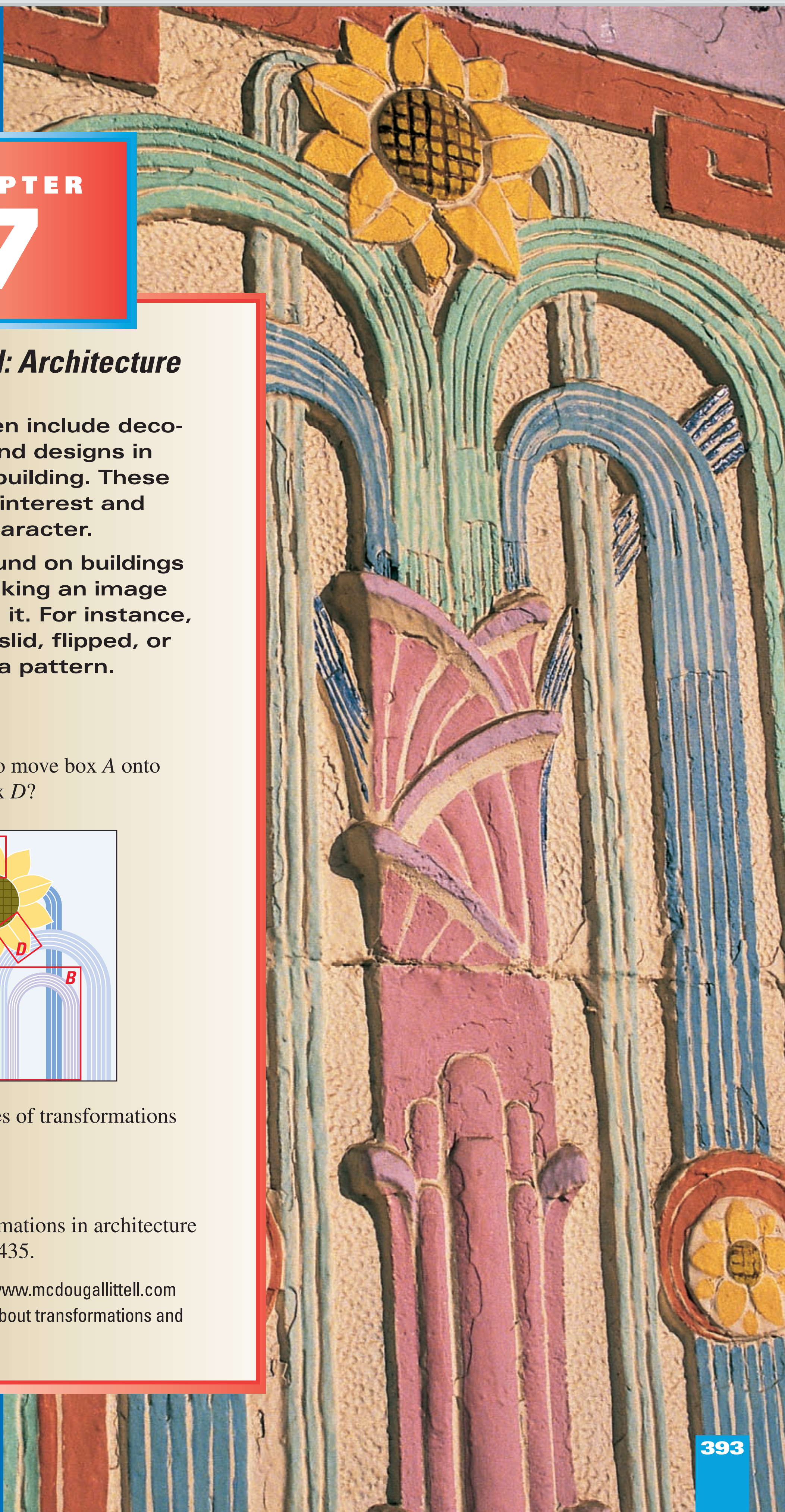
TRANSFORMATIONS

How do architects use transformations?



CHAPTER 7

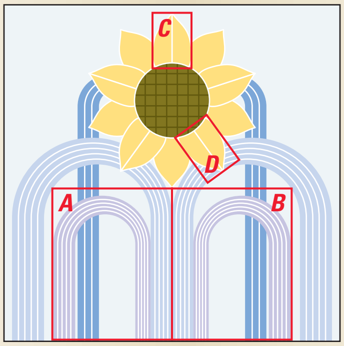
APPLICATION: Architecture

Architects often include decorative patterns and designs in their plans for a building. These adornments add interest and give a building character.

Some designs found on buildings are created by taking an image and transforming it. For instance, an image can be slid, flipped, or turned to create a pattern.

Think & Discuss

1. What motion is used to move box *A* onto box *B*? box *C* onto box *D*?



2. Describe any other uses of transformations in the design.

Learn More About It

You will identify transformations in architecture in Exercises 35–37 on p. 435.

APPLICATION LINK Visit www.mcdougallittell.com for more information about transformations and patterns in architecture.

CHAPTER 7

Study Guide

PREVIEW

What's the chapter about?

Chapter 7 is about **transformations**. Transformations describe how geometric figures of the same shape are related to one another. In Chapter 7, you'll learn

- three ways to describe motion of geometric figures in the plane.
- how to use transformations in real-life situations, such as making a kaleidoscope or designing a border pattern.

KEY VOCABULARY

Review
- Distance Formula, p. 19
- parallel lines, p. 129
- congruent figures, p. 202
- corresponding sides, p. 202
- corresponding angles, p. 202

New
- image, p. 396
- preimage, p. 396
- transformation, p. 396
- reflection, p. 404
- rotation, p. 412
- translation, p. 421
- vector, p. 423
- glide reflection, p. 430
- frieze pattern, p. 437

PREPARE

Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you'll apply in this chapter. See the given **reference page** if there is something you don't understand.

STUDENT HELP

Study Tip
"Student Help" boxes throughout the chapter give you study tips and tell you where to look for extra help in this book and on the Internet.

Use the Distance Formula to decide whether $\overline{AB} \cong \overline{BC}$. (Review p. 19)

1. $A(-6, 4)$
$B(1, 3)$
$C(8, 4)$

2. $A(0, 3)$
$B(3, 1)$
$C(7, 4)$

3. $A(1, 1)$
$B(4, 6)$
$C(7, 1)$

Complete the statement, given that $\triangle PQR \cong \triangle XYZ$. (Review p. 202)

4. $XZ = \underline{\ ?\ }$

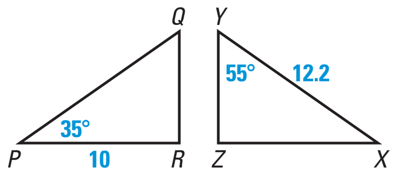
5. $m\angle X = \underline{\ ?\ }$

6. $m\angle Q = \underline{\ ?\ }$

7. $m\angle Z = \underline{\ ?\ }$

8. $\overline{YZ} \cong \underline{\ ?\ }$

9. $QR = \underline{\ ?\ }$



STUDY STRATEGY

Here's a study strategy!

Making Sample Exercises

Writing your own exercises can test what you have learned in this chapter. After each lesson, follow these steps:

- Write a summary of the lesson.
- Write at least three exercises that test the lesson's goals.

7.1 Rigid Motion in a Plane

What you should learn

GOAL 1 Identify the three basic rigid transformations.

GOAL 2 Use transformations in **real-life** situations, such as building a kayak in **Example 5**.

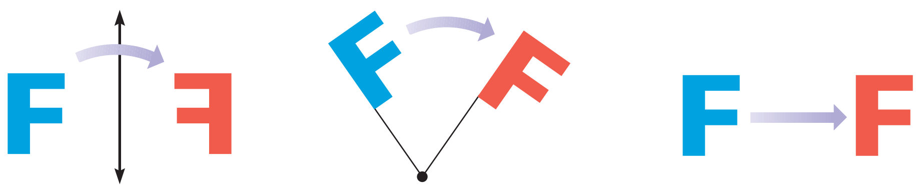
Why you should learn it

▼ Transformations help you when planning a stenciled design, such as on the wall below and the stencil in **Ex. 41**.

GOAL 1 IDENTIFYING TRANSFORMATIONS

Figures in a plane can be reflected, rotated, or translated to produce new figures. The new figure is called the **image**, and the original figure is called the **preimage**. The operation that *maps*, or moves, the preimage onto the image is called a **transformation**.

In this chapter, you will learn about three basic transformations—*reflections*, *rotations*, and *translations*—and combinations of these. For each of the three transformations below, the blue figure is the preimage and the red figure is the image. This color convention will be used throughout this book.



Reflection in a line **Rotation about a point** **Translation**

Some transformations involve labels. When you name an image, take the corresponding point of the preimage and add a prime symbol. For instance, if the preimage is A, then the image is A', read as "*A prime*."

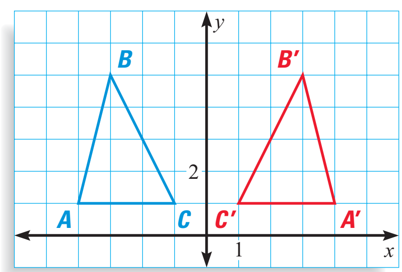
EXAMPLE 1 *Naming Transformations*

Use the graph of the transformation at the right.

a. Name and describe the transformation.

b. Name the coordinates of the vertices of the image.

c. Is $\triangle ABC$ congruent to its image?



SOLUTION

a. The transformation is a reflection in the y-axis. You can imagine that the image was obtained by flipping $\triangle ABC$ over the y-axis.

b. The coordinates of the vertices of the image, $\triangle A'B'C'$, are $A'(4, 1)$, $B'(3, 5)$, and $C'(1, 1)$.

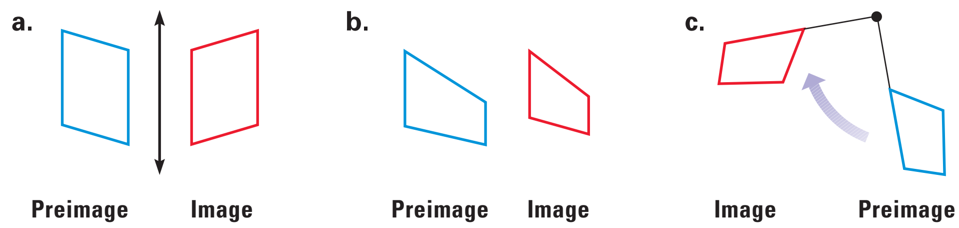
c. Yes, $\triangle ABC$ is congruent to its image $\triangle A'B'C'$. One way to show this would be to use the Distance Formula to find the lengths of the sides of both triangles. Then use the SSS Congruence Postulate.

Study Tip
The term **isometry** comes from the Greek phrase *isos metrom*, meaning *equal measure*.

An **isometry** is a transformation that preserves lengths. Isometries also preserve angle measures, parallel lines, and distances between points. Transformations that are isometries are called *rigid transformations*.

EXAMPLE 2 *Identifying Isometries*

Which of the following transformations appear to be isometries?



SOLUTION

a. This transformation appears to be an isometry. The blue parallelogram is reflected in a line to produce a congruent red parallelogram.

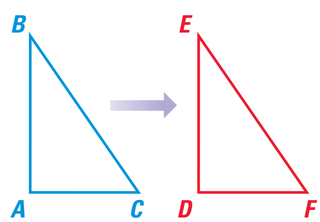
b. This transformation is not an isometry. The image is not congruent to the preimage.

c. This transformation appears to be an isometry. The blue parallelogram is rotated about a point to produce a congruent red parallelogram.

. . . .

MAPPINGS You can describe the transformation in the diagram by writing "$\triangle ABC$ is *mapped onto* $\triangle DEF$." You can also use arrow notation as follows:

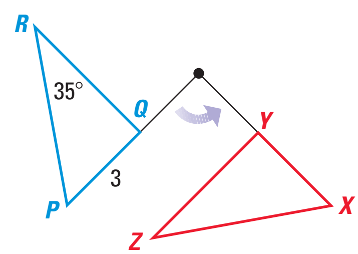
$$\triangle ABC \rightarrow \triangle DEF$$



The order in which the vertices are listed specifies the correspondence. Either of the descriptions implies that $A \rightarrow D$, $B \rightarrow E$, and $C \rightarrow F$.

EXAMPLE 3 *Preserving Length and Angle Measure*

In the diagram, $\triangle PQR$ is mapped onto $\triangle XYZ$. The mapping is a rotation. Given that $\triangle PQR \rightarrow \triangle XYZ$ is an isometry, find the length of $\overline{XY}$ and the measure of $\angle Z$.



SOLUTION

The statement "$\triangle PQR$ is mapped onto $\triangle XYZ$" implies that $P \rightarrow X$, $Q \rightarrow Y$, and $R \rightarrow Z$. Because the transformation is an isometry, the two triangles are congruent.

▶ So, $XY = PQ = 3$ and $m\angle Z = m\angle R = 35°$.

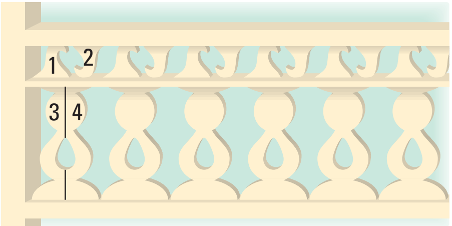
FOCUS ON APPLICATIONS

CARPENTER GOTHIC The woodwork of carpenter gothic houses contains decorative patterns. Notice the translations in the patterns of the carpenter gothic house above.

GOAL 2 USING TRANSFORMATIONS IN REAL LIFE

EXAMPLE 4 *Identifying Transformations*

CARPENTRY You are assembling pieces of wood to complete a railing for your porch. The finished railing should resemble the one below.



a. How are pieces 1 and 2 related? pieces 3 and 4?

b. In order to assemble the rail as shown, explain why you need to know how the pieces are related.

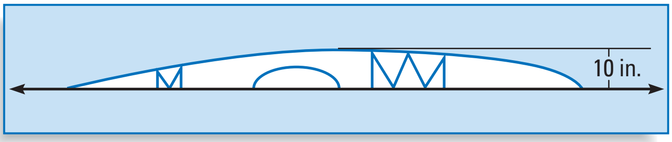
SOLUTION

a. Pieces 1 and 2 are related by a rotation. Pieces 3 and 4 are related by a reflection.

b. Knowing how the pieces are related helps you manipulate the pieces to create the desired pattern.

EXAMPLE 5 *Using Transformations*

BUILDING A KAYAK Many building plans for kayaks show the layout and dimensions for only half of the kayak. A plan of the top view of a kayak is shown below.



a. What type of transformation can a builder use to visualize plans for the entire body of the kayak?

b. Using the plan above, what is the maximum width of the entire kayak?

SOLUTION

a. The builder can use a reflection to visualize the entire kayak. For instance, when one half of the kayak is reflected in a line through its center, you obtain the other half of the kayak.

b. The two halves of the finished kayak are congruent, so the width of the entire kayak will be 2(10), or 20 inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. An operation that maps a preimage onto an image is called a __?__.

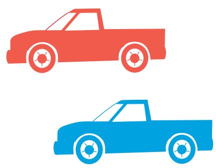
Concept Check ✓

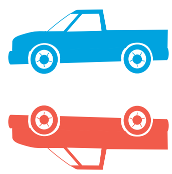
Complete the statement with *always*, *sometimes*, or *never*.

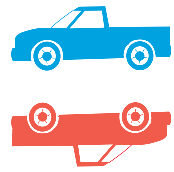
2. The preimage and the image of a transformation are __?__ congruent.
3. A transformation that is an isometry __?__ preserves length.
4. An isometry __?__ maps an acute triangle onto an obtuse triangle.

Skill Check ✓

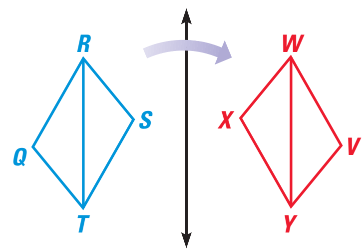
Name the transformation that maps the blue pickup truck (preimage) onto the red pickup (image).

5. 

6. 

7. 

Use the figure shown, where figure *QRST* is mapped onto figure *VWXY*.



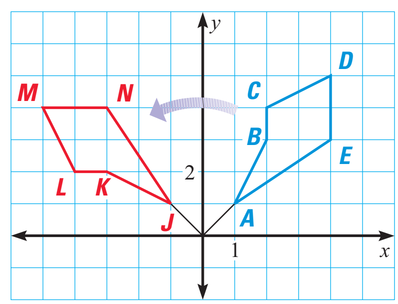
8. Name the preimage of $\overline{XY}$.
9. Name the image of $\overline{QR}$.
10. Name two angles that have the same measure.
11. Name a triangle that appears to be congruent to $\triangle RST$.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 815.

NAMING TRANSFORMATIONS **Use the graph of the transformation below.**



12. Figure *ABCDE* → Figure __?__
13. Name and describe the transformation.
14. Name two sides with the same length.
15. Name two angles with the same measure.
16. Name the coordinates of the preimage of point *L*.
17. Show two corresponding sides have the same length, using the Distance Formula.

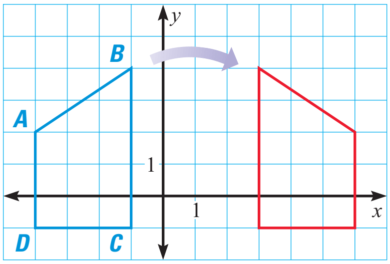
STUDENT HELP

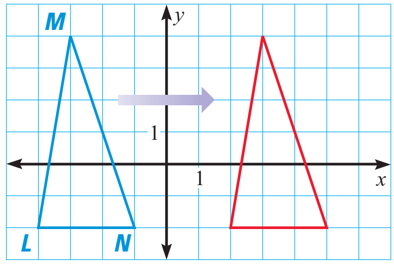
HOMEWORK HELP
Example 1: Exs. 12–22
Example 2: Exs. 23–25
Example 3: Exs. 26–31
Example 4: Exs. 36–39
Example 5: Ex. 41

ANALYZING STATEMENTS **Is the statement *true* or *false*?**

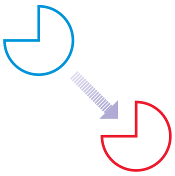
18. Isometries preserve angle measures and parallel lines.
19. Transformations that are *not* isometries are called rigid transformations.
20. A reflection in a line is a type of transformation.

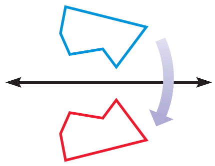
DESCRIBING TRANSFORMATIONS Name and describe the transformation. Then name the coordinates of the vertices of the image.

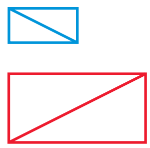
21. 

22. 

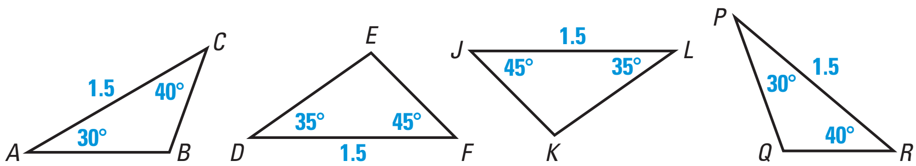
ISOMETRIES Does the transformation appear to be an isometry? Explain.

23. 

24. 

25. 

COMPLETING STATEMENTS Use the diagrams to complete the statement.



26. $\triangle ABC \to \triangle$ _?_

27. $\triangle DEF \to \triangle$ _?_

28. $\triangle$ _?_ $\to \triangle EFD$

29. $\triangle$ _?_ $\to \triangle ACB$

30. $\triangle LJK \to \triangle$ _?_

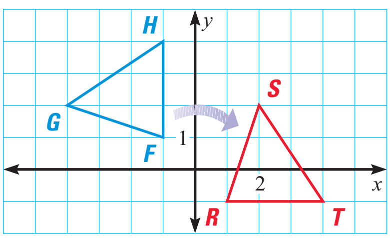
31. $\triangle$ _?_ $\to \triangle CBA$

STUDENT HELP

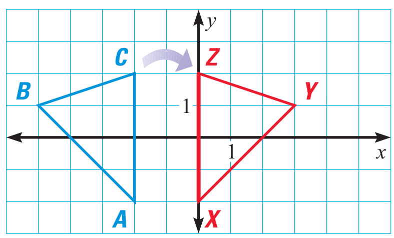
INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 32 and 33.

SHOWING AN ISOMETRY Show that the transformation is an isometry by using the Distance Formula to compare the side lengths of the triangles.

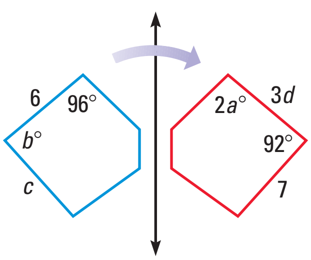
32. $\triangle FGH \to \triangle RST$

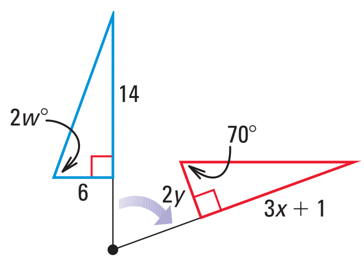


33. $\triangle ABC \to \triangle XYZ$



USING ALGEBRA Find the value of each variable, given that the transformation is an isometry.

34. 

35. 

FOOTPRINTS In Exercises 36–39, name the transformation that will map footprint *A* onto the indicated footprint.

36. Footprint *B*

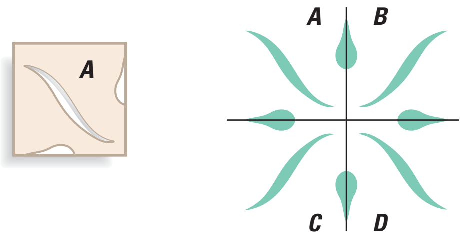
37. Footprint *C*

38. Footprint *D*

39. Footprint *E*

40. *Writing* Can a point or a line segment be its own preimage? Explain and illustrate your answer.

41. **STENCILING** You are stenciling the living room of your home. You want to use the stencil pattern below on the left to create the design shown. What type of transformation will you use to manipulate the stencil from *A* to *B*? from *A* to *C*? from *A* to *D*?



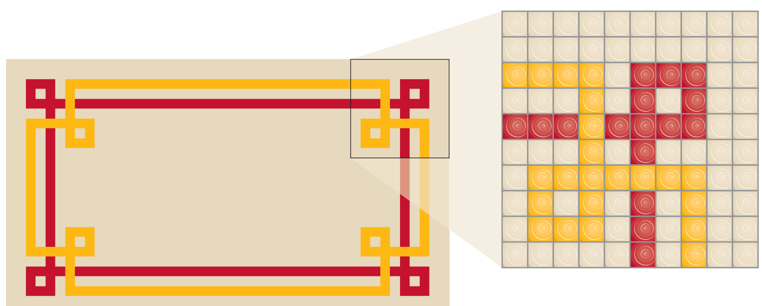
FOCUS ON APPLICATIONS

EMBROIDERY Before machines, all stitching was done by hand. Completing samplers, such as the one above, served as practice for those learning how to stitch.

42. **MACHINE EMBROIDERY** Computerized embroidery machines are used to sew letters and designs on fabric. A computerized embroidery machine can use the same symbol to create several different letters. Which of the letters below are rigid transformations of other letters? Explain how a computerized embroidery machine can create these letters from one symbol.

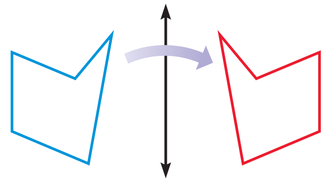
a b c d e f g h i j k l m
n o p q r s t u v w x y z

43. **TILING A FLOOR** You are tiling a kitchen floor using the design shown below. You use a plan to lay the tile for the upper right corner of the floor design. Describe how you can use the plan to complete the other three corners of the floor.



Test Preparation

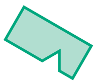
44. **MULTIPLE CHOICE** What type of transformation is shown?



(A) slide (B) reflection

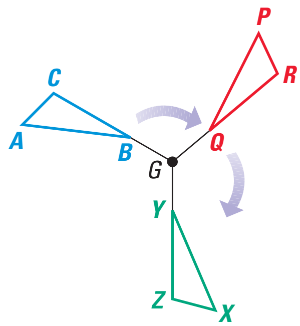
(C) translation (D) rotation

45. **MULTIPLE CHOICE** Which of the following is *not* a rotation of the figure at right?

(A)  (B)  (C)  (D) 

★ Challenge

46. **TWO-COLUMN PROOF** Write a two-column proof using the given information and the diagram.



GIVEN ▶ $\triangle ABC \to \triangle PQR$ and $\triangle PQR \to \triangle XYZ$ are isometries.

PROVE ▶ $\triangle ABC \to \triangle XYZ$ is an isometry.

Plan for Proof Show that $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, and $\overline{AC} \cong \overline{XZ}$.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

USING THE DISTANCE FORMULA **Find the distance between the two points.** (Review 1.3 for 7.2)

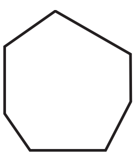
47. $A(3, 10)$, $B(-2, -2)$

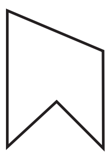
48. $C(5, -7)$, $D(-11, 6)$

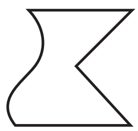
49. $E(0, 8)$, $F(-8, 3)$

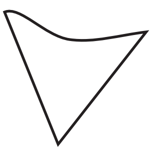
50. $G(0, -7)$, $H(6, 3)$

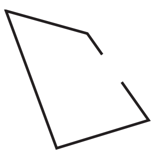
IDENTIFYING POLYGONS **Determine whether the figure is a polygon. If it is not, explain why not.** (Review 6.1 for 7.2)

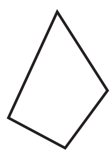
51. 

52. 

53. 

54. 

55. 

56. 

USING COORDINATE GEOMETRY **Use two different methods to show that the points represent the vertices of a parallelogram.** (Review 6.3)

57. $P(0, 4)$, $Q(7, 6)$, $R(8, -2)$, $S(1, -4)$

58. $W(1, 5)$, $X(9, 5)$, $Y(6, -1)$, $Z(-2, -1)$

7.2 Reflections

What you should learn

GOAL 1 Identify and use reflections in a plane.

GOAL 2 Identify relationships between reflections and line symmetry.

Why you should learn it

▼ Reflections and line symmetry can help you understand how mirrors in a kaleidoscope create interesting patterns, as in **Example 5**.

REAL LIFE

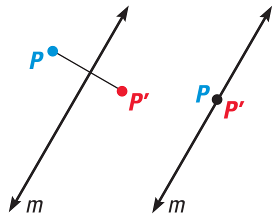
GOAL 1 USING REFLECTIONS IN A PLANE

One type of transformation uses a line that acts like a mirror, with an image reflected in the line. This transformation is a **reflection** and the mirror line is the **line of reflection**.

A reflection in a line m is a transformation that maps every point P in the plane to a point P', so that the following properties are true:

1. If P is not on m, then m is the perpendicular bisector of $\overline{PP'}$.

2. If P is on m, then $P = P'$.

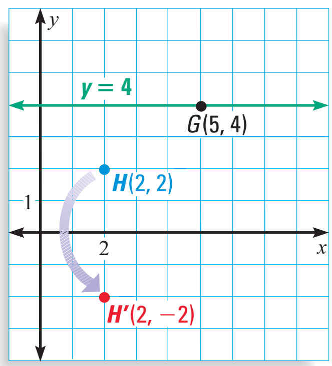


EXAMPLE 1 *Reflections in a Coordinate Plane*

Graph the given reflection.

a. $H(2, 2)$ in the x-axis

b. $G(5, 4)$ in the line $y = 4$



SOLUTION

a. Since H is two units above the x-axis, its reflection, H', is two units below the x-axis.

b. Start by graphing $y = 4$ and G. From the graph, you can see that G is on the line. This implies that $G = G'$.

Reflections in the coordinate axes have the following properties:

1. If (x, y) is reflected in the x-axis, its image is the point $(x, -y)$.

2. If (x, y) is reflected in the y-axis, its image is the point $(-x, y)$.

In Lesson 7.1, you learned that an isometry preserves lengths. Theorem 7.1 relates isometries and reflections.

THEOREM

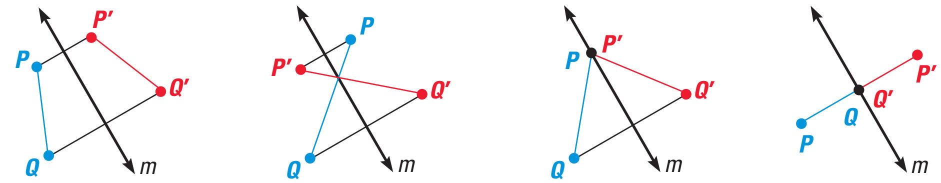
THEOREM 7.1 ***Reflection Theorem***

A reflection is an isometry.

STUDENT HELP

Study Tip
Some theorems have more than one case, such as the Reflection Theorem. To fully prove this type of theorem, all of the cases must be proven.

To prove the Reflection Theorem, you need to show that a reflection preserves the length of a segment. Consider a segment $\overline{PQ}$ that is reflected in a line m to produce $\overline{P'Q'}$. The four cases to consider are shown below.



Case 1
P and Q are on the same side of m.

Case 2
P and Q are on opposite sides of m.

Case 3
One point lies on m and $\overline{PQ}$ is not perpendicular to m.

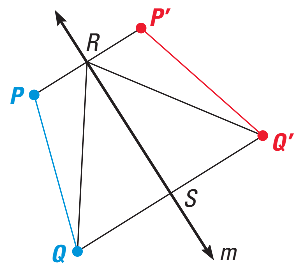
Case 4
Q lies on m and $\overline{PQ} \perp m$.

EXAMPLE 2 *Proof of Case 1 of Theorem 7.1*

Proof

GIVEN ▶ A reflection in m maps P onto P' and Q onto Q'.

PROVE ▶ $PQ = P'Q'$



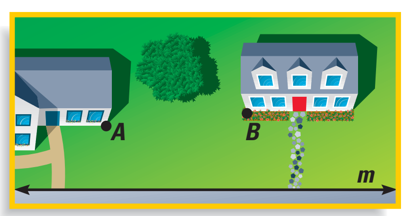
Paragraph Proof For this case, P and Q are on the same side of line m. Draw $\overline{PP'}$ and $\overline{QQ'}$, intersecting line m at R and S. Draw $\overline{RQ}$ and $\overline{RQ'}$.

By the definition of a reflection, $m \perp \overline{QQ'}$ and $\overline{QS} \cong \overline{Q'S}$. It follows that $\triangle RSQ \cong \triangle RSQ'$ using the SAS Congruence Postulate. This implies $\overline{RQ} \cong \overline{RQ'}$ and $\angle QRS \cong \angle Q'RS$. Because $\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{PP'}$, you have enough information to apply SAS to conclude that $\triangle RQP \cong \triangle RQ'P'$. Because corresponding parts of congruent triangles are congruent, $PQ = P'Q'$.

EXAMPLE 3 *Finding a Minimum Distance*

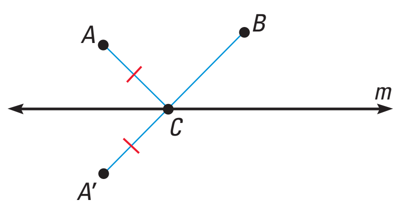
REAL LIFE

SURVEYING Two houses are located on a rural road m, as shown at the right. You want to place a telephone pole on the road at point C so that the length of the telephone cable, $AC + BC$, is a minimum. Where should you locate C?



SOLUTION

Reflect A in line m to obtain A'. Then, draw $\overline{A'B}$. Label the point at which this segment intersects m as C. Because $\overline{A'B}$ represents the shortest distance between A' and B, and $AC = A'C$, you can conclude that at point C a minimum length of telephone cable is used.



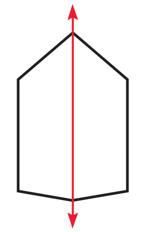
GOAL 2 REFLECTIONS AND LINE SYMMETRY

A figure in the plane has a **line of symmetry** if the figure can be mapped onto itself by a reflection in the line.

EXAMPLE 4 *Finding Lines of Symmetry*

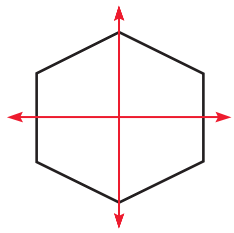
Hexagons can have different lines of symmetry depending on their shape.

a.



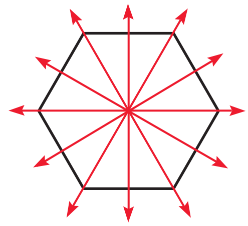
This hexagon has only one line of symmetry.

b.



This hexagon has two lines of symmetry.

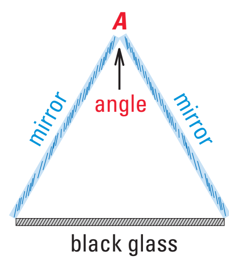
c.



This hexagon has six lines of symmetry.

EXAMPLE 5 *Identifying Reflections*

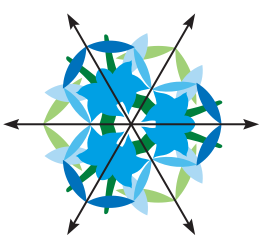
KALEIDOSCOPES Inside a kaleidoscope, two mirrors are placed next to each other to form a V, as shown at the right. The angle between the mirrors determines the number of lines of symmetry in the image. The formula below can be used to calculate the angle between the mirrors, A, or the number of lines of symmetry in the image, n.



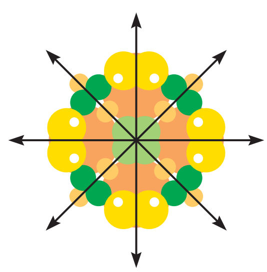
$$n(m\angle A) = 180°$$

Use the formula to find the angle that the mirrors must be placed for the image of a kaleidoscope to resemble the design.

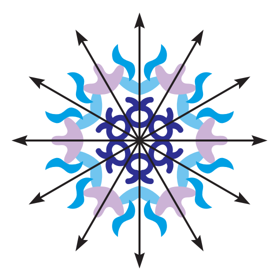
a.



b.



c.



SOLUTION

a. There are 3 lines of symmetry. So, you can write $3(m\angle A) = 180°$. The solution is $m\angle A = 60°$.

b. There are 4 lines of symmetry. So, you can write $4(m\angle A) = 180°$. The solution is $m\angle A = 45°$.

c. There are 6 lines of symmetry. So, you can write $6(m\angle A) = 180°$. The solution is $m\angle A = 30°$.

FOCUS ON PEOPLE

REAL LIFE **KALEIDOSCOPES** Sue and Bob Rioux design and make kaleidoscopes. The kaleidoscope in front of Sue is called *Sea Angel*.

INTERNET APPLICATION LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe what a *line of symmetry* is.

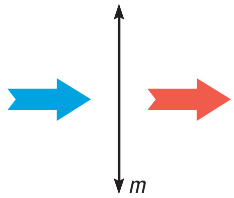
Concept Check ✓

2. When a point is reflected in the x-axis, how are the coordinates of the image related to the coordinates of the preimage?

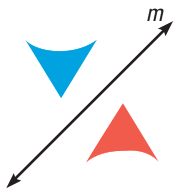
Skill Check ✓

Determine whether the blue figure maps onto the red figure by a reflection in line *m*.

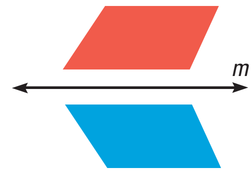
3.



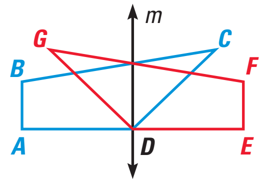
4.



5.



Use the diagram at the right to complete the statement.



6. $\overline{AB} \rightarrow$ ___?___

7. ___?___ $\rightarrow \angle DEF$

8. $C \rightarrow$ ___?___

9. $D \rightarrow$ ___?___

10. ___?___ $\rightarrow \angle GFE$

11. ___?___ $\rightarrow \overline{DG}$

FLOWERS **Determine the number of lines of symmetry in the flower.**

12.

13.

14.

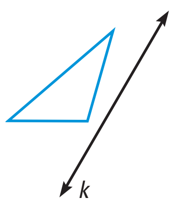
PRACTICE AND APPLICATIONS

STUDENT HELP

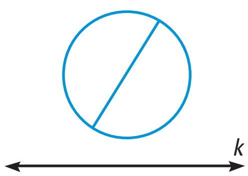
Extra Practice
to help you master skills is on pp. 815 and 816.

DRAWING REFLECTIONS **Trace the figure and draw its reflection in line *k*.**

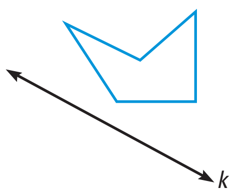
15.



16.



17.



STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 15–30
Example 2: Exs. 33–35
Example 3: Exs. 36–40
Example 4: Exs. 31, 32
Example 5: Exs. 44–46

ANALYZING STATEMENTS **Decide whether the conclusion is *true* or *false*. Explain your reasoning.**

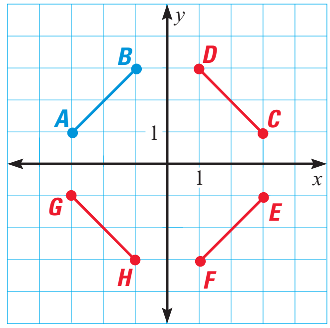
18. If $N(2, 4)$ is reflected in the line $y = 2$, then N' is $(2, 0)$.

19. If $M(6, -2)$ is reflected in the line $x = 3$, then M' is $(0, -2)$.

20. If $W(-6, -3)$ is reflected in the line $y = -2$, then W' is $(-6, 1)$.

21. If $U(5, 3)$ is reflected in the line $x = 1$, then U' is $(-3, 3)$.

REFLECTIONS IN A COORDINATE PLANE Use the diagram at the right to name the image of $\overline{AB}$ after the reflection.



22. Reflection in the x-axis

23. Reflection in the y-axis

24. Reflection in the line $y = x$

25. Reflection in the y-axis, followed by a reflection in the x-axis.

REFLECTIONS In Exercises 26–29, find the coordinates of the reflection without using a coordinate plane. Then check your answer by plotting the image and preimage on a coordinate plane.

26. $S(0, 2)$ reflected in the x-axis

27. $T(3, 8)$ reflected in the x-axis

28. $Q(-3, -3)$ reflected in the y-axis

29. $R(7, -2)$ reflected in the y-axis

30. **CRITICAL THINKING** Draw a triangle on the coordinate plane and label its vertices. Then reflect the triangle in the line $y = x$. What do you notice about the coordinates of the vertices of the preimage and the image?

LINES OF SYMMETRY Sketch the figure, if possible.

31. An octagon with exactly two lines of symmetry

32. A quadrilateral with exactly four lines of symmetry

PARAGRAPH PROOF In Exercises 33–35, write a paragraph proof for each case of Theorem 7.1. (Refer to the diagrams on page 405.)

33. In Case 2, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, $\overline{PQ}$ intersects m at point R.

PROVE ▶ $PQ = P'Q'$

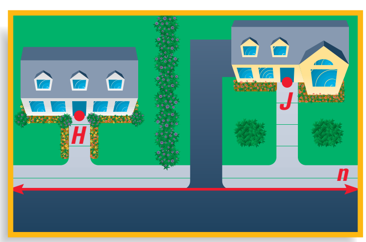
34. In Case 3, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, P lies on line m and $\overline{PQ}$ is not perpendicular to m.

PROVE ▶ $PQ = P'Q'$

35. In Case 4, it is given that a reflection in m maps P onto P' and Q onto Q'. Also, Q lies on line m and $\overline{PQ}$ is perpendicular to line m.

PROVE ▶ $PQ = P'Q'$

36. **DELIVERING PIZZA** You park your car at some point K on line n. You deliver a pizza to house H, go back to your car, and deliver a pizza to house J. Assuming that you cut across both lawns, explain how to estimate K so the distance that you travel is as small as possible.



STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 37–40.

MINIMUM DISTANCE Find point C on the x-axis so $AC + BC$ is a minimum.

37. $A(1, 5), B(7, 1)$

38. $A(2, -2), B(11, -4)$

39. $A(-1, 4), B(6, 3)$

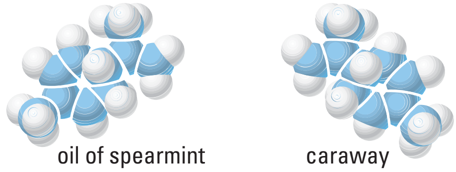
40. $A(-4, 6), B(3.5, 9)$

FOCUS ON CAREERS

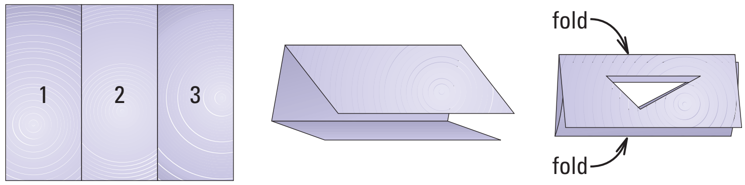
CHEMIST Some chemists study the molecular structure of living things. The research done by these chemists has led to important discoveries in the field of medicine.

CAREER LINK
www.mcdougallittell.com

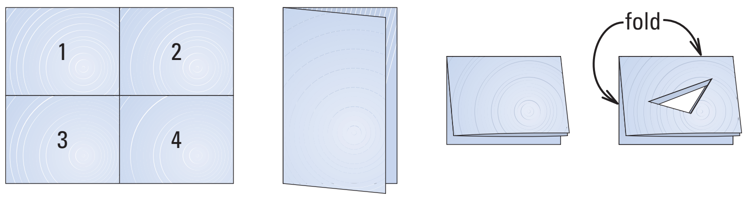
41. **CHEMISTRY CONNECTION** The figures at the right show two versions of the carvone molecule. One version is oil of spearmint and the other is caraway. How are the structures of these two molecules related?



42. **PAPER FOLDING** Fold a piece of paper and label it as shown. Cut a scalene triangle out of the folded paper and unfold the paper. How are triangle 2 and triangle 3 related to triangle 1?

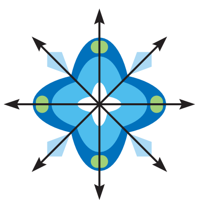


43. **PAPER FOLDING** Fold a piece of paper and label it as shown. Cut a scalene triangle out of the folded paper and unfold the paper. How are triangles 2, 3, and 4 related to triangle 1?

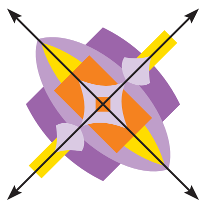


KALEIDOSCOPES **In Exercises 44–46, calculate the angle at which the mirrors must be placed for the image of a kaleidoscope to resemble the given design. (Use the formula in Example 5 on page 406.)**

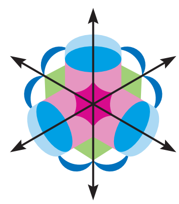
44.



45.



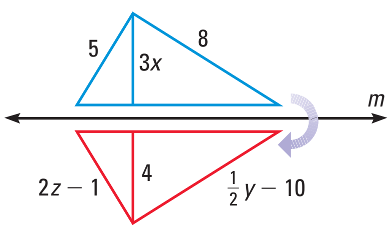
46.



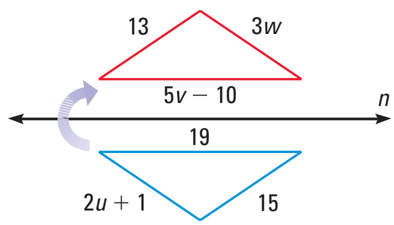
47. **TECHNOLOGY** Use geometry software to draw a polygon reflected in line m. Connect the corresponding vertices of the preimage and image. Measure the distance between each vertex and line m. What do you notice about these measures?

USING ALGEBRA **Find the value of each variable, given that the diagram shows a reflection in a line.**

48.

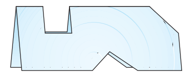


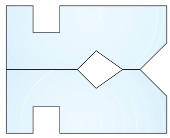
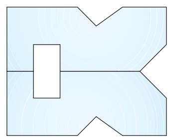
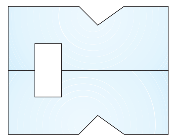
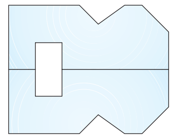
49.



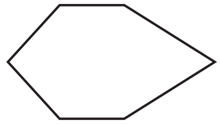
Test Preparation

50. MULTIPLE CHOICE A piece of paper is folded in half and some cuts are made, as shown. Which figure represents the piece of paper unfolded?



Ⓐ  Ⓑ  Ⓒ  Ⓓ 

51. MULTIPLE CHOICE How many lines of symmetry does the figure at the right have?



Ⓐ 0 Ⓑ 1 Ⓒ 2

Ⓓ 3 Ⓔ 6

★ Challenge

WRITING AN EQUATION **Follow the steps to write an equation for the line of reflection.**

52. Graph $R(2, 1)$ and $R'(-2, -1)$. Draw a segment connecting the two points.

53. Find the midpoint of $\overline{RR'}$ and name it Q.

54. Find the slope of $\overline{RR'}$. Then write the slope of a line perpendicular to $\overline{RR'}$.

55. Write an equation of the line that is perpendicular to $\overline{RR'}$ and passes through Q.

56. Repeat Exercises 52–55 using $R(-2, 3)$ and $R'(3, -2)$.

EXTRA CHALLENGE
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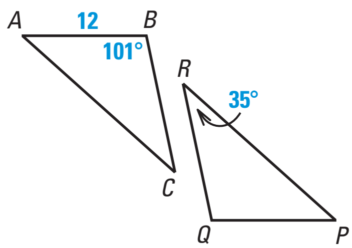
MIXED REVIEW

CONGRUENT TRIANGLES **Use the diagram, in which $\triangle ABC \cong \triangle PQR$, to complete the statement.** (Review 4.2 for 7.3)



57. $\angle A \cong$ __?__ **58.** $PQ =$ __?__

59. $\overline{QR} \cong$ __?__ **60.** $m\angle C =$ __?__

61. $m\angle Q =$ __?__ **62.** $\angle R \cong$ __?__

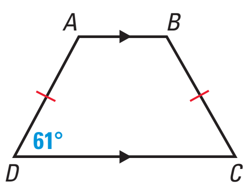
FINDING SIDE LENGTHS OF A TRIANGLE **Two side lengths of a triangle are given. Describe the length of the third side, *c*, with an inequality.** (Review 5.5)

63. $a = 7, b = 17$ **64.** $a = 9, b = 21$ **65.** $a = 12, b = 33$

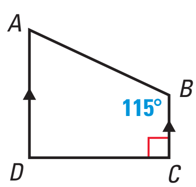
66. $a = 26, b = 6$ **67.** $a = 41.2, b = 15.5$ **68.** $a = 7.1, b = 11.9$

FINDING ANGLE MEASURES **Find the angle measures of *ABCD*.** (Review 6.5)

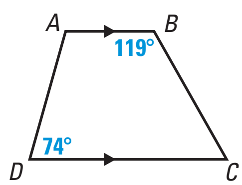
69.



70.



71.



7.3 Rotations

What you should learn

GOAL 1 Identify rotations in a plane.

GOAL 2 Use rotational symmetry in **real-life** situations, such as the logo designs in **Example 5**.

Why you should learn it

▼ Rotations and rotational symmetry can be used to create a design, as in the wheel hubs below and in **Exs. 36–38**.

REAL LIFE

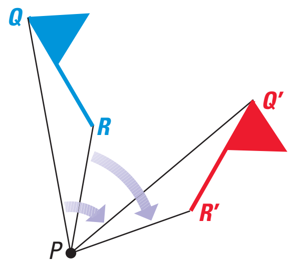
GOAL 1 USING ROTATIONS

A **rotation** is a transformation in which a figure is turned about a fixed point. The fixed point is the **center of rotation**. Rays drawn from the center of rotation to a point and its image form an angle called the **angle of rotation**.

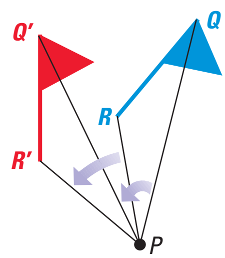
A rotation about a point P through x degrees ($x°$) is a transformation that maps every point Q in the plane to a point Q', so that the following properties are true:

1. If Q is not point P, then $QP = Q'P$ and $m\angle QPQ' = x°$.
2. If Q is point P, then $Q = Q'$.

Rotations can be clockwise or counterclockwise, as shown below.



Clockwise rotation of 60°



Counterclockwise rotation of 40°

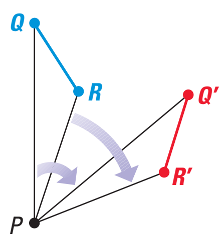
THEOREM

THEOREM 7.2 ***Rotation Theorem***

A rotation is an isometry.

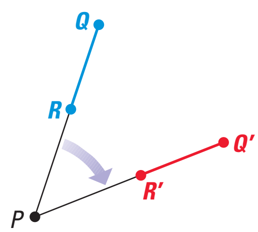
To prove the Rotation Theorem, you need to show that a rotation preserves the length of a segment. Consider a segment $\overline{QR}$ that is rotated about a point P to produce $\overline{Q'R'}$. The three cases are shown below. The first case is proved in Example 1.

CASE 1



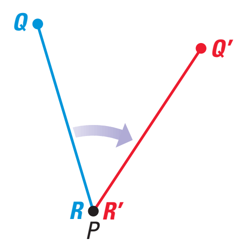
***R*, *Q*, and *P* are noncollinear.**

CASE 2



***R*, *Q*, and *P* are collinear.**

CASE 3

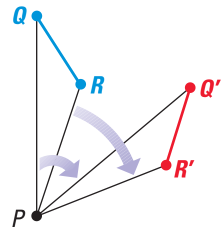


***P* and *R* are the same point.**

EXAMPLE 1 *Proof of Theorem 7.2*

Proof

Write a paragraph proof for Case 1 of the Rotation Theorem.



GIVEN ▶ A rotation about P maps Q onto Q' and R onto R'.

PROVE ▶ $\overline{QR} \cong \overline{Q'R'}$

SOLUTION

Paragraph Proof By the definition of a rotation, $PQ = PQ'$ and $PR = PR'$. Also, by the definition of a rotation, $m\angle QPQ' = m\angle RPR'$.

You can use the Angle Addition Postulate and the subtraction property of equality to conclude that $m\angle QPR = m\angle Q'PR'$. This allows you to use the SAS Congruence Postulate to conclude that $\triangle QPR \cong \triangle Q'PR'$. Because corresponding parts of congruent triangles are congruent, $\overline{QR} \cong \overline{Q'R'}$.

....

STUDENT HELP

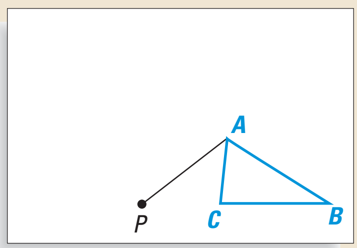
Look Back
For help with using a protractor, see p. 27.

You can use a compass and a protractor to help you find the images of a polygon after a rotation. The following construction shows you how.

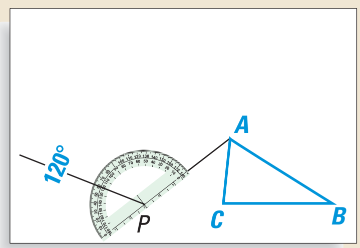
ACTIVITY
Construction

Rotating a Figure

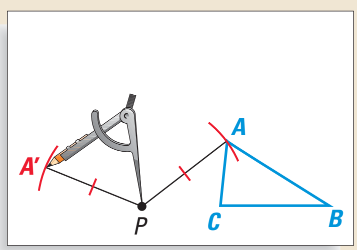
Use the following steps to draw the image of $\triangle ABC$ after a 120° counterclockwise rotation about point P.



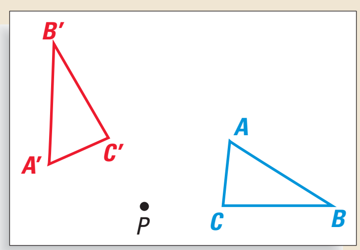
1. Draw a segment connecting vertex A and the center of rotation point P.



2. Use a protractor to measure a 120° angle counterclockwise and draw a ray.



3. Place the point of the compass at P and draw an arc from A to locate A'.



4. Repeat **Steps 1–3** for each vertex. Connect the vertices to form the image.

EXAMPLE 2 *Rotations in a Coordinate Plane*

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

In a coordinate plane, sketch the quadrilateral whose vertices are $A(2, -2)$, $B(4, 1)$, $C(5, 1)$, and $D(5, -1)$. Then, rotate $ABCD$ $90°$ counterclockwise about the origin and name the coordinates of the new vertices. Describe any patterns you see in the coordinates.

SOLUTION

Plot the points, as shown in blue. Use a protractor, a compass, and a straightedge to find the rotated vertices. The coordinates of the preimage and image are listed below.

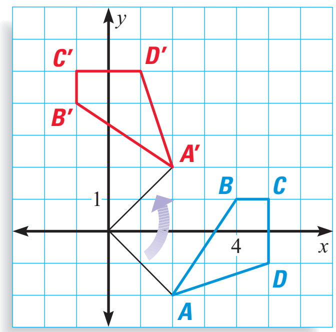


Figure *ABCD*	Figure *A′B′C′D′*
$A(2, -2)$	$A'(2, 2)$
$B(4, 1)$	$B'(-1, 4)$
$C(5, 1)$	$C'(-1, 5)$
$D(5, -1)$	$D'(1, 5)$

In the list above, the x-coordinate of the image is the opposite of the y-coordinate of the preimage. The y-coordinate of the image is the x-coordinate of the preimage.

▶ This transformation can be described as $(x, y) \to (-y, x)$.

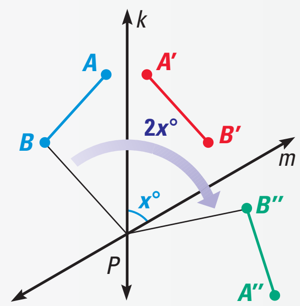
THEOREM

THEOREM 7.3

If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is a rotation about point P.

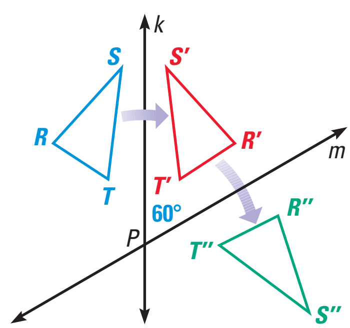
The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

$$m\angle BPB'' = 2x°$$



EXAMPLE 3 *Using Theorem 7.3*

In the diagram, $\triangle RST$ is reflected in line k to produce $\triangle R'S'T'$. This triangle is then reflected in line m to produce $\triangle R''S''T''$. Describe the transformation that maps $\triangle RST$ to $\triangle R''S''T''$.

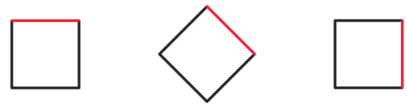


SOLUTION

The acute angle between lines k and m has a measure of $60°$. Applying Theorem 7.3 you can conclude that the transformation that maps $\triangle RST$ to $\triangle R''S''T''$ is a clockwise rotation of $120°$ about point P.

GOAL 2 ROTATIONS AND ROTATIONAL SYMMETRY

A figure in the plane has **rotational symmetry** if the figure can be mapped onto itself by a rotation of 180° or less. For instance, a square has rotational symmetry because it maps onto itself by a rotation of 90°.

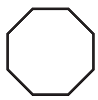


0° rotation **45° rotation** **90° rotation**

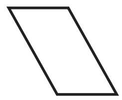
EXAMPLE 4 *Identifying Rotational Symmetry*

Which figures have rotational symmetry? For those that do, describe the rotations that map the figure onto itself.

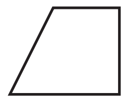
a. Regular octagon



b. Parallelogram



c. Trapezoid



SOLUTION

a. This octagon has rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 45°, 90°, 135°, or 180° about its center.

b. This parallelogram has rotational symmetry. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180° about its center.

c. The trapezoid does not have rotational symmetry.

FOCUS ON CAREERS

REAL LIFE **LOGO DESIGNERS** create symbols that represent the name of a company or organization. The logos appear on packaging, letterheads, and Web sites.

INTERNET **CAREER LINK**
www.mcdougallittell.com

EXAMPLE 5 *Using Rotational Symmetry*

LOGO DESIGN A music store called Ozone is running a contest for a store logo. The winning logo will be displayed on signs throughout the store and in the store's advertisements. The only requirement is that the logo include the store's name. Two of the entries are shown below. What do you notice about them?

a.

b.

SOLUTION

a. This design has rotational symmetry about its center. It can be mapped onto itself by a clockwise or counterclockwise rotation of 180°.

b. This design also has rotational symmetry about its center. It can be mapped onto itself by a clockwise or counterclockwise rotation of 90° or 180°.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is a *center of rotation*?

Concept Check ✓

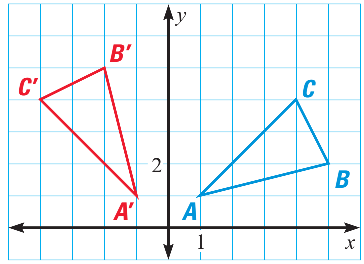
Use the diagram, in which $\triangle ABC$ is mapped onto $\triangle A'B'C'$ by a rotation of 90° about the origin.

2. Is the rotation clockwise or counterclockwise?

3. Does $AB = A'B'$? Explain.

4. Does $AA' = BB'$? Explain.

5. If the rotation of $\triangle ABC$ onto $\triangle A'B'C'$ was obtained by a reflection of $\triangle ABC$ in some line k followed by a reflection in some line m, what would be the measure of the acute angle between lines k and m? Explain.



Skill Check ✓

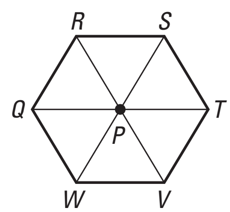
The diagonals of the regular hexagon below form six equilateral triangles. Use the diagram to complete the sentence.

6. A clockwise rotation of 60° about P maps R onto __?__.

7. A counterclockwise rotation of 60° about __?__ maps R onto Q.

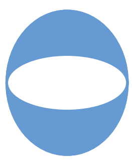
8. A clockwise rotation of 120° about Q maps R onto __?__.

9. A counterclockwise rotation of 180° about P maps V onto __?__.



Determine whether the figure has rotational symmetry. If so, describe the rotations that map the figure onto itself.

10.



11.

12.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 816.

DESCRIBING AN IMAGE **State the segment or triangle that represents the image. You can use tracing paper to help you visualize the rotation.**

13. 90° clockwise rotation of $\overline{AB}$ about P

14. 90° clockwise rotation of $\overline{KF}$ about P

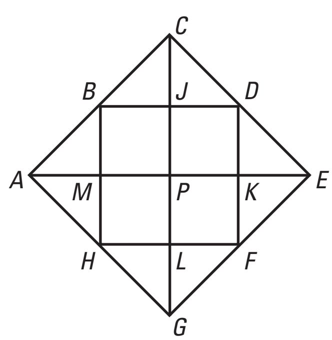
15. 90° counterclockwise rotation of $\overline{CE}$ about E

16. 90° counterclockwise rotation of $\overline{FL}$ about H

17. 180° rotation of $\triangle KEF$ about P

18. 180° rotation of $\triangle BCJ$ about P

19. 90° clockwise rotation of $\triangle APG$ about P



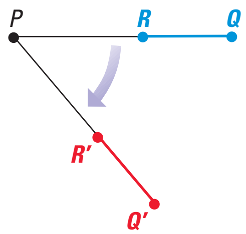
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 13–21
Example 2: Exs. 22–29
Example 3: Exs. 30–33
Example 4: Exs. 36–38
Example 5: Exs. 39–42

PARAGRAPH PROOF **Write a paragraph proof for the case of Theorem 7.2.**

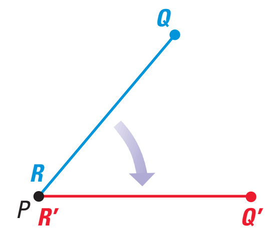
20. GIVEN ▶ A rotation about P maps Q onto Q' and R onto R'.

PROVE ▶ $\overline{QR} \cong \overline{Q'R'}$



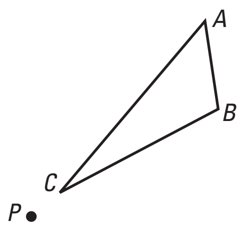
21. GIVEN ▶ A rotation about P maps Q onto Q' and R onto R'. P and R are the same point.

PROVE ▶ $\overline{QR} \cong \overline{Q'R'}$

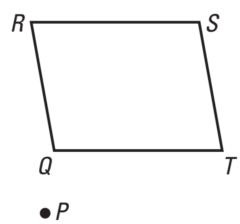


ROTATING A FIGURE **Trace the polygon and point *P* on paper. Then, use a straightedge, compass, and protractor to rotate the polygon clockwise the given number of degrees about *P*.**

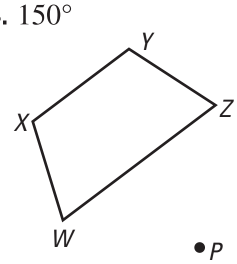
22. 60°



23. 135°

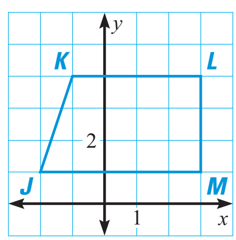


24. 150°

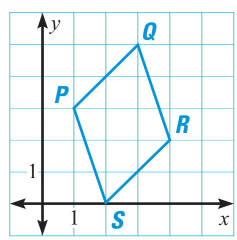


ROTATIONS IN A COORDINATE PLANE **Name the coordinates of the vertices of the image after a clockwise rotation of the given number of degrees about the origin.**

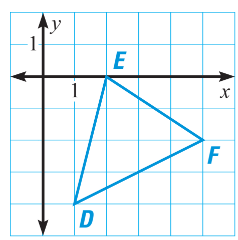
25. 90°



26. 180°

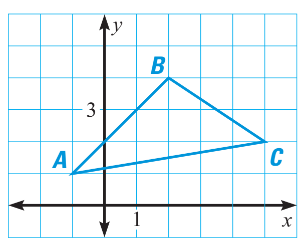


27. 270°

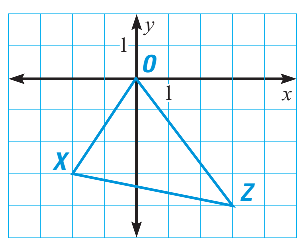


FINDING A PATTERN **Use the given information to rotate the triangle. Name the vertices of the image and compare with the vertices of the preimage. Describe any patterns you see.**

28. 90° clockwise about origin

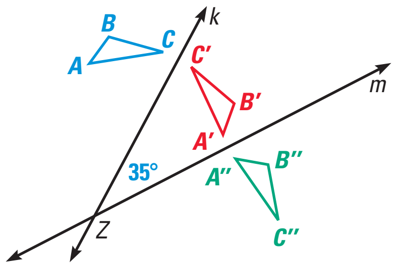


29. 180° clockwise about origin

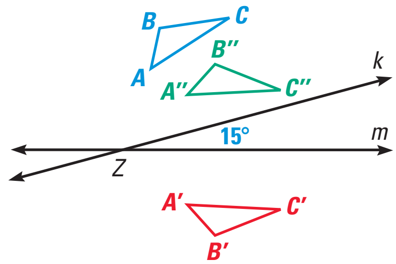


USING THEOREM 7.3 **Find the angle of rotation that maps $\triangle ABC$ onto $\triangle A''B''C''$.**

30.



31.



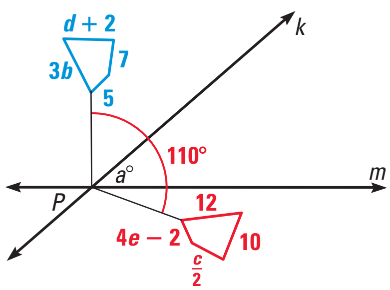
LOGICAL REASONING **Lines m and n intersect at point D. Consider a reflection of $\triangle ABC$ in line m followed by a reflection in line n.**

32. What is the angle of rotation about D, when the measure of the acute angle between lines m and n is 36°?

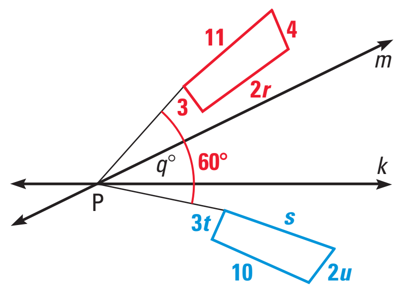
33. What is the measure of the acute angle between lines m and n, when the angle of rotation about D is 162°?

USING ALGEBRA **Find the value of each variable in the rotation of the polygon about point P.**

34.



35.



WHEEL HUBS **Describe the rotational symmetry of the wheel hub.**

36.

37.

38.

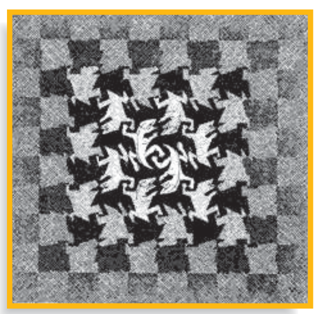
ROTATIONS IN ART **In Exercises 39–42, refer to the image below by M.C. Escher. The piece is called *Development I* and was completed in 1937.**

39. Does the piece have rotational symmetry? If so, describe the rotations that map the image onto itself.

40. Would your answer to Exercise 39 change if you disregard the shading of the figures? Explain your reasoning.

41. Describe the center of rotation.

42. Is it possible that this piece could be hung upside down? Explain.



M.C. Escher's *"Development I"* © 1999 Cordon Art B.V. - Baarn - Holland. All rights reserved.

FOCUS ON PEOPLE

REAL LIFE **M.C. ESCHER** is a Dutch graphic artist whose works include optical illusions and geometric patterns. (M.C. Escher's *"Hand with Reflecting Sphere"* © 1999 Cordon Art B.V. - Baarn - Holland. All rights reserved.)

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

Test Preparation

43. MULTI-STEP PROBLEM Follow the steps below.

a. Graph $\triangle RST$ whose vertices are $R(1, 1)$, $S(4, 3)$, and $T(5, 1)$.

b. Reflect $\triangle RST$ in the y-axis to obtain $\triangle R'S'T'$. Name the coordinates of the vertices of the reflection.

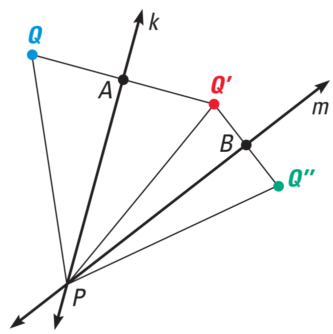
c. Reflect $\triangle R'S'T'$ in the line $y = -x$ to obtain $\triangle R''S''T''$. Name the coordinates of the vertices of the reflection.

d. Describe a single transformation that maps $\triangle RST$ onto $\triangle R''S''T''$.

e. *Writing* Explain how to show a 90° counterclockwise rotation of any polygon about the origin using two reflections of the figure.

★ **Challenge**

44. PROOF Use the diagram and the given information to write a paragraph proof for Theorem 7.3.



GIVEN ▶ Lines k and m intersect at point P, Q is any point not on k or m.

PROVE ▶ **a.** If you reflect point Q in k, and then reflect its image Q' in m, Q'' is the image of Q after a rotation about point P.

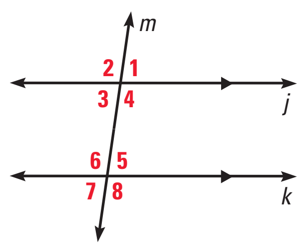
b. $m\angle QPQ'' = 2(m\angle APB)$.

Plan for Proof First show $k \perp \overline{QQ'}$ and $\overline{QA} \cong \overline{Q'A}$. Then show $\triangle QAP \cong \triangle Q'AP$. Use a similar argument to show $\triangle Q'BP \cong \triangle Q''BP$. Use the congruent triangles and substitution to show that $\overline{QP} \cong \overline{Q''P}$. That proves part (a) by the definition of a rotation. You can use the congruent triangles to prove part (b).

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

PARALLEL LINES Find the measure of the angle using the diagram, in which $j \parallel k$ and $m\angle 1 = 82°$. (Review 3.3 for 7.4)



45. $m\angle 5$

46. $m\angle 7$

47. $m\angle 3$

48. $m\angle 6$

49. $m\angle 4$

50. $m\angle 8$

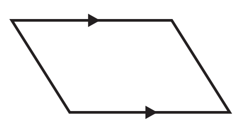
DRAWING TRIANGLES In Exercises 51–53, draw the triangle. (Review 5.2)

51. Draw a triangle whose circumcenter lies outside the triangle.

52. Draw a triangle whose circumcenter lies on the triangle.

53. Draw a triangle whose circumcenter lies inside the triangle.

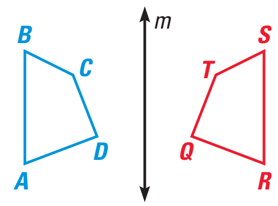
54. PARALLELOGRAMS Can it be proven that the figure at the right is a parallelogram? If not, explain why not. (Review 6.2)



QUIZ 1

Self-Test for Lessons 7.1–7.3

Use the transformation at the right. (Lesson 7.1)



1. Figure $ABCD \rightarrow$ Figure __?__

2. Name and describe the transformation.

3. Is the transformation an isometry? Explain.

In Exercises 4–7, find the coordinates of the reflection without using a coordinate plane. (Lesson 7.2)

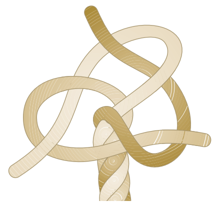
4. $L(2, 3)$ reflected in the x-axis

5. $M(-2, -4)$ reflected in the y-axis

6. $N(-4, 0)$ reflected in the x-axis

7. $P(8.2, -3)$ reflected in the y-axis

8. **KNOTS** The knot at the right is a *wall knot*, which is generally used to prevent the end of a rope from running through a pulley. Describe the rotations that map the knot onto itself and describe the center of rotation. **(Lesson 7.3)**



MATH & History

History of Decorative Patterns

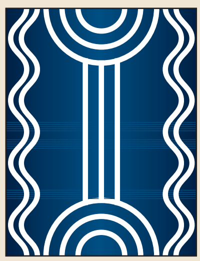
APPLICATION LINK www.mcdougallittell.com

THEN

FOR THOUSANDS OF YEARS, people have adorned their buildings, pottery, clothing, and jewelry with decorative patterns. Simple patterns were created by using a transformation of a shape.

NOW

TODAY, you are likely to find computer generated patterns decorating your clothes, CD covers, sports equipment, computer desktop, and even textbooks.

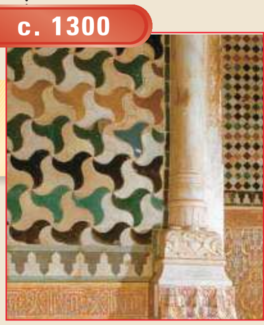


1. The design at the right is based on a piece of pottery by Marsha Gomez. How many lines of symmetry does the design have?

2. Does the design have rotational symmetry? If so, describe the rotation that maps the pattern onto itself.

c. 1300 B.C.

Egyptian jewelry is decorated with patterns.

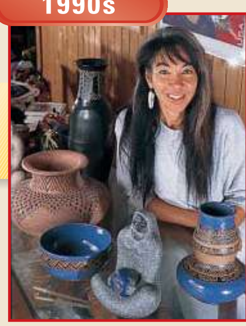


c. 1300

Tiles are arranged in symmetric patterns in the Alhambra in Spain.

1899

Painted textile pattern called 'Bulow Birds'



1990s

Marsha Gomez decorates pottery with symmetrical patterns.

7.4 Translations and Vectors

What you should learn

GOAL 1 Identify and use translations in the plane.

GOAL 2 Use vectors in **real-life** situations, such as navigating a sailboat in **Example 6**.

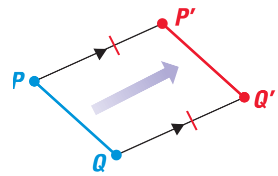
Why you should learn it

▼ You can use translations and vectors to describe the path of an aircraft, such as the hot-air balloon in **Exs. 53–55**.

REAL LIFE

GOAL 1 USING PROPERTIES OF TRANSLATIONS

A **translation** is a transformation that maps every two points P and Q in the plane to points P' and Q', so that the following properties are true:



1. $PP' = QQ'$

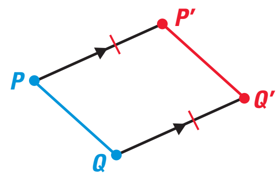
2. $\overline{PP'} \parallel \overline{QQ'}$, or $\overline{PP'}$ and $\overline{QQ'}$ are collinear.

THEOREM

THEOREM 7.4 ***Translation Theorem***

A translation is an isometry.

Theorem 7.4 can be proven as follows.

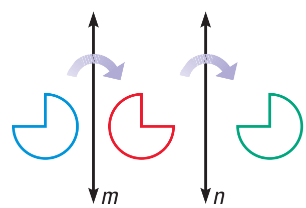


GIVEN ▶ $PP' = QQ'$, $\overline{PP'} \parallel \overline{QQ'}$

PROVE ▶ $PQ = P'Q'$

Paragraph Proof The quadrilateral $PP'Q'Q$ has a pair of opposite sides that are congruent and parallel, which implies $PP'Q'Q$ is a parallelogram. From this you can conclude $PQ = P'Q'$. (Exercise 43 asks for a coordinate proof of Theorem 7.4, which covers the case where $\overline{PQ}$ and $\overline{P'Q'}$ are collinear.)

You can find the image of a translation by gliding a figure in the plane. Another way to find the image of a translation is to complete one reflection after another in two parallel lines, as shown. The properties of this type of translation are stated below.



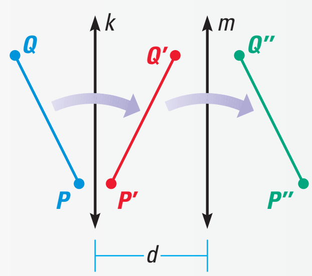
THEOREM

THEOREM 7.5

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is a translation. If P'' is the image of P, then the following is true:

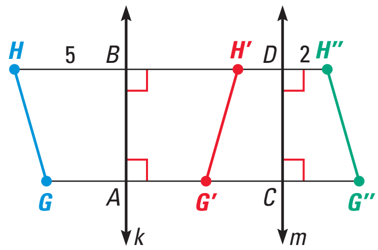
1. $\overleftrightarrow{PP''}$ is perpendicular to k and m.

2. $PP'' = 2d$, where d is the distance between k and m.



EXAMPLE 1 *Using Theorem 7.5*

In the diagram, a reflection in line k maps $\overline{GH}$ to $\overline{G'H'}$, a reflection in line m maps $\overline{G'H'}$ to $\overline{G''H''}$, $k \parallel m$, $HB = 5$, and $DH'' = 2$.



a. Name some congruent segments.

b. Does $AC = BD$? Explain.

c. What is the length of $\overline{GG''}$?

> **STUDENT HELP**
>
> **Study Tip**
>
> In Lesson 7.2, you learned that the line of reflection is the perpendicular bisector of the segment connecting a point and its image. In Example 1, you can use this property to conclude that figure *ABDC* is a rectangle.

SOLUTION

a. Here are some sets of congruent segments: $\overline{GH}$, $\overline{G'H'}$, and $\overline{G''H''}$; $\overline{HB}$ and $\overline{H'B}$; $\overline{H'D}$ and $\overline{H''D}$.

b. Yes, $AC = BD$ because $\overline{AC}$ and $\overline{BD}$ are opposite sides of a rectangle.

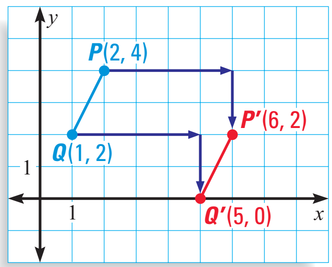
c. Because $GG'' = HH''$, the length of $\overline{GG''}$ is $5 + 5 + 2 + 2$, or 14 units.

. . . .

Translations in a coordinate plane can be described by the following coordinate notation:

$$(x, y) \rightarrow (x + a, y + b)$$

where a and b are constants. Each point shifts a units horizontally and b units vertically. For instance, in the coordinate plane at the right, the translation $(x, y) \rightarrow (x + 4, y - 2)$ shifts each point 4 units to the right and 2 units down.



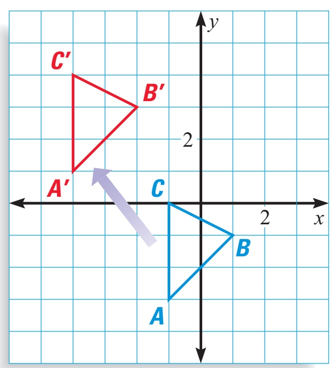
EXAMPLE 2 *Translations in a Coordinate Plane*

Sketch a triangle with vertices $A(-1, -3)$, $B(1, -1)$, and $C(-1, 0)$. Then sketch the image of the triangle after the translation $(x, y) \rightarrow (x - 3, y + 4)$.

SOLUTION

Plot the points as shown. Shift each point 3 units to the left and 4 units up to find the translated vertices. The coordinates of the vertices of the preimage and image are listed below.

$\triangle ABC$	$\triangle A'B'C'$
$A(-1, -3)$	$A'(-4, 1)$
$B(1, -1)$	$B'(-2, 3)$
$C(-1, 0)$	$C'(-4, 4)$



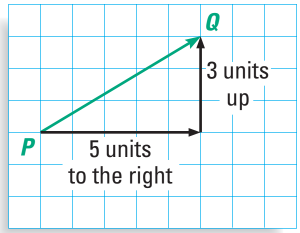
Notice that each x-coordinate of the image is 3 units less than the x-coordinate of the preimage and each y-coordinate of the image is 4 units more than the y-coordinate of the preimage.

Study Tip
When writing a vector in component form, use the correct brackets. The brackets used to write the component form of a vector are different than the parentheses used to write an ordered pair.

GOAL 2 TRANSLATIONS USING VECTORS

Another way to describe a translation is by using a vector. A **vector** is a quantity that has both direction and *magnitude*, or size, and is represented by an arrow drawn between two points.

The diagram shows a vector. The **initial point**, or starting point, of the vector is P and the **terminal point**, or ending point, is Q. The vector is named $\overrightarrow{PQ}$, which is read as "vector PQ." The *horizontal component* of $\overrightarrow{PQ}$ is 5 and the *vertical component* is 3.

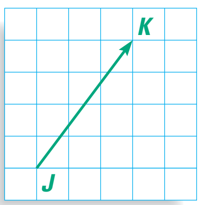


The **component form** of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{PQ}$ is $\langle 5, 3\rangle$.

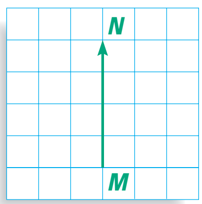
EXAMPLE 3 *Identifying Vector Components*

In the diagram, name each vector and write its component form.

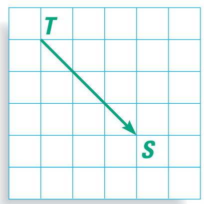
a.



b.



c.



SOLUTION

a. The vector is $\overrightarrow{JK}$. To move from the initial point J to the terminal point K, you move 3 units to the right and 4 units up. So, the component form is $\langle 3, 4\rangle$.

b. The vector is $\overrightarrow{MN} = \langle 0, 4\rangle$.

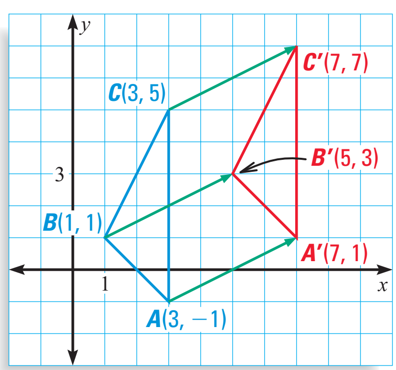
c. The vector is $\overrightarrow{TS} = \langle 3, -3\rangle$.

EXAMPLE 4 *Translation Using Vectors*

The component form of $\overrightarrow{GH}$ is $\langle 4, 2\rangle$. Use $\overrightarrow{GH}$ to translate the triangle whose vertices are $A(3, -1)$, $B(1, 1)$, and $C(3, 5)$.

SOLUTION

First graph $\triangle ABC$. The component form of $\overrightarrow{GH}$ is $\langle 4, 2\rangle$, so the image vertices should all be 4 units to the right and 2 units up from the preimage vertices. Label the image vertices as $A'(7, 1)$, $B'(5, 3)$, and $C'(7, 7)$. Then, using a straightedge, draw $\triangle A'B'C'$. Notice that the vectors drawn from preimage to image vertices are parallel.

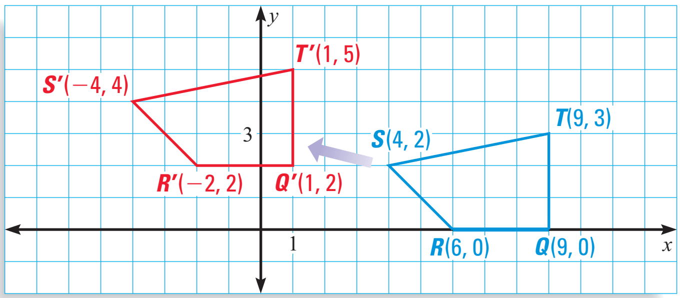


STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 5 *Finding Vectors*

In the diagram, *QRST* maps onto *Q′R′S′T′* by a translation. Write the component form of the vector that can be used to describe the translation.



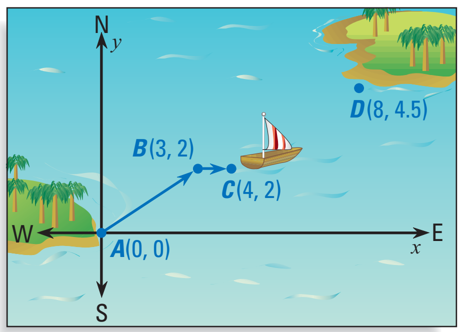
SOLUTION

Choose any vertex and its image, say *R* and *R′*. To move from *R* to *R′*, you move 8 units to the left and 2 units up. The component form of the vector is $\langle -8, 2\rangle$.

✓**CHECK** To check the solution, you can start any where on the preimage and move 8 units to the left and 2 units up. You should end on the corresponding point of the image.

EXAMPLE 6 *Using Vectors*

NAVIGATION A boat travels a straight path between two islands, *A* and *D*. When the boat is 3 miles east and 2 miles north of its starting point it encounters a storm at point *B*. The storm pushes the boat off course to point *C*, as shown.



a. Write the component forms of the two vectors shown in the diagram.

b. The final destination is 8 miles east and 4.5 miles north of the starting point. Write the component form of the vector that describes the path the boat can follow to arrive at its destination.

SOLUTION

a. The component form of the vector from $A(0, 0)$ to $B(3, 2)$ is

$$\overrightarrow{AB} = \langle 3 - 0, 2 - 0\rangle = \langle 3, 2\rangle.$$

The component form of the vector from $B(3, 2)$ to $C(4, 2)$ is

$$\overrightarrow{BC} = \langle 4 - 3, 2 - 2\rangle = \langle 1, 0\rangle.$$

b. The boat needs to travel from its current position, point *C*, to the island, point *D*. To find the component form of the vector from $C(4, 2)$ to $D(8, 4.5)$, subtract the corresponding coordinates:

$$\overrightarrow{CD} = \langle 8 - 4, 4.5 - 2\rangle = \langle 4, 2.5\rangle.$$

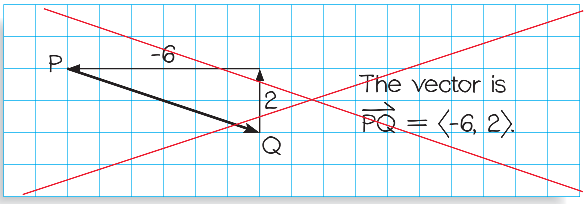
GUIDED PRACTICE

Vocabulary Check ✓

1. A __?__ is a quantity that has both __?__ and magnitude.

Concept Check ✓

2. ERROR ANALYSIS Describe Jerome's error.



Skill Check ✓

Use coordinate notation to describe the translation.

3. 6 units to the right and 2 units down

4. 3 units up and 4 units to the right

5. 7 units to the left and 1 unit up

6. 8 units down and 5 units to the left

Complete the statement using the description of the translation. In the description, points (0, 2) and (8, 5) are two vertices of a pentagon.

7. If (0, 2) maps onto (0, 0), then (8, 5) maps onto (_?_, _?_).

8. If (0, 2) maps onto (_?_, _?_), then (8, 5) maps onto (3, 7).

9. If (0, 2) maps onto $(-3, -5)$, then (8, 5) maps onto (_?_, _?_).

10. If (0, 2) maps onto (_?_, _?_), then (8, 5) maps onto (0, 0).

Draw three vectors that can be described by the given component form.

11. $\langle 3, 5 \rangle$ **12.** $\langle 0, 4 \rangle$ **13.** $\langle -6, 0 \rangle$ **14.** $\langle -5, -1 \rangle$

PRACTICE AND APPLICATIONS

STUDENT HELP

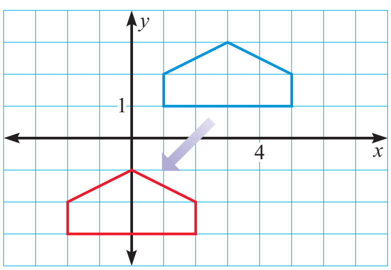
Extra Practice
to help you master skills is on p. 816.

STUDENT HELP

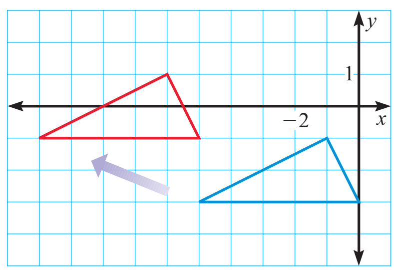
HOMEWORK HELP
Example 1: Exs. 20–24
Example 2: Exs. 15, 16, 25–34
Example 3: Exs. 15–19
Example 4: Exs. 39–42
Example 5: Exs. 44–47
Example 6: Exs. 53–55

DESCRIBING TRANSLATIONS Describe the translation using (a) coordinate notation and (b) a vector in component form.

15.

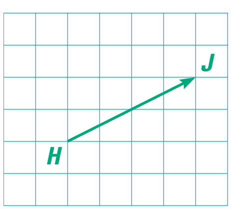


16.

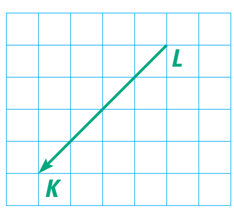


IDENTIFYING VECTORS Name the vector and write its component form.

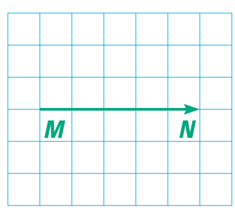
17.



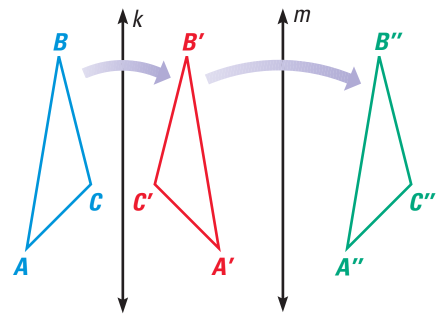
18.



19.



USING THEOREM 7.5 **In the diagram, $k \parallel m$, $\triangle ABC$ is reflected in line k, and $\triangle A'B'C'$ is reflected in line m.**



20. A translation maps $\triangle ABC$ onto which triangle?

21. Which lines are perpendicular to $\overleftrightarrow{AA''}$?

22. Name two segments parallel to $\overline{BB''}$.

23. If the distance between k and m is 1.4 inches, what is the length of $\overline{CC''}$?

24. Is the distance from B' to m the same as the distance from B'' to m? Explain.

IMAGE AND PREIMAGE **Consider the translation that is defined by the coordinate notation $(x, y) \rightarrow (x + 12, y - 7)$.**

25. What is the image of (5, 3)?

26. What is the image of $(-1, -2)$?

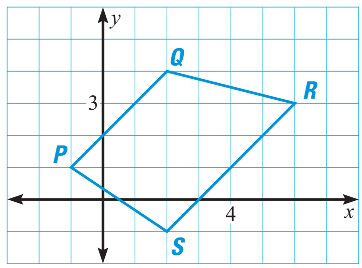
27. What is the preimage of $(-2, 1)$?

28. What is the preimage of $(0, -6)$?

29. What is the image of (0.5, 2.5)?

30. What is the preimage of $(-5.5, -5.5)$?

DRAWING AN IMAGE **Copy figure *PQRS* and draw its image after the translation.**



31. $(x, y) \rightarrow (x + 1, y - 4)$

32. $(x, y) \rightarrow (x - 6, y + 7)$

33. $(x, y) \rightarrow (x + 5, y - 2)$

34. $(x, y) \rightarrow (x - 1, y - 3)$

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 35–38.

LOGICAL REASONING **Use a straightedge and graph paper to help determine whether the statement is true.**

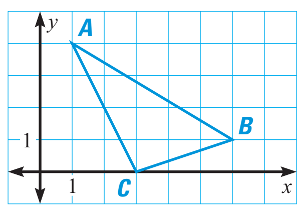
35. If line p is a translation of a different line q, then p is parallel to q.

36. It is possible for a translation to map a line p onto a perpendicular line q.

37. If a translation maps $\triangle ABC$ onto $\triangle DEF$ and a translation maps $\triangle DEF$ onto $\triangle GHK$, then a translation maps $\triangle ABC$ onto $\triangle GHK$.

38. If a translation maps $\triangle ABC$ onto $\triangle DEF$, then $AD = BE = CF$.

TRANSLATING A TRIANGLE **In Exercises 39–42, use a straightedge and graph paper to translate $\triangle ABC$ by the given vector.**



39. $\langle 2, 4 \rangle$

40. $\langle 3, -2 \rangle$

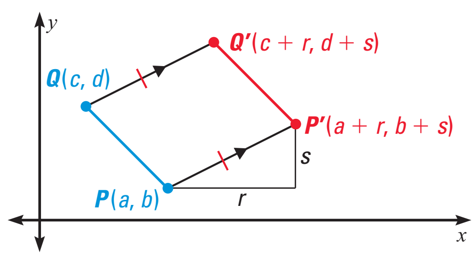
41. $\langle -1, -5 \rangle$

42. $\langle -4, 1 \rangle$

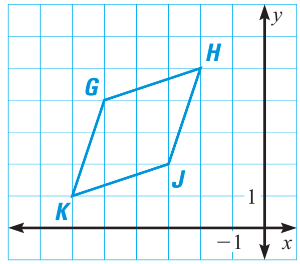
43. **PROOF** Use coordinate geometry and the Distance Formula to write a paragraph proof of Theorem 7.4.

GIVEN ▶ $PP' = QQ'$ and $\overline{PP'} \parallel \overline{QQ'}$

PROVE ▶ $PQ = P'Q'$



VECTORS **The vertices of the image of *GHJK* after a translation are given. Choose the vector that describes the translation.**



A. $\overrightarrow{PQ} = \langle 1, -3 \rangle$ **B.** $\overrightarrow{PQ} = \langle 0, 1 \rangle$

C. $\overrightarrow{PQ} = \langle -1, -3 \rangle$ **D.** $\overrightarrow{PQ} = \langle 6, -1 \rangle$

44. $G'(-6, 1), H'(-3, 2), J'(-4, -1), K'(-7, -2)$

45. $G'(1, 3), H'(4, 4), J'(3, 1), K'(0, 0)$

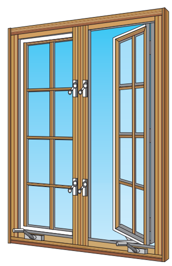
46. $G'(-4, 1), H'(-1, 2), J'(-2, -1), K'(-5, -2)$

47. $G'(-5, 5), H'(-2, 6), J'(-3, 3), K'(-6, 2)$

WINDOW FRAMES **In Exercises 48–50, decide whether "opening the window" is a translation of the moving part.**

48. Double hung

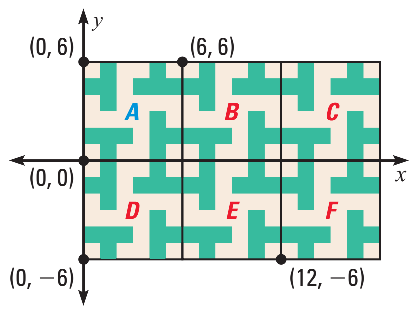
49. Casement



50. Sliding

51. **DATA COLLECTION** Look through some newspapers and magazines to find patterns containing translations.

52. **COMPUTER-AIDED DESIGN** Mosaic floors can be designed on a computer. An example is shown at the right. On the computer, the design in square A is copied to cover an entire floor. The translation $(x, y) \rightarrow (x + 6, y)$ maps square A onto square B. Use coordinate notation to describe the translations that map square A onto squares C, D, E, and F.

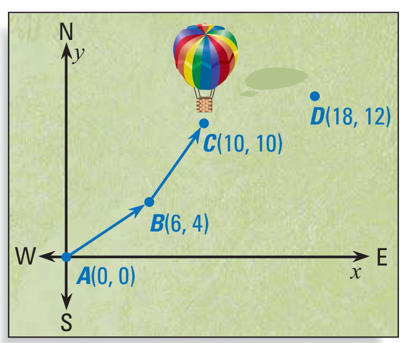


NAVIGATION **A hot-air balloon is flying from town *A* to town *D*. After the balloon leaves town *A* and travels 6 miles east and 4 miles north, it runs into some heavy winds at point *B*. The balloon is blown off course as shown in the diagram.**

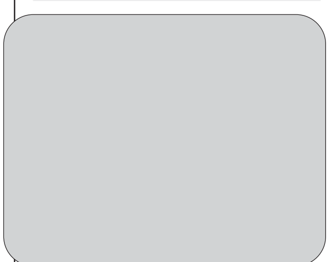
53. Write the component forms of the two vectors in the diagram.

54. Write the component form of the vector that describes the path the balloon can take to arrive in town D.

55. Suppose the balloon was not blown off course. Write the component form of the vector that describes this journey from town A to town D.



FOCUS ON APPLICATIONS



REAL LIFE

HOT-AIR BALLOONS
Bertrand Piccard and Brian Jones journeyed around the world in their hot-air balloon in 19 days.

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 56–59, choose the statement that is true about the given quantities.**

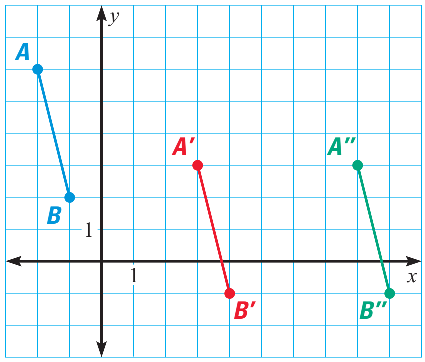
Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

The translation $(x, y) \to (x + 5, y - 3)$ maps $\overline{AB}$ to $\overline{A'B'}$, and the translation $(x, y) \to (x + 5, y)$ maps $\overline{A'B'}$ to $\overline{A''B''}$.



	Column A	Column B
56.	AB	$A'B'$
57.	AB	AA'
58.	BB'	$A'A''$
59.	$A'B''$	$A''B'$

★ Challenge

USING ALGEBRA **A translation of $\overline{AB}$ is described by $\overrightarrow{PQ}$. Find the value of each variable.**

60. $\overrightarrow{PQ} = \langle 4, 1 \rangle$

$A(-1, w), A'(2x + 1, 4)$

$B(8y - 1, 1), B'(3, 3z)$

61. $\overrightarrow{PQ} = \langle 3, -6 \rangle$

$A(r - 1, 8), A'(3, s + 1)$

$B(2t - 2, u), B'(5, -2u)$

EXTRA CHALLENGE

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MIXED REVIEW

FINDING SLOPE **Find the slope of the line that passes through the given points.** (Review 3.6)

62. $A(0, -2), B(-7, -8)$

63. $C(2, 3), D(-1, 18)$

64. $E(-10, 1), F(-1, 1)$

65. $G(-2, 12), H(-1, 6)$

66. $J(-6, 0), K(0, 10)$

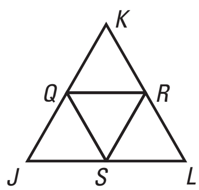
67. $M(-3, -3), N(9, 6)$

COMPLETING THE STATEMENT **In $\triangle JKL$, points Q, R, and S are midpoints of the sides.** (Review 5.4)

68. If $JK = 12$, then $SR =$ __?__.

69. If $QR = 6$, then $JL =$ __?__.

70. If $RL = 6$, then $QS =$ __?__.



REFLECTIONS IN A COORDINATE PLANE **Decide whether the statement is *true* or *false*.** (Review 7.2 for 7.5)

71. If $N(3, 4)$ is reflected in the line $y = -1$, then N' is $(3, -6)$.

72. If $M(-5, 3)$ is reflected in the line $x = -2$, then M' is $(3, 1)$.

73. If $W(4, 3)$ is reflected in the line $y = 2$, then W' is $(1, 4)$.

7.5 Glide Reflections and Compositions

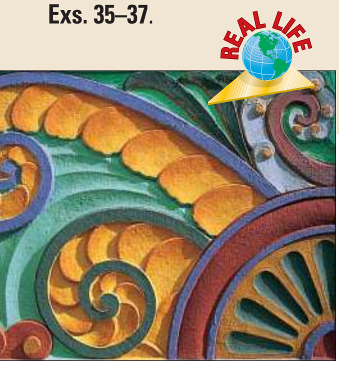
What you should learn

GOAL 1 Identify glide reflections in a plane.

GOAL 2 Represent transformations as compositions of simpler transformations.

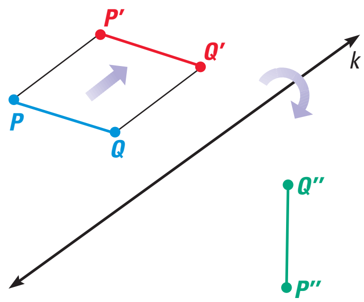
Why you should learn it

▼ Compositions of transformations can help when creating patterns in **real life**, such as the decorative pattern below and in **Exs. 35–37**.



GOAL 1 USING GLIDE REFLECTIONS

A translation, or glide, and a reflection can be performed one after the other to produce a transformation known as a *glide reflection*. A **glide reflection** is a transformation in which every point P is mapped onto a point P'' by the following steps:



1. A translation maps P onto P'.
2. A reflection in a line k parallel to the direction of the translation maps P' onto P''.

As long as the line of reflection is parallel to the direction of the translation, it does not matter whether you glide first and then reflect, or reflect first and then glide.

EXAMPLE 1 *Finding the Image of a Glide Reflection*

Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

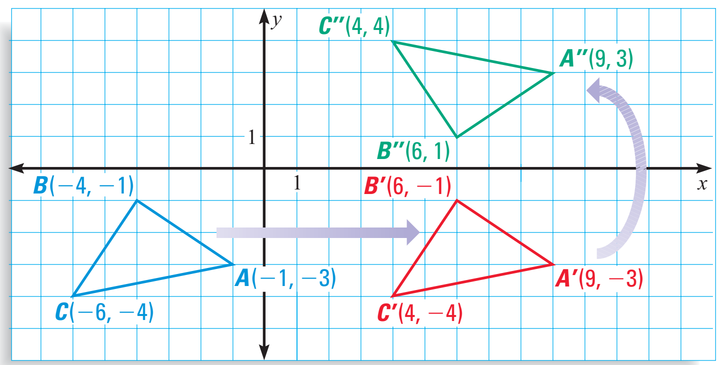
$A(-1, -3)$, $B(-4, -1)$, $C(-6, -4)$

Translation: $(x, y) \rightarrow (x + 10, y)$

Reflection: in the x-axis

SOLUTION

Begin by graphing $\triangle ABC$. Then, shift the triangle 10 units to the right to produce $\triangle A'B'C'$. Finally, reflect the triangle in the x-axis to produce $\triangle A''B''C''$.



....

In Example 1, try reversing the order of the transformations. Notice that the resulting image will have the same coordinates as $\triangle A''B''C''$ above. This is true because the line of reflection is parallel to the direction of the translation.

GOAL 2 USING COMPOSITIONS

When two or more transformations are combined to produce a single transformation, the result is called a **composition** of the transformations.

THEOREM

THEOREM 7.6 ***Composition Theorem***

The composition of two (or more) isometries is an isometry.

Because a glide reflection is a composition of a translation and a reflection, this theorem implies that glide reflections are isometries. In a glide reflection, the order in which the transformations are performed does not affect the final image. For other compositions of transformations, the order may affect the final image.

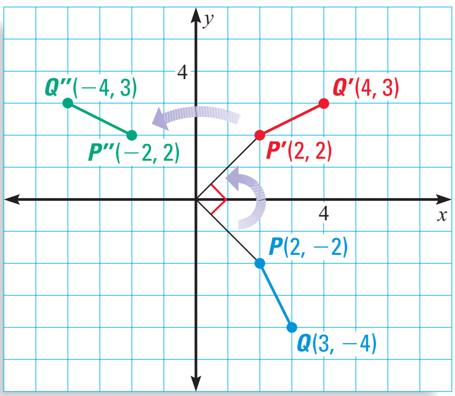
EXAMPLE 2 *Finding the Image of a Composition*

Sketch the image of $\overline{PQ}$ after a composition of the given rotation and reflection.

$P(2, -2)$, $Q(3, -4)$

Rotation: 90° counterclockwise about the origin

Reflection: in the y-axis



SOLUTION

Begin by graphing $\overline{PQ}$. Then rotate the segment 90° counterclockwise about the origin to produce $\overline{P'Q'}$. Finally, reflect the segment in the y-axis to produce $\overline{P''Q''}$.

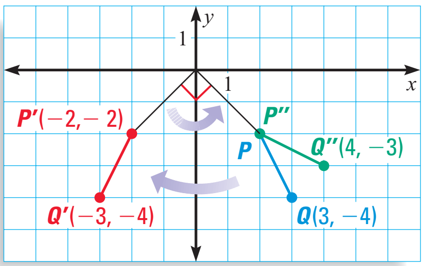
EXAMPLE 3 *Comparing Orders of Compositions*

Repeat Example 2, but switch the order of the composition by performing the reflection first and the rotation second. What do you notice?

SOLUTION

Graph $\overline{PQ}$. Then reflect the segment in the y-axis to obtain $\overline{P'Q'}$. Rotate $\overline{P'Q'}$ 90° counterclockwise about the origin to obtain $\overline{P''Q''}$. Instead of being in Quadrant II, as in Example 2, the image is in Quadrant IV.

▶ The order which the transformations are performed affects the final image.

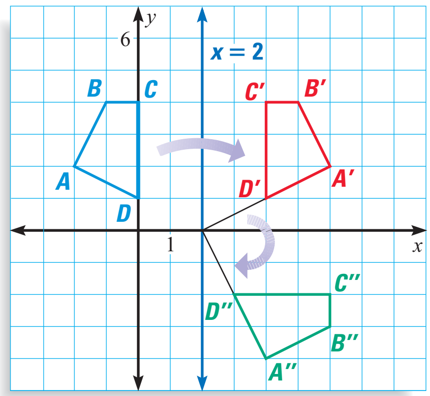


STUDENT HELP

Study Tip
Unlike the addition or multiplication of real numbers, the composition of transformations is not generally commutative.

EXAMPLE 4 *Describing a Composition*

Describe the composition of transformations in the diagram.



SOLUTION

Two transformations are shown. First, figure $ABCD$ is reflected in the line $x = 2$ to produce figure $A'B'C'D'$. Then, figure $A'B'C'D'$ is rotated 90° clockwise about the point (2, 0) to produce figure $A''B''C''D''$.

EXAMPLE 5 *Describing a Composition*

PUZZLES The mathematical game pentominoes is a tiling game that uses twelve different types of tiles, each composed of five squares. The tiles are referred to by the letters they resemble. The object of the game is to pick up and arrange the tiles to create a given shape. Use compositions of transformations to describe how the tiles below will complete the 6 × 5 rectangle.

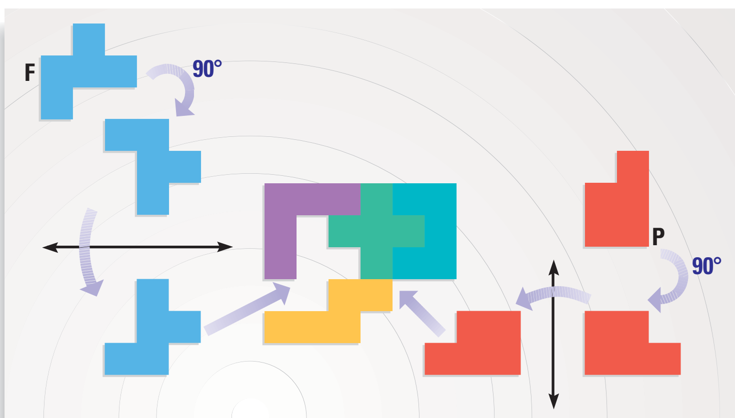
STUDENT HELP

Study Tip
You can make your own pentomino tiles by cutting the shapes out of graph paper.

SOLUTION

To complete part of the rectangle, rotate the F tile 90° clockwise, reflect the tile over a horizontal line, and translate it into place.

To complete the rest of the rectangle, rotate the P tile 90° clockwise, reflect the tile over a vertical line, and translate it into place.



GUIDED PRACTICE

Vocabulary Check ✓

1. In a glide reflection, the direction of the __?__ must be parallel to the line of __?__.

Concept Check ✓

Complete the statement with *always*, *sometimes*, or *never*.

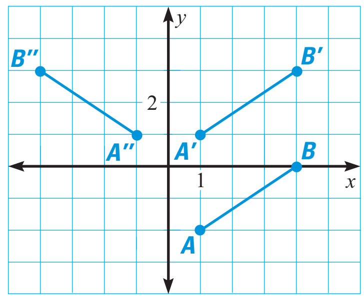
2. The order in which two transformations are performed __?__ affects the resulting image.

3. In a glide reflection, the order in which the two transformations are performed __?__ matters.

4. A composition of isometries is __?__ an isometry.

Skill Check ✓

In the diagram, $\overline{AB}$ is the preimage of a glide reflection.



5. Which segment is a translation of $\overline{AB}$?

6. Which segment is a reflection of $\overline{A'B'}$?

7. Name the line of reflection.

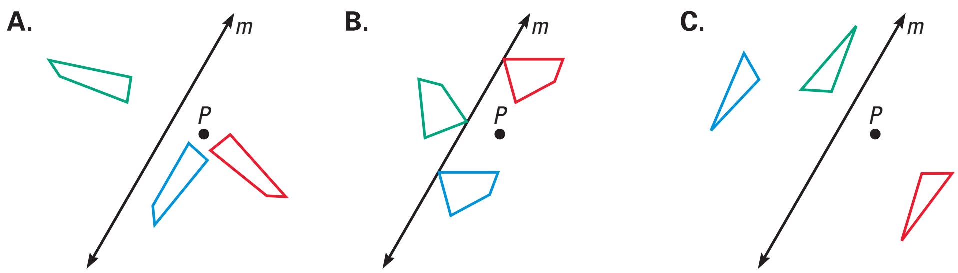
8. Use coordinate notation to describe the translation.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 816.

LOGICAL REASONING **Match the composition with the diagram, in which the blue figure is the preimage of the red figure and the red figure is the preimage of the green figure.**



9. Rotate about point P, then reflect in line m.

10. Reflect in line m, then rotate about point P.

11. Translate parallel to line m, then rotate about point P.

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 9–15
Example 2: Exs. 16–19
Example 3: Exs. 20, 21
Example 4: Exs. 22–25
Example 5: Ex. 38

FINDING AN IMAGE **Sketch the image of $A(-3, 5)$ after the described glide reflection.**

12. Translation: $(x, y) \rightarrow (x, y - 4)$
Reflection: in the y-axis

13. Translation: $(x, y) \rightarrow (x + 4, y + 1)$
Reflection: in $y = -2$

14. Translation: $(x, y) \rightarrow (x - 6, y - 1)$
Reflection: in $x = -1$

15. Translation: $(x, y) \rightarrow (x - 3, y - 3)$
Reflection: in $y = x$

Student Help

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for help with Exs. 16–19.

SKETCHING COMPOSITIONS **Sketch the image of $\triangle PQR$ after a composition using the given transformations in the order they appear.**

16. $P(4, 2), Q(7, 0), R(9, 3)$
Translation: $(x, y) \to (x - 2, y + 3)$
Rotation: $90°$ clockwise about $T(0, 3)$

17. $P(4, 5), Q(7, 1), R(8, 8)$
Translation: $(x, y) \to (x, y - 7)$
Reflection: in the y-axis

18. $P(-9, -2), Q(-9, -5), R(-5, -4)$
Translation: $(x, y) \to (x + 14, y + 1)$
Translation: $(x, y) \to (x - 3, y + 8)$

19. $P(-7, 2), Q(-6, 7), R(-2, -1)$
Reflection: in the x-axis
Rotation: $90°$ clockwise about origin

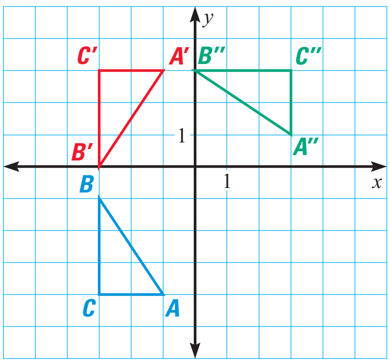
REVERSING ORDERS **Sketch the image of $\overline{FG}$ after a composition using the given transformations in the order they appear. Then, perform the transformations in reverse order. Does the order affect the final image?**

20. $F(4, -4), G(1, -2)$
Rotation: $90°$ clockwise about origin
Reflection: in the y-axis

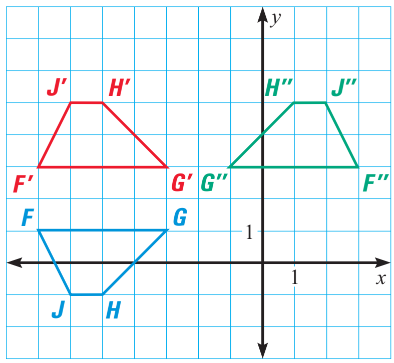
21. $F(-1, -3), G(-4, -2)$
Reflection: in the line $x = 1$
Translation: $(x, y) \to (x + 2, y + 10)$

DESCRIBING COMPOSITIONS **In Exercises 22–25, describe the composition of the transformations.**

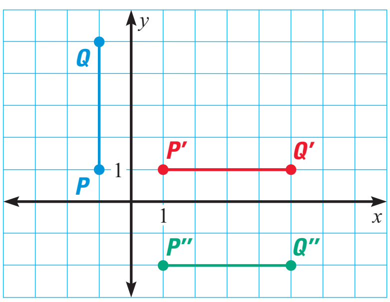
22.



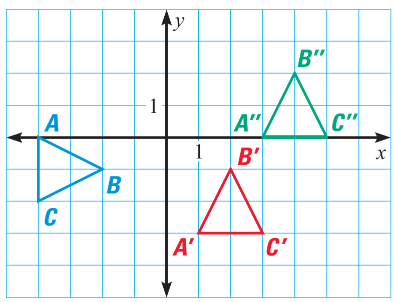
23.



24.



25.



26. *Writing* Explain why a glide reflection is an isometry.

27. **LOGICAL REASONING** Which are preserved by a glide reflection?

A. distance **B.** angle measure **C.** parallel lines

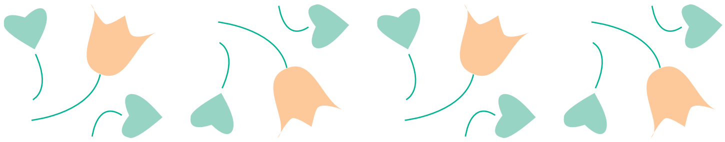
28. **TECHNOLOGY** Use geometry software to draw a polygon. Show that if you reflect the polygon and then translate it in a direction that is *not* parallel to the line of reflection, then the final image is *different* from the final image if you perform the translation first and the reflection second.

CRITICAL THINKING In Exercises 29 and 30, the first translation maps J to J' and the second maps J' to J''. Find the translation that maps J to J''.

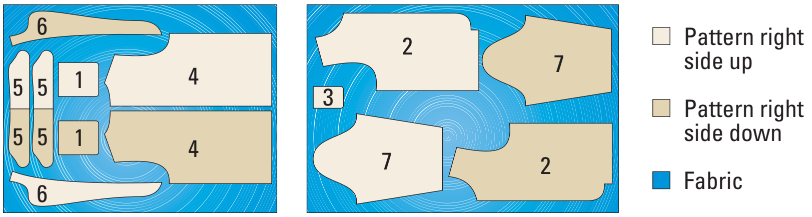
29. Translation 1: $(x, y) \to (x + 7, y - 2)$
Translation 2: $(x, y) \to (x - 1, y + 3)$
Translation: $(x, y) \to (\underline{\ ?\ }, \underline{\ ?\ })$

30. Translation 1: $(x, y) \to (x + 9, y + 4)$
Translation 2: $(x, y) \to (x + 6, y - 4)$
Translation: $(x, y) \to (\underline{\ ?\ }, \underline{\ ?\ })$

31. **STENCILING A BORDER** The border pattern below was made with a stencil. Describe how the border was created using one stencil four times.



CLOTHING PATTERNS The diagram shows the pattern pieces for a jacket arranged on some blue fabric.



32. Which pattern pieces are translated?

33. Which pattern pieces are reflected?

34. Which pattern pieces are glide reflected?

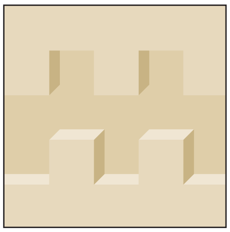
FOCUS ON CAREERS

REAL LIFE **ARCHITECTURAL HISTORIANS** play an important role in the renovation of an old building. Their expertise is used to help restore a building to its original splendor.

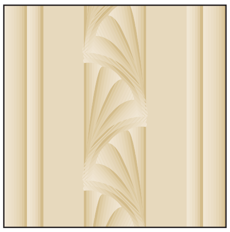
INTERNET **CAREER LINK**
www.mcdougallittell.com

ARCHITECTURE In Exercises 35–37, describe the transformations that are combined to create the pattern in the architectural element.

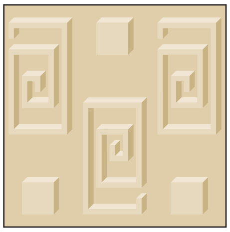
35.



36.



37.



38. **PENTOMINOES** Use compositions of transformations to describe how to pick up and arrange the tiles to complete the 6×10 rectangle.

Test Preparation

39. **MULTI-STEP PROBLEM** Follow the steps below.

   a. On a coordinate plane, draw a point and its image after a glide reflection that uses the x-axis as the line of reflection.

   b. Connect the point and its image. Make a conjecture about the midpoint of the segment.

   c. Use the coordinates from part (a) to prove your conjecture.

   d. **CRITICAL THINKING** Can you extend your conjecture to include glide reflections that do not use the x-axis as the line of reflection?

★ Challenge

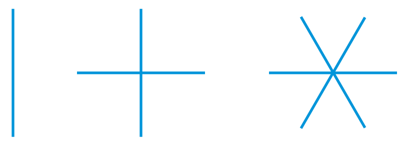
40. **USING ALGEBRA** Solve for the variables in the glide reflection of $\triangle JKL$ described below.

$J(-2, -1)$	**Translate**	$J'(c + 1, -1)$	**Reflect**	$J''(1, -f)$
$K(-4, 2a)$	$(x, y) \rightarrow (x + 3, y)$	$K'(5d - 11, 4)$	**in x-axis**	$K''(-1, 3g + 5)$
$L(b - 6, 6)$		$L'(2, 4e)$		$L''(h + 4, -6)$

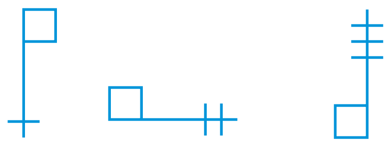
EXTRA CHALLENGE
www.mcdougallittell.com

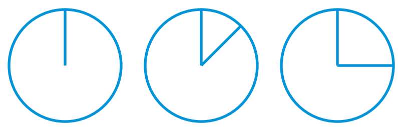
MIXED REVIEW

ANALYZING PATTERNS **Sketch the next figure in the pattern.** (Review 1.1 for 7.6)

41. 

42. 

43. 

44. 

COORDINATE GEOMETRY **In Exercises 45–47, decide whether ▱$PQRS$ is a *rhombus*, a *rectangle*, or a *square*. Explain your reasoning.** (Review 6.4)

45. $P(1, -2)$, $Q(5, -1)$, $R(6, -5)$, $S(2, -6)$

46. $P(10, 7)$, $Q(15, 7)$, $R(15, 1)$, $S(10, 1)$

47. $P(8, -4)$, $Q(10, -7)$, $R(8, -10)$, $S(6, -7)$

48. **ROTATIONS** A segment has endpoints $(3, -8)$ and $(7, -1)$. If the segment is rotated 90° counterclockwise about the origin, what are the endpoints of its image? (Review 7.3)

STUDYING TRANSLATIONS **Sketch $\triangle ABC$ with vertices $A(-9, 7)$, $B(-9, 1)$, and $C(-5, 6)$. Then translate the triangle by the given vector and name the vertices of the image.** (Review 7.4)

49. $\langle 3, 2\rangle$
50. $\langle -1, -5\rangle$
51. $\langle 6, 0\rangle$
52. $\langle -4, -4\rangle$
53. $\langle 0, 2.5\rangle$
54. $\langle 1.5, -4.5\rangle$

7.6 Frieze Patterns

What you should learn

GOAL 1 Use transformations to classify frieze patterns.

GOAL 2 Use frieze patterns to design border patterns in **real life**, such as the tiling pattern in **Example 4**.

Why you should learn it

▼ You can use frieze patterns to create decorative borders for **real-life** objects, such as the pottery below and the pottery in **Exs. 35–37**.

REAL LIFE

GOAL 1 CLASSIFYING FRIEZE PATTERNS

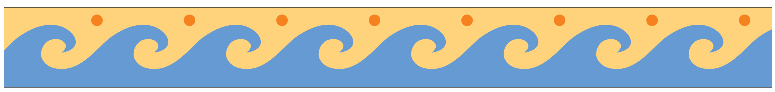
A **frieze pattern** or **border pattern** is a pattern that extends to the left and right in such a way that the pattern can be mapped onto itself by a horizontal translation. In addition to being mapped onto itself by a horizontal translation, some frieze patterns can be mapped onto themselves by other transformations.

1. Translation	T
2. 180° rotation	R
3. Reflection in a horizontal line	H
4. Reflection in a vertical line	V
5. Horizontal glide reflection	G

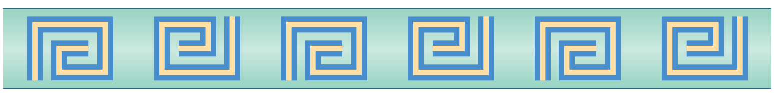
EXAMPLE 1 *Describing Frieze Patterns*

Describe the transformations that will map each frieze pattern onto itself.

a.

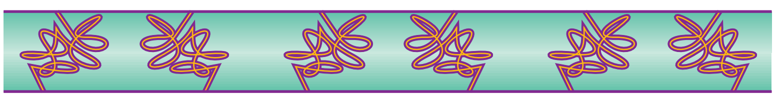


b.



c.

d.



SOLUTION

a. This frieze pattern can be mapped onto itself by a horizontal translation (T).

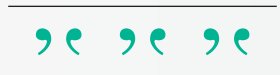
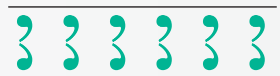
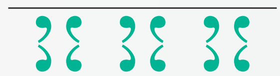
b. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a 180° rotation (R).

c. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a horizontal glide reflection (G).

d. This frieze pattern can be mapped onto itself by a horizontal translation (T) or by a reflection in a vertical line (V).

CONCEPT SUMMARY

CLASSIFICATIONS OF FRIEZE PATTERNS

T	Translation	
TR	Translation and 180° rotation	
TG	Translation and horizontal glide reflection	
TV	Translation and vertical line reflection	
THG	Translation, horizontal line reflection, and horizontal glide reflection	
TRVG	Translation, 180° rotation, vertical line reflection, and horizontal glide reflection	
TRHVG	Translation, 180° rotation, horizontal line reflection, vertical line reflection, and horizontal glide reflection	

STUDENT HELP

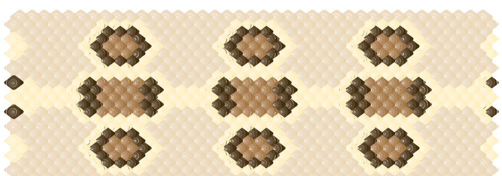
Study Tip
To help classify a frieze pattern, you can use a process of elimination. This process is described at the right and in the tree diagram in **Ex. 53**.

To classify a frieze pattern into one of the seven categories, you first decide whether the pattern has 180° rotation. If it does, then there are three possible classifications: TR, TRVG, and TRHVG.

If the frieze pattern does not have 180° rotation, then there are four possible classifications: T, TV, TG, and THG. Decide whether the pattern has a line of reflection. By a process of elimination, you will reach the correct classification.

EXAMPLE 2 *Classifying a Frieze Pattern*

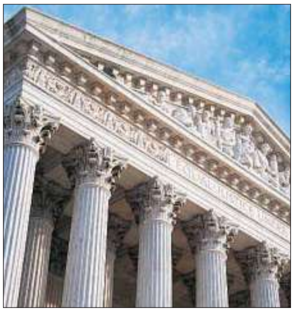
SNAKES Categorize the snakeskin pattern of the mountain adder.



SOLUTION

This pattern is a TRHVG. The pattern can be mapped onto itself by a translation, a 180° rotation, a reflection in a horizontal line, a reflection in a vertical line, and a horizontal glide reflection.

FOCUS ON APPLICATIONS

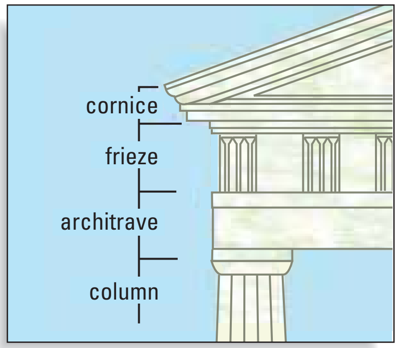


ARCHITECTURE Features of classical architecture from Greece and Rome are seen in "neo-classical" buildings today, such as the Supreme Court building shown.

GOAL 2 USING FRIEZE PATTERNS IN REAL LIFE

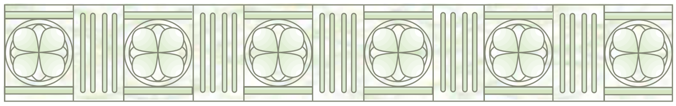
EXAMPLE 3 *Identifying Frieze Patterns*

ARCHITECTURE The frieze patterns of ancient Doric buildings are located between the cornice and the architrave, as shown at the right. The frieze patterns consist of alternating sections. Some sections contain a person or a symmetric design. Other sections have simple patterns of three or four vertical lines.

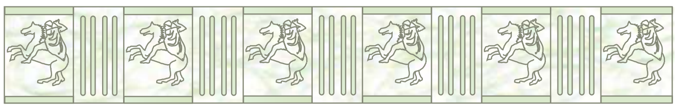


Portions of two frieze patterns are shown below. Classify the patterns.

a.



b.



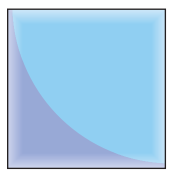
SOLUTION

a. Following the diagrams on the previous page, you can see that this frieze pattern has rotational symmetry, line symmetry about a horizontal line and a vertical line, and that the pattern can be mapped onto itself by a glide reflection. So, the pattern can be classified as TRHVG.

b. The only transformation that maps this pattern onto itself is a translation. So, the pattern can be classified as T.

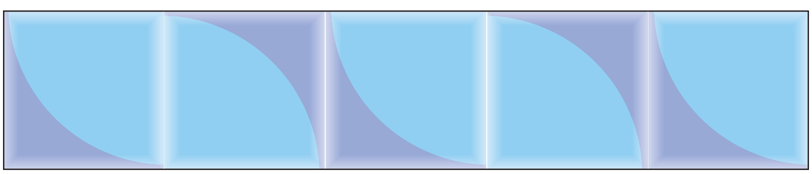
EXAMPLE 4 *Drawing a Frieze Pattern*

TILING A border on a bathroom wall is created using the decorative tile at the right. The border pattern is classified as TR. Draw one such pattern.



SOLUTION

Begin by rotating the given tile 180°. Use this tile and the original tile to create a pattern that has rotational symmetry. Then translate the pattern several times to create the frieze pattern.



GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the term *frieze pattern* in your own words.

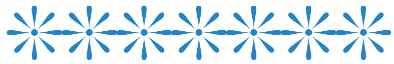
Concept Check ✓

2. **ERROR ANALYSIS** Describe Lucy's error below.

Skill Check ✓

In Exercises 3–6, describe the transformations that map the frieze pattern onto itself.

3. 

4. 

5. 

6. 

7. List the five possible transformations, along with their letter abbreviations, that can be found in a frieze pattern.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 816.

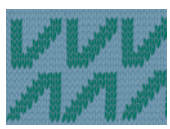
SWEATER PATTERN **Each row of the sweater is a frieze pattern. Match the row with its classification.**

A. TRHVG **B.** TR **C.** TRVG **D.** THG

8. 

9. 

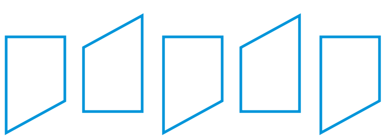
10. 

11. 

CLASSIFYING PATTERNS **Name the isometries that map the frieze pattern onto itself.**

12. 

13. 

14. 

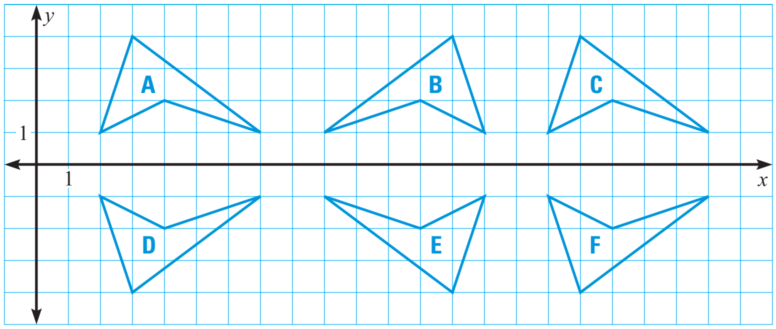
15. 

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 8–15
Example 2: Exs. 16–23
Example 3: Exs. 32–39
Example 4: Exs. 40–43

DESCRIBING TRANSFORMATIONS **Use the diagram of the frieze pattern.**



16. Is there a reflection in a vertical line? If so, describe the reflection(s).

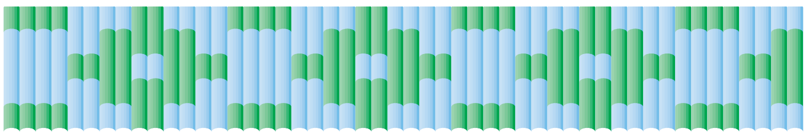
17. Is there a reflection in a horizontal line? If so, describe the reflection(s).

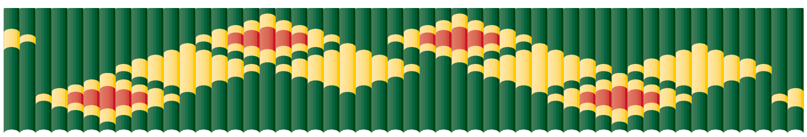
18. Name and describe the transformation that maps A onto F.

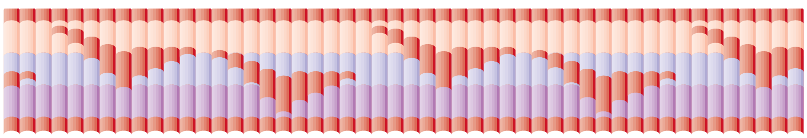
19. Name and describe the transformation that maps D onto B.

20. Classify the frieze pattern.

PET COLLARS **In Exercises 21–23, use the chart on page 438 to classify the frieze pattern on the pet collars.**

21. 

22. 

23. 

24. **TECHNOLOGY** Pick one of the seven classifications of patterns and use geometry software to create a frieze pattern of that classification. Print and color your frieze pattern.

25. **DATA COLLECTION** Use a library, magazines, or some other reference source to find examples of frieze patterns. How many of the seven classifications of patterns can you find?

CREATING A FRIEZE PATTERN **Use the design below to create a frieze pattern with the given classification.**

26. TR

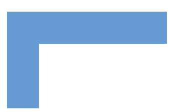
27. TV

28. TG

29. THG

30. TRVG

31. TRHVG



FOCUS ON APPLICATIONS

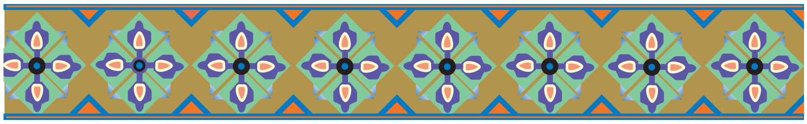
NIKKO MEMORIAL

The building shown is a memorial to Tokugawa Ieyasu (1543–1616), the founder of the Tokugawa Shogunate.

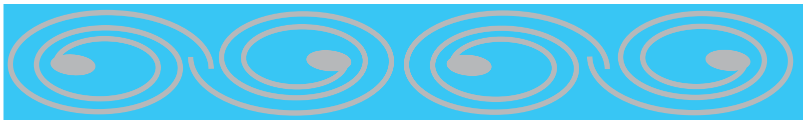
JAPANESE PATTERNS **The patterns shown were used in Japan during the Tokugawa Shogunate. Classify the frieze patterns.**

32.

33.



34.



POTTERY **In Exercises 35–37, use the pottery shown below. This pottery was created by the Acoma Indians. The Acoma pueblo is America's oldest continually inhabited city.**

35. Identify any frieze patterns on the pottery.

36. Classify the frieze pattern(s) you found in Exercise 35.

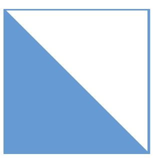
37. Create your own frieze pattern similar to the patterns shown on the pottery.

38. Look back to the southwestern pottery on page 437. Describe and classify one of the frieze patterns on the pottery.

39. **LOGICAL REASONING** You are decorating a large circular vase and decide to place a frieze pattern around its base. You want the pattern to consist of ten repetitions of a design. If the diameter of the base is about 9.5 inches, how wide should each design be?

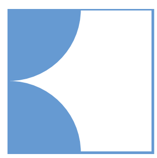
TILING **In Exercises 40–42, use the tile to create a border pattern with the given classification. Your border should consist of one row of tiles.**

40. TR



41. TRVG

42. TRHVG



43. *Writing* Explain how the design of the tiles in Exercises 40–42 is a factor in the classification of the patterns. For instance, could the tile in Exercise 40 be used to create a single row of tiles classified as THG?

CRITICAL THINKING **Explain why the combination is not a category for frieze pattern classification.**

44. TVG

45. THV

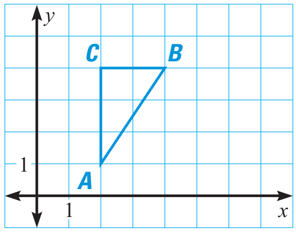
46. TRG

STUDENT HELP

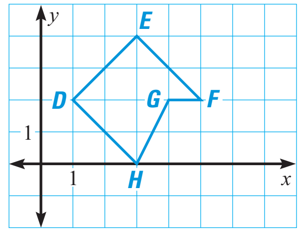
HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Exs. 47 and 48.

USING THE COORDINATE PLANE **The figure shown in the coordinate plane is part of a frieze pattern with the given classification. Copy the graph and draw the figures needed to complete the pattern.**

47. TR



48. TRVG

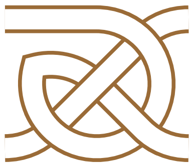


Test Preparation

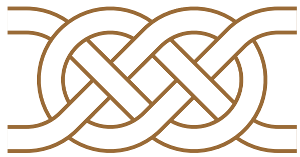
MULTI-STEP PROBLEM **In Exercises 49–52, use the following information.**

In Celtic art and design, border patterns are used quite frequently, especially in jewelry. Three different designs are shown.

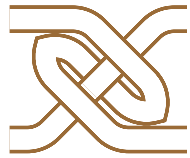
A.



B.



C.



49. Use translations to create a frieze pattern of each design.

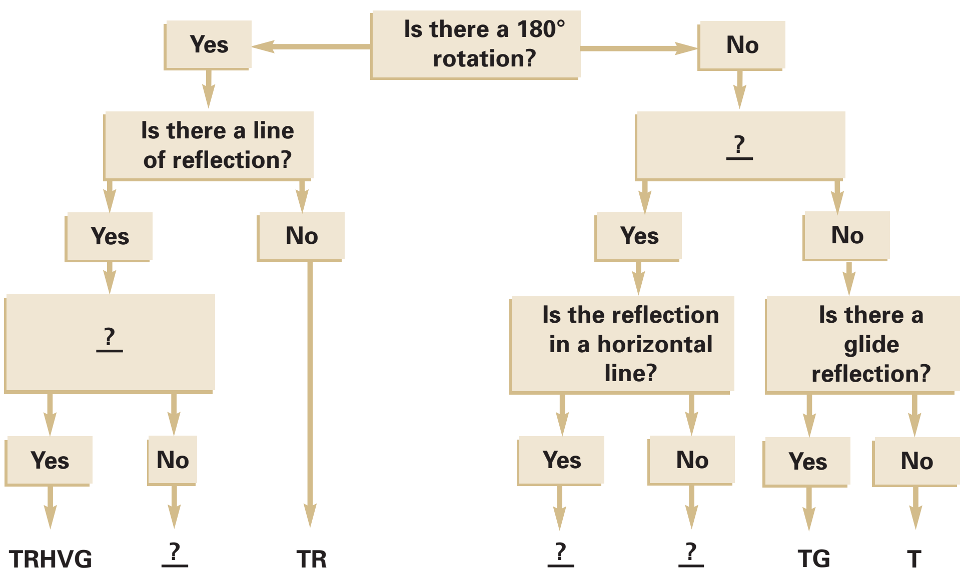
50. Classify each frieze pattern that you created.

51. Which design does not have rotational symmetry? Use rotations to create a new frieze pattern of this design.

52. *Writing* If a design has 180° rotational symmetry, it cannot be used to create a frieze pattern with classification *T*. Explain why not.

★ **Challenge**

53. **TREE DIAGRAM** The following tree diagram can help classify frieze patterns. Copy the tree diagram and fill in the missing parts.



Mixed Review

RATIOS **Find the ratio of girls to boys in a class, given the number of boys and the total number of students.** **(Skills Review for 8.1)**

54. 12 boys, 23 students

55. 8 boys, 21 students

56. 3 boys, 13 students

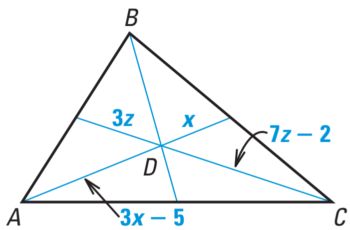
57. 19 boys, 35 students

58. 11 boys, 18 students

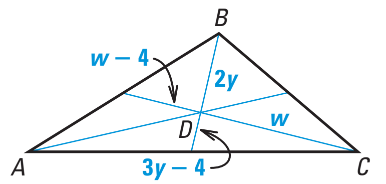
59. 10 boys, 20 students

PROPERTIES OF MEDIANS **Given that D is the centroid of $\triangle ABC$, find the value of each variable.** **(Review 5.3)**

60.

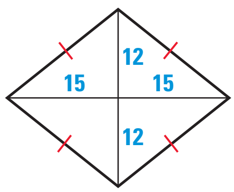


61.

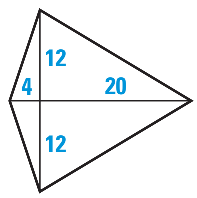


FINDING AREA **Find the area of the quadrilateral.** **(Review 6.7)**

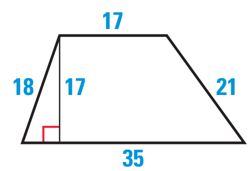
62.



63.



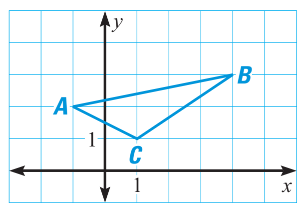
64.



Quiz 2

Self-Test for Lessons 7.4–7.6

Write the coordinates of the vertices A', B', and C' after $\triangle ABC$ is translated by the given vector. **(Lesson 7.4)**



1. $\langle 1, 3\rangle$

2. $\langle -3, 4\rangle$

3. $\langle -2, -4\rangle$

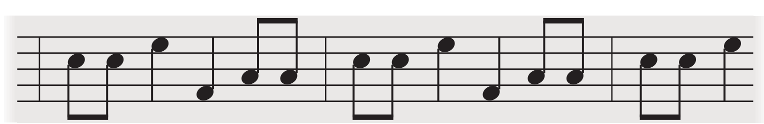
4. $\langle 5, 2\rangle$

In Exercises 5 and 6, sketch the image of $\triangle PQR$ after a composition using the given transformations in the order they appear. **(Lesson 7.5)**

5. $P(5, 1)$, $Q(3, 4)$, $R(0, 1)$
Translation: $(x, y) \rightarrow (x - 2, y - 4)$
Reflection: in the y-axis

6. $P(7, 2)$, $Q(3, 1)$, $R(6, -1)$
Translation: $(x, y) \rightarrow (x - 4, y + 3)$
Rotation: 90° clockwise about origin

7. **MUSICAL NOTES** Do the notes shown form a frieze pattern? If so, classify the frieze pattern. **(Lesson 7.6)**



CHAPTER 7

Chapter Summary

WHAT did you learn?	WHY did you learn it?
Identify types of rigid transformations. (7.1)	Plan a stencil pattern, using one design repeated many times. (p. 401)
Use properties of reflections. (7.2)	Choose the location of a telephone pole so that the length of the cable is a minimum. (p. 405)
Relate reflections and line symmetry. (7.2)	Understand the construction of the mirrors in a kaleidoscope. (p. 406)
Relate rotations and rotational symmetry. (7.3)	Use rotational symmetry to design a logo. (p. 415)
Use properties of translations. (7.4)	Use vectors to describe the path of a hot-air balloon. (p. 427)
Use properties of glide reflections. (7.5)	Describe the transformations in patterns in architecture. (p. 435)
Classify frieze patterns. (7.6)	Identify the frieze patterns in pottery. (p. 442)

How does Chapter 7 fit into the BIGGER PICTURE of geometry?

In this chapter, you learned that the basic rigid transformations in the plane are reflections, rotations, translations, and glide reflections. Rigid transformations are closely connected to the concept of congruence. That is, two plane figures are congruent if and only if one can be mapped onto the other by exactly one rigid transformation or by a composition of rigid transformations. In the next chapter, you will study transformations that are not rigid. You will learn that some nonrigid transformations are closely connected to the concept of similarity.

STUDY STRATEGY

How did making sample exercises help you?

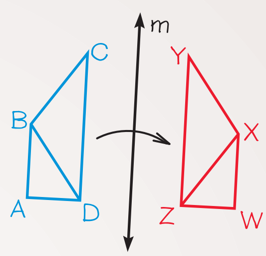
Some sample exercises you made, following the **Study Strategy** on p. 394, may resemble these.

Lesson 7.1 Rigid Motion in a Plane

Summary: This lesson is about the three basic rigid transformations—reflections, rotations, and translations.

Sample Exercises:

1. Name the preimage of $\overline{XW}$.
2. Name the image of $\overline{BC}$.
3. Name a triangle that appears to be congruent to $\triangle DBC$.



CHAPTER 7

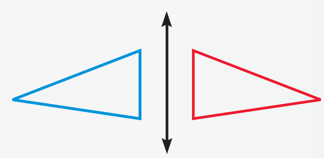
Chapter Review

- image, p. 396
- preimage, p. 396
- transformation, p. 396
- isometry, p. 397
- reflection, p. 404
- line of reflection, p. 404
- line of symmetry, p. 406
- rotation, p. 412
- center of rotation, p. 412
- angle of rotation, p. 412
- rotational symmetry, p. 415
- translation, p. 421
- vector, p. 423
- initial point, p. 423
- terminal point, p. 423
- component form, p. 423
- glide reflection, p. 430
- composition, p. 431
- frieze pattern, or border pattern, p. 437

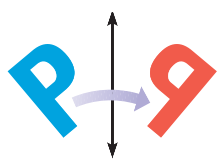
7.1 RIGID MOTION IN A PLANE

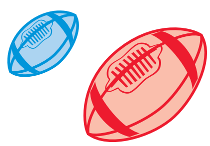
Examples on pp. 396–398

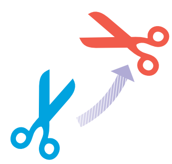
EXAMPLE The blue triangle is reflected to produce the congruent red triangle, so the transformation is an isometry.



Does the transformation appear to be an isometry? Explain.

1. 

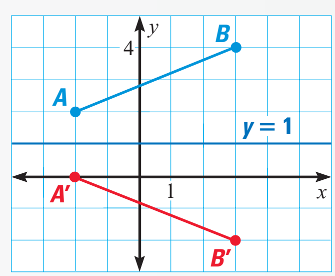
2. 

3. 

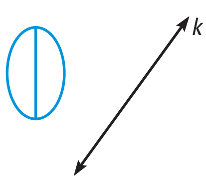
7.2 REFLECTIONS

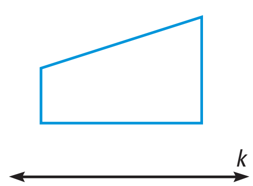
Examples on pp. 404–406

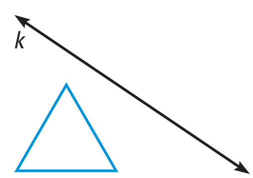
EXAMPLE In the diagram, $\overline{AB}$ is reflected in the line $y = 1$, so $\overline{A'B'}$ has endpoints $A'(-2, 0)$ and $B'(3, -2)$.



Copy the figure and draw its reflection in line *k*.

4. 

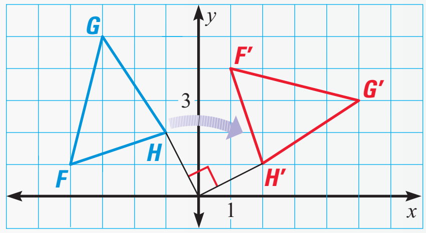
5. 

6. 

7.3 ROTATIONS

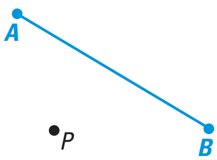
Examples on pp. 412–415

EXAMPLE In the diagram, $\triangle FGH$ is rotated 90° clockwise about the origin.

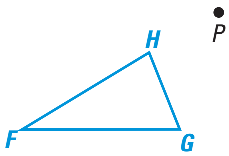


Copy the figure and point P. Then, use a straightedge, a compass, and a protractor to rotate the figure 60° counterclockwise about P.

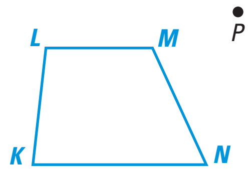
7.



8.



9.

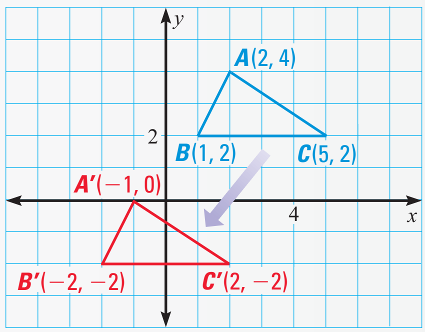


7.4 TRANSLATIONS AND VECTORS

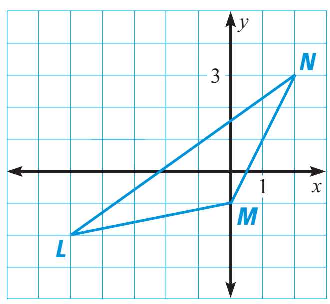
Examples on pp. 421–424

EXAMPLE Using the vector $\langle -3, -4\rangle$, $\triangle ABC$ can be translated to $\triangle A'B'C'$.

$A(2, 4)$	$A'(-1, 0)$
$B(1, 2)$	$B'(-2, -2)$
$C(5, 2)$	$C'(2, -2)$



The vertices of the image of $\triangle LMN$ after a translation are given. Choose the vector that describes the translation.



10. $L'(-1, -3), M'(4, -2), N'(6, 2)$

11. $L'(-5, 1), M'(0, 2), N'(2, 6)$

12. $L'(-3, 2), M'(2, 3), N'(4, 7)$

13. $L'(-7, 3), M'(-2, 4), N'(0, 8)$

A. $\overrightarrow{PQ} = \langle 0, 3\rangle$

B. $\overrightarrow{PQ} = \langle -2, 5\rangle$

C. $\overrightarrow{PQ} = \langle 4, -1\rangle$

D. $\overrightarrow{PQ} = \langle 2, 4\rangle$

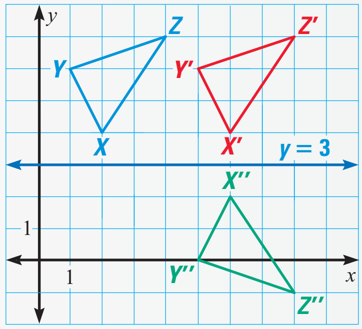
7.5 GLIDE REFLECTIONS AND COMPOSITIONS

Examples on pp. 430–432

EXAMPLE The diagram shows the image of $\triangle XYZ$ after a glide reflection.

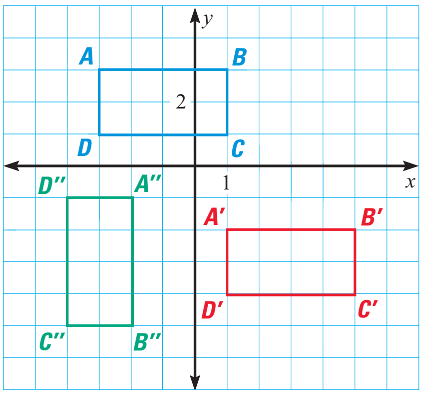
Translation: $(x, y) \rightarrow (x + 4, y)$

Reflection: in the line $y = 3$

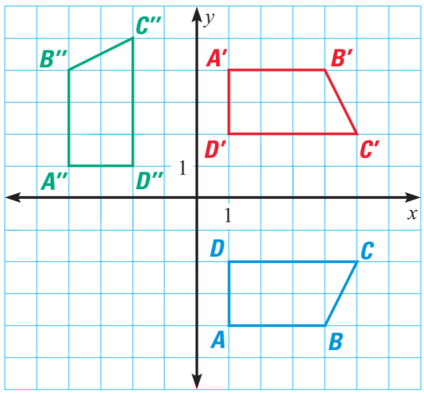


Describe the composition of the transformations.

14.



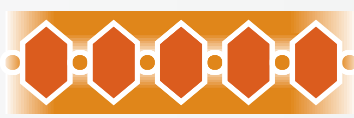
15.



7.6 FRIEZE PATTERNS

Examples on pp. 437–439

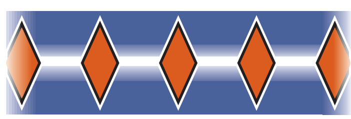
EXAMPLE The corn snake frieze pattern at the right can be classified as TRHVG because the pattern can be mapped onto itself by a translation, 180° rotation, horizontal line reflection, vertical line reflection, and glide reflection.



Classify the snakeskin frieze pattern.

16. Rainbow boa

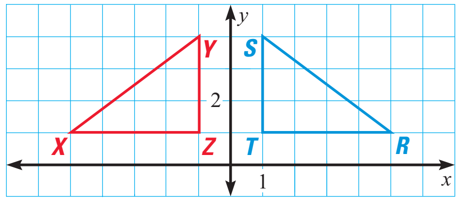
17. Gray-banded kingsnake



CHAPTER 7

Chapter Test

In Exercises 1–4, use the diagram.



1. Identify the transformation $\triangle RST \rightarrow \triangle XYZ$.

2. Is $\overline{RT}$ congruent to $\overline{XZ}$?

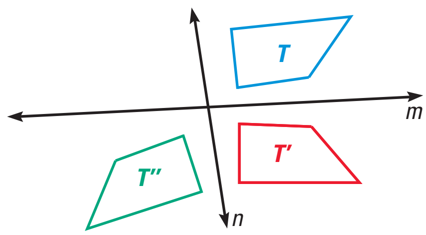
3. What is the image of T?

4. What is the preimage of Y?

5. Sketch a polygon that has line symmetry, but not rotational symmetry.

6. Sketch a polygon that has rotational symmetry, but not line symmetry.

Use the diagram, in which lines *m* and *n* are lines of reflection.

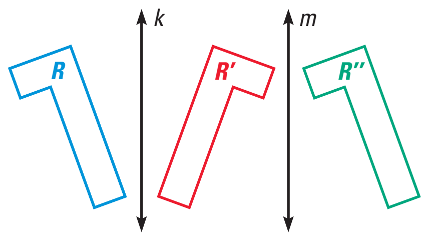


7. Identify the transformation that maps figure T onto figure T'.

8. Identify the transformation that maps figure T onto figure T''.

9. If the measure of the acute angle between m and n is 85°, what is the angle of rotation from figure T to figure T''?

In Exercises 10–12, use the diagram, in which $k \parallel m$.



10. Identify the transformation that maps figure R onto figure R'.

11. Identify the transformation that maps figure R onto figure R''.

12. If the distance between k and m is 5 units, what is the distance between corresponding parts of figure R and figure R''?

13. What type of transformation is a composition of a translation followed by a reflection in a line parallel to the translation vector?

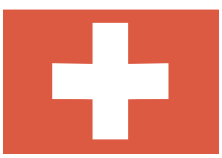
Give an example of the described composition of transformations.

14. The order in which two transformations are performed affects the final image.

15. The order in which two transformations are performed does not affect the final image.

FLAGS Identify any symmetry in the flag.

16. Switzerland

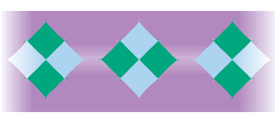


17. Jamaica

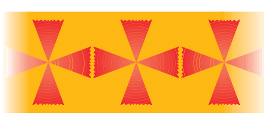
18. United Kingdom

Name all of the isometries that map the frieze pattern onto itself.

19.



20.



21.