

Chapter 11 Vibrations and Waves

Section 1 Simple Harmonic Motion

Simple harmonic motion

Vibration = oscillation = back and forth motion of an object.

Periodic motion = Vibration or Oscillation that regularly repeats itself.

Motion of an object that *regularly returns* to a given position after a fixed time interval.

- A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position

–If the force is always directed toward the equilibrium position, the motion is called *simple harmonic motion*

- Simplest form of periodic motion:
Mass **m** attached to an ideal spring, spring constant **k** (Hooke’s “Law”: **$F = -kx$**) moving in one dimension (**x**).

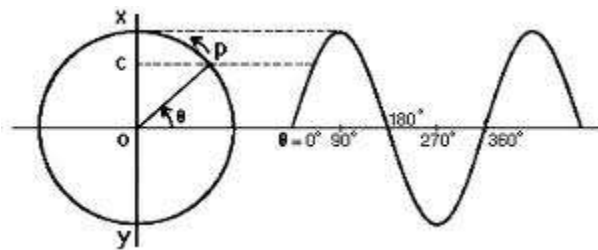
A spring-mass system on a horizontal frictionless surface.

Block is in Equilibrium position when the spring is neither stretched nor compressed
Equilibrium position: at $x = 0$.

Any object that exhibits simple harmonic motion is a “simple harmonic oscillator”

Sinusoidal motion represents a mass oscillating on a spring and a simple pendulum moving in a small angle.

Hooke's Law $F = -kX$



Sine curve:

Cosine curve: 90° out of phase with sine curve.

Amplitude = A = maximum displacement

One cycle = one complete round trip.

Period = T = time for one complete round trip
measured in seconds

Frequency $f = 1/T$ = number of cycles per
second measured in Hz or 1/sec

Angular frequency $\omega = 2\pi f = 2\pi/T$
measured in radians/sec

The angular frequency is the “natural resonant frequency” of the system.

The spring is displaced a distance A and released and then the mass oscillates from displacement of $+A$ to $-A$.

At the maximum displacements, all of the energy is stored elastic energy, and the velocity is zero.

At the equilibrium point, $x=0$, all of the energy is kinetic energy and the velocity is maximum.

Acceleration of a mass-spring system.

$$\Sigma F = ma$$

$$-kx = ma$$

$$a = - [k/m] x$$

*direction of a is **opposite** displacement*

An object moves with simple harmonic motion SHM whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium

Acceleration is *NOT* constant.

You CANNOT use kinematics equations to describe SHM

$$\omega^2 = k/m \quad \text{and} \quad \omega = \sqrt{k/m}$$

k is the spring constant,
m is the mass of the block

Displacement $x = A \sin \omega t$
maximum displacement

$$x_{\max} \text{ at } \omega t = \pi/2 \quad y_{\max} = A$$

Velocity $v = \omega A \cos \omega t$
maximum velocity

$$v_{\max} \text{ at } \omega t = 0$$
$$v_{\max} = \omega A = A \sqrt{k/m}$$

Acceleration $a = -\omega^2 A \sin \omega t$
maximum acceleration

$$a_{\max} \text{ at } \omega t = \pi/2 \quad a_{\max} = -\omega^2 A = -Ak/m$$

Summary: mass spring system

A simple mass-spring system has a resonant frequency ω

Angular frequency $\omega = 2\pi f = \sqrt{k/m}$

frequency $f = [1/2\pi] \sqrt{k/m}$

period $T = 2\pi \sqrt{m/k}$

Frequency and period **only** depend on k and m
NOT on amplitude and velocity

As k increases, f increases

As m increases, f decreases

Simple pendulum: a point mass m , suspended from a string of length L

A simple pendulum has a single resonant frequency, ω .

Angular frequency $\omega = \sqrt{g/L}$

For small displacements:

Period $T = 2\pi\sqrt{L/g}$

frequency $f = [1/2\pi] \sqrt{g/L}$

1) A certain spring is relaxed on a horizontal table. One end is anchored to the table. The spring is observed to stretch 3.2 cm when a force of 2 Newtons is applied to the free end. *What is the value of the force constant?*

2) *How much will the spring stretch if a force of 4.3 Newtons is applied?*

3) When the spring is released is the acceleration of the free end of the spring constant as the spring contracts?

4) define: displacement, amplitude, cycle, period, frequency, angular frequency,

5) What are the units used for frequency?

6) In mechanical systems the motion is SHM if the magnitude of the restoring acceleration is proportional to the displacement. A spring satisfies this requirement $\mathbf{F} = -k\mathbf{X}$ so $\mathbf{a} = -(k/m)\mathbf{X}$

7) *Why can't we use the four kinematic equations to analyze SHM?*

Section 2 Energy of a Harmonic Oscillator

A system undergoing simple harmonic motion has both potential and kinetic energy. At one point in the cycle all the energy is KE. Later, all the energy is PE. If the motion is not damped (no energy is lost during each cycle),

$$KE + PE = \text{constant.}$$

$$\frac{1}{2} m V^2 + \frac{1}{2} k A^2 = \text{Total Energy, } E \quad E \text{ is proportional to } A^2$$

8) For a spring: *What is the maximum velocity if $k = 20 \text{ N/m}$, $A = 5 \text{ cm}$ and $m = 3 \text{ kg}$?*

Section 3 The Period and Sinusoidal Nature of SHM

The shadow of an object has simple harmonic motion if the object is moving with uniform circular motion. The study of uniform circular motion leads to the following formula for the natural period of an object moving with simple harmonic motion:

10) $T =$

The natural frequency f is $f = 1/T$.

11) *If 2 kg is placed on the end of the spring discussed in section 1, what is the frequency of the motion?*

12) *TRUE or FALSE If for the same spring the amplitude of the motion is made larger then the period of simple harmonic motion will be greater.*

Section 4 The Simple Pendulum

For small angles ($<15^\circ$) , a pendulum moves approximately with simple harmonic motion.

13) $T =$ $f =$

14) *What is the period of a simple pendulum that has a length of 9.8 meters?*

15) On a certain hypothetical planet an astronaut sets up a pendulum that has a length of 2 meters. She records that the pendulum has a frequency of 0.3 Hz. *What is the value of the local acceleration due to gravity?*

16) TRUE or FALSE A pendulum could be used to determine the altitude of an aircraft.

Section 5 Damped Harmonic Motion

17) What happens to the amplitude of a system that is losing energy?

Section 6 Forced Vibrations; Resonance

A forced vibration is produced by applying a force that has its own particular frequency. The frequency of the external force is the frequency of the vibration.

18) *TRUE or FALSE It is possible for a small force to create a large amplitude for harmonic motion.*

19) The natural frequency $f =$

Resonance exists if the frequency of the driving force is equal to the natural frequency of the system. *Name some situations in which resonance is desired and some in which it is destructive.*

Section 7 Wave Motion

A pulse is a single, localized disturbance that travels in a medium. *A periodic wave is a repeated propagation of a disturbance through a medium (except for light) without any net displacement of the medium.

20) TRUE or FALSE Each particle of a water wave merely oscillates about an equilibrium point.

21) define wave amplitude, wavelength, frequency

22) The fundamental equation of wave propagation is $v = \lambda f$. It is true for all waves. For waves of small amplitude on a string $v =$

Section 8 Types of Waves: Transverse and Longitudinal

For a longitudinal wave the medium oscillates in SHM parallel to the wave velocity direction. For a transverse wave the medium oscillates in SHM perpendicular to the wave velocity direction.

23) List two transverse waves: _____ List two longitudinal waves: _____

24) TRUE or FALSE A water wave is a transverse wave.

Sections 9 & 10 Energy Transported by Waves

The energy transmitted by a wave is proportional to the square of the frequency and the square of the

amplitude of the wave. The intensity of a wave is a measure of the amount of power transmitted through a unit of area. $I = P/A$ e.g. watts/m² The intensity of sunlight at the top of the earth's atmosphere is 1,370 watts/m² .

25) What minimum area of a solar collector is needed if the solar collector is to be the sole power source for a 100 watt light bulb?

26) What is the intensity of sunlight at the top of the Mar's atmosphere? Mars is 2.28×10^6 km from the sun.

Section 11 Reflection and Interference of Waves

When a pulse on a string hits a wall it reflects with a 180° phase shift.

27) Write down the definitions for wave front, ray, plane wave

*The **principle of superposition**: If two waves occupy the same space at the same time then the net displacement of the medium is the sum of the individual wave displacements. (This assumes that the medium does not exceed its elastic limit.)

If both waves have the same sign of displacement then the net displacement will be larger than each individual displacement. *This situation is called **constructive interference**. The two waves are said to be **in phase**. If the displacements have opposite signs then the net displacement will be smaller than the magnitude of the largest displacement. *This situation is called **destructive interference**. The two waves are said to be **out of phase**.

Section 12 Standing Waves; Resonance

If two interfering waves have the same wavelengths and amplitudes and opposite directions of travel a standing wave is produced.

There will be locations (nodes) at which the displacement of the medium is always zero. The distance between adjacent nodes is $\frac{1}{2}$ wavelength. Between the nodes are antinodes. The medium has its maximum vibration at the antinodes. There is $\frac{1}{4}$ wavelength between a node and antinode.

*The lowest frequency of a standing wave is called the **fundamental frequency**. For a string anchored at both ends the fundamental frequency is

$$f_1 = V / (2L)$$

The harmonic frequencies are integer multiples of the fundamental frequency. $f_n = n * f_1$

28) The speed of a transverse wave on a string is given by $V =$

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