Chapter 1 Essentials of Geometry

Prerequisite Skills for the chapter "Essentials of Geometry"

1. The distance around a polygon is called its *perimeter*, and the distance around a circle is called its *circumference*.
2. The number of square units covered by a figure is called its *area*.
3. $|4 - 6| = |-2| = 2$
4. $|3 - 11| = |-8| = 8$
5. $|-4 + 5| = |1| = 1$
6. $|-8 - 10| = |-18| = 18$
7. $5x = 5(2) = 10$
8. $20 - 8x = 20 - 8(2) = 20 - 16 = 4$
9. $-18 + 3x = -18 + 3(2) = -18 + 6 = -12$
10. $-5x - 4 + 2x = -5(2) - 4 + 2(2)$
    $= -10 - 4 + 4 = -10$

11. $274 = -2z$
    $-137 = z$

12. $8x + 12 = 60$
    $8x = 48$
    $x = 6$

13. $2y - 5 + 7y = -32$
    $9y - 5 = -32$
    $9y = -27$
    $y = -3$

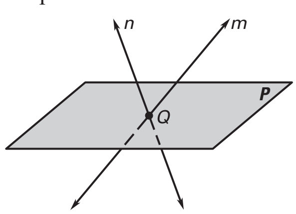
14. $6p + 11 + 3p = -7$
    $9p + 11 = -7$
    $9p = -18$
    $p = -2$

15. $4 + \frac{m}{7} = 10$
    $\frac{m}{7} = 6$
    $m = 42$

16. $5n - 8 = 47$
    $5n = 55$
    $n = 11$

Lesson 1.1 Identify Points, Lines, and Planes

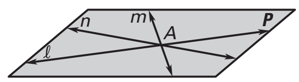
Guided Practice for the lesson "Identify Points, Lines, and Planes"

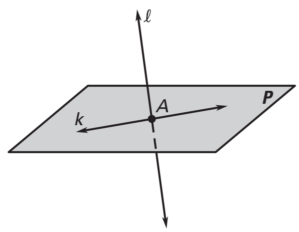
1. Other names for $\overleftrightarrow{ST}$ are $\overleftrightarrow{TS}$ and line *m*. Point *V* is not coplanar with points *Q*, *S*, and *T*.
2. Another name for $\overline{EF}$ is $\overline{FE}$.
3. $\overrightarrow{HJ}$ and $\overrightarrow{JH}$ are not the same ray. They have different endpoints and continue in different directions. $\overrightarrow{HJ}$ and $\overrightarrow{HG}$ are the same ray because they have the same endpoint and continue in the same direction.
4. 

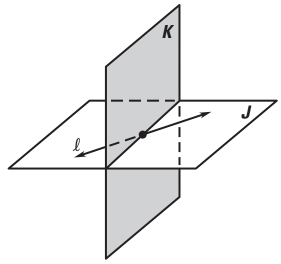
5. $\overleftrightarrow{PQ}$ intersects line *k* at point *M*.
6. Plane *A* intersects plane *B* at line *k*.
7. Line *k* intersects plane *A* at line *k*.

Exercises for the lesson "Identify Points, Lines, and Planes"

Skill Practice

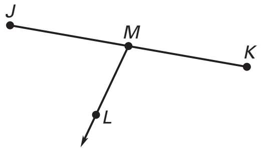
1. **a.** Point *Q* **b.** Segment *MN*
   c. Ray *ST* **d.** Line *FG*
2. Collinear points lie on the same line, so they are also coplanar. Coplanar points lie in the same plane, but not necessarily on the same line, so they may not be collinear.
3. Other names for $\overleftrightarrow{WQ}$ are $\overleftrightarrow{QW}$ and line *g*.
4. Another name for plane *V* is plane *QST*.
5. Points *R*, *Q*, and *S* are collinear. Point *T* is not collinear with those points.
6. Point *W* is not coplanar with points *R*, *S*, and *T*.
7. Point *W* is coplanar with points *Q* and *R* because there is only one plane through any 3 points not on the same line.
8. Another name for $\overline{ZY}$ is $\overline{YZ}$.
9. $\overrightarrow{VY}$, $\overrightarrow{VX}$, $\overrightarrow{VZ}$, $\overrightarrow{VW}$
10. $\overrightarrow{VY}$ and $\overrightarrow{VZ}$; $\overrightarrow{VX}$ and $\overrightarrow{VW}$
11. Another name for $\overrightarrow{WV}$ is $\overrightarrow{WX}$.
12. $\overrightarrow{VW}$ and $\overrightarrow{VZ}$ do have the same endpoints, but points *W* and *Z* are not on the same line, so the rays are not opposite.
13. B; *C*, *D*, *E*, and *G* are coplanar.
14. 

15. 

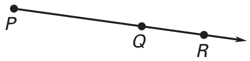
16. A; $\overrightarrow{EC}$ and $\overrightarrow{ED}$ are opposite rays.
17. $\overleftrightarrow{PR}$ intersects $\overleftrightarrow{HR}$ at point *R*.
18. Plane *EFG* and plane *FGS* intersect at $\overleftrightarrow{FG}$.
19. Plane *PQS* and plane *HGS* intersect at $\overleftrightarrow{RS}$.
20. *P*, *Q*, and *F* are not collinear, but they are coplanar.
21. *P* and *G* are neither collinear nor coplanar.
22. Planes *HEF*, *PEF*, and *PEH*, intersect at point *E*.
23. 

24. *Sample answer:* $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{AC}$, $\overrightarrow{AB}$, $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{BE}$, $\overrightarrow{CB}$, $\overrightarrow{EB}$, $\overrightarrow{DC}$; $\overrightarrow{CA}$ and $\overrightarrow{CD}$ are opposite rays and $\overrightarrow{BA}$ and $\overrightarrow{BE}$ are opposite rays.

25.



26.



27. $y = x - 4$; $A(5, 1)$

$1 \stackrel{?}{=} 5 - 4$

$1 = 1$ ✓

$A(5, 1)$ is on the line.

28. $y = x + 1$; $A(1, 0)$

$0 \stackrel{?}{=} 1 + 1$

$0 \neq 2$

$A(1, 0)$ is not on the line.

29. $y = 3x + 4$; $A(7, 1)$

$1 \stackrel{?}{=} 3(7) + 4$

$1 \neq 25$

$A(7, 1)$ is not on the line.

30. $y = 4x + 2$; $A(1, 6)$

$6 \stackrel{?}{=} 4(1) + 2$

$6 = 6$ ✓

$A(1, 6)$ is on the line.

31. $y = 3x - 2$; $A(-1, -5)$

$-5 \stackrel{?}{=} 3(-1) - 2$

$-5 = -5$ ✓

$A(-1, -5)$ is on the line.

32. $y = -2x + 8$; $A(-4, 0)$

$0 \stackrel{?}{=} -2(-4) + 8$

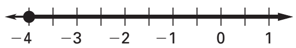
$0 \neq 16$

$A(-4, 0)$ is not on the line.

33. $x \le 3$

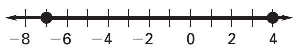
The graph is a ray.

34. $x \ge -4$



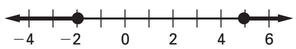
The graph is a ray.

35. $-7 \le x \le 4$



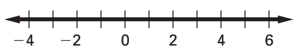
The graph is a segment.

36. $x \ge 5$ or $x \le -2$



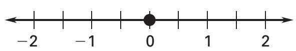
The graph is a pair of rays.

37. $x \ge -1$ or $x \le 5$



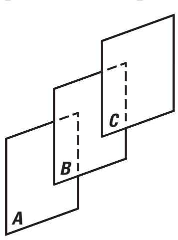
The graph is a line.

38. $|x| \le 0$

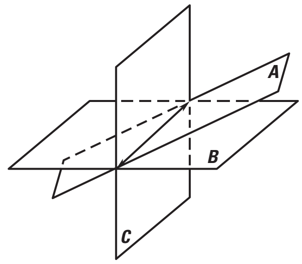


The graph is a point.

39. **a.** Three planes that do not intersect are possible if the planes are all parallel.

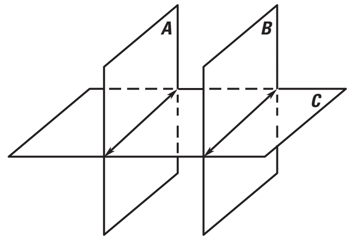


b. It is possible to have three planes that intersect in one line. The planes intersect at $\overleftrightarrow{AB}$.



c. This is not possible because planes intersect at a line, not a point.

d. It is possible for a third plane to intersect two planes that do not intersect, if the two planes are parallel and the other plane intersects those two.



e. This is not possible because the third plane cannot be parallel to both of the other intersecting planes, so it must intersect at least one of them.

Problem Solving

40. Intersections of several lines

41. Intersection of a line with a plane

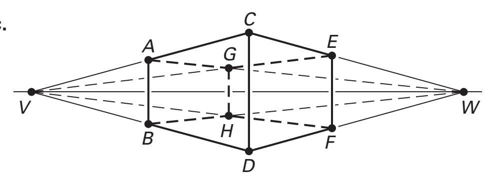
42. Planes intersecting with planes

43. A four-legged table may rock from side to side because four points are not necessarily coplanar. A three-legged table would not rock because three points determine a unique plane.

44. **a.** When the tripod is on a level surface, the tips of the legs are coplanar.

b. The tips of the legs are still coplanar because three points determine a unique plane.

45. **a–c.**



46. **a.** If there are 5 streets, there must be 10 traffic lights. If there are 6 streets, there must be 15 traffic lights.

b. Each time a street is added, the number of additional traffic lights that are needed is equal to the previous number of streets.

Lesson 1.2 Use Segments and Congruence

Guided Practice for the lesson "Use Segments and Congruence"

1. $1\frac{5}{8}$ inches

2. $1\frac{3}{8}$ inches

3. $XY + YZ = XZ$

$23 + 50 = 73$

The length of $\overline{XZ}$ is 73 units.

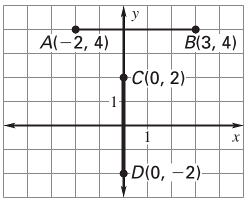
4. You cannot use the Segment Addition Postulate to find the length of $\overline{WZ}$ given $WY = 30$ because Y is not between W and Z.

5. $VW + WX = VX$

$37 + WX = 144$

$WX = 107$

6.



$\overline{AB}$ and $\overline{CD}$ are not congruent, because $AB = |3 - (-2)| = 5$ and $CD = |-2 - 2| = 4$.

Exercises for the lesson "Use Segments and Congruence"

Skill Practice

1. $\overline{MN}$ means the line segment MN, and MN means the distance between M and N.

2. You can find PN by adding PQ and QN. You can find PN by subtracting MP from MN.

3. 2.1 cm

4. 3.2 cm

5. 3.5 cm

6. $MN + NP = MP$

$5 + 18 = MP$

$23 = MP$

7. $RS + ST = RT$

$22 + 22 = RT$

$44 = RT$

8. $UV + VW = UW$

$39 + 26 = UW$

$65 = UW$

9. $XY + YZ = XZ$

$XY + 7 = 30$

$XY = 23$

10. $AB + BC = AC$

$27 + BC = 42$

$BC = 15$

11. $DE + EF = DF$

$DE + 50 = 63$

$DE = 13$

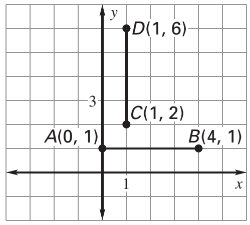
12. The Segment Addition Postulate was used incorrectly.

$AB + BC = AC$

$9 + BC = 14$

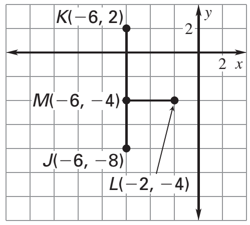
$BC = 5$

13.



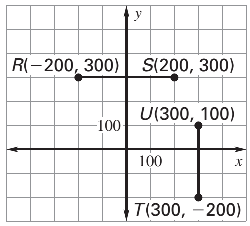
$AB = |4 - 0| = 4$ and $CD = |6 - 2| = 4$, so $\overline{AB} \cong \overline{CD}$.

14.



Because $JK = |-8 - 2| = 10$ and $LM = |-2 - (-6)| = 4$, $\overline{JK}$ is not congruent to $\overline{LM}$.

15.



Because $RS = |200 - (-200)| = 400$ and $TU = |-200 - 100| = 300$, $\overline{RS}$ is not congruent to $\overline{TU}$.

16. $JK = |-3 - (-6)| = |-3 + 6| = 3$

17. $JL = |1 - (-6)| = |1 + 6| = 7$

18. $JM = |6 - (-6)| = |6 + 6| = 12$

19. $KM = |6 - (-3)| = |6 + 3| = 9$

20. Yes, it is possible to show that $FB > CB$ using the Segment Addition Postulate. $FC + CB = FB$, so FB must be greater than FC and CB individually.
It is not possible to show that $AC > DB$ using the Segment Addition Postulate because B is not between A and C.

21. $XY = YZ = WX$

$XY + YZ = XZ$

$WX + WX = 20$

$2(WX) = 20$

$WX = 10$

22. $VW + WX + XZ = VZ$

$VW + 10 + 20 = 52$

$VW = 22$

23. $WY = WX + XY$

$WY = 10 + 10$

$WY = 20$

24. $VX + XZ = VZ$

$VX + 20 = 52$

$VX = 32$

25. $WZ = WX + XY + YZ$

$WZ = 10 + 10 + 10 = 30$

26. $VY = VW + WY$

$VY = 22 + 20 = 42$

27. C; $EF + FG = EG$

$$6 + x = 1.6x$$
$$6 = 0.6x$$
$$10 = x$$

28.

$$RS + ST = RT$$
$$2x + 10 + x - 4 = 21$$
$$3x + 6 = 21$$
$$3x = 15$$
$$x = 5$$
$$RS = 2(5) + 10 = 20$$
$$ST = 5 - 4 = 1$$

29.

$$RS + ST = RT$$
$$3x - 16 + 4x - 8 = 60$$
$$7x - 24 = 60$$
$$7x = 84$$
$$x = 12$$
$$RS = 3(12) - 16 = 20$$
$$ST = 4(12) - 8 = 40$$

30.

$$RS + ST = RT$$
$$2x - 8 + 3x - 10 = 17$$
$$5x - 18 = 17$$
$$5x = 35$$
$$x = 7$$
$$RS = 2(7) - 8 = 6$$
$$ST = 3(7) - 10 = 11$$

31. $AC + CD = 12$

$AC = CD = 6$

$AB + BC = 6$

$AB = BC = 3$

$AB = 3$, $BC = 3$, $AC = 6$, $CD = 6$, $BD = 9$, $AD = 12$

Because 4 of the 6 segments in the figure are longer than 3 units, the probability of choosing one of these is $\frac{4}{6}$ or $\frac{2}{3}$.

Problem Solving

32. Abdomen $= \left|2\frac{1}{4} - 0\right| = 2\frac{1}{4}$

thorax $= \left|4 - 2\frac{1}{4}\right| = 1\frac{3}{4}$

Its abdomen is $2\frac{1}{4} - 1\frac{3}{4} = \frac{1}{2}$ inch longer than its thorax.

33. a. $AB + BC = AC$

$1282 + 601 = 1883$

The total distance was 1883 miles.

b. $d = rt$

$1883 = r(38)$

$49.6 \approx r$

The airplane's average speed was about 50 miles per hour.

34. a. 2003: $11 - 6 = 5$

2004: $12 - 7 = 5$

2005: $13 - 8 = 5$

The length of the yellow bar represents the number of losses in that year.

b. 2003: $\frac{5}{11} = 0.45 = 45\%$

The team lost 45% of their games in 2003.

2004: $\frac{5}{12} = 0.42 = 42\%$

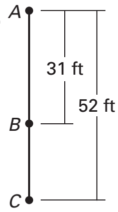
The team lost 42% of their games in 2004.

2005: $\frac{5}{13} = 0.38 = 38\%$

The team lost 38% of their games in 2005.

c. You apply the Segment Addition Postulate by subtracting one color of the stacked bar from the entire length of the bar, as you would subtract the length of a short line segment from the length of a longer segment that contains it.

35. a.



b. $AC - AB = BC$

$52 - 31 = BC$

$21 = BC$

The climber must descend 21 feet farther to reach the bottom.

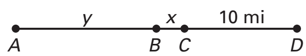
36.

	City *A*	**City *B***	**City *C***	**City *D***
City *A*		12.5 mi	15 mi	25 mi
City *B*	12.5 mi		2.5 mi	12.5 mi
City *C*	15 mi	2.5 mi		10 mi
City *D*	25 mi	12.5 mi	10 mi	

$AB = 5(BC)$

$AD = 2(AB)$

$CD = 10$



$$y = 5x$$
$$y + x + 10 = 2y$$
$$x + 10 = y$$
$$x + 10 = 5x$$
$$10 = 4x$$
$$2.5 = x$$
$$y = 5(2.5) = 12.5$$
$$AC = 12.5 + 2.5 = 15$$
$$AD = 15 + 10 = 25$$
$$BD = 2.5 + 10 = 12.5$$

Lesson 1.3 Use Midpoint and Distance Formulas

Guided Practice for the lesson "Use Midpoint and Distance Formulas"

1. $\overrightarrow{MN}$ is a segment bisector of $\overline{PQ}$.

$$PQ = 2\left(1\frac{7}{8}\right) = 3\frac{3}{4}$$

2. Line ℓ is a segment bisector of $\overline{PQ}$.

$5x - 7 = 11 - 2x$

$7x - 7 = 11$

$7x = 18$

$x = \frac{18}{7}$

$5\left(\frac{18}{7}\right) - 7 = \frac{41}{7}$

$PQ = 2\left(\frac{41}{7}\right) = \frac{82}{7}$

3. $M\left(\frac{1+7}{2}, \frac{2+8}{2}\right) = (4, 5)$

4. $\frac{4+x}{2} = -1$ $\qquad$ $\frac{4+y}{2} = -2$

$4 + x = -2$ $\qquad$ $4 + y = -4$

$x = -6$ $\qquad$ $y = -8$

The coordinates of endpoint V are $(-6, -8)$.

5. It does not matter which ordered pair you substitute for (x_1, y_1) or which you substitute for (x_2, y_2) because the distance between the two points is the same no matter which you start with.

6. B; $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$

$= \sqrt{(1 - (-3))^2 + (-4 - 2)^2}$

$= \sqrt{16 + 36}$

$= \sqrt{52} \approx 7.2$

The approximate length of $\overline{AB}$ is 7.2 units.

Exercises for the lesson "Use Midpoint and Distance Formulas"

Skill Practice

1. To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the *distance formula*.

2. To bisect a segment means to intersect a segment at its midpoint. You cannot bisect a line because it continues forever in both directions and, therefore, has no midpoint.

3. $RS = ST = 5\frac{1}{8}$ in.

$RS + ST = RT$

$5\frac{1}{8} + 5\frac{1}{8} = RT$

$RT = 10\frac{1}{4}$ in.

4. $UV = VW = \frac{5}{8}$ in.

$UV + VW = UW$

$\frac{5}{8} + \frac{5}{8} = UW$

$UW = 1\frac{1}{4}$ in.

5. $EF = FG = 13$ cm

$EF + FG = EG$

$13 + 13 = EG$

$EG = 26$ cm

6. $AB = BC = \frac{1}{2}(AC)$

$BC = \frac{1}{2}(19) = 9\frac{1}{2}$ cm

7. $PQ = QR = \frac{1}{2}(PR)$

$QR = \frac{1}{2}\left(9\frac{1}{2}\right) = 4\frac{3}{4}$ in.

8. $LM = MN = \frac{1}{2}(LN)$

$LM = \frac{1}{2}(137) = 68\frac{1}{2}$ mm

9. $RQ = \frac{1}{2}(PQ)$

$RQ = \frac{1}{2}\left(4\frac{3}{4}\right) = 2\frac{3}{8}$ in.

10. $UV = 2(UT)$

$UV = 2\left(2\frac{7}{8}\right) = 5\frac{3}{4}$ m

11. $x + 5 = 2x$

$5 = x$

$AM = x + 5$

$= 5 + 5 = 10$

12. $7x = 8x - 6$

$-x = -6$

$x = 6$

$EM = 7x = 7(6) = 42$

13. $6x + 7 = 4x + 5$

$2x + 7 = 5$

$2x = -2$

$x = -1$

$JM = 6x + 7$

$= 6(-1) + 7 = 1$

14. $6x - 11 = 10x - 51$

$-11 = 4x - 51$

$40 = 4x$

$10 = x$

$PR = 6x - 11 + 10x - 51$

$= 6(10) - 11 + 10(10) - 51$

$= 60 - 11 + 100 - 51 = 98$

15. $x + 15 = 4x - 45$

$60 = 3x$

$20 = x$

$SU = x + 15 + 4x - 45$

$= 20 + 15 + 4(20) - 45 = 70$

16. $2x + 35 = 5x - 22$

$35 = 3x - 22$

$57 = 3x$

$19 = x$

$XZ = 2x + 35 + 5x - 22$

$= 2(19) + 35 + 5(19) - 22 = 146$

17. $M\left(\frac{3+7}{2}, \frac{5+5}{2}\right) = M(5, 5)$

18. $M\left(\frac{0+4}{2}, \frac{4+3}{2}\right) = M(2, 3.5)$

19. $M\left(\frac{-4+6}{2}, \frac{4+4}{2}\right) = M(1, 4)$

20. $M\left(\frac{-7+(-3)}{2}, \frac{-5+7}{2}\right) = M(-5, 1)$

21. $M\left(\frac{-8+11}{2}, \frac{-7+5}{2}\right) = M(1.5, -1)$

22. $M\left(\frac{-3+(-8)}{2}, \frac{3+6}{2}\right) = M(-5.5, 4.5)$

23. Substitute the given numbers and variables into the midpoint formula.

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M\left(\frac{0 + m}{2}, \frac{0 + n}{2}\right)$$

Simplify the expression.

$$M\left(\frac{m}{2}, \frac{n}{2}\right)$$

24. When calculating the averages of the x-coordinates and of the y-coordinates, x_1 and x_2 and y_1 and y_2 should have been added, not subtracted.

$$\left(\frac{8 + 2}{2}, \frac{3 + (-1)}{2}\right) = (5, 1)$$

25. $B(3, 0)$, $M(0, 5)$

$\frac{3 + x}{2} = 0$ $\qquad$ $\frac{0 + y}{2} = 5$

$x = -3$ $\qquad$ $y = 10$

The other endpoint is $S(-3, 10)$.

26. $B(5, 1)$, $M(1, 4)$

$\frac{5 + x}{2} = 1$ $\qquad$ $\frac{1 + y}{2} = 4$

$5 + x = 2$ $\qquad$ $1 + y = 8$

$x = -3$ $\qquad$ $y = 7$

The other endpoint is $S(-3, 7)$.

27. $R(6, -2)$, $M(5, 3)$

$\frac{6 + x}{2} = 5$ $\qquad$ $\frac{-2 + y}{2} = 3$

$6 + x = 10$ $\qquad$ $-2 + y = 6$

$x = 4$ $\qquad$ $y = 8$

The other endpoint is $S(4, 8)$.

28. $R(-7, 11)$, $M(2, 1)$

$\frac{-7 + x}{2} = 2$ $\qquad$ $\frac{11 + y}{2} = 1$

$-7 + x = 4$ $\qquad$ $11 + y = 2$

$x = 11$ $\qquad$ $y = -9$

The other endpoint is $S(11, -9)$.

29. $R(4, -6)$, $M(-7, 8)$

$\frac{4 + x}{2} = -7$ $\qquad$ $\frac{-6 + y}{2} = 8$

$4 + x = -14$ $\qquad$ $-6 + y = 16$

$x = -18$ $\qquad$ $y = 22$

The other endpoint is $S(-18, 22)$.

30. $R(-4, -6)$, $M(3, -4)$

$\frac{-4 + x}{2} = 3$ $\qquad$ $\frac{-6 + y}{2} = -4$

$-4 + x = 6$ $\qquad$ $-6 + y = -8$

$x = 10$ $\qquad$ $y = -2$

The other endpoint is $S(10, -2)$.

31. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 1)^2 + (4 - 2)^2}$

$= \sqrt{16 + 4} = \sqrt{20} \approx 4.5$ units

32. $QR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - (-3))^2 + (3 - 5)^2}$

$= \sqrt{25 + 4} = \sqrt{29} \approx 5.4$ units

33. $ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - (-1))^2 + (-2 - 2)^2}$

$= \sqrt{16 + 16} = \sqrt{32} \approx 5.7$ units

34. D; $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(4 - (-3))^2 + (8 - (-9))^2}$

$= \sqrt{49 + 289} = \sqrt{338} \approx 18.4$ units.

35. The length of the segment is $|3 - (-4)| = |7| = 7$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-4 + 3}{2} = -\frac{1}{2}$.

36. The length of the segment is $|2 - (-6)| = |8| = 8$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-6 + 2}{2} = -2$.

37. The length of the segment is $|25 - (-15)| = |40| = 40$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-15 + 25}{2} = 5$.

38. The length of the segment is $|-5 - (-20)| = |15| = 15$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-20 + (-5)}{2} = -12.5$.

39. The length of the segment is $|1 - (-8)| = 9$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-8 + 1}{2} = -3.5$.

40. The length of the segment is $|-2 - (-7)| = 5$ units.

The midpoint has the coordinate $\frac{x_1 + x_2}{2} = \frac{-7 + (-2)}{2} = -4.5$.

41. B; $LF = \sqrt{(3 - (-2))^2 + (1 - 2)^2}$

$= \sqrt{25 + 1} = \sqrt{26} \approx 5.10$

$JR = \sqrt{(2 - 1)^2 + (-3 - (-1))^2}$

$= \sqrt{1 + 4} = \sqrt{5} \approx 2.24$

The approximate difference in lengths is $5.10 - 2.24 = 2.86$.

42. Substitute the coordinates of point P and of the midpoint into the distance formula to find the length of the segment from P to M.

$PM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1-(-2))^2 + (0-4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

If the distance from point P to the midpoint is 5 units, then the length of $\overline{PQ}$ is twice that, or $2(5) = 10$ units.

43. $AB = \sqrt{(-3-0)^2 + (8-2)^2}$

$= \sqrt{9+36} = \sqrt{45} \approx 6.71$

$CD = \sqrt{(0-(-2))^2 + (-4-2)^2}$

$= \sqrt{4+36} = \sqrt{40} \approx 6.32$

The segments are not congruent.

44. $EF = \sqrt{(5-1)^2 + (1-4)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$GH = \sqrt{(1-(-3))^2 + (6-1)^2}$

$= \sqrt{16+25} = \sqrt{41} \approx 6.40$

The segments are not congruent.

45. $JK = \sqrt{(4-(-4))^2 + (8-0)^2}$

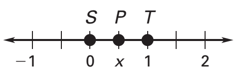
$= \sqrt{64+64} = \sqrt{128} \approx 11.31$

$LM = \sqrt{(3-(-4))^2 + (-7-2)^2}$

$= \sqrt{49+81} = \sqrt{130} \approx 11.40$

The segments are not congruent.

46. $SP = PT$, so P is the midpoint of $\overline{ST}$.



$SP = PT$

$x - 0 = 1 - x$

$2x = 1$

$x = \frac{1}{2}$

47. $2(JM) = JK$

$2\left(\frac{x}{8}\right) = \frac{3x}{4} - 6$

$4\left(\frac{x}{4}\right) = \left(\frac{3x}{4} - 6\right)4$

$x = 3x - 24$

$-2x = -24$

$x = 12$

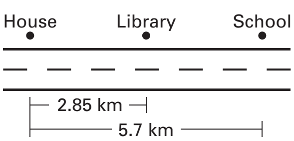
$JM = MK = \frac{x}{8} = \frac{12}{8} = 1\frac{1}{2}$

Problem Solving

48. $MR = QM = 18\frac{1}{2}$ feet

$QR = 2(QM) = 2\left(18\frac{1}{2}\right) = 37$ feet

49.



The library is $\frac{5.7}{2} = 2.85$ kilometers from the house.

50. a. $A(1, 1)$, $B(4, 2)$

$AB = \sqrt{(4-1)^2 + (2-1)^2}$

$= \sqrt{9+1} = \sqrt{10} \approx 3.2$ m

b. $B(4, 2)$, $C(2, 6)$

$BC = \sqrt{(2-4)^2 + (6-2)^2}$

$= \sqrt{4+16} = \sqrt{20} \approx 4.5$ m

c. $C(2, 6)$, $D(5, 4)$

$CD = \sqrt{(5-2)^2 + (4-6)^2}$

$= \sqrt{9+4} = \sqrt{13} \approx 3.6$ m

d. $A(1, 1)$, $D(5, 4)$

$AD = \sqrt{(5-1)^2 + (4-1)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$ m

e. $B(4, 2)$, $D(5, 4)$

$BD = \sqrt{(5-4)^2 + (4-2)^2}$

$= \sqrt{1+4} = \sqrt{5} \approx 2.2$ m

f. $A(1, 1)$, $C(2, 6)$

$AC = \sqrt{(2-1)^2 + (6-1)^2}$

$= \sqrt{1+25} = \sqrt{26} \approx 5.1$ m

51. The objects at B and D are closest together. The objects at A and C are farthest apart.

52. $AB = \sqrt{(18-8)^2 + (7-4)^2}$

$= \sqrt{100+9} = \sqrt{109} \approx 10.4$

Player A threw the ball about 10.4 meters.

$BC = \sqrt{(24-18)^2 + (14-7)^2}$

$= \sqrt{36+49} = \sqrt{85} \approx 9.2$

Player B threw the ball about 9.2 meters.

$AC = \sqrt{(24-8)^2 + (14-4)^2}$

$= \sqrt{256+100} = \sqrt{356} \approx 18.9$

Player A would have thrown the ball about 18.9 meters to player C.

53. a. $P(10, 50)$, $Q(10, 10)$, $R(80, 10)$

$PQ = \sqrt{(10-10)^2 + (10-50)^2} = \sqrt{1600} = 40$

$QR = \sqrt{(80-10)^2 + (10-10)^2} = \sqrt{4900} = 70$

$RP = \sqrt{(80-10)^2 + (10-50)^2} = \sqrt{6500} \approx 80.62$

$40 + 70 + 81 = 191$

The distance around the park is about 191 yards.

b. $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$M\left(\frac{10+80}{2}, \frac{50+10}{2}\right)$

$M(45, 30)$

$QM = \sqrt{(45-10)^2 + (30-10)^2}$

$= \sqrt{1225+400} = \sqrt{1625} \approx 40.3$

QM is about 40 yards.

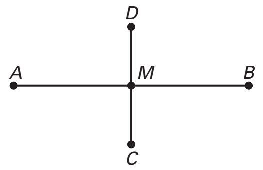
c. It takes him about 1.5 minutes.

$PQ = 40, QM = 40, MR = \frac{1}{2}(80) = 40, RQ = 70$

$40 + 40 + 40 + 70 + 40 = 230$

Divide the total distance, about 230 yards, by 150 yards per minute.

54.



$AB = 2AM$

$CM = \frac{1}{2}CD$

$AB = 4 \cdot CM$

$2AM = 4\left(\frac{1}{2}\right)CD$

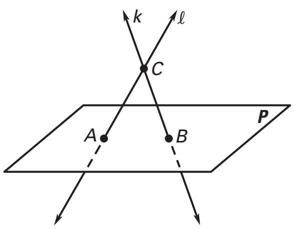
$2AM = 2CD$

$AM = CD$

AM and CD are equal.

Quiz for the lessons "Identify Points, Lines, and Planes", "Use Segments and Congruence", and "Use Midpoint and Distance Formulas"

1. *Sample answer:*



2. $DE = AE - AD$

$DE = 26 - 15 = 11$

3. $AB = \frac{AD}{3} = \frac{15}{3} = 5$

4. $AC = 2AB = 2(5) = 10$

5. $BD = AC = 10$

6. $CE = CD + DE = 5 + 11 = 16$

7. $BE = BD + DE = 10 + 11 = 21$

8. $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$M\left(\frac{-2 + 2}{2}, \frac{-1 + 3}{2}\right) = M(0, 1)$

The coordinates of the midpoint are (0, 1). The distance between R and S is about 5.7 units.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(2 - (-2))^2 + (3 - (-1))^2}$

$= \sqrt{16 + 16} \approx 5.66$ units

Mixed Review of Problem Solving for the lessons "Identify Points, Lines, and Planes", "Use Segments and Congruence", and "Use Midpoint and Distance Formulas"

1. a. $B(1, 7), C(7, 7), D(10, 7), E(10, 3)$

$BD = \sqrt{(10 - 1)^2 + (7 - 7)^2} = \sqrt{81} = 9$

$DE = \sqrt{(10 - 10)^2 + (3 - 7)^2} = \sqrt{16} = 4$

$9 + 4 = 13$

You travel 13 miles using existing roads.

b. $BC = \sqrt{(7 - 1)^2 + (7 - 7)^2} = \sqrt{36} = 6$

$CE = \sqrt{(10 - 7)^2 + (3 - 7)^2} = \sqrt{9 + 16} = 5$

$6 + 5 = 11$

You travel 11 miles using the new road.

c. $13 - 11 = 2$

The trip is 2 miles shorter.

2. $23x + 5 = 25x - 4$

$5 = 2x - 4$

$9 = 2x$

$4.5 = x$

$PQ = 23(4.5) + 5 + 25(4.5) - 4$

$= 103.5 + 5 + 112.5 - 4 = 217$

$= 217$

3. $d = rt$

$r = 2.4$ km/h

$t = 45$ min $= 0.75$ h

$d = 2.4(0.75) = 1.8$

$5.4 - 1.8 = 3.6$

You need to hike 3.6 kilometers to reach the end of the trail.

4. $FH = 2.8$ m $= 280$ cm

$\frac{280}{2} = 140$

The midpoint of $\overline{FH}$ is 140 cm from F. $\overline{EG}$ intersects $\overline{FM}$ 143 cm from F, so it is not a bisector.

5. Using the midpoint formula.

$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$M\left(\frac{-4 + 6}{2}, \frac{5 + (-5)}{2}\right)$

$M(1, 0)$

These are the coordinates of point $E(1, 0)$.

To find point D, substitute the coordinates of point C into the midpoint formula, and set each coordinate equal to the corresponding coordinate from the midpoint E.

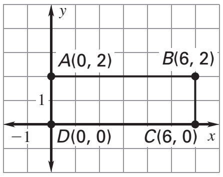
$\frac{2 + x}{2} = 1$ $\qquad$ $\frac{8 + y}{2} = 0$

$2 + x = 2$ $\qquad$ $8 + y = 0$

$x = 0$ $\qquad$ $y = -8$

$(0, -8)$ are the coordinates for point D.

6. *Sample answer:*



Perimeter = $AB + BC + CD + AD$

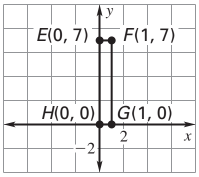
$AB = \sqrt{(6-0)^2 + (2-2)^2} = \sqrt{36} = 6$

$BC = \sqrt{(6-6)^2 + (2-0)^2} = \sqrt{4} = 2$

$CD = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36} = 6$

$AD = \sqrt{(0-0)^2 + (2-0)^2} = \sqrt{4} = 2$

Perimeter = $6 + 2 + 6 + 2 = 16$



Perimeter = $EF + FG + GM + EH$

$EF = \sqrt{(1-0)^2 + (7-7)^2} = \sqrt{1} = 1$

$FG = \sqrt{(1-1)^2 + (7-0)^2} = \sqrt{49} = 7$

$GH = \sqrt{(1-0)^2 + (0-0)^2} = \sqrt{1} = 1$

$EH = \sqrt{(0-0)^2 + (7-0)^2} = \sqrt{49} = 7$

Perimeter = $1 + 7 + 1 + 7 = 16$

7. The plane that contains B, F, and C can be called plane BFG, plane FGC, plane GCB, or plane CBF. Plane ABC intersects plane BFE at $\overleftrightarrow{AB}$.

8. a. $AB = 18.7$ km

$BC = 2AB = 2(18.7) = 37.4$ km

$AC = AB + BC = 18.7 + 37.4 = 56.1$ km

$AB + BC + CA = 18.7 + 37.4 + 56.1 = 112.2$ km

Jill travels 112.2 kilometers.

b. $d = rt$

$112.2 = 70t$

$1.6 = t$

She spends about 1.6 hours driving.

c. No. *Sample answer:* If she spends 2.5 hours in each town. $2.5(3) = 7.5$ hours spent in towns. The time spent in the three towns plus the total driving time is $7.5 + 1.6 = 9.1$ hours.

Lesson 1.4 Measure and Classify Angles

Guided Practice for the lesson "Measure and Classify Angles"

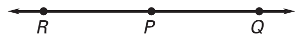
1. $\angle PQR$

$\angle RQS$

$\angle PQS$

$\angle PQS$ is a right angle.

2.



The rays form a straight angle.

3. $m\angle KLN + m\angle NLM = 180°$

$10x - 5 + 4x + 3 = 180$

$14x - 2 = 180$

$14x = 182$

$x = 13$

$10(13) - 5 = 125$

$m\angle KLN = 125°$

$14(13) + 3 = 55$

$m\angle NLM = 55°$

4. $m\angle EFM + m\angle HFG = 90°$

$2x + 2 + x + 1 = 90$

$3x + 3 = 90$

$3x = 87$

$x = 29$

$m\angle EFM = 2(29) + 2 = 60°$

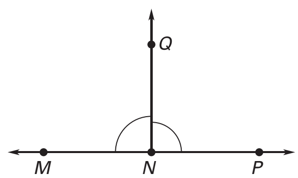
$m\angle MFG = 29 + 1 = 30°$

5. $\angle QPT \cong \angle QRS$

$\angle PTS \cong \angle RST$

6. If $m\angle QRS = 84°$ and $\angle QRS \cong \angle QPT$, then $m\angle QPT = 84°$. If $m\angle TSR = 121°$ and $\angle TSR \cong \angle STP$, then $m\angle STP = 121°$.

7.



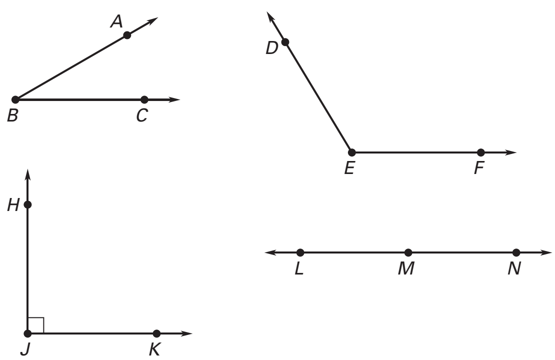
$m\angle MNQ = 90°$

$m\angle PNQ = 90°$

Exercises for the lesson "Measure and Classify Angles"

Skill Practice

1. Answers may vary. *Sample answers:*



2. The measure of $\angle PQR$ is equal to the absolute value of the difference between the degree measures of $\overrightarrow{QP}$ and $\overrightarrow{QR}$.

3. $\angle ABC$, $\angle CBA$, $\angle B$

B is the vertex, and $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are the sides.

4. $\angle NQT$, $\angle TQN$, $\angle Q$

Q is the vertex, and $\overrightarrow{QN}$ and $\overrightarrow{QT}$ are the sides.

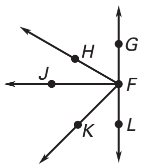
5. $\angle MTP$, $\angle PTM$, $\angle T$

T is the vertex, and $\overrightarrow{TM}$ and $\overrightarrow{TP}$ are the sides.

6. $\angle QRS$, $\angle TRS$, $\angle QRT$

7. Straight **8.** Acute **9.** Right **10.** Obtuse

11–14.



11. $m\angle JFL = 90°$, Right

12. $m\angle GFM = 60°$, Acute

13. $m\angle GFK = 135°$, Obtuse

14. $m\angle GFL = 180°$, Straight

15. Another name for $\angle ACB$ is $\angle BCA$. The angle is a right angle because it is labeled with a red square.

16. Another name for $\angle ABC$ is $\angle CBA$. The angle is an acute angle because its measure is less than 90°.

17. Another name for $\angle BFD$ is $\angle DFB$. The angle is a straight angle because its measure is 180°

18. Another name for $\angle AEC$ is $\angle CEA$. The angle is an obtuse angle because its measure is between 90° and 180°.

19. Another name for $\angle BDC$ is $\angle CDB$. The angle is an acute angle because its measure is less than 90°.

20. Another name for $\angle BEC$ is $\angle CEB$. The angle is an acute angle because its measure is less than 90°.

21. B; $m\angle ACD$ is between 90° and 180°.

22. $m\angle QST = m\angle QSR + m\angle RST = 52° + 47° = 99°$

23. $m\angle ADC = m\angle ADB + m\angle CDB = 21° + 44° = 65°$

24. $m\angle NPM = m\angle LPM - m\angle LPN = 180° - 79° = 101°$

25. $x + 5 + 3x - 5 = 80$

$4x = 80$

$x = 20$

$m\angle YXZ = 3(20) - 5 = 55°$

26. $6x - 15 + x + 8 = 168$

$7x - 7 = 168$

$7x = 175$

$x = 25$

$m\angle FJG = 6(25) - 15 = 135°$

27. A; $2x + 6 + 80 = 140$

$2x = 54$

$x = 27$

28. $\angle AED \cong \angle ADE \cong \angle BDC \cong \angle BCD \cong \angle DAB \cong \angle ABD$; $\angle EAD \cong \angle DBC \cong \angle ADB$

$m\angle BDC = 34°$

$m\angle ADB = 112°$

29. $m\angle ZWY = m\angle XWY = 52°$

$m\angle XWY = 2(52) = 104°$

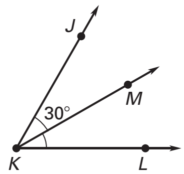
30. $m\angle ZWX = \frac{1}{2}m\angle XWY = 68°$

$m\angle XWY = 2(68) = 136°$

31. $m\angle ZWY = \frac{1}{2}\angle XWY = 35.5°$

$m\angle XWZ = 71° - 35.5° = 35.5°$

32.



$m\angle JKL$ is twice the measure of $\angle JKM$, not half of it.
$m\angle JKL = 60°$

33. $a° = 180 - 142 = 38°$

34. $b° \cong a° = 38°$

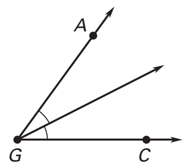
35. $c° = 142°$

36. $d° = 180 - 53 - 90 = 37°$

37. $e° = 53°$

38. $f° \cong d \cong 37°$

39. For a ray to bisect $\angle AGC$, the endpoint of the ray must be at point G.



40. $4x - 2 = 3x + 18$

$x = 20$

$m\angle ABC = 4(20) - 2 + 3(20) + 18 = 156°$

41. $2x + 20 = 4x$

$20 = 2x$

$10 = x$

$m\angle ABC = 2(10) + 20 + 4(10) = 80°$

42. $\frac{x}{2} + 17 = x - 33$

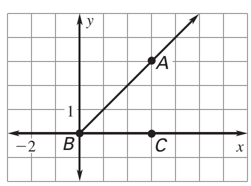
$x + 34 = 2x - 66$

$100 = x$

$m\angle ABC = \frac{100}{2} + 17 + 100 - 33 = 134°$

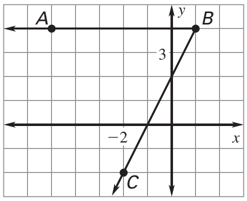
43. $\overrightarrow{QP}$ lines up with the 75° mark. The new mark for $\overrightarrow{QR}$ is 5° less than before. The difference between the marks that $\overrightarrow{QR}$ and $\overrightarrow{QP}$ line up with on the protractor must remain the same.

44.



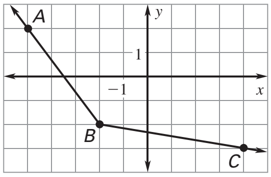
The angle is acute.

Sample answer: point (2, 1) lies on the interior of the angle.

45. 

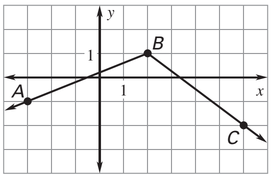
$\angle ABC$ is acute.

Sample answer: point $(-2, 2)$ lies in the interior of the angle.

46. 

$\angle ABC$ is obtuse.

Sample answer: point $(-1, -1)$ lies in the interior of the angle.

47. 

$\angle ABC$ is obtuse.

Sample answer: Point $(2, -1)$ lies in the interior of the angle.

48. $0 < (2x - 12)° < 90°$

$0 + 12 < 2x < 90 + 12$

$12 < 2x < 112$

$6 < x < 51$

49. 68°

Sample answer: Since $m\angle VSP = 17°$, $m\angle RSP = 34°$.

Since $m\angle RSP = 34°$, $m\angle RSQ = 68°$, which is equal to $m\angle TSQ$.

50. $m\angle AEB = \frac{1}{2} \cdot m\angle CED$

$\frac{1}{2} \cdot m\angle CED + 90 + m\angle CED = 180$

$1\frac{1}{2} \cdot m\angle CED + 90 = 180$

$1\frac{1}{2} \cdot m\angle CED = 90$

$m\angle CED = 60°$

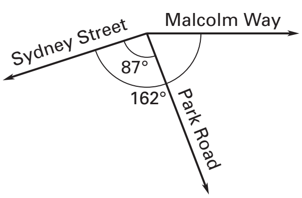
$m\angle AEB = \frac{1}{2}(60) = 30°$

Problem Solving

51. $m\angle LMP = m\angle LMN - m\angle PMN$

$m\angle LMP = 79 - 47$

$m\angle LMP = 32°$

52. 

$162° - 87° = 75°$

Malcom Way intersects Park Road at an angle of 75°.

53. **a.** $m\angle DEF = m\angle ABC = 112°$

b. Because $\overrightarrow{BG}$ bisects $\angle ABC$, $m\angle ABG = \frac{1}{2} \cdot m\angle ABC$

$= \frac{1}{2}(112°) = 56°$.

c. Because $\overrightarrow{BG}$ bisects $\angle ABC$, $m\angle CBG = \frac{1}{2} \cdot m\angle ABC$

$= \frac{1}{2}(112°) = 56°$.

d. Because $\overrightarrow{BG}$ bisects $\angle DEF$, $m\angle DEG = \frac{1}{2} \cdot m\angle DEF$

$= \frac{1}{2}(112°) = 56°$.

54. Because $\angle DGF$ is a straight angle, $m\angle DGF = 180°$.

$m\angle DGE = 90°$

$m\angle FGE = 90°$

55. *Sample answer:* Acute: $\angle ABG$, Obtuse: $\angle ABC$, Right: $\angle FGE$, Straight: $\angle DGF$

56. about 158° **57.** about 140° **58.** about 167°

59. about 62° **60.** about 39° **61.** about 107°

62. **a.** $\angle AFB$, $\angle BFC$, $\angle CFD$, $\angle DFE$ are acute.

$\angle AFD$, $\angle BFD$, $\angle BFE$ are obtuse.

$\angle AFC$ and $\angle CFE$ are right.

b. $\angle BFC \cong \angle DFC$, $\angle AFC \cong \angle EFC$,

$\angle AFB \cong \angle EFD$, $\angle AFD \cong \angle EFB$

c. $m\angle DFE = 26°$, $m\angle BFC = 64°$, $m\angle CFD = 64°$,

$m\angle AFC = 90°$, $m\angle AFD = 154°$, $m\angle BFD = 128°$

$\angle AFE$ is a straight angle, so $m\angle AFC + m\angle CFE = 180°$. $\overrightarrow{FC}$ bisects $\angle AFE$, so $m\angle AFC = m\angle CFE$. Therefore, $m\angle AFC = m\angle CFE = 90°$.

$m\angle BFC = m\angle AFC - m\angle AFB = 90° - 26° = 64°$

$\overrightarrow{FC}$ bisects $\angle BFD$, so $m\angle CFD = m\angle BFC = 64°$

$m\angle DFE = m\angle CFE - m\angle CFD = 90° - 64° = 26°$

$m\angle AFD = m\angle AFC + m\angle CFD = 90° + 64° = 154°$

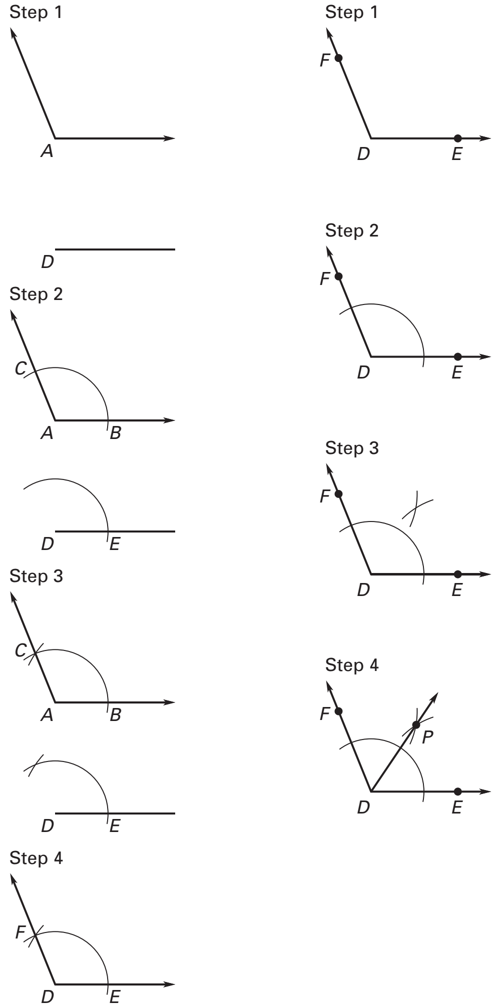
$m\angle BFD = m\angle BFC + m\angle CFD = 64° + 64° = 128°$

63. *Sample answer:* In your drawer you have 4 pairs of brown socks, 4 pairs of black socks, 4 pairs of gray socks, 6 pairs of white socks, and 6 pairs of blue socks. The brown, black and gray socks each represent $\frac{1}{6}$, and the white and blue socks each represent $\frac{1}{4}$.

Investigating Geometry Construction for the lesson "Measure and Classify Angles"

1. *Sample answer:* Draw a segment more than twice as long as the given segment. Set your compass to the length of the given segment. Using your compass, mark off two adjacent line segments on the line segment you drew.

2. *Sample answer:*



Lesson 1.5 Describe Angle Pair Relationships

Guided Practice for the lesson "Describe Angle Pair Relationships"

1. Because $41° + 49° = 90°$, $\angle FGK$ and $\angle GKL$ are complementary.

Because $131° + 49° = 180°$, $\angle HGK$ and $\angle GKL$ are supplementary.

Because $\angle FGK$ and $\angle HGK$ share a common vertex and side, they are adjacent.

2. No, they do not share a common vertex.

No, they do have common interior points.

3. $m\angle 1 + m\angle 2 = 90°$

$m\angle 1 + 8° = 90°$

$m\angle 1 = 82°$

4. $m\angle 3 + m\angle 4 = 80°$

$117° + m\angle 4 = 180°$

$m\angle 4 = 63°$

5. $m\angle LMN + m\angle PQR = 90°$

$(4x - 2)° + (9x + 1)° = 90°$

$13x - 1 = 90$

$13x = 91$

$x = 7$

$m\angle LMN = 4(7) - 2 = 26°$

$m\angle PQR = 9(7) + 1 = 64°$

6. No, adjacent angles have their noncommon sides as opposite rays.

Angles 1 and 4, 2 and 5, 3 and 6 are vertical angles because each pair of sides form two pairs of opposite rays.

7. Let $x°$ be the measure of the angle's complement.

$2x° + x° = 90°$

$3x = 90$

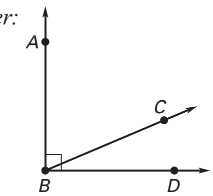
$x = 30$

One angle measures 30° and the other angle measures $2(30°) = 60°$.

Exercises for the lesson "Describe Angle Pair Relationships"

Skill Practice

1. *Sample answer:*



No, two angles could have angle measures that add up to 90° without sharing a common vertex and side.

2. All linear pairs are supplementary angles because their noncommon sides are opposite rays which form a straight angle. All supplementary angles are not linear pairs. Two angles can have angle measurements that add up to 180° without their noncommon sides being opposite rays.

3. $\angle ABD$ and $\angle DBC$ are adjacent.

4. $\angle WXY$ and $\angle XYZ$ are not adjacent.

5. $\angle LQM$ and $\angle NQM$ are adjacent.

6. Because $60° + 30° = 90°$, $\angle STR$ and $\angle VWU$ are complementary.

Because $150° + 30° = 180°$ $\angle QTS$ and $\angle VWU$, are supplementary.

7. $\angle GLH$ and $\angle HLJ$ are complementary because their measures add up to 90°. $\angle GLJ$ and $\angle JLK$ are supplementary because their measures add up to 180°.

8. $m\angle 1 + m\angle 2 = 90°$

$43° + m\angle 2 = 90°$

$m\angle 2 = 47°$

9. $m\angle 1 + m\angle 2 = 90°$

$21° + m\angle 2 = 90°$

$m\angle 2 = 69°$

10. $m\angle 1 + m\angle 2 = 90°$

$89° + m\angle 2 = 90°$

$m\angle 2 = 1°$

11. $m\angle 1 + m\angle 2 = 90°$

$5° + m\angle 2 = 90°$

$m\angle 2 = 85°$

12. $m\angle 1 + m\angle 2 = 180°$
$60° + m\angle 2 = 180°$
$m\angle 2 = 120°$

13. $m\angle 1 + m\angle 2 = 180°$
$155° + m\angle 2 = 180°$
$m\angle 2 = 25°$

14. $m\angle 1 + m\angle 2 = 180°$
$130° + m\angle 2 = 180°$
$m\angle 2 = 50°$

15. $m\angle 1 + m\angle 2 = 180°$
$27° + m\angle 2 = 180°$
$m\angle 2 = 153°$

16. B; $90° - 37° = 53°$

17. $m\angle DEG + m\angle GEF = 180°$
$(18x - 9)° + (4x + 13)° = 180°$
$22x + 4 = 180$
$22x = 176$
$x = 8$
$m\angle DEG = 18(8) - 9 = 135°$
$m\angle GEF = 4(8) + 13 = 45°$

18. $m\angle DEG + m\angle GEF = 180°$
$(7x - 3)° + (12x - 7)° = 180°$
$19x - 10 = 180$
$19x = 190$
$x = 10$
$m\angle DEF = 7(10) - 3 = 67°$
$m\angle GEF = 12(10) - 7 = 113°$

19. $m\angle DEG + m\angle GEF = 90°$
$6x° + 4x° = 90°$
$10x = 90$
$x = 9$
$m\angle DEG = 6(9) = 54°$
$m\angle GEF = 4(9) = 36°$

20. $\angle 1$ and $\angle 4$ are vertical angles.

21. $\angle 1$ and $\angle 2$ are a linear pair.

22. $\angle 3$ and $\angle 5$ are neither.

23. $\angle 2$ and $\angle 3$ are vertical angles.

24. $\angle 7$, $\angle 8$, and $\angle 9$ are neither.

25. $\angle 5$ and $\angle 6$ are a linear pair.

26. $\angle 6$ and $\angle 7$ are neither.

27. $\angle 5$ and $\angle 9$ are neither.

28. Use the fact that angles in a linear pair are supplementary angles.
$x° + 4x° = 180°$
$5x = 180$
$x = 36$
$4(36) = 144$
One angle is 36°, and the other angle is 144°.

29. The angles are complementary so the sum of their measures equals 90°.
$x° + 3x° = 90°$
$4x = 90$
$x = 22.5$

30. C; $x° + (x + 24)° = 90°$
$2x = 66$
$x = 33$
$33 + 24 = 57$

31. $7x° + (9x + 20)° = 180°$
$16x = 160$
$x = 10$
$7x° = 2y°$
$7(10) = 2y$
$70 = 2y$
$35 = y$

32. $3x° + (8x + 26)° = 180°$
$11x = 154$
$x = 14$
$(5y + 38)° = (8x + 26)°$
$5y + 38 = 8(14) + 26$
$5y + 38 = 138$
$5y = 100$
$y = 20$

33. $2y° = (x + 5)°$
$3y + 30 = 4x - 100$
$2y - 5 = x$
$3y + 30 = 4(2y - 5) - 100$
$3y + 30 = 8y - 20 - 100$
$3y + 30 = 8y - 120$
$150 = 5y$
$30 = y$
$2(30) - 5 = x$
$55 = x$

34. Never; the measure of an obtuse angle is greater than 90°, so its angle measurement cannot be added to the measurement of another angle to equal 90°.

35. Never; the measure of a straight angle is 180°, so its measurement cannot be added to the measurement of another angle to equal 90°.

36. Sometimes; an angle that measures less than 180° has a supplement.

37. Always; for the measurements of two angles to add up to 90°, they must be be acute.

38. Always; An acute angle measures less than 90° so its supplement must measure between 90° and 180° for the two to add up to 180°.

39. $m\angle A + m\angle B = 90°$
$(3x + 2)° + (x - 4)° = 90°$
$4x - 2 = 90$
$4x = 92$
$x = 23$
$m\angle A = 3(23) + 2 = 71°$
$m\angle B = 23 - 4 = 19°$

40. $m\angle A + m\angle B = 90°$

$(15x + 3)° + (5x - 13)° = 90°$

$20x - 10 = 90$

$20x = 100$

$x = 5$

$m\angle A = 15(5) + 3 = 78°$

$m\angle B = 5(5) - 13 = 12°$

41. $m\angle A + m\angle B = 90°$

$(11x + 24)° + (x + 18)° = 90°$

$12x + 42 = 90$

$12x = 48$

$x = 4$

$m\angle A = 11(4) + 24 = 68°$

$m\angle B = 4 + 18 = 22°$

42. $m\angle A + m\angle B = 180°$

$(8x + 100)° + (2x + 50)° = 180°$

$10x + 150 = 180$

$10x = 30$

$x = 3$

$m\angle A = 8(3) + 100 = 124°$

$m\angle B = 2(3) + 50 = 56°$

43. $m\angle A + m\angle B = 180°$

$(2x - 20)° + (3x + 5)° = 180°$

$5x - 15 = 180$

$5x = 195$

$x = 39$

$m\angle A = 2(39) - 20 = 58°$

$m\angle B = 3(39) + 5 = 122°$

44. $m\angle A + m\angle B = 180°$

$(6x + 72)° + (2x + 28)° = 180°$

$8x + 100 = 180$

$8x = 80$

$x = 10$

$m\angle A = 6(10) + 72 = 132°$

$m\angle B = 2(10) + 28 = 48°$

45. Given $\angle GHJ$ is a complement of $\angle RST$;
$m\angle GMJ + m\angle RST = 90°$

$x° + m\angle RST = 90°$

$m\angle RST = 90° - x°$

Given $\angle RST$ is a supplement of $\angle ABC$

$m\angle RST + m\angle ABC = 180°$,
so $90° - x° + m\angle ABC = 180°$

$m\angle ABC = 90° + x°$

Problem Solving

46. The angles are complementary because the sum of their measures is 90°.

47. The angles are neither complementary nor supplementary because the sum of their measures is greater than 180°.

48. The angles are supplementary because the sum of their measure is 180°.

49. *Sample answer:* $\angle FGA$ and $\angle AGC$ are supplementary.

50. *Sample answer:* $\angle AGB$ and $\angle EGD$ are vertical angles.

51. *Sample answer:* $\angle FGE$ and EGC are a linear pair.

52. *Sample answer:* $\angle CGD$ and $\angle DGE$ are adjacent angles.

53. *Sample answer:* Because $\angle FGB$ and $\angle BGC$ are supplementary angles, $m\angle BGC = 180° - m\angle FGB$

$m\angle BGC = 180° - 120°$

$m\angle BGC = 60°$.

54. As the sun rises, the shadow becomes shorter and the angle increases.

55. a. $y_1 = 90 - x$

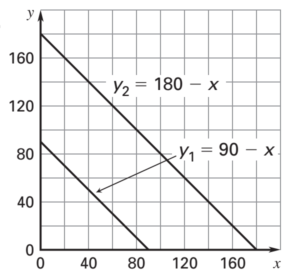
Domain: $0 < x < 90$

$y_2 = 180 - x$

Domain: $0 < x < 180$

The measure of a complement must be less than 90° and the measure of its supplement must be less than 180°.

b.



Range of y_1: $0 < y < 90$

Range of y_2: $0 < y < 180$

56. Let $x°$ be the measure of one angle and let $y°$ be the measure of the other angle.

$x° + y° = 90°$

$x° + y° = x° - y° + 86°$

$x = 90 - y$

$90 - y + y = 90 - y - y + 86$

$90 = 176 - 2y$

$-86 = -2y$

$43 = y$

$x + 43 = 90$

$x = 47$

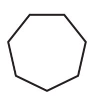
One angle measures 43° and the other angle measures 47°.

Lesson 1.6 Classify Polygons

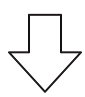
Guided Practice for the lesson "Classify Polygons"

1. *Sample answers:*

Convex heptagon



Concave heptagon



2. Quadrilateral; each of the sides is 2 meters long and all the angles are right angles.

3.
$$8y^\circ = (9y - 15)^\circ$$
$$-y = -15$$
$$y = 15$$
$$8(15) = 120$$
$$9(15) - 15 = 120$$

Each angle measures 120°.

Exercises for the lesson "Classify Polygons"

Skill Practice

1. The term *n*-gon is used to name a polygon, where *n* is the number of sides of the polygon.
2. Yes, the string will lie on the sides of the figure so it will match the distance around the polygon.

   No, because the string cannot lie on the concave sides, the length of the string will be less than the distance around the polygon.
3. The figure is a concave polygon.
4. Part of the figure is not a segment, so it is not a polygon.
5. The figure is a convex polygon.
6. Some segments intersect more than two segments, so it is not a polygon.
7. C; the figure is a polygon and is not convex.
8. The polygon has 8 sides. It is equilateral and equiangular, so it is a regular octogon.
9. The polygon has 5 sides. It is equilateral and equiangular, so it is a regular pentagon.
10. The polygon has 3 sides so the figure is a triangle. It is equilateral and equiangular, so it is regular.
11. The polygon has 3 sides, so the figure is a triangle. It is not equilateral or equiangular, so it is not regular.
12. The polygon has 4 sides, so it is a quadrilateral. It is equilateral but not equiangular, so it is not regular.
13. The polygon is a quadrilateral because it has 4 sides. It is equiangular but not equilateral, so it is not regular.
14. Student A: The error is the hexagon must be convex.

    Student B: The error is the hexagon does not have congruent sides.

15.
$$5x - 27 = 2x - 6$$
$$3x = 21$$
$$x = 7$$
$$2(7) - 6 = 8$$

A side of the pentagon is 8 inches.

16.
$$(9x + 5)^\circ = (11x - 25)^\circ$$
$$30 = 2x$$
$$15 = x$$
$$9(15) + 5 = 140$$

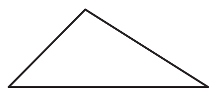
An angle of the nonagon measures 140°.

17.
$$3x - 9 = 23 - 5x$$
$$8x = 32$$
$$x = 4$$
$$3(4) - 9 = 3$$

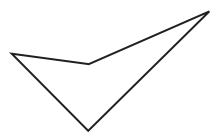
A side of the triangle is 3 feet.

18. A triangle is always convex, because no line that contains a side of the triangle contains a point in the interior of the triangle.
19. A decagon is sometimes regular, because all of its sides and all of its angles can be congruent, but they don't have to be.
20. A regular polygon is always equiangular, because all of its angles in the interior of the polygon are congruent.
21. A circle is never a polygon, because a circle does not have sides.
22. A polygon is always a plane figure, because a polygon is a closed plane figure.
23. A concave polygon is never regular, because a regular polygon is not concave.

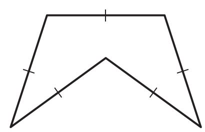
24. *Sample answer:* (Equilateral, not equiangular)



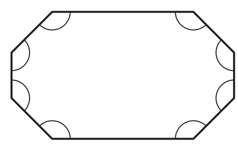
25. *Sample answer:* (Equilateral, not equiangular)



26. *Sample answer:* (Equilateral, not equiangular)



27. *Sample answer:* (Equiangular, not equilateral)



28.
$$x^2 + 4 = x^2 + x$$
$$4 = x$$

29.
$$x^2 + 3x = x^2 + x + 2$$
$$2x = 2$$
$$x = 1$$

30.
$$x^2 + 2x + 40 = x^2 - x + 190$$
$$3x = 150$$
$$x = 50$$

31. $m\angle BAC = 36^\circ$, $m\angle ABC = 72^\circ$, $m\angle ACB = 72^\circ$; *Sample answer:* Because the pentagonal tiles are regular, all of their interior angles are congruent. By setting the given expressions for angle measures equal to each other and solving for *x*, we can find the measure of each angle.

$$(20x + 48)^\circ = (33x + 9)^\circ$$
$$39 = 13x$$
$$3 = x$$
$$20(3) + 48 = 108$$

Each interior angle of the pentagonal tiles measures 108°. So, $m\angle CAD = 108°$. Because $\angle BAE$ is a straight angle, $m\angle BAC + m\angle CAD + m\angle DAE = 180°$. Since $\angle BAC \cong \angle DAE$,

$$2 \cdot m\angle BAC + 108° = 180°$$
$$2 \cdot m\angle BAC = 72°$$
$$m\angle BAC = 36°$$

$\angle ACB$ is supplementary to one of the 108° interior angles of a pentagon. So $180° - 108° = 72° = m\angle ACB$. Because the angles of a triangle must add up to 180°, $m\angle ABC = 180° - 72° - 36° = 72°$.

Problem Solving

32. **a.** The red polygon is convex because no line that contains a side of the polygon contains a point in the interior of the polygon.

b. The polygon has 8 sides. It appears to be equilateral and equiangular, so it is a regular octagon.

33. The polygon has 3 sides. It appears to be equilateral and equiangular, so it is a regular triangle.

34. The polygon has 4 sides so it is a quadrilateral. It appears to be equiangular but not equilateral, so it is not regular.

35. The polygon has 8 sides. It appears to be equilateral and equiangular, so it is a regular octagon.

36. The polygon has 12 sides, so it is a decagon. It is concave, so it is not regular.

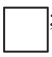
37. C; $AB = \sqrt{(0 - 0)^2 + (-4 - 4)^2} = \sqrt{64} = 8$

$CD = \sqrt{(8 - 8)^2 + (-4 - 4)^2} = \sqrt{64} = 8$

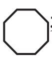
$AD = \sqrt{(8 - 0)^2 + (4 - 4)^2} = \sqrt{64} = 8$

$BC = \sqrt{(8 - 0)^2 + (-4 - (-4))^2} = \sqrt{64} = 8$

$C(8, -4)$ and $D(8, 4)$

38. **a.** ; The polygon is a quadrilateral.

b. It appears to be regular, and it is convex.

a. ; The polygon is an octagon.

b. It appears to be regular, and it is convex.

a. ; The polygon is a pentagon.

b. It is not equilateral or equiangular, and it is convex.

a. ; The polygon is a heptagon.

b. It is not equilateral or equiangular, and it is concave.

a.  The polygon is a dudecagon.

b. It appears to be equilateral but it is concave, so it is not regular.

39. 105 mm; Because the button is a regular polygon, set the expressions given for the length of the sides equal to each other and solve for x.

$$3x + 12 = 20 - 5x$$
$$8x = 8$$
$$x = 1$$

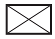
Substitute the value for x back into one of the expressions to find the length of one side.

$3(1) + 12 = 15$ mm

Because there are 7 sides, multiply the length of one side by 7 to find the length of silver wire needed.

$15(7) = 105$

40. **a.**

Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon		5	5
Hexagon		6	9
Heptagon		7	14

Sample answer: Every time you add another side, you increase the number of diagonals by the amount added to the previous polygon plus 1.

b. An octagon has 20 diagonals. A nonagon has 27 diagonals. *Sample answer:* The pattern described continues.

c. $\frac{60(60 - 3)}{2} = \frac{3420}{2} = 1710$

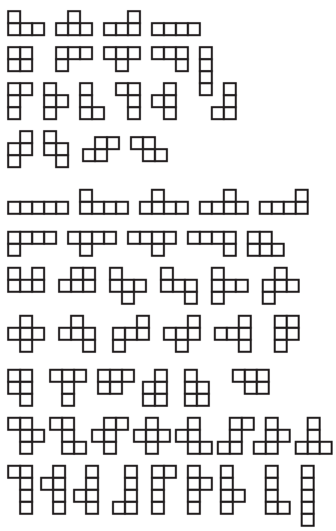
A 60-gon has 1710 diagonals.

41. **a.** A regular triangle has 3 lines of symmetry.

b. A regular pentagon has 5 lines of symmetry.

c. A regular hexagon has 6 lines of symmetry.

d. A regular octagon has 8 lines of symmetry.

42. 

43. *Sample answer:* The scrap paper side length matches all of the diagram side lengths and the angle made by the scrap paper matches all of the diagram angles, so the diagram is regular.

44. *Sample answer:* The scrap paper side length matches all of the diagram side lengths and the angle made by the scrap paper matches all of the diagram angles, so the diagram is regular.

45. Sample answer: The scrap paper side length does not match all of the diagram side lengths and the angle made by the scrap paper does not match all of the angles, so the diagram is neither equilateral, equiangular, nor regular.

46. Sample answer: The scrap paper side length does not match all of the diagram side lengths, but the angle made by the scrap paper matches all of the diagram angles, so the diagram is equiangular.

Quiz for the lessons "Measure and Classify Angles" and "Describe Angle Pair Relationships"

1. $m\angle ABD = m\angle DBC$

$(x + 20)° = (3x - 4)°$

$24 = 2x$

$12 = x$

$m\angle ABD = 12 + 20 = 32°$

$m\angle DBC = 3(12) - 4 = 32°$

2. $m\angle ABD = m\angle DBC$

$(10x - 42)° = (6x + 10)°$

$4x = 52$

$x = 13$

$m\angle ABD = 10(13) - 42 = 88°$

$m\angle DBC = 6(13) + 10 = 88°$

3. $m\angle ABD = m\angle DBC$

$(18x + 27)° = (9x + 36)°$

$9x = 9$

$x = 1$

$m\angle ABD = 18(1) + 27 = 45°$

$m\angle DBC = 9(1) + 36 = 45°$

4. a. $90° - 47° = 43°$

The measure of the complement of $\angle 1$ is 43°.

b. $180° - 47° = 133°$

The measure of the supplement of $\angle 1$ is 133°

5. a. $90° - 19° = 71°$

The measure of the complement of $\angle 1$ is 71°.

b. $180° - 19° = 161°$

The measure of the supplement of $\angle 1$ is 161°.

6. a. $90° - 75° = 15°$

The measure of the complement of $\angle 1$ is 15°.

b. $180° - 75° = 105°$

The measure of the supplement of $\angle 1$ is 105°.

7. a. $90° - 2° = 88°$

The measure of the complement of $\angle 1$ is 88°.

b. $180° - 2° = 178°$.

The measure of the supplement of $\angle 1$ is 178°.

8. The figure is a concave polygon.

9. The figure is not a polygon because part of it is not a segment.

10. The figure is a convex polygon.

Problem Solving Workshop for the lesson "Classify Polygons"

1. Sample answer: The scrap paper side length matches all of the diagram side lengths and the angle made by the scrap paper matches all of the diagram angles, so the diagram is regular.

2. Sample answer: The scrap paper side length matches all of the diagram side lengths and the angle made by the scrap paper matches all of the diagram angles, so the diagram is regular.

3. Sample answer: The scrap paper side length does not match all of the diagram side lengths and the angle made by the scrap paper does not match all of the angles, so the diagram is neither equilateral, equiangular, nor regular.

4. Sample answer: The scrap paper side length does not match all of the diagram side lengths, but the angle made by the scrap paper matches all of the diagram angles, so the diagram is equiangular.

Mixed Review of Problem Solving for the lessons "Measure and Classify Angles", "Describe Angle Pair Relationships", "Classify Polygons", and "Find Perimeter, Circumference, and Area"

1. a. $\ell = 4$ yd $= 12$ ft

$w = 3$ yd $= 9$ ft

$A = \ell w = 12(9) = 108$

The area of the roof is 108 square feet.

b. asphalt shingles: $0.75(108) = \$81$

Wood shingles: $1.15(108) = \$124.20$

c. $\$124.20 - \$81 = \$43.20$

You will pay \$43.20 more to use wood shingles.

2. *Sample answer: GAB*, *GABCDEF*; triangle, heptagon

3. a. $\angle DGB$ and $\angle BGH$ are complementary. $\angle HGF$ and $\angle CGF$ are complementary. $\angle DGB$ and $\angle BGF$, $\angle DGH$ and $\angle HGF$, $\angle CGF$ and $\angle CGD$, $\angle DBG$ and $\angle GBH$, $\angle HCG$ and $\angle GCF$ are supplementary.

b. $m\angle FGC = 21°$ because $m\angle DGB = 21°$ and $\angle FGC \cong \angle DGB$. $m\angle BGH = 69°$ because it is complementary to a 21° angle. $m\angle HGC = 69°$ because it is complementary to a 21° angle.

c. $m\angle HCG = 55°$ because $m\angle HBG$ is 55° and $\angle HCG \cong \angle HBG$.

$m\angle DBG = 125°$ because it is supplementary to a 55° angle. $m\angle FCG = 125°$ because it is supplementary to a 55° angle.

4. $m\angle 1 + m\angle 2 = 180°$

$m\angle 1 + m\angle 3 = 90°$

$m\angle 1 = m\angle 2 - 28°$

$m\angle 2 - 28° + m\angle 2 = 180°$

$2(m\angle 2) = 208°$

$m\angle 2 = 104°$

$m\angle 1 = 104° - 28° = 76°$

$76° + m\angle 3 = 90°$

$m\angle 3 = 14°$

5. a. $P = 4s = 4(22.5) = 90$ ft

$C = \pi d \approx 3.14(26) \approx 81.64$ ft

For the square garden you need enough bricks for 90 ft = 1080 inches of perimeter. $\frac{1080}{10} = 108$ bricks.

You need 108 bricks for the square garden.

The circumference of the circular garden in inches is 81.64 ft(12) ≈ 979.68 inches.

$\frac{979.68}{10} \approx 97.97$ bricks

You need 98 bricks for the circular garden.

b. You need a total of 108 + 98 = 206 bricks, so 3 bundles are needed.

6. $7x - 3 = 4x + 6$

$3x = 9$

$x = 3$

$7(3) - 3 = 18$

$18(5) = 90$

90 inches = 7.5 feet

7.5 feet of bamboo are used in the frame.

7. $(2x + 5)^\circ + (9x - 1)^\circ = 180^\circ$

$11x + 4 = 180$

$11x = 176$

$x = 16$

$2(16) + 5 = 37$

$m\angle ZWX = 37^\circ$

8. a. Side length $= \sqrt{(4-0)^2 + (0-2)^2} = \sqrt{20} = 2\sqrt{5}$

$P = 4(2\sqrt{5}) = 8\sqrt{5}$

The perimeter is $8\sqrt{5}$ units.

b. Area of $\triangle ABC = \frac{1}{2}bh = \frac{1}{2}(8)(2) = 8$

Area of $\triangle ADC = \frac{1}{2}(8)(2) = 8$

The area of quadrilateral *ABCD* is the sum of the areas of triangles *ABC* and *ADC*.

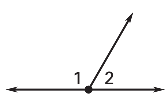
$8 + 8 = 16$

The area of quadrilateral *ABCD* is 16 square units.

Chapter Review for the chapter "Essentials of Geometry"

1. Points *A* and *B* are the *endpoints* of $\overline{AB}$.

2. *Sample answer:*



3. If *Q* is between points *P* and *R* on $\overleftrightarrow{PR}$, and $PQ = QR$, then *Q* is the *midpoint* of $\overline{PR}$.

4. *Sample answer:* Another name for line *y* is $\overleftrightarrow{XY}$.

5. *Sample answer:* Points *P*, *X*, and *N* are not collinear.

6. *Sample answer:* Points *N*, *X*, *Y*, and *Z* are coplanar.

7. $\overrightarrow{YX}$ and $\overrightarrow{YZ}$ are opposite rays.

8. The intersection of line *h* and plane *M* is point *y*.

9. $AB = AC - BC$

$AB = 3.2 - 2$

$AB = 1.2$

10. $NP = NM + MP$

$NP = 22 + 8$

$NP = 30$

11. $XY = XZ - YZ$

$XY = 16 - 9$

$XY = 7$

12. $DE = |11 - (-13)| = 24$

$GH = |-14 - (-9)| = 5$

$\overline{DE}$ and $\overline{GH}$ are not congruent because they have different lengths.

13. $JM = MK$

$6x - 7 = 2x + 3$

$4x = 10$

$x = 2.5$

$6(2.5) - 7 = 8$

$2(8) = 16$

JK is 16 units.

14. $AB = \sqrt{(4-2)^2 + (3-5)^2} = \sqrt{4+4} \approx 2.8$

The length is about 2.8 units.

$M\left(\frac{2+4}{2}, \frac{5+3}{2}\right)$

$M(3, 4)$

The coordinates of the midpoint are (3, 4).

15. $FG = \sqrt{(6-1)^2 + (0-7)^2} = \sqrt{25+49} \approx 8.6$

The length is about 8.6 units.

$M\left(\frac{6+1}{2}, \frac{0+7}{2}\right)$

$M(3.5, 3.5)$

The coordinates of the mid point are (3.5, 3.5).

16. $HJ = \sqrt{(5-(-3))^2 + (4-9)^2} = \sqrt{64+25} \approx 9.4$

The length is about 9.4 units.

$M\left(\frac{-3+5}{2}, \frac{9+4}{2}\right)$

$M(1, 6.5)$

The coordinates of the midpoint are (1, 6.5).

17. $KL = \sqrt{(0-10)^2 + (-7-6)^2} = \sqrt{100+169} \approx 16.4$

The length is about 16.4 units.

$M\left(\frac{10+0}{2}, \frac{6+(-7)}{2}\right)$

$M(5, -0.5)$

The coordinates of the midpoint are (5, −0.5).

18. $\frac{-1+x}{2} = 3$ $\frac{5+y}{2} = 8$

$-1 + x = 6$ $5 + y = 16$

$x = 7$ $y = 11$

The coordinates of endpopint *B* are (7, 11).

19. $EF = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{36+64} = 10$

$\overline{EF}$ is 10 units, so $\overline{EM}$ is 5 units.

20. $m\angle LMP + m\angle PMN = m\angle LMN$

$(11x - 9)° + (5x + 5)° = 140°$

$16x - 4 = 140$

$16x = 144$

$x = 9$

$m\angle PMN = 5(9) + 5 = 50°$

21. $m\angle UVW = 2 \cdot m\angle UVZ$

$m\angle UVW = 2(81°)$

$m\angle UVW = 162°$

The angle is obtuse.

22. $m\angle 2 = 90° - m\angle 1 = 90° - 12° = 78°$

23. $m\angle 2 = 90° - m\angle 1 = 90° - 83° = 7°$

24. $m\angle 2 = 90° - m\angle 1 = 90° - 46° = 44°$

25. $m\angle 2 = 90° - m\angle 1 = 90° - 2° = 88°$

26. $m\angle 4 = 180 - m\angle 3 = 180° - 116° = 64°$

27. $m\angle 4 = 180° - m\angle 3 = 180° - 56° = 124°$

28. $m\angle 4 = 180° - m\angle 3 = 180° - 89° = 91°$

29. $m\angle 4 = 180° - m\angle 3 = 180° - 12° = 168°$

30. $m\angle 1 + m\angle 2 = 90°$

$(x - 10)° + (2x + 40)° = 90°$

$3x + 30 = 90$

$3x = 60$

$x = 20$

$m\angle 1 = 20 - 10 = 10°$

$m\angle 2 = 2(20) + 40 = 80°$

31. $m\angle 1 + m\angle 2 = 180°$

$(3x + 50)° + (4x + 32)° = 180°$

$7x + 82 = 180$

$7x = 98$

$x = 14$

$m\angle 1 = 3(14) + 50 = 92°$

$m\angle 2 = 4(14) + 32 = 88°$

$\angle 1$ is obtuse.

32. The polygon has 3 sides. It is equilateral and equiangular, so it is a regular triangle.

33. The polygon has 4 sides. Its angles are all the same, so it is an equiangular quadrilateral.

34. The polygon has 8 sides, so it is an octagon. It is equilateral but not regular because it is concave.

35. $\overline{BC} \cong \overline{DE}$, so $BC = DE$

$5x - 4 = 2x + 11$

$3x = 15$

$x = 5$

$5(5) - 4 = 21$

$\overline{BC} \cong \overline{AB}$, so $BC = AB = 21$ units.

Chapter Test for the chapter "Essentials of Geometry"

1. True
2. False; point D lies on line ℓ.
3. False; Points B, C, and E are coplanar but point Q is not.
4. False; Points C and E are on line ℓ, but point B is not.
5. False; Point Q is not on the plane G.
6. $HJ = HK - JK$

$HJ = 52 - 30$

$HJ = 22$

7. $BC = AC - AB$

$BC = 18 - 7$

$BC = 11$

8. $XZ = XY + YZ$

$XZ = 26 + 45$

$XZ = 71$

9. $TW = \sqrt{(2-3)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10} \approx 3.2$

10. $CD = \sqrt{(6-5)^2 + (-1-10)^2}$

$= \sqrt{1+121} = \sqrt{122} \approx 11.0$

11. $MN = \sqrt{(-1-(-8))^2 + (3-0)^2} = \sqrt{49+9}$

$= \sqrt{58} \approx 7.6$

12. $\frac{3+x}{2} = 9 \qquad \frac{9+y}{2} = 7$

$3 + x = 18 \qquad 9 + y = 14$

$x = 15 \qquad y = 5$

The coordinates of endpoint B are (15, 5).

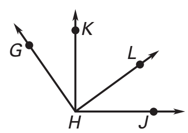
13. $CM = MD$

$3x = 27$

$x = 9$

$CD = 3(9) + 27 = 54$

14.



$m\angle GMJ = 125°$; $\angle GMJ$ is obtuse.

15. $m\angle KHL + m\angle CMJ = m\angle KHJ$

$(8x - 1)° + (4x + 7)° = 90°$

$12x + 6 = 90$

$12x = 84$

$x = 7$

$m\angle LMJ = 4(7) + 7 = 35°$

16. $m\angle QRT = m\angle SRT = \frac{1}{2}m\angle QRT$

$m\angle QRT = \frac{1}{2}(154°) = 77°$

$m\angle QRS = m\angle SRT = 77°$

17. $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 1$ and $\angle 4$, $\angle 1$ and $\angle 2$ are all linear pairs.

18. $\angle 2$ and $\angle 4$, $\angle 1$ and $\angle 3$ are vertical angles.

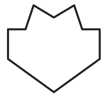
19. $90° - 64° = 26°$

Its complement is 26°.

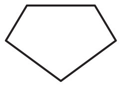
$180° - 64° = 116°$

Its supplement is 116°.

20. *Sample answer:*



concave decagon



convex pentagon

Algebra Review for the chapter "Essentials of Geometry"

1. $9y + 1 - y = 49$

$8y = 48$

$y = 6$

2. $5z + 7 + z = -8$

$6z = -15$

$z = -2\frac{1}{2}$

3. $-4(2 - t) = -16$

$-8 + 4t = -16$

$4t = -8$

$t = -2$

4. $7a - 2(a - 1) = 17$

$7a - 2a + 2 = 17$

$5a = 15$

$a = 3$

5. $\frac{4x}{3} + 2(3 - x) = 5$

$\frac{4x}{3} + 6 - 2x = 5$

$4x + 18 - 6x = 15$

$-2x = -3$

$x = 1\frac{1}{2}$

6. $\frac{2x - 5}{7} = 4$

$2x - 5 = 28$

$2x = 33$

$x = 16\frac{1}{2}$

7. $9c - 11 = -c + 29$

$10c = 40$

$c = 4$

8. $2(0.3r + 1) = 23 - 0.1r$

$0.6r + 2 = 23 - 0.1r$

$0.7r = 21$

$r = 30$

9. $5(k + 2) = 3(k - 4)$

$5k + 10 = 3k - 12$

$2x = -22$

$x = -11$

10. Let x represent the number of boxes of stationary.

(Cost of each box × Number of boxes) + Cost of book = Gift certificate

$4.59x + 8.99 = 50$

$4.59x = 41.01$

$x = 8.93$

You can buy 8 boxes of stationary.

11. Let x represent the number of people.

(Charge per person × Number of people) + Cost to rent the room = Amount to spend

$8.75x + 350 = 500$

$8.75x = 150$

$x = 17.14$

17 people can come to the party.

12. Let x represent the number of smaller beads.

(Length of smaller bead × Number of smaller beads) + Length of larger bead = Length of necklace

$\frac{3}{4}x + 1\frac{1}{2} = 18$

$\frac{3}{4}x = 16\frac{1}{2}$

$x = 22$

You need 22 smaller beads.

Extra Practice

For the chapter "Essentials of Geometry"

1. *Sample answer:* Points A, F, and B are collinear. A name for the line is $\overleftrightarrow{AB}$.

2. The intersection of plane ABC and $\overleftrightarrow{EG}$ is F.

3. *Sample answer:* $\overrightarrow{FE}$ and $\overrightarrow{FG}$ are opposite rays, $\overrightarrow{FA}$ and $\overrightarrow{FB}$ are opposite rays.

4. Yes; because A, C, and G are noncollinear, there is a plane that contains them.

5. *Sample answer:* $\overleftrightarrow{AB}$ intersects plane AFD at more than one point.

6. $PQ + QT = PT$

$PQ + 42 = 54$

$PQ = 54 - 42$

$PQ = 12$

7. $PS = PQ + QS$

$PS = 12 + 31$

$PS = 43$

8. $QR + RS = QS$

$QR + 17 = 31$

$QR = 31 - 17$

$QR = 14$

9. $PR = PQ + QR$

$PR = 12 + 14$

$PR = 26$

10. $QS + ST = QT$

$31 + ST = 42$

$ST = 42 - 31$

$ST = 11$

11. $RT = RS + ST$

$RT = 17 + 11$

$RT = 28$

12. $AB + BC = AC$

$(x + 3) + (2x + 1) = 10$

$3x + 4 = 10$

$3x = 6$

$x = 2$

$AB = x + 3 = 2 + 3 = 5$

$BC = 2x + 1 = 2(2) + 1 = 5$

Because $AB = 5$ and $BC = 5$, $\overline{AB}$ and $\overline{BC}$ are congruent.

13. $AB + BC = AC$

$(3x - 7) + (3x - 1) = 16$

$6x - 8 = 16$

$6x = 24$

$x = 4$

$AB = 3x - 7 = 3(4) - 7 = 5$

$BC = 3x - 1 = 3(4) - 1 = 11$

Because $AB = 5$ and $BC = 11$, $\overline{AB}$ and $\overline{BC}$ are not congruent.

14. $AB + BC = AC$

$(11x - 16) + (8x - 1) = 78$

$19x - 17 = 78$

$19x = 95$

$x = 5$

$AB = 11x - 16 = 11(5) - 16 = 39$

$BC = 8x - 1 = 8(5) - 1 = 39$

Because $AB = 39$ and $BC = 39$, $\overline{AB}$ and $\overline{BC}$ are congruent.

15. $AB + BC = AC$

$(4x - 5) + (2x - 7) = 54$

$6x - 12 = 54$

$6x = 66$

$x = 11$

$AB = 4x - 5 = 4(11) - 5 = 39$

$BC = 2x - 7 = 2(11) - 7 = 15$

Because $AB = 39$ and $BC = 15$, $\overline{AB}$ and $\overline{BC}$ are not congruent.

16. $AB + BC = AC$

$(14x + 5) + (10x + 15) = 80$

$24x + 20 = 80$

$24x = 60$

$x = 2.5$

$AB = 14x + 5 = 14(2.5) + 5 = 40$

$BC = 10x + 15 = 10(2.5) + 15 = 40$

Because $AB = 40$ and $BC = 40$, $\overline{AB}$ and $\overline{BC}$ are congruent.

17. $AB + BC = AC$

$(3x - 7) + (2x + 5) = 108$

$5x - 2 = 108$

$5x = 110$

$x = 22$

$AB = 3x - 7 = 3(22) - 7 = 59$

$BC = 2x + 5 = 2(22) + 5 = 49$

Because $AB = 59$ and $BC = 49$, $\overline{AB}$ and $\overline{BC}$ are not congruent.

18. $M\left(\frac{2 + 7}{2}, \frac{-4 + 1}{2}\right) = M\left(\frac{9}{2}, -\frac{3}{2}\right)$

19. $M\left(\frac{-3 - 8}{2}, \frac{-2 + 4}{2}\right) = M\left(-\frac{11}{2}, 1\right)$

20. $M\left(\frac{-2.3 + 3.1}{2}, \frac{-1.9 - 9.7}{2}\right) = M(0.4, -5.8)$

21. $M\left(\frac{3 - 1}{2}, \frac{-7 + 9}{2}\right) = M(1, 1)$

22. $M\left(\frac{4 + 2}{2}, \frac{3 + 2}{2}\right) = M\left(3, \frac{5}{2}\right)$

23. $M\left(\frac{1.7 + 8.5}{2}, \frac{-7.9 - 8.2}{2}\right) = M(5.1, -8.05)$

24. $ZM = \sqrt{(7 - 0)^2 + (1 - 1)^2} = \sqrt{49} = 7$

So, the length of the segment with endpoint Z and midpoint M is $2 \cdot ZM = 2 \cdot 7 = 14$.

25. $YM = \sqrt{(1 - 4)^2 + (7 - 3)^2} = \sqrt{25} = 5$

So, the length of the segment with endpoint Z and midpoint M is $2 \cdot YM = 2 \cdot 5 = 10$.

26. $XM = \sqrt{(12 - 0)^2 + (4 - [-1])^2} = \sqrt{169} = 13$

So, the length of the segment with endpoint X and midpoint M is $2 \cdot XM = 2 \cdot 13 = 26$.

27. $WM = \sqrt{(-10 - 5)^2 + (-5 - 3)^2} = \sqrt{289} = 17$

So, the length of the segment with endpoint W and midpoint M is $2 \cdot WM = 2 \cdot 17 = 34$.

28. $VM = \sqrt{[9 - (-3)]^2 + [5 - (-4)]^2} = \sqrt{225} = 15$

So, the length of the segment with endpoint V and midpoint M is $2 \cdot VM = 2 \cdot 15 = 30$.

29. $UM = \sqrt{(11 - 3)^2 + (-4 - 2)^2} = \sqrt{100} = 10$

So, the length of the segment with endpoint U and midpoint M is $2 \cdot UM = 2 \cdot 10 = 20$.

30. $m\angle QPS = m\angle QPR + m\angle RPS = 57° + 64° = 121°$

31. $m\angle LMN = m\angle LMJ + m\angle JMN = 36° + 68° = 104°$

32. $\angle WXY \cong \angle ZWY$, so $m\angle ZWY = 43°$.

$m\angle XWZ = m\angle XWY + m\angle ZWY = 43° + 43° = 86°$

33. $m\angle ABC = m\angle ABD + m\angle CBD$

$133° = (7x + 4)° + (3x + 9)°$

$133 = 10x + 13$

$120 = 10x$

$12 = x$

So, $m\angle ABD = (7x + 4)° = (7[12] + 4)° = 88°$.

34. $m\angle GHK = 17°$

$4x - 3 = 17$

$4x = 20$

$x = 5$

So, $m\angle KHJ = (3x + 2)° = (3[5] + 2)° = 17°$.

35. Because they share a common vertex and side, but have no common interior points, $\angle 1$ and $\angle 2$ are adjacent angles.

36. $41° + m\angle 1 + 49° + m\angle 2 = 180°$

$90° + m\angle 1 + m\angle 2 = 180°$

$m\angle 1 + m\angle 2 = 90°$

So, $\angle 1$ and $\angle 2$ are complementary.

37. The sides of $\angle 1$ and $\angle 2$ form two pairs of opposite rays. So $\angle 1$ and $\angle 2$ are vertical angles. Also, $\angle 1$ forms a linear pair with a right angle. The angles in a linear pair are supplementary, so $m\angle 1 + 90° = 180°$, or $m\angle 1 = 90°$. Similarly, $m\angle 2 = 90°$. So $\angle 1$ and $\angle 2$ are supplementary because $m\angle 1 + m\angle 2 = 90° + 90° = 180°$.

38. *Sample answer:* $\angle ACE$ and $\angle ACB$ form a linear pair, so they are supplementary. $\angle ACB \cong \angle BCD$, so $m\angle ACB = m\angle BCD$. Using substitution, $m\angle ACE + m\angle BCD = 180°$. So, $\angle ACE$ and $\angle BCD$ are supplementary.

39. *Sample answer:* $\angle ACE$ and $\angle BCF$ are vertical angles that cannot be complementary, because each angle is obtuse.

40. Because $\angle ACF$ is a straight angle, $m\angle DCF = 90°$, and $\angle ACB \cong \angle BCD$, $\angle ACB$ and $\angle BCD$ are complementary angles and adjacent angles. Because $\angle BCE$ is a straight angle, $m\angle DCF = 90°$, and $\angle ECF \cong \angle BCD$, $\angle ECF$ and $\angle BCD$ are complementary angles, and neither vertical nor adjacent angles. Because $\angle ECF \cong \angle BCD$, $\angle ECF$ and $\angle ACB$ are complementary angles and vertical angles.

41. The figure is a concave polygon.

42. The figure is not a polygon because at least one side intersects more than two other sides.

43. The figure is not a polygon because part of the figure is not a line segment.

44. The figure is a convex polygon.

45. *DFHKB* and *ABCDEFGHJK* are equilateral polygons. *DFHKB* is a pentagon because it has five sides. *ABCDEFGHJK* is a decagon because it has ten sides.

46. *Sample answer:*

ABK is a triangle.

AEJK is a quadrilateral.

BDFHK is a pentagon.

ADGHJK is a hexagon.

JCDEFGH is a heptagon.