XYZ$, $XY = 10$, $YZ = 8$, and $XZ = 6$

TWO-COLUMN PROOF **Write a two-column proof.** **(3.3, 6.2)**

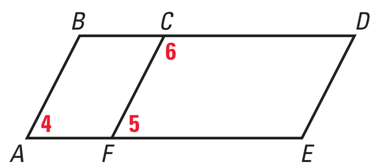
6. GIVEN ▶ $j \parallel k$, $m\angle 1 = 73°$

PROVE ▶ $m\angle 2 = 107°$



7. GIVEN ▶ *ABDE* and *CDEF* are ▱s.

PROVE ▶ $\angle 4$ and $\angle 6$ are supplementary.



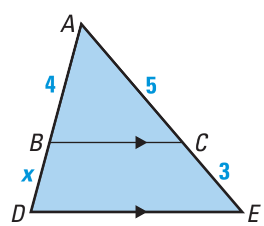
In Exercises 8 and 9, use *always, sometimes,* or *never* to complete the statement.

8. The sides of a rhombus are __?__ congruent. **(6.4)**

9. A trapezoid __?__ has rotational symmetry. **(6.5, 7.3)**

10. A segment has endpoints $X(-3, 3)$ and $Y(-5, 8)$. The segment is reflected in the x-axis and then in the y-axis. Sketch the image of the segment after the two reflections. Describe a single transformation that would map $\overline{XY}$ to the final image. **(7.2, 7.3, 7.5)**

In Exercises 11–13, use the diagram.



11. Show that $\triangle ABC \sim \triangle ADE$. **(8.4)**

12. Find the value of x. **(8.6)**

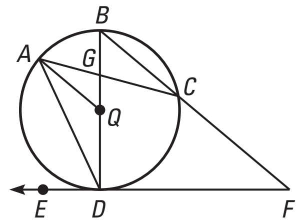
13. Find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle ADE$. Then find the ratio of their areas. **(11.3)**

A right triangle has legs of lengths 7 and 24.

14. Find the lengths of the hypotenuse and the altitude to the hypotenuse. **(9.1, 9.2)**

15. Find the measures of the acute angles of the triangle. **(9.6)**

In $\odot Q$, $\overline{EF} \perp \overline{DB}$, $m\angle AQB = 50°$, and $m\angle F = 40°$. Find the measure of the angle or the arc. (10.2, 10.3, 10.4)



16. $\widehat{AD}$ **17.** $\angle ADB$ **18.** $\angle ACB$

19. $\angle EDB$ **20.** $\angle EDA$ **21.** $\widehat{DC}$

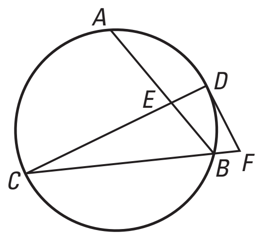
22. $\widehat{BC}$ **23.** $\widehat{ABD}$ **24.** $\angle BGC$

Suppose A is in the exterior of $\odot P$, and $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot P$.

25. What can you conclude about $\angle BAC$ and $\angle BPC$? Explain. (10.1)

26. What special kind of quadrilateral is $BACP$? Explain. (6.5, 10.1)

In Exercises 27–29, use the diagram at the right. (10.5)



27. Find AE if $EB = 6$, $CE = 18$, and $ED = 4$.

28. Find BC if $DF = 6$ and $FB = 4$.

29. Find CE if $AE = 10$, $EB = 7$, and $ED = 3.5$.

In Exercises 30 and 31, the endpoints of a diameter of a circle are $(-2, 1)$ and $(6, -5)$.

30. Write the standard equation of the circle. (10.6)

31. Find the circumference of the circle. Use $\pi \approx 3.14$. (11.4)

In Exercises 32 and 33, describe the locus of points in a plane. (10.7)

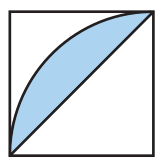
32. Points that are equidistant from the vertices of a regular hexagon.

33. Points that are equidistant from two perpendicular lines, j and k.

34. What is the sum of the measures of the interior angles of a convex polygon with 25 sides? (11.1)

35. Find the area of a regular octagon whose perimeter is 240 centimeters. (11.2)

36. A quarter circle and a diagonal are drawn inside a square, shown at the right. Find the probability that a randomly chosen point in the interior of the square lies in the shaded region. (11.6)



Ex. 36

37. Find the volume of a cone that is 7 feet in diameter and 6 feet high. (12.5)

38. Two right rectangular prisms are similar. The dimensions of the smaller prism are 4 inches, 5 inches, and 5 inches, and the volume of the larger prism is 1562.5 cubic inches. What is the scale factor of the two prisms? (12.4, 12.7)

39. **TABLE** A square table has hinged leaves that can be raised to enlarge the table. When all four leaves are up, the table top is a circle. If the area of the square table is 16 square feet, what is the area of the round table? (9.4, 11.5)

40. **HONEYCOMB** A cell of a honeycomb is a right regular hexagonal prism with base edges of 0.25 inch and a height of 0.75 inch. Find the lateral area of one cell. (12.2)

41. **TABLE TENNIS** The diameter of a table tennis ball is 1.5 inches. Find its surface area to the nearest tenth. (12.6)