Warm Up

Find the unknown side lengths in each special right triangle.

1. a 30°-60°-90° triangle with hypotenuse 2 ft

1 ft; $\sqrt{3}$ ft

2. a 45°-45°-90° triangle with leg length 4 in.

4 in; $4\sqrt{2}$ in.

3. a 30°-60°-90° triangle with longer leg length 3m

$\sqrt{3}$ m; $2\sqrt{3}$ m

Objectives

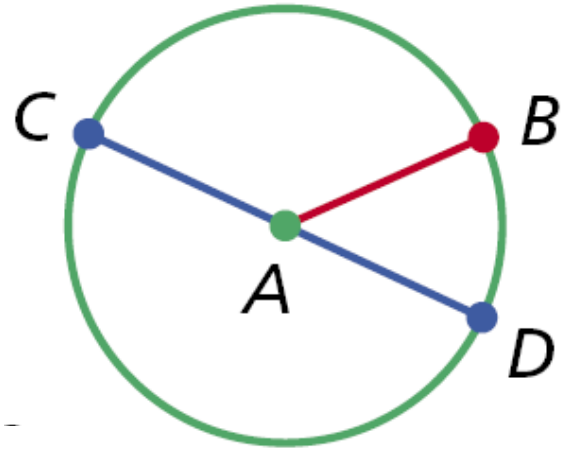
Develop and apply the formulas for the area and circumference of a circle.

Develop and apply the formula for the area of a regular polygon.

9-2 Developing Formulas for Circles and Regular Polygons

A **circle** is the locus of points in a plane that are a fixed distance from a point called the **center of the circle**. A circle is named by the symbol ⊙ and its center. ⊙*A* has radius ***r*** = ***AB*** and diameter ***d*** = ***CD***.

The irrational number π is defined as the ratio of the circumference C to the diameter d, or $\pi = \frac{C}{d}$.

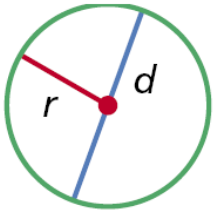


Solving for C gives the formula $C = \pi d$. Also $d = 2r$, so $C = 2\pi r$.

9-2 Developing Formulas for Circles and Regular Polygons

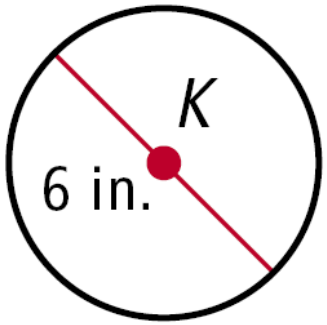
Circumference and Area — Circle

A circle with diameter d and radius r has circumference $C = \pi d$ or $C = 2\pi r$ and area $A = \pi r^2$.



Example 1A: Finding Measurements of Circles

Find the area of ⊙K in terms of π.



$A = \pi r^2$ *Area of a circle.*

$A = \pi(3)^2$ *Divide the diameter by 2 to find the radius, 3.*

$A = 9\pi \text{ in}^2$ *Simplify.*

Example 1B: Finding Measurements of Circles

Find the radius of ⊙*J* if the circumference is $(65x + 14)\pi$ m.

$C = 2\pi r$ *Circumference of a circle*

$(65x + 14)\pi = 2\pi r$ *Substitute* $(65x + 14)\pi$ *for C.*

$r = (32.5x + 7)$ m *Divide both sides by* 2π.

Example 1C: Finding Measurements of Circles

Find the circumference of ⊙*M* if the area is $25x^2\pi$ ft^2

Step 1 Use the given area to solve for *r*.

$A = \pi r^2$ — *Area of a circle*

$25x^2\pi = \pi r^2$ — *Substitute $25x^2\pi$ for A.*

$25x^2 = r^2$ — *Divide both sides by π.*

$5x = r$ — *Take the square root of both sides.*

Example 1C Continued

Step 2 Use the value of r to find the circumference.

$C = 2\pi r$

$C = 2\pi(5x)$ *Substitute 5x for r.*

$C = 10x\pi$ ft *Simplify.*

Example 2: Cooking Application

A pizza-making kit contains three circular baking stones with diameters 24 cm, 36 cm, and 48 cm. Find the area of each stone. Round to the nearest tenth.

24 cm diameter

$r = \frac{24}{2} = 12$

$A = \pi(12)^2$

$\approx 452.4 \text{ cm}^2$

36 cm diameter

$r = \frac{36}{2} = 18$

$A = \pi(18)^2$

$\approx 1017.9 \text{ cm}^2$

48 cm diameter

$r = \frac{48}{2} = 24$

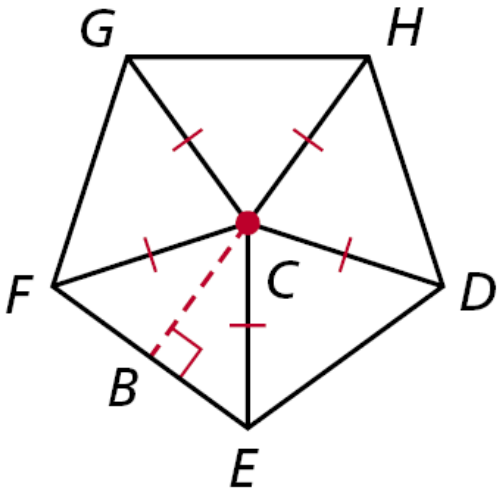
$A = \pi(24)^2$

$\approx 1809.6 \text{ cm}^2$

1. The **center of a regular polygon** is equidistant from the vertices.
2. The **apothem** is the distance from the center to a side.
3. A **central angle of a regular polygon** has its vertex at the center, and its sides pass through consecutive vertices.

a. Each central angle measure of a regular n-gon is $\frac{360^\circ}{n}$.

Regular pentagon *DEFGH* has a center *C*, apothem *BC*, and central angle $\angle DCE$.



9-2 Developing Formulas for Circles and Regular Polygons

To find the area of a regular *n*-gon with side length *s* and apothem *a*, divide it into *n* congruent isosceles triangles.

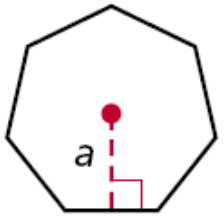
area of each triangle: $\frac{1}{2}as$

total area of the polygon: $A = n\left(\frac{1}{2}as\right)$, or $A = \frac{1}{2}aP$

The perimeter is P = ns.

Area **Regular Polygon**

The area of a regular polygon with apothem *a* and perimeter *P* is $A = \frac{1}{2}aP$.



Example 3A: Finding the Area of a Regular Polygon

Find the area of regular heptagon with side length 2 ft to the nearest tenth.

Step 1 Draw the heptagon. Draw an isosceles triangle with its vertex at the center of the heptagon. The central angle is $\frac{360^\circ}{7} \approx 51.4^\circ$.

Draw a segment that bisects the central angle and the side of the polygon to form a right triangle.

Example 3A Continued

Step 2 Use the tangent ratio to find the apothem.

$\tan(25.7°) = \frac{1}{a}$ *The tangent of an angle is* $\frac{\text{opp. leg}}{\text{adj. leg}}$.

$a = \frac{1}{\tan(25.7°)}$ *Solve for a.*

Example 3A Continued

Step 3 Use the apothem and the given side length to find the area.

$A = \frac{1}{2}aP$ *Area of a regular polygon*

$A = \frac{1}{2}\left(\frac{1}{\tan(25.7°)}\right)(14)$ *The perimeter is 2(7) = 14ft.*

$A \approx 14.5 \text{ ft}^2$ *Simplify. Round to the nearest tenth.*

Example 3B: Finding the Area of a Regular Polygon

Find the area of a regular dodecagon with side length 5 cm to the nearest tenth.

Step 1 Draw the dodecagon. Draw an isosceles triangle with its vertex at the center of the dodecagon. The central angle is $\frac{360^\circ}{12} = 30^\circ$.

Draw a segment that bisects the central angle and the side of the polygon to form a right triangle.

Example 3B Continued

Step 2 Use the tangent ratio to find the apothem.

$$\tan(15^\circ) = \frac{2.5}{a}$$

The tangent of an angle is $\frac{\text{opp. leg}}{\text{adj. leg}}$.

$$a = \frac{2.5}{\tan(15^\circ)}$$

Solve for a.

Example 3B Continued

Step 3 Use the apothem and the given side length to find the area.

$$A = \frac{1}{2}aP$$

Area of a regular polygon

$$A = \frac{1}{2}\left(\frac{2.5}{\tan(15°)}\right)(60)$$

The perimeter is 5(12) = 60 ft.

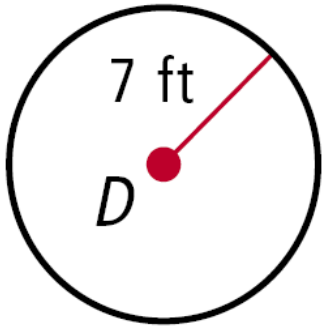
$A \approx 279.9$ cm^2

Simplify. Round to the nearest tenth.

Lesson Quiz: Part I

Find each measurement.

1. the area of $\odot D$ in terms of π



$A = 49\pi \text{ ft}^2$

2. the circumference of $\odot T$ in which $A = 16\pi \text{ mm}^2$

$C = 8\pi \text{ mm}$

Lesson Quiz: Part II

Find each measurement.

3. Speakers come in diameters of 4 in., 9 in., and 16 in. Find the area of each speaker to the nearest tenth.

$A_1 \approx 12.6$ in^2 ; $A_2 \approx 63.6$ in^2 ; $A_3 \approx 201.1$ in^2

Find the area of each regular polygon to the nearest tenth.

4. a regular nonagon with side length 8 cm

$A \approx 395.6$ cm^2

5. a regular octagon with side length 9 ft

$A \approx 391.1$ ft^2

homework

Re-teach

Worksheets

9-2