AP Calculus Chapter 1 Test Review

1. Find all *x*-values where discontinuities exist. Classify as removable or non-removable.

a) $f(x)=\dfrac{1}{x^2+16}$ b) $f(x)=\begin{cases} x^2-4, & x\neq 1 \\ 2, & x=1 \end{cases}$ c) $f(x)=\cos x$

2. Evaluate each limit. Choose any method, but show all work.

a) $\lim\limits_{x\to 0^+}\dfrac{x+1}{\sqrt{x}}$ b) $\lim\limits_{x\to 6^+}\dfrac{|x-6|}{x-6}$ c) $\lim\limits_{x\to -8}[[x]]$

3. Find the following using the given information. $\lim\limits_{x\to 2} f(x)=3$ and $\lim\limits_{x\to 2} g(x)=4$

a) $\lim\limits_{x\to 2}\sqrt{32+g(x)}$ b) $\lim\limits_{x\to 2}\left(x^4 f(x)\right)$

4. Find each limit analytically a) $\lim\limits_{x\to 4}\dfrac{x^2-6x+8}{x-4}$ b) $\lim\limits_{x\to 0}\dfrac{\dfrac{1}{x+8}-\dfrac{1}{8}}{x}$

5. Sketch the graph of a function with the following conditions.

$\lim\limits_{x\to 0^-} f(x)=-4$ $\lim\limits_{x\to 0^+} f(x)=4$
$\lim\limits_{x\to 4^-} f(x)=1$ $\lim\limits_{x\to 4^+} f(x)=-3$
$f(2)=2$ $f(-2)$ is undefined

6. Does the Intermediate Value Theorem guarantee the existence of a zero for $f(x)=x^2-4$ on [1, 3]? Why or why not?

7. Find the limit. Make sure to show all work and/or explain your answer. $\lim\limits_{x\to -10^+}\dfrac{8-x}{10+x}$

8. Find all vertical asymptotes if they exist. a) $f(x)=\dfrac{x-2}{x^2-12x+20}$ b) $f(x)=\dfrac{x+5}{x^2-25}$

9. Determine whether each statement is true/false. If false, explain why or give a counterexample.

a) If $\lim\limits_{x\to d} f(x)=M$, then $f(d)=M$ b) $\lim\limits_{x\to 7} f(x)=1$ if $f(x)=\begin{cases} x-6, & x\leq 7 \\ -x^2+2x+10, & x>7 \end{cases}$

Chapter 1 Test Review Key

1. a) none b) $x = 1$, hole (removable) c) none 2. a) ∞ b) 1 c) DNE
3. a) 6 b) 48
4. a) 2 b) -1/64 5. On the board

6. $f(1)=-3,\ f(3)=5$, so yes, b/c -3 < 0 < 5
7. Since $x = -10$ makes the bottom zero, it is a vertical asymptote. The limit must be ∞ or $-\infty$. If you substitute -9.999 for x (b/c -9.999 is right of -10), the fraction is 17.999/0.01, which is positive. So, the limit must be ∞.
8. a) $x = 10$ is an asymptote, $x = 2$ is a hole b) $x = 5$ is an asymptote, $x = -5$ is a hole
9. a) False, limit is a behavior NEAR a point, it doesn't have to pass thru (d, M).
b) False, $7 - 6 = 1$, $-7^2 + 2(7) + 10 = -25$, that is a jump.