ARE YOU READY? PAGE 515

1. D **2.** C

3. A **4.** E

5. $\frac{PR}{RT} = \frac{10}{5} = 2$; $\frac{QR}{RS} = \frac{12}{6} = 2$
$\angle PRQ \cong \angle TRS$ by Vert. $\angle$s Thm.
yes; $\triangle PRQ \sim \triangle TRS$ by SAS $\sim$

6. $\frac{AB}{FE} = \frac{6}{4} = \frac{3}{2}$; $\frac{BC}{ED} = \frac{15}{10} = \frac{3}{2}$
$\angle B \cong \angle E$ by Rt. $\angle \cong$ Thm.
yes; $\triangle ABC \sim \triangle FED$ by SAS $\sim$

7. $x = 5\sqrt{2}$

8. $16 = x\sqrt{2}$
$16\sqrt{2} = 2x$
$x = 8\sqrt{2}$

9. $x = 4\sqrt{3}$

10. $x = 2(3) = 6$

11. $3(x - 1) = 12$
$x - 1 = 4$
$x = 5$

12. $-2(y + 5) = -1$
$y + 5 = 0.5$
$y = -4.5$

13. $6 = 8(x - 3)$
$6 = 8x - 24$
$30 = 8x$
$x = 3.75$

14. $2 = -1(z + 4)$
$2 = -z - 4$
$z = -6$

15. $\frac{4}{y} = \frac{6}{18}$
$\frac{4}{y} = \frac{1}{3}$
$4(3) = y(1)$
$y = 12$

16. $\frac{5}{8} = \frac{x}{32}$
$5(32) = 8(x)$
$160 = 8x$
$x = 20$

17. $\frac{m}{9} = \frac{8}{12} = \frac{2}{3}$
$3m = 9(2) = 18$
$m = 6$

18. $\frac{y}{4} = \frac{9}{y}$
$y(y) = 4(9)$
$y^2 = 36$
$y = \pm 6$

19. $13.118 \approx 13.12$

20. $37.91 \approx 37.9$

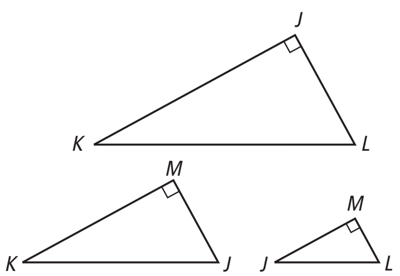
21. $15.992 \approx 16.0$

22. $173.05 \approx 173$

8-1 SIMILARITY IN RIGHT TRIANGLES, PAGES 518–523

CHECK IT OUT! PAGES 518–520

1. Sketch the 3 rt. $\triangle$s with $\angle$s of $\triangle$s in corr. positions.



By Thm. 8-1-1, $\triangle LJK \sim \triangle JMK \sim \triangle LMJ$.

2a. $x^2 = (2)(8) = 16$
$x = 4$

b. $x^2 = (10)(30) = 300$
$x = \sqrt{300} = 10\sqrt{3}$

c. $x^2 = (8)(9) = 72$
$x = \sqrt{72} = 6\sqrt{2}$

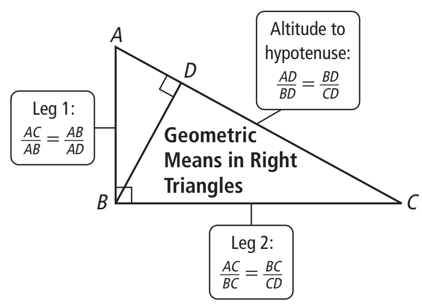
3. $9^2 = (u)(3)$
$81 = 3u$
$u = 27$
$v^2 = (3)(3 + u)$
$v^2 = (3)(30) = 90$
$v = \sqrt{90} = 3\sqrt{10}$
$w^2 = (u)(3 + u)$
$w^2 = (27)(30) = 810$
$w = \sqrt{810} = 9\sqrt{10}$

4. Let x be height of cliff above eye level.
$(28)^2 = 5.5x$
$x \approx 142.5$ ft
Cliff is about 142.5 + 5.5, or 148 ft high.

THINK AND DISCUSS, PAGE 520

1. Set up the proportion $\frac{7}{x} = \frac{x}{21}$, and solve for x.
$x^2 = 7(21) = 147$
$x = \sqrt{147} = 7\sqrt{3}$

2.

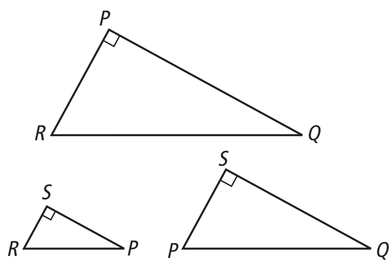


EXERCISES, PAGES 521–523

GUIDED PRACTICE, PAGE 521

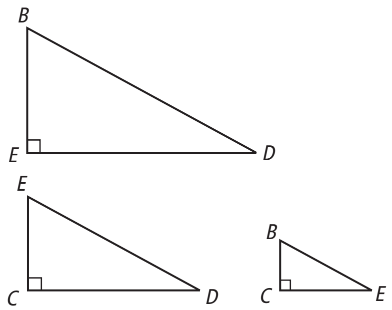
1. 8 is geometric mean of 2 and 32.

2. Sketch the 3 rt. $\triangle$s with $\angle$s of $\triangle$s in corr. positions.



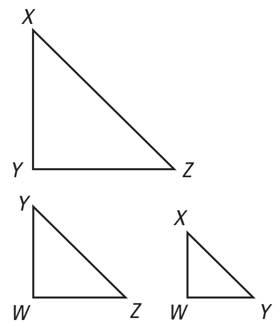
By Thm. 8-1-1, $\triangle RPQ \sim \triangle PSQ \sim \triangle RSP$.

3. Sketch the 3 rt. △s with ∠s of △s in corr. positions.



By Thm. 8-1-1, $\triangle BED \sim \triangle ECD \sim \triangle BCE$.

4. Sketch the 3 rt. △s with ∠s of △s in corr. positions.



By Thm. 8-1-1, $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$.

5. $x^2 = (2)(50) = 100$
$x = 10$

6. $x^2 = (4)(16) = 64$
$x = 8$

7. $x^2 = \left(\frac{1}{2}\right)(8) = 4$
$x = 2$

8. $x^2 = (9)(12) = 108$
$x = \sqrt{108} = 6\sqrt{3}$

9. $x^2 = (16)(25) = 400$
$x = 20$

10. $x^2 = (7)(11) = 77$
$x = \sqrt{77}$

11. $x^2 = (10)(6) = 60$
$x = \sqrt{60} = 2\sqrt{15}$
$y^2 = (6)(4) = 24$
$y = \sqrt{24} = 2\sqrt{6}$
$z^2 = (10)(4) = 40$
$z = \sqrt{40} = 2\sqrt{10}$

12. $10^2 = 100 = 20x$
$x = 5$
$y^2 = (20)(20 + 5) = 500$
$y = \sqrt{500} = 10\sqrt{5}$
$z^2 = (5)(20 + 5) = 125$
$z = \sqrt{125} = 5\sqrt{5}$

13. $\left(6\sqrt{13}\right)^2 = (18)(18 + z)$
$468 = 324 + 18z$
$144 = 18z$
$z = 8$
$x^2 = (8)(18) = 144$
$x = 12$
$y^2 = (8)(18 + 8) = 208$
$y = \sqrt{208} = 4\sqrt{13}$

14. $RS^2 = (64)(60) = 3840$
$RS = \sqrt{3840} \approx 62.0$ m

PRACTICE AND PROBLEM SOLVING, PAGES 521–523

15. By Thm. 8-1-1, $\triangle MPN \sim \triangle PQN \sim \triangle MQP$.

16. By Thm. 8-1-1, $\triangle CAB \sim \triangle ADB \sim \triangle CDA$.

17. By Thm. 8-1-1, $\triangle RSU \sim \triangle RTS \sim \triangle STU$.

18. $x^2 = (5)(45) = 225$
$x = 15$

19. $x^2 = (3)(15) = 45$
$x = 3\sqrt{5}$

20. $x^2 = (5)(8) = 40$
$x = 2\sqrt{10}$

21. $x^2 = \left(\frac{1}{4}\right)(80) = 20$
$x = 2\sqrt{5}$

22. $x^2 = (1.5)(12) = 18$
$x = 3\sqrt{2}$

23. $x^2 = \left(\frac{2}{3}\right)\left(\frac{27}{40}\right) = \frac{9}{20}$
$x = \frac{3}{2\sqrt{5}} = \frac{3\sqrt{5}}{10}$

24. $12^2 = 4(4 + x)$
$144 = 16 + 4x$
$128 = 4x$
$x = 32$
$y^2 = 4(32) = 128$
$y = 8\sqrt{2}$
$z^2 = 32(4 + 32) = 1152$
$z = 24\sqrt{2}$

25. $x^2 = (30)(40) = 1200$
$x = 20\sqrt{3}$
$y^2 = (30)(70) = 2100$
$y = 10\sqrt{21}$
$z^2 = (70)(40) = 2800$
$z = 20\sqrt{7}$

26. $9.6^2 = (z)(12.8)$
$92.16 = 12.8z$
$z = 7.2$
$y^2 = (12.8)(12.8 + 7.2) = 256$
$y = 16$
$x^2 = (7.2)(12.8 + 7.2) = 144$
$x = 12$

27. Let h represent height of tower above eye level.
91 ft 3 in. = 91.25 ft
$(91.25)^2 = 5h$
$h \approx 1665$ ft
Tower is about 1665 + 5 = 1670 ft high.

28. $8^2 = 64 = 2x$
$x = 32$

29. $\left(2\sqrt{5}\right)^2 = 20 = 6x$
$x = \frac{10}{3} = 3\frac{1}{3}$

30. y; $\frac{x}{z} = \frac{z}{y}$

31. $x + y$; $\frac{x + y}{u} = \frac{u}{x}$

32. y; $\frac{x + y}{v} = \frac{v}{y}$

33. z; $\frac{y}{z} = \frac{z}{x}$

34. v; $v^2 = y(x + y)$

35. x; $u^2 = (x + y)x$

36. $BD^2 = (AD)(CD)$
$= (12)(8) = 96$
$BD = 4\sqrt{6}$

37. $BC^2 = (AC)(CD)$
$= (16)(5) = 80$
$BC = 4\sqrt{5}$

38. $BD^2 = (AC)(CD)$
$= (\sqrt{2})(\sqrt{2}) = 2$
$BD = \sqrt{2}$

39. $BC^2 = (AC)(CD)$
$5 = CD\sqrt{10}$
$5\sqrt{10} = 10CD$
$CD = \dfrac{\sqrt{10}}{2}$

40. $\sqrt{(0.1)\,(0.03)}$ X 100% ≈ 5.5%

41. B is incorrect; proportion should be $\dfrac{12}{EF} = \dfrac{EF}{8}$.

42. $a^2 = (2)(5) = 10$
$a = \sqrt{10} \approx 3.2$
Altitude is about 3.2 cm long.

43. By Corollary 8-1-3, $a^2 = x(x + y)$ and $b^2 = y(x + y)$. So $a^2 + b^2 = x(x + y) + y(x + y)$. By Distrib. Prop., this expression simplifies to $(x + y)(x + y) = (x + y)^2 = c^2$. So $a^2 + b^2 = c^2$.

44a. $SW^2 = (RS)(ST) = (4)(3) = 12$
$SW = \sqrt{12} \approx 3.46$ ft, or 3 ft 6 in.

b. $RW^2 = (RS)(RT) = (4)(7) = 28$
$RW = \sqrt{28} \approx 5.29$ ft, or 5 ft 3 in.

45. Area of rect. is ab, and area of square is s^2. It is given that $s^2 = ab$, so s is geometric mean of a and b.

46. Let z be geometric mean of x and y, where $x = a^2$ and $y = b^2$. So $z = \sqrt{a^2b^2} = ab$, which is a whole number.

TEST PREP, PAGE 523

47. D
$XY^2 = (8)(11) = 88$
$XY \approx 9.4$ ft

48. H
$BD^2 = (9)(4) = 36$
$BD = 6$
Area $= \dfrac{1}{2}(BD)(AC)$
$= \dfrac{1}{2}(6)(13) = 39\text{ m}^2$

49. A
$RS^2 = (1)(y + 1) = y + 1$
$RS = \sqrt{y + 1}$

CHALLENGE AND EXTEND, PAGE 523

50. Let x be length of shorter seg.
$8^2 = (x)(4x) = 4x^2$
$8 = \sqrt{4x^2} = 2x$
$x = 4$
Lengths of segs. are 4 in. and 4(4) = 16 in.

51. $(2\sqrt{21})^2 = (x)(x + 5)$
$84 = x^2 + 5x$
$0 = x^2 + 5x - 84$
$0 = (x - 7)(x + 12)$
$x = 7$ (since $x > 0$)
$y^2 = (7)(5) = 35$
$y = \sqrt{35}$
$z^2 = 5(5 + 7) = 60$
$z = 2\sqrt{15}$

52. Let $AD = DC = a$. By Corollary 8-1-3, $AB^2 = (a)(2a) = 2a^2$, and $BC^2 = (a)(2a) = 2a^2$. So $AB = BC = a\sqrt{2}$. Therefore $\triangle ABC$ is isosc., so it is a 45°-45°-90° $\triangle$.

53. **Step 1** Apply Cor. 8-1-3 in $\triangle BDE$ to find BF and BD.
$EF^2 = (BF)(FD)$
$3.28^2 = 4.86BF$
$BF \approx 2.214$
$BD \approx 7.074$
Step 2 Apply Cor. 8-1-3 in $\triangle BDE$ to find BE.
$BE^2 = (BF)(BD) \approx 15.662$
$BE \approx 3.958$
Step 3 Apply Cor. 8-1-3 in $\triangle BCD$ to find BC.
$BD^2 = (BE)(BC)$
$7.074^2 \approx 3.958BC$
$BC \approx 12.643$
Step 4 Apply Cor. 8-1-3 in $\triangle BCD$ to find CD.
$CD^2 = (BC)(EC) \approx 109.806$
$CD \approx 10.479$
Step 5 Apply Cor. 8-1-3 in $\triangle ABC$ to find AC.
$BC^2 = (AC)(CD)$
$12.643^2 \approx 10.479AC$
$AC \approx 15.26$ cm
Step 6 Apply Pyth. Thm. in $\triangle ABD$ to find AB.
$AB^2 = BD^2 + AD^2$
$AB^2 \approx 7.07^2 + (15.26 - 10.48)^2$
$AB \approx 8.53$ cm

SPIRAL REVIEW, PAGE 523

54. at x-intercept, $y = 0$
$3(0) + 4 = 4 = 6x$
$x = \dfrac{4}{6} = \dfrac{2}{3}$

at y-intercept, $x = 0$
$3y + 4 = 6(0)$
$3y = -4$
$y = -\dfrac{4}{3}$

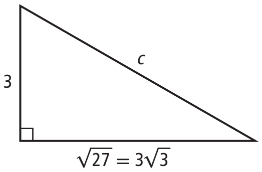
55. at x-intercept, $y = 0$
$x + 4 = 2(0)$
$x = -4$

at y-intercept, $x = 0$
$0 + 4 = 2y$
$y = 2$

56. at x-intercept, $y = 0$
$3(0) - 15 = -15 = 15x$
$x = -1$

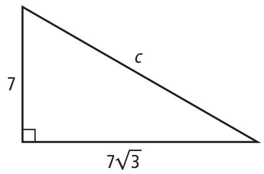
at y-intercept, $x = 0$
$3y - 15 = 15(0)$
$3y = 15$
$y = 5$

57.



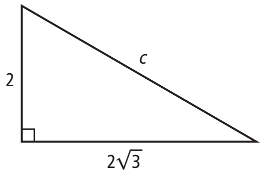
$c = 2(3) = 6$

58.



$c = 2(7) = 14$

59.



$c = 2(2) = 4$

60. $\angle DEC$ is a rt. $\angle$, so $30y = 90 \rightarrow y = 3$
$m\angle EDC = 8(3) + 15 = 39°$

61. $\overrightarrow{DB}$ bisects $\angle ADC$, so $m\angle EDA = m\angle EDC = 39°$

62.
$AB = BC$
$2x + 8 = 4x$
$8 = 2x$
$x = 4$
$AB = 2(4) + 8 = 16$

TECHNOLOGY LAB: EXPLORE TRIGONOMETRIC RATIOS, PAGE 524

TRY THIS, PAGE 524

1. $m\angle A$ stays the same. Each $\triangle$ will have 2 ∡ ($\angle DEA$ and $\angle A$) $\cong$ to 2 ∡ in every other $\triangle$, so by AA $\sim$ Post., these ⨻ are $\sim$ to each other.
2. Values of ratios do not change, because ratios of side lengths are equal in $\sim$ ⨻.
3. As C moves, $m\angle A$ changes. Once C is in a new position, moving D does not change the ratios.
4. $\frac{DE}{AE} = 1$, and $m\angle A = 45°$.
   If $\frac{DE}{AD} = \frac{AE}{AD}$, then $DE = AE$, so $\frac{DE}{AE} = 1$. Since 2 sides are $=$, $\triangle DEA$ is an isosc. $\triangle$. It is also a rt. $\triangle$. Thus $\triangle DEA$ must be a 45°-45°-90° special rt. $\triangle$, so $m\angle A = 45°$.

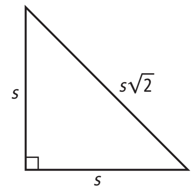
8-2 TRIGONOMETRIC RATIOS, PAGES 525–532

CHECK IT OUT! PAGES 526–528

1a. $\cos A = \frac{24}{25} = 0.96$ **b.** $\tan B = \frac{24}{7} \approx 3.43$

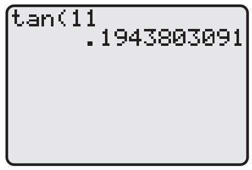
c. $\sin B = \frac{24}{25} = 0.96$

2.



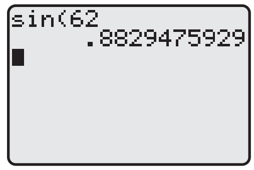
$\tan 45° = \frac{s}{s} = 1$

3a.



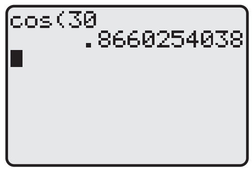
$\tan 11° \approx 0.19$

b.



$\sin 62° \approx 0.88$

c.



$\cos 30° \approx 0.87$

4a. $\overline{DF}$ is the hyp. Given: EF, opp. to given $\angle D$. Since opp. side and hyp. are involved, use a sine ratio.

$\sin D = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{EF}{DF}$

$\sin 51° = \frac{17}{DF}$

$DF = \frac{17}{\sin 51°} \approx 21.87$ m

b. $\overline{ST}$ is adj. to the given $\angle$. Given: TU, the hyp. Since adj. side and hyp. are involved, use a cosine ratio.

$\cos T = \frac{\text{adj. leg}}{\text{hyp.}} = \frac{ST}{TU}$

$\cos 42° = \frac{ST}{9.5}$

$9.5(\cos 42°) = ST$

$ST \approx 7.06$ in.

c. $\overline{BC}$ is adj. to the given $\angle$. Given: AC, opp. $\angle B$. Since opp. side and adj. side are involved, use a tangent ratio.

$\tan B = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{AC}{BC}$

$\tan 18° = \frac{12}{BC}$

$BC = \frac{12}{\tan 18°} \approx 36.93$ ft

d. $\overline{JL}$ is opp. the given ∠. Given: KL, the hyp. Since opp. side and hyp. are involved, use a sine ratio.

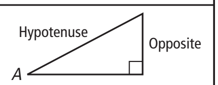
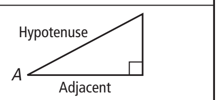
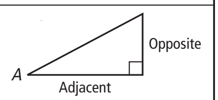
$$\sin K = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{JL}{KL}$$
$$\sin 27° = \frac{JL}{13.6}$$
$$13.6(\sin 27°) = JL$$
$$JL \approx 6.17 \text{ cm}$$

5. 1 Understand the Problem
Make a sketch. The **answer** is AC.
2 Make a Plan
$\overline{AC}$ is the hyp. You are given AB, the leg opp. ∠C. Since opp. leg and hyp. are involved, write an equation using a sine ratio.
3 Solve
$$\sin C = \frac{AB}{AC}$$
$$\sin 4.8° = \frac{1.2}{AC}$$
$$AC = \frac{1.2}{\sin 4.8°} \approx 14.34 \text{ ft}$$
4 Look Back
Problem asks for AC rounded to nearest hundredth, so round the length to 14.34. Length AC of ramp is 14.34 ft.

THINK AND DISCUSS, PAGE 528

1. Solve $\sin 32° = \frac{4}{AB}$. **2.** Solve $\cos 32° = \frac{6.4}{AB}$.

3.

Abbreviation	Words	Diagram
$\sin = \frac{\text{opp. leg}}{\text{hyp.}}$	The sine of an ∠ is the ratio of the length of the opp. leg to the length of the hyp.	
$\cos = \frac{\text{adj. leg}}{\text{hyp.}}$	The cosine of an ∠ is the ratio of the length of the adj. leg to the length of the hyp.	
$\tan = \frac{\text{opp. leg}}{\text{adj. leg}}$	The tangent of an ∠ is the ratio of the length of the opp. leg to the length of the adj. leg.	

EXERCISES, PAGES 529–532

GUIDED PRACTICE, PAGE 529

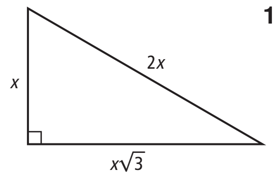
1. $\sin J = \frac{LK}{JL}$ **2.** $\tan N = \frac{MP}{MN}$

3. $\sin C = \frac{4}{5} = 0.8$ **4.** $\tan A = \frac{3}{4} = 0.75$

5. $\cos A = \frac{4}{5} = 0.8$ **6.** $\cos C = \frac{3}{5} = 0.6$

7. $\tan C = \frac{4}{3} \approx 1.33$ **8.** $\sin A = \frac{3}{5} = 0.6$

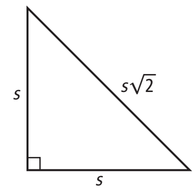
9.



$\cos 60° = \frac{x}{2x} = \frac{1}{2}$

10. $\tan 30° = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3}$

11.



$\sin 45° = \frac{s}{s\sqrt{2}} = \frac{\sqrt{2}}{2}$

12.
```
tan(67
     2.355852366
```
$\tan 67° \approx 2.36$

13.
```
sin(23
     .3907311285
```
$\sin 23° \approx 0.39$

14.
```
sin(49
     .7547095802
```
$\sin 49° \approx 0.75$

15.
```
cos(88
     .0348994967
```
$\cos 88° \approx 0.03$

16.
```
cos(12
     .9781476007
```
$\cos 12° \approx 0.98$

17.
```
tan(9
     .1583844403
```
$\tan 9° \approx 0.16$

18. $\overline{BC}$ is opp. the given ∠. Given: AC, the hyp. Since opp. side and hyp. are involved, use a sine ratio.
$$\sin A = \frac{\text{opp. leg}}{\text{hyp.}} = \frac{BC}{AC}$$
$$\sin 23° = \frac{BC}{4}$$
$$4(\sin 23°) = BC$$
$$BC \approx 1.56 \text{ in.}$$

19. $\overline{QR}$ is opp. the given ∠. Given: PQ, adj. to given ∠. Since opp. and adj. sides are involved, use a tangent ratio.
$$\tan P = \frac{\text{opp. leg}}{\text{adj. leg}} = \frac{QR}{PQ}$$
$$\tan 50° = \frac{QR}{8.1}$$
$$8.1(\tan 50°) = QR$$
$$QR \approx 9.65 \text{ m}$$

20. $\overline{KL}$ is adj. to the given ∠. Given: JL, the hyp. Since adj. side and hyp. are involved, use a cosine ratio.
$$\cos L = \frac{\text{adj. leg}}{\text{hyp.}} = \frac{KL}{JL}$$
$$\cos 61° = \frac{KL}{2.5}$$
$$2.5(\cos 61°) = KL$$
$$KL \approx 1.21 \text{ cm}$$

21. 1 Understand the Problem
The **answer** is XY, opp. the given $\angle$.
2 Make a Plan
You are given WZ, which is twice WY, the leg adj. to $\angle W$. First, calculate WY. Then, since opp. and adj. legs are involved, write an equation using a tangent ratio.
3 Solve

$$WY = \frac{1}{2}WZ$$
$$= \frac{1}{2}(56) = 28 \text{ ft}$$
$$\tan W = \frac{XY}{WY}$$
$$\tan 15° = \frac{XY}{28}$$
$$XY = 28(\tan 15°) \approx 7.5028 \text{ ft}$$

4 Look Back
Problem asks for XY rounded to nearest inch. Height XY of pediment is 7 ft 6 in.

PRACTICE AND PROBLEM SOLVING, PAGES 529–531

22. $\cos D = \frac{8}{17} \approx 0.47$

23. $\tan D = \frac{15}{8} \approx 1.88$

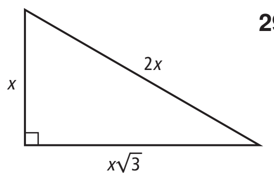
24. $\tan F = \frac{8}{15} \approx 0.53$

25. $\cos F = \frac{15}{17} \approx 0.88$

26. $\sin F = \frac{8}{17} \approx 0.47$

27. $\sin D = \frac{15}{17} \approx 0.88$

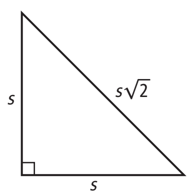
28.



$\tan 60° = \frac{x\sqrt{3}}{x} = \sqrt{3}$

29. $\sin 30° = \frac{x}{2x} = \frac{1}{2}$

30.



$\cos 45° = \frac{s}{s\sqrt{2}} = \frac{\sqrt{2}}{2}$

31. $\tan 51° \approx 1.23$

32. $\sin 80° \approx 0.98$

33. $\cos 77° \approx 0.22$

34. $\tan 14° \approx 0.25$

35. $\sin 55° \approx 0.82$

36. $\cos 48° \approx 0.67$

37. $PQ = 11 \sin 19°$
≈ 3.58 cm

38. $\cos 46° = \frac{19.2}{AC}$
$AC = \frac{19.2}{\cos 46°}$
≈ 27.64 in.

39. $\sin 34° = \frac{11}{GH}$
$GH = \frac{11}{\sin 34°}$
≈ 19.67 ft

40. $\cos 25° = \frac{33}{XZ}$
$XZ = \frac{33}{\cos 25°}$
≈ 36.41 in.

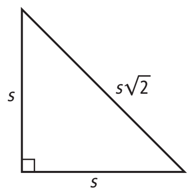
41. $\tan 61° = \frac{9.5}{KL}$
$KL = \frac{9.5}{\tan 61°}$
≈ 5.27 ft

42. $EF = 83.1 \tan 12°$
≈ 17.66 m

43. $\sin 15° = \frac{1.58}{\ell}$
$\ell = \frac{1.58}{\sin 15°} \approx 6.10$ m

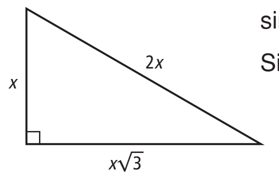
44. If a and b are opp. and adj. leg lengths,
$\tan(m\angle) = \frac{a}{b} = 1$
$a = b$
$\triangle$ is 45°-45°-90°, so $m\angle = 45°$

45.



$\sin 45° = \frac{s}{s\sqrt{2}} = \cos 45°$
So sine and cosine ratios are =.

46.



$\sin 30° = \frac{x}{2x} = \frac{1}{2}$
Sine of a 30° $\angle$ is 0.5.

47. $\cos 30° = \frac{x\sqrt{3}}{2x} = \sin 60°$
$\cos 30°$ = sine of a 60° $\angle$

48. $h = 10 \sin 75.5° \approx 9.7$ ft

49. $BC = AD$
$= 3 \tan(90 - 68)°$
≈ 1.2 ft

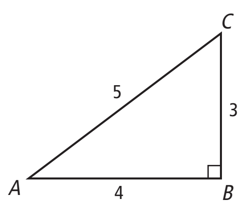
50. $SU = \frac{RS}{\cos 49°}$
$= \frac{UT}{\cos 49°}$
$= \frac{9.4}{\cos 49°} \approx 14.3$ in.

51. The tangent ratio is < 1 for $\angle$s measuring < 45° and > 1 for $\angle$s measuring > 45°. In a 45°-45°-90° $\triangle$, both legs have same length, so tan 45° = 1. If the acute $\angle$ measure increases, opp. leg length also increases, so tangent ratio is > 1. If the acute $\angle$ measure decreases, the opp. leg length also decreases, so tangent ratio is < 1.

52a. $AC = \frac{AB}{\sin C}$
$= \frac{25}{\sin 65°}$
≈ 27.58 ft
≈ 27 ft 7 in.

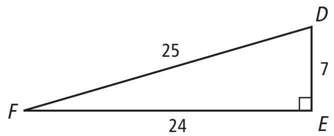
b. $AD = AB \sin\angle ABD$
$= 25 \sin(90 - 28)°$
≈ 22.07 ft
≈ 22 ft 1 in.

53. From diagram,
$\sin A = \frac{3}{5} = 0.6$.



54. From diagram,

$\tan D = \frac{24}{7} \approx 3.43.$



55. Let 1 face be $\triangle ABC$ with A the apex; let M be mdpt. of $\overline{BC}$.

$$\begin{aligned} BC &= 2BM \\ &= \frac{2h}{\tan 52°} \\ &= \frac{2(482)}{\tan 52°} \approx 753 \text{ ft} \end{aligned}$$

56a, b. Check students' work.

c. 20°

d. $\sin 20° = 0.34$
$\cos 20° = 0.94$
$\tan 20° = 0.36$

e. Values in part **d** should be close to estimate-based values in part **b**.

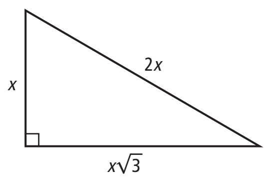
57a. $\tan 30° = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 30° = \frac{x}{2x} = \frac{1}{2}$

$\cos 30° = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

So $\tan 30° = \frac{\sin 30°}{\cos 30°}$



b. $\tan A = \frac{a}{b}$, $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$

c. $\frac{\sin A}{\cos A} = \frac{\frac{a}{c}}{\frac{b}{c}} = \frac{a}{c} \cdot \frac{c}{b} = \frac{a}{b} = \tan A$

58. $$\begin{aligned} (\sin 45°)^2 + (\cos 45°)^2 &= \left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{2}{4} + \frac{2}{4} = 1 \end{aligned}$$

59. $$\begin{aligned} (\sin 30°)^2 + (\cos 30°)^2 &= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} + \frac{3}{4} = 1 \end{aligned}$$

60. $$\begin{aligned} (\sin 60°)^2 + (\cos 60°)^2 &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \\ &= \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$$

61a. $\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$

b. $$\begin{aligned} (\sin A)^2 + (\cos A)^2 &= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 \\ &= \frac{a^2 + b^2}{c^2} \\ &= \frac{c^2}{c^2} = 1 \end{aligned}$$

c. Derivation of identity uses fact that in a rt. $\triangle$, $a^2 + b^2 = c^2$, which is Pyth. Thm.
$\sin A = \frac{BC}{AB}$; $\cos B = \frac{BC}{AB}$; $\sin A = \cos B$; $\angle A$ and $\angle B$ are comp.; the sine of an $\angle$ is equal to the cosine of its comp.

62. $P = 2 + 2\tan 24° + 2/\cos 24° \approx 5.08$ m
$A = \frac{1}{2}(2)(2\tan 24°) \approx 0.89$ m^2

63. $P = 7.2 + \frac{\frac{7.2}{2}}{\cos 51°} + \frac{\frac{7.2}{2}}{\cos 51°} \approx 18.64$ cm
$A = \frac{1}{2}(7.2)\left(\frac{7.2}{2}\tan 51°\right) \approx 16.00$ cm^2

64. $P = 4 + \frac{4}{\sin 58°} + \frac{4}{\tan 58°} \approx 11.22$ ft
$A = \frac{1}{2}(4)\left(\frac{4}{\tan 58°}\right) \approx 5.00$ ft^2

65. $P = 10 + 10\sin 72° + 10\cos 72° \approx 22.60$ in.
$A = \frac{1}{2}(10\sin 72°)(10\cos 72°) \approx 14.69$ in.^{2}

66. $\sin A = \frac{BC}{AB}$; $\cos B = \frac{BC}{AB}$; $\sin A = \cos B$; $\angle A$ and $\angle B$ are comp.; sine of an $\angle$ is = to cosine of its comp.

67. Tangent of an acute $\angle$ increases as measure of the $\angle$ increases.

TEST PREP, PAGE 532

68. A

69. H
$17\tan 65° \approx 36$ ft

70. C
$\cos N = \frac{NP}{MN} = \sin M$

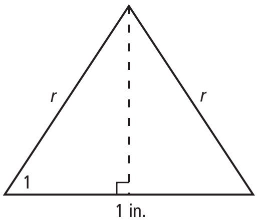
CHALLENGE AND EXTEND, PAGE 532

71. $$\begin{aligned} AB\tan A &= BC \\ (4x)\tan 42° &= 3x + 3 \\ (4\tan 42° - 3)(x) &= 3 \\ x &= \frac{3}{4\tan 42° - 3} \approx 5 \end{aligned}$$
$AB \approx 4(5) \approx 20$
$BC \approx 3(5) + 3 \approx 18$
$AC \approx \sqrt{20^2 + 18^2} \approx 27$

72. $$\begin{aligned} AC\cos A &= AB \\ (15x)\cos 21° &= 5x + 27 \\ (15\cos 21° - 5)(x) &= 27 \\ x &= \frac{27}{15\cos 21° - 5} \approx 3 \end{aligned}$$
$AB \approx 5(3) + 27 \approx 42$
$AC \approx 15(3) \approx 45$
$BC \approx \sqrt{45^2 - 42^2} \approx 16$

73. $$\begin{aligned} (\tan A)^2 + 1 &= \left(\frac{\sin A}{\cos A}\right)^2 + 1 \\ &= \frac{(\sin A)^2 + (\cos A)^2}{(\cos A)^2} \\ &= \frac{1}{(\cos A)^2} \end{aligned}$$

74. Int. ∠s of a reg. pentagon measure $\left(\frac{5-2}{5}\right)(180) = 108°$.
In the diagram,
$m\angle 1 = \frac{1}{2}(108) = 54°$.
Therefore,
$r = \frac{0.5}{\cos 54°} \approx 0.85$ in.



75. $\csc Y = \frac{1}{\sin Y}$
$= \frac{1}{\frac{XZ}{YZ}}$
$= \frac{YZ}{XZ}$
$= \frac{5}{4} = 1.25$

76. $\sec Z = \frac{1}{\cos Z}$
$= \frac{1}{\frac{XZ}{YZ}}$
$= \frac{YZ}{XZ}$
$= \frac{5}{4} = 1.25$

77. $\cot Y = \frac{1}{\tan Y}$
$= \frac{1}{\frac{XZ}{XY}}$
$= \frac{XY}{XZ}$
$= \frac{3}{4} = 0.75$

SPIRAL REVIEW, PAGE 532

78.–80. Possible answers given.

78. (−3, −15), (−1, −9), (0, −6)

79. (−2, 11), (0, 10), (2, 9)

80. (−2, 14), (0, 2), (4, 2)

81. Trans. Prop. of ≅

82. Reflex. Prop. of ≅

83. Symm. Prop. of ≅

84. $\sqrt{3 \cdot 27} = \sqrt{81} = 9$

85. $\sqrt{6 \cdot 24} = \sqrt{144} = 12$

86. $\sqrt{8 \cdot 32} = \sqrt{256} = 16$

CONNECTING GEOMETRY TO ALGEBRA: INVERSE FUNCTIONS, PAGE 533

TRY THIS, PAGE 533

1. $x = 0°$
$\sin^{-1}(0) = 0°$

2. $x = 60°$
$\cos^{-1}\left(\frac{1}{2}\right) = 60°$

3. $x = 45°$
$\tan^{-1}(1) = 45°$

4. $x = 90°$
$\cos^{-1}(0) = 90°$

5. $x = 0°$
$\tan^{-1}(0) = 0°$

6. $x = 30°$
$\sin^{-1}\left(\frac{1}{2}\right) = 30°$

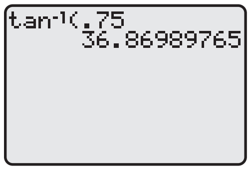
8-3 SOLVING RIGHT TRIANGLES, PAGES 534–541

CHECK IT OUT! PAGES 534–536

1a. $\frac{14.4}{30.6} = \frac{8}{17} = \sin A$
$\rightarrow \angle A$ is $\angle 2$

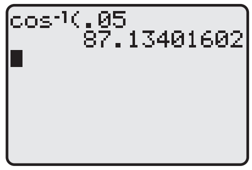
b. $\frac{27}{14.4} = 1.875 = \tan A$
$\rightarrow \angle A$ is $\angle 1$

2a.



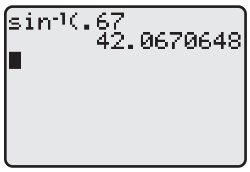
$\tan^{-1}(0.75) \approx 37°$

b.



$\cos^{-1}(0.05) \approx 87°$

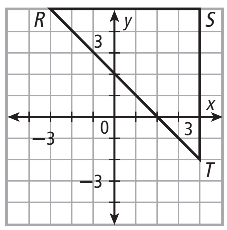
c.



$\sin^{-1}(0.67) \approx 42°$

3. $DF = \frac{DE}{\sin F}$
$= \frac{14}{\sin 58°} \approx 16.51$
$EF = \frac{DE}{\tan F}$
$= \frac{14}{\tan 58°} \approx 8.75$
Acute ∠s of a rt. △ are comp. So,
$m\angle D = 90 - 58 = 32°$

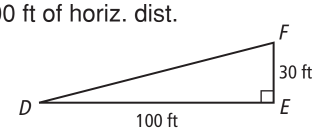
4. Step 1 Find side lengths.
Plot pts. *R*, *S*, and *T*.
$RS = 7 \quad ST = 7$
RT
$= \sqrt{(-3-4)^2 + (5+2)^2}$
$= \sqrt{(-7)^2 + 7^2}$
$= \sqrt{98} \approx 9.90$



Step 2 Find ∠ measures.
$m\angle S = 90°$
$m\angle R = \tan^{-1}\left(\frac{7}{7}\right) = 45°$
$m\angle T = 90 - 45 = 45°$

5. $38\% = \frac{38}{100}$

A 38% grade means Baldwin St. rises 38 ft for every 100 ft of horiz. dist.



$m\angle D = \tan^{-1}\left(\frac{38}{100}\right) \approx 21°$

THINK AND DISCUSS, PAGE 537

1. Find RS using Pyth. Thm. Then find $m\angle R$ using $m\angle R = \sin^{-1}\left(\frac{3.5}{4.1}\right)$, and find $m\angle T$ using either $m\angle T = \cos^{-1}\left(\frac{3.5}{4.1}\right)$ or $m\angle T = 90 - m\angle R$.

2. $\cos^{-1}(0.35) = m\angle Z$

3.

	Trigonometric Ratio	Inverse Trigonometric Function
Sine	$\sin A = \frac{3}{5}$	$\sin^{-1}\left(\frac{3}{5}\right) = m\angle A$
Cosine	$\cos A = \frac{4}{5}$	$\cos^{-1}\left(\frac{4}{5}\right) = m\angle A$
Tangent	$\tan A = \frac{3}{4}$	$\tan^{-1}\left(\frac{3}{4}\right) = m\angle A$

EXERCISES, PAGES 537–541

GUIDED PRACTICE, PAGES 537–538

1. $\frac{8}{10} = \frac{4}{5} = \sin A$
$\rightarrow \angle A$ is $\angle 1$

2. $\frac{8}{6} = 1\frac{1}{3} = \tan A$
$\rightarrow \angle A$ is $\angle 1$

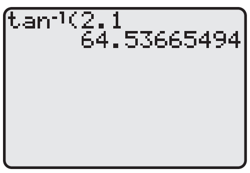
3. $\frac{6}{10} = 0.6 = \cos A$
$\rightarrow \angle A$ is $\angle 1$

4. $\frac{8}{10} = 0.8 = \cos A$
$\rightarrow \angle A$ is $\angle 2$

5. $\frac{6}{8} = 0.75 = \tan A$
$\rightarrow \angle A$ is $\angle 2$

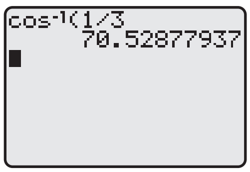
6. $\frac{6}{10} = 0.6 = \sin A$
$\rightarrow \angle A$ is $\angle 2$

7.



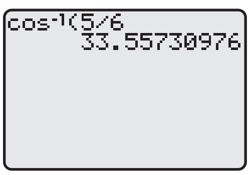
$\tan^{-1}(2.1) \approx 65°$

8.



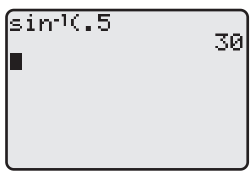
$\cos^{-1}\left(\frac{1}{3}\right) \approx 71°$

9.



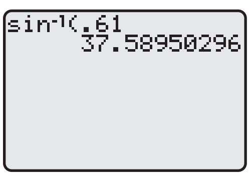
$\cos^{-1}\left(\frac{5}{6}\right) \approx 34°$

10.



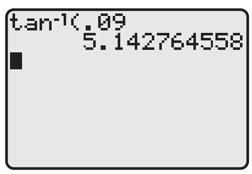
$\sin^{-1}(0.5) = 30°$

11.



$\sin^{-1}(0.61) \approx 38°$

12.



$\tan^{-1}(0.09) \approx 5°$

13. $\tan P = \frac{3.1}{8.9}$

$m\angle P = \tan^{-1}\left(\frac{3.1}{8.9}\right) \approx 19°$

Acute $\angle$s of a rt. $\triangle$ are comp. So $m\angle R \approx 90 - 19 \approx 71°$.

$RP = \frac{3.1}{\sin P}$

$= \frac{3.1}{\sin\left(\tan^{-1}\left(\frac{3.1}{8.9}\right)\right)} \approx 9.42$

14. $AB = 7.4\cos 32° \approx 6.28$

$BC = 7.4\sin 32° \approx 3.92$

Acute $\angle$s of a rt. $\triangle$ are comp. So $m\angle C = 90 - 32 = 58°$.

15. By Pyth. Thm.,

$YZ = \sqrt{11^2 + 8.6^2}$

$= \sqrt{194.96} \approx 13.96$

$\tan Y = \frac{8.6}{11}$

$m\angle Y = \tan^{-1}\left(\frac{8.6}{11}\right) \approx 38°$

$\tan Z = \frac{11}{8.6}$

$m\angle Z = \tan^{-1}\left(\frac{11}{8.6}\right) \approx 52°$

16. Step 1 Find side lengths.

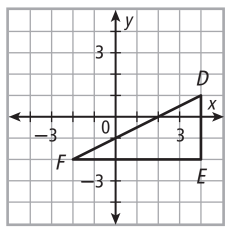
Plot pts. D, E, and F.

$DE = 3 \quad EF = 6$

$DF = \sqrt{(-2-4)^2 + (-2-1)^2}$

$= \sqrt{(-6)^2 + (-3)^2}$

$= \sqrt{45} \approx 6.71$



Step 2 Find $\angle$ measures.

$m\angle E = 90°$

$m\angle D = \tan^{-1}\left(\frac{6}{3}\right) \approx 63°$

$m\angle F \approx 90 - 63 \approx 27°$

17. Step 1 Find side lengths.

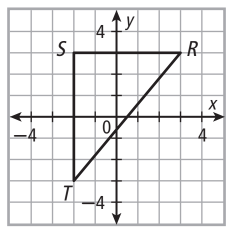
Plot pts. R, S, and T.

$RS = 5 \quad ST = 6$

$RT = \sqrt{(-2-3)^2 + (-3-3)^2}$

$= \sqrt{(-5)^2 + (-6)^2}$

$= \sqrt{61} \approx 7.81$



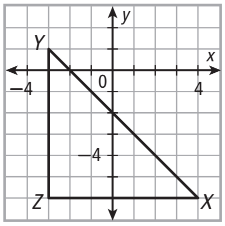
Step 2 Find $\angle$ measures.

$m\angle S = 90°$

$m\angle R = \tan^{-1}\left(\frac{6}{5}\right) \approx 50°$

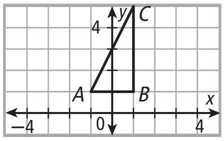
$m\angle T \approx 90 - 50 \approx 40°$

18. Step 1 Find side lengths.
Plot pts. X, Y, and Z.
$XZ = 7 \quad YZ = 7$
$XY = \sqrt{(-3-4)^2 + (1+6)^2}$
$= \sqrt{(-7)^2 + 7^2}$
$= \sqrt{98} \approx 9.90$



Step 2 Find ∠ measures.
$m\angle Z = 90°$
$m\angle X = \tan^{-1}\left(\frac{7}{7}\right) = 45°$
$m\angle Y = 90 - 45 = 45°$

19. Step 1 Find side lengths.
Plot pts. A, B, and C.
$AB = 2 \quad BC = 4$
$AC = \sqrt{(1+1)^2 + (5-1)^2}$
$= \sqrt{2^2 + 4^2}$
$= \sqrt{20} \approx 4.47$



Step 2 Find ∠ measures.
$m\angle B = 90°$
$m\angle A = \tan^{-1}\left(\frac{4}{2}\right) \approx 63°$
$m\angle C = 90 - 63 \approx 27°$

20. $8\% = \frac{8}{100}$
An 8% grade means hill rises 8 m for every 100 m of horiz. dist.
$m\angle = \tan^{-1}\left(\frac{8}{100}\right) \approx 5°$

PRACTICE AND PROBLEM SOLVING, PAGES 538–540

21. $\frac{5}{12} = \frac{7.5}{18} = \tan\angle 2$
$\angle A = \angle 2$

22. $2.4 = \frac{18}{7.5} = \tan\angle 1$
$\angle A = \angle 1$

23. $\frac{12}{13} = \frac{18}{19.5} = \sin\angle 1$
$\angle A = \angle 1$

24. $\frac{5}{13} = \frac{7.5}{19.5} = \sin\angle 2$
$\angle A = \angle 2$

24. $\frac{12}{13} = \frac{18}{19.5} = \cos\angle 2$
$\angle A = \angle 2$

26. $\frac{5}{13} = \frac{7.5}{19.5} = \cos\angle 1$
$\angle A = \angle 1$

27. $\sin^{-1}(0.31) \approx 18°$

28. $\tan^{-1}(1) = 45°$

29. $\cos^{-1}(0.8) \approx 37°$

30. $\cos^{-1}(0.72) \approx 44°$

31. $\tan^{-1}(1.55) \approx 57°$

32. $\sin^{-1}\left(\frac{9}{17}\right) \approx 32°$

33. Step 1 Find unknown side lengths.
$JK = 3.2\cos 26° \approx 2.88$
$LK = 3.2\sin 26° \approx 1.40$
Step 2 Find unknown ∠ measure.
$m\angle L = 90 - 26 = 64°$

34. Step 1 Find unknown side length.
$DF = \sqrt{\left(\sqrt{5}\right)^2 + \left(\sqrt{2}\right)^2}$
$= \sqrt{5+2} = \sqrt{7} \approx 2.65$
Step 2 Find unknown ∠ measures.
$m\angle D = \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{5}}\right) \approx 32°$
$m\angle F = \tan^{-1}\left(\frac{\sqrt{5}}{\sqrt{2}}\right) \approx 58°$

35. Step 1 Find unknown ∠ measures.
$m\angle P = \cos^{-1}\left(\frac{6.7}{8.3}\right) \approx 36°$
$m\angle R = \sin^{-1}\left(\frac{6.7}{8.3}\right) \approx 54°$
Step 2 Find unknown side length.
$QR = 8.3\sin P$
$= 8.3\sin\left(\cos^{-1}\left(\frac{6.7}{8.3}\right)\right) \approx 4.90$

36. Step 1 Find side lengths.
$\overline{AB}$ is vert., $AB = 5$; $\overline{BC}$ is horiz., $BC = 1$
By Pyth. Thm., $AC = \sqrt{5^2 + 1^2} = \sqrt{26} \approx 5.10$
Step 2 Find ∠ measures.
$m\angle B = 90°$
$m\angle A = \tan^{-1}\left(\frac{BC}{AB}\right) = \sin^{-1}\left(\frac{1}{5}\right) \approx 11°$
$m\angle C \approx 90 - 11 \approx 79°$

37. Step 1 Find side lengths.
$\overline{MN}$ is vert., $MN = 4$; $\overline{NP}$ is horiz., $NP = 4$
By Pyth. Thm., $MP = \sqrt{4^2 + 4^2} = \sqrt{32} \approx 5.66$
Step 2 Find ∠ measures.
$m\angle N = 90°$
$m\angle M = \tan^{-1}\left(\frac{NP}{MN}\right) = \tan^{-1}\left(\frac{4}{4}\right) = 45°$
$m\angle P = 90 - 45 = 45°$

38. Range of ∠ measures is between $\tan^{-1}\left(\frac{1}{20}\right) \approx 3°$ and $\tan^{-1}\left(\frac{1}{16}\right) \approx 4°$.

39. $\tan^{-1}(3.5) \approx 74°$
$\tan \mathbf{74°} \approx 3.5$

40. $\sin^{-1}\left(\frac{2}{3}\right) \approx 42°$
$\sin \mathbf{42°} \approx \frac{2}{3}$

41. $\mathbf{cos}\ 42° \approx 0.74$

42. $\cos 12° \approx 0.98$
$\cos^{-1}(\mathbf{0.98}) \approx 12°$

43. $\sin 69° \approx 0.93$
$\sin^{-1}(\mathbf{0.93}) \approx 69°$

44. $\mathbf{cos}\ 60° = \frac{1}{2}$

45. Assume square has sides of length a. Then either rt. △ formed by a diag. has legs of length a. So measure of ∠ formed by diag. and a side is $\tan^{-1}\left(\frac{a}{a}\right) = \tan^{-1}(1) = 45°$.

46a. Possible answer: $m\angle P \approx 40°$

b. $RQ \approx 2.2$ cm, $PQ \approx 3.1$ cm

c. $m\angle P = \tan^{-1}\left(\frac{RQ}{PQ}\right)$
$\approx \tan^{-1}\left(\frac{2.2}{3.1}\right) \approx 35°$

d. Possible answer: Answer in part **c** is likely more accurate, since it is easier to measure lengths to the nearest tenth than to measure ∠s to the nearest degree.

47a. $m\angle 1 = \tan^{-1}\left(\frac{8}{100}\right) \approx 5°$

b. $m\angle 1 \approx 90 - 5 \approx 85°$

c. $h = \frac{31}{\sin\left(90 - \tan^{-1}\left(\frac{8}{100}\right)\right)} \approx 31.10$ ft, or 31 ft 1 in.

48. $\sin^{-1}\left(\frac{3}{5}\right) \approx 37°$, $\sin^{-1}\left(\frac{4}{5}\right) \approx 53°$

49. $\sin^{-1}\left(\frac{5}{13}\right) \approx 23°$, $\sin^{-1}\left(\frac{12}{13}\right) \approx 67°$

50. $\sin^{-1}\left(\frac{8}{17}\right) \approx 28°$, $\sin^{-1}\left(\frac{15}{17}\right) \approx 62°$

51. $\tan^{-1}\left(\frac{45}{28}\right) \approx 58°$, $\tan^{-1}\left(\frac{90}{28}\right) \approx 73°$

Acute ∠ measure changes from about 58° to about 73°, an increase by a factor of 1.26.

52. $m\angle = \tan^{-1}\left(\frac{28}{100}\right) \approx 16°$

53a.
$$\begin{aligned} AB &= \sqrt{(6+1)^2 + (1-0)^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{50} = 5\sqrt{2} \\ BC &= \sqrt{(0-6)^2 + (3-1)^2} \\ &= \sqrt{6^2 + 2^2} \\ &= \sqrt{40} = 2\sqrt{10} \\ AC &= \sqrt{(0+1)^2 + (3-0)^2} \\ &= \sqrt{1^2 + 3^2} = \sqrt{10} \end{aligned}$$

b.
$$\begin{aligned} AC^2 + BC^2 &= 10 + 40 \\ &= 50 = AB^2 \end{aligned}$$

So $\triangle ABC$ is a rt. △, and C is the rt. ∠.

c.
$$\begin{aligned} m\angle A &= \sin^{-1}\left(\frac{BC}{AB}\right) \\ &= \sin^{-1}\left(\frac{2\sqrt{10}}{5\sqrt{2}}\right) \\ &= \sin^{-1}\left(\frac{2\sqrt{5}}{5}\right) \approx 63° \end{aligned}$$

$m\angle B = 90 - m\angle A \approx 27°$

54. $m\angle BDC = \tan^{-1}\left(\frac{2}{7}\right) \approx 16°$

55. $m\angle STV = \tan^{-1}\left(\frac{3.2}{4.5}\right) \approx 35°$

56. $m\angle DGF = 2m\angle DGH = 2\sin^{-1}\left(\frac{2.4}{4.4}\right) \approx 66°$

57. $m\angle LKN = \tan^{-1}\left(\frac{9}{4.8}\right) \approx 62°$

58. tan 70° > tan 60°; possible answer: consider 2 rt. △s, 1 with a 60° ∠ and 1 with a 70° ∠. Suppose that legs adj. to these ∠s have length 1 unit. Leg opp. 70° ∠ will be longer than leg opp. 60° ∠. So tan 70° is greaater than tan 60°.

59. $\tan^{-1}(m) = \tan^{-1}(3) \approx 72°$

60. $\tan^{-1}(m) = \tan^{-1}\left(\frac{2}{3}\right) \approx 34°$

61.
$$\begin{aligned} 5y &= 4x + 3 \\ y &= \frac{4}{5}x + \frac{3}{5} \end{aligned}$$
$\tan^{-1}(m) = \tan^{-1}\left(\frac{4}{5}\right) \approx 39°$

62. Since △ is not a rt. △., trig. ratios do not apply.

63. No; possible answer: you only need to know 2 side lengths. You can use Pyth. Thm. to find 3rd side length or use trig. ratios to find acute ∠ measures.

64.
$$\begin{aligned} AD &= AC\cos A \\ &= 10\cos\left(\tan^{-1}\left(\frac{6}{10}\right)\right) \approx 8.57 \\ BD &= BC\cos B \\ &= 6\cos\left(\tan^{-1}\left(\frac{10}{6}\right)\right) \approx 3.09 \\ CD &= BC\sin B \\ &= 6\sin\left(\tan^{-1}\left(\frac{10}{6}\right)\right) \approx 5.14 \end{aligned}$$
$(8.57)(3.09) \approx 26 \approx (5.14)^2$
$(8.57)(8.57 + 3.09) \approx 100 = (10)^2$
$(3.09)(8.57 + 3.09) \approx 36 = (6)^2$

TEST PREP, PAGE 540

65. D

66. J

67. A
$\tan^{-1}\left(\frac{1.4}{2.7}\right) \approx 27°$

68. 9°
$\tan^{-1}\left(\frac{3}{20}\right) \approx 9°$

CHALLENGE AND EXTEND, PAGES 540–541

69.
$$\begin{aligned} LH = 10\sin J &= 20\sin 25° \\ \sin J &= 2\sin 25° \\ m\angle J &= \sin^{-1}(2\sin 25°) \approx 58° \end{aligned}$$

70.
$$\begin{aligned} BD = 3.2\tan A &= 8\cos 64° \\ \tan A &= 2.5\cos 64° \\ m\angle A &= \tan^{-1}(2.5\cos 64°) \approx 48° \end{aligned}$$

71. Let ∠A be an acute ∠ with $m\angle A = \cos^{-1}(\cos 34°)$. Then $\cos A = \cos 34°$. Since cos is a 1-to-1 function on acute ∠ measures, $m\angle A = 34°$.

72. Since tan is a 1-to-1 function on acute ∠ measures, $x = \tan[\tan^{-1}(1.5)] \rightarrow x = 1.5$

73. Since sin is a 1-to-1 function on acute ∠ measures, $y = \sin\left(\sin^{-1} x\right) \rightarrow y = x$

74. $y = 40\sin\left(\tan^{-1}\left(\frac{6}{100}\right)\right) \approx 2.40$ ft

75. Possible answer: The expression $\sin^{-1}(1.5)$ represents an ∠ measure that has a sine of 1.5. The sine of an acute ∠ of a rt. △ must be between 0 and 1, so the expression $\sin^{-1}(1.5)$ is undefined.

76. Let $\overline{BD}$ be altitude. Then
$$\begin{aligned} \text{Area} &= \frac{1}{2}(\text{base})(\text{height}) \\ &= \frac{1}{2}(AC)(BD) \\ &= \frac{1}{2}(b)(c\sin A) \\ &= \frac{1}{2}bc\sin A. \end{aligned}$$

SPIRAL REVIEW, PAGE 541

77. false; $6.8 \not> 2 + 2.5 + 3.3 = 7.8$

78. true; $\dfrac{2 + 2.5 + 3.3 + 6.8 + 3.6}{5} \approx 3.5$ in.

79. False; rainfall decreased from April to May.

80. $\angle B \cong \angle E$
$$37 = 2x + 11$$
$$26 = 2x$$
$$x = 13$$

81. $\dfrac{AB}{DE} = \dfrac{AC}{DF}$
$$\frac{3y+7}{2y+6} = \frac{1.4+1.6}{1.4+1.6} = 1$$
$$3y + 7 = 2y + 6$$
$$y = -1$$

82. $DF = 1.4 + 1.6 = 3$

83. $\sin 63° \approx 0.89$

84. $\cos 27° \approx 0.89$

85. $\tan 64° \approx 2.05$

USING TECHNOLOGY, PAGE 541

1.–5. Check students' work.

MULTI-STEP TEST PREP, PAGE 542

1. $(DB)^2 = (DA)(DC)$
$$= (30 - 6)(6)$$
$$= 144$$
$$DB = \sqrt{144} = 12 \text{ ft}$$

2. $(AB)^2 = (AD)(AC)$
$$= (24)(30)$$
$$= 720$$
$$AB = \sqrt{720}$$
$$\approx 26.83 \text{ ft}$$
or 26 ft 10 in.

3. $m\angle ABD = \tan^{-1}\left(\dfrac{AD}{DB}\right)$
$$= \tan^{-1}\left(\frac{24}{12}\right)$$
$$= \tan^{-1}(2) = 63°$$

4. $\angle A$ and $\angle ABD$ are comp.
$$m\angle A = 90 - m\angle ABD$$
$$\approx 90 - 63 \approx 27°$$

5. grade $= \dfrac{DB}{DC} \cdot 100\% = \dfrac{6}{12} \cdot 100\% = 50\%$

READY TO GO ON? PAGE 543

1. $x = \sqrt{(5)(12)}$
$$= \sqrt{60} = 2\sqrt{15}$$

2. $x = \sqrt{(2.75)(44)}$
$$= \sqrt{121} = 11$$

3. $x = \sqrt{\left(\dfrac{5}{2}\right)\left(\dfrac{15}{8}\right)}$
$$= \sqrt{\frac{75}{16}} = \frac{5\sqrt{3}}{4}$$

4. $x^2 = (4)(8) = 32$
$$x = \sqrt{32} = 4\sqrt{2}$$
$$y^2 = (4)(12) = 48$$
$$y = \sqrt{48} = 4\sqrt{3}$$
$$z^2 = (8)(12) = 96$$
$$z = \sqrt{96} = 4\sqrt{6}$$

5. $\left(12\sqrt{5}\right)^2 = (24)(x + 24)$
$$720 = 24x + 576$$
$$144 = 24x$$
$$x = 6$$
$$y^2 = 24x$$
$$= 24(6) = 144$$
$$y = 12$$
$$z^2 = (24 + x)(x)$$
$$= (30)(6) = 180$$
$$z = \sqrt{180} = 6\sqrt{5}$$

6. $6^2 = 12x$
$$36 = 12x$$
$$x = 3$$
$$y^2 = 12(12 + x)$$
$$= (12)(15) = 180$$
$$y = \sqrt{180} = 6\sqrt{5}$$
$$z^2 = (12 + x)x$$
$$= (15)(3) = 45$$
$$z = \sqrt{45} = 3\sqrt{5}$$

7. $(AB)^2 = (BC)(BD)$
$$= (22)(30) = 660$$
$$AB = \sqrt{660} \approx 25.7 \text{ m}$$

8. Let legs of 45°-45°-90° △ have length x.
$\tan 45° = \dfrac{x}{x} = 1$

9. Let 30°-60°-90° △ have side lengths x, $x\sqrt{3}$, $2x$.
$\sin 30° = \dfrac{x}{2x} = \dfrac{1}{2}$

10. $\cos 30° = \dfrac{x\sqrt{3}}{2x} = \dfrac{\sqrt{3}}{2}$

11. $\sin 16° \approx 0.28$

12. $\cos 79° \approx 0.19$

13. $\tan 27° \approx 0.51$

14. $QR = \dfrac{14}{\tan 31°} \approx 23.30$ in.

15. $AB = 6 \cos 50° \approx 3.86$ m

16. $LM = 4.2 \sin 62° \approx 3.71$ cm

17. $m\angle A = 90 - 32 = 58°$
$$BC = \frac{22}{\tan 32°} \approx 35.21$$
$$AC = \frac{22}{\sin 32°} \approx 41.52$$

18. $HJ = \sqrt{7^2 + 10.5^2}$
$$= \sqrt{159.25} \approx 12.62$$
$$m\angle H = \tan^{-1}\left(\frac{10.5}{7}\right) \approx 56°$$
$$m\angle J = \tan^{-1}\left(\frac{7}{10.5}\right) \approx 34°$$

19. $m\angle Z = 90 - 28 = 62°$
$$XY = 5.1 \cos 28° \approx 4.50$$
$$YZ = 5.1 \sin 28° \approx 2.39$$

20. $\tan^{-1}\left(\dfrac{1}{18}\right) \approx 3°$

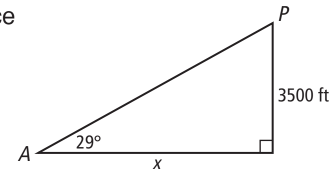
8-4 ANGLES OF ELEVATION AND DEPRESSION, PAGES 544–549

CHECK IT OUT! PAGES 544–546

1a. $\angle 5$ is formed by a horiz. line and a line of sight to a pt. below the line. It is an $\angle$ of depression.

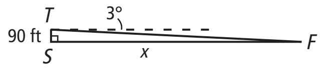
b. $\angle 6$ is formed by a horiz. line and a line of sight to a pt. above the line. It is an $\angle$ of elevation.

2. Let A represent airport and P represent plane. Let x be horiz. distance between plane and airport.



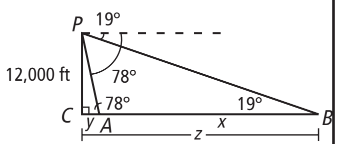
$$\tan 29° = \frac{3500}{x}$$
$$x = \frac{3500}{\tan 29°}$$
$$\approx 6314 \text{ ft}$$

3. Let T represent top of tower and F represent fire. Let x be horiz. distance between tower and fire.



By Alt. Int. $\angle$s Thm., $m\angle F = 3°$.
$$\tan 3° = \frac{90}{x}$$
$$x = \frac{90}{\tan 3°} \approx 1717 \text{ ft}$$

4. Step 1 Let P represent plane, and A and B represent two airports. Let x be distance between airports.



Step 2 Find y.
By Alt. Int. $\angle$s Thm., $m\angle CAP = 78°$. In $\triangle APC$,
$$\tan 78° = \frac{12{,}000}{y}$$
$$y = \frac{12{,}000}{\tan 78°} \approx 2550.7 \text{ ft}$$

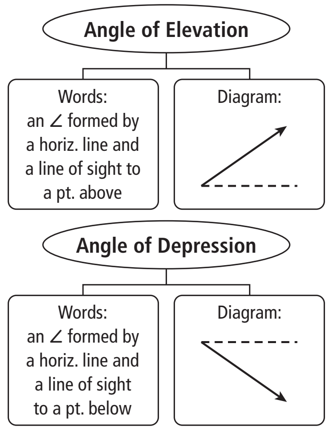
Step 3 Find z.
By Alt. Int. $\angle$s Thm., $m\angle CBP = 19°$. In $\triangle BPC$,
$$\tan 19° = \frac{12{,}000}{z}$$
$$z = \frac{12{,}000}{\tan 19°} \approx 34{,}850.6 \text{ ft}$$

Step 4 Find x.
$$x = z - y$$
$$\approx 34{,}850.6 - 2550.7 \approx 32{,}300 \text{ ft}$$

THINK AND DISCUSS, PAGE 546

1. It increases, because height of skyscraper is constant and horiz. dist. is decreasing.

2.



EXERCISES, PAGES 547–549

GUIDED PRACTICE, PAGE 547

1. elevation **2.** depression

3. $\angle 1$ is formed by a horiz. line and a line of sight to a pt. above the line. It is an $\angle$ of elevation.

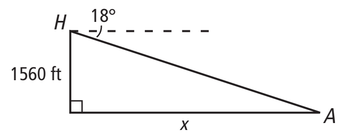
4. $\angle 2$ is formed by a horiz. line and a line of sight to a pt. below the line. It is an $\angle$ of depression.

5. $\angle 3$ is formed by a horiz. line and a line of sight to a pt. above the line. It is an $\angle$ of elevation.

6. $\angle 4$ is formed by a horiz. line and a line of sight to a pt. below the line. It is an $\angle$ of depression.

7. Let h be height of flagpole.
$$\tan 37° = \frac{h}{24.2}$$
$$h = 24 \tan 37° \approx 18 \text{ ft}$$

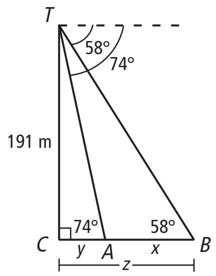
8. Let H represent helicopter and A represent accident. Let x be horiz. dist. between helicopter and accident.



By Alt. Int. $\angle$s Thm., $m\angle A = 18°$.
$$\tan 18° = \frac{1560}{x}$$
$$x = \frac{1560}{\tan 18°} \approx 4801 \text{ ft}$$

9. Step 1 Let T represent top of canyon, and A and B represent near and far sides of river. Let w be width of river.

Step 2 Find y.
By Alt. Int. $\measuredangle$ Thm., $m\angle CAT = 74°$. In $\triangle ATC$,



$$\tan 74° = \frac{191}{y}$$

$$y = \frac{191}{\tan 74°} \approx 54.77 \text{ m}$$

Step 3 Find z.
By Alt. Int. $\measuredangle$ Thm., $m\angle CBT = 58°$. In $\triangle BTC$,

$$\tan 58° = \frac{191}{z}$$

$$z = \frac{191}{\tan 58°} \approx 119.35 \text{ m}$$

Step 4 Find w.

$$w = z - y$$
$$\approx 119.35 - 54.77 \approx 64.6 \text{ m}$$

PRACTICE AND PROBLEM SOLVING, PAGES 547–548

10. $\angle 1$: $\angle$ of depression **11.** $\angle 2$: $\angle$ of elevation

12. $\angle 3$: $\angle$ of elevation **13.** $\angle 4$: $\angle$ of depression

14. $h = 1.5 + 100 \tan 67° \approx 237$ m

15. $x = \dfrac{120}{\tan 3.5°} \approx 1962$ ft **16.** $z = y - x$
$= 1 \tan 74° - 1 \tan 16°$
≈ 3.2 mi

17. true **18.** true

19. false; $\angle$ of elevation gets closer to 90°

20. true **21.** $\angle 1$ and $\angle 3$

22. $m\angle 2 = 30°$ (given)
$m\angle 1 = 90 - m\angle 2$
$= 90 - 30 = 60°$ (comp. $\measuredangle$)
$m\angle 3 = m\angle 1 = 60°$,
$m\angle 4 = m\angle 2 = 30°$ (Alt. Int. $\measuredangle$ Thm.)

23. Possible answer: As a hot air ballon descends vertically, $\angle$ of depression to an object on the ground decreases.

24. By Alt. Int. $\measuredangle$ Thm.,
$\angle$ of depression $= \tan^{-1}\left(\dfrac{165}{50}\right) \approx 73°$

25a. $x = \dfrac{1000}{\tan 67°} \approx 424$ ft **b.** $z = y - x$
$= \dfrac{1000}{\tan 55°} - \dfrac{1000}{\tan 67°}$
≈ 276 ft

26. When the $\angle$ of elevation is exactly 45°, the length of the shadow will be the same as the length of telephone pole, since a rt. isosc. $\triangle$ is formed and $\tan 45° = 1$.

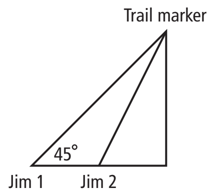
27a. $x = \dfrac{1250}{\tan 31°} \approx 2080$ ft **b.** $v = \dfrac{s}{t}$
$t = \dfrac{s}{v}$
$\approx \dfrac{2080}{150} \approx 14$ s

TEST PREP, PAGE 549

28. D
dist. $= \dfrac{1600}{\tan 35°} \approx 2285$ ft

29. J
height $= 93 \tan 60° \approx 161$ ft

30.



The $\angle$ of elevation increases as Jim moves closer to trail marker.

CHALLENGE AND EXTEND, PAGE 549

31. Let x and y be dists. from Jorge and from Susan to foot of Big Ben; let h be height of Big Ben.
Jorge: $h = x \tan 49.5°$
Susan: $h = y \tan 65°$
$y = x - 38$
$h = x \tan 49.5° = (x - 38) \tan 65°$
$38 \tan 65° = x(\tan 65° - \tan 49.5°)$
$x = \dfrac{38 \tan 65°}{(\tan 65° - \tan 49.5°)}$
$h = x \tan 49.5°$
$= \dfrac{38 \tan 65°}{(\tan 65° - \tan 49.5°)}(\tan 49.5°) \approx 98$ m

32. Speed $= 500 \dfrac{\text{mi}}{\text{h}} \bullet \dfrac{1 \text{ h}}{60 \text{ min}} \bullet \dfrac{5280 \text{ ft}}{1 \text{ mi}} = 44{,}000$ ft/min
Let time until over lake be t. Then horiz. dist to lake is
$s = 44{,}000t = \dfrac{14{,}000}{\tan 6°}$
$t = \dfrac{14{,}000}{44{,}000 \tan 6°} \approx 3$ min.

33. $h = x \tan 5° = (10 - x) \tan 2°$
$x(\tan 5° + \tan 2°) = 10 \tan 2°$
$x = \dfrac{10 \tan 2°}{\tan 5° + \tan 2°}$
$h = x \tan 5°$
$= \dfrac{10 \tan 2°}{\tan 5° + \tan 2°}(\tan 5°)$
≈ 0.2496 mi ≈ 1318 ft

34. $h = y - x$
$= 46 \tan 42° - 46 \tan 18°$
≈ 26.47 ft or 26 ft 6 in.

SPIRAL REVIEW, PAGE 549

35. Let x and y be dists. run by Emma and mother in time t. When they meet,
$x + y = 1$
$6t + 4t = 1$
$10t = 1$
$t = 0.1$ h or 6 min

36. Let p be original price.
discounted price $= 0.7p$
price after coupon $= 0.85(0.7p) = 17.85$
$p = \dfrac{17.85}{0.85(0.7)} = 30$
Original price was \$30.

37. rhombus, square **38.** rectangle, square

39. rectangle, rhombus, square

40. rectangle, rhombus, square

41. $x^2 + 3^2 = 5^2$
$x^2 + 9 = 25$
$x^2 = 16$
$x = 4$

42. $\frac{y}{x} = \frac{5}{3}$
$y = \frac{5x}{3}$
$= \frac{5(4)}{3} = \frac{20}{3}$

43. $x^2 = 3z$
$4^2 = 3z$
$z = \frac{16}{3}$

GEOMETRY LAB: INDIRECT MEASUREMENT USING TRIGONOMETRY, PAGE 550

TRY THIS, PAGE 550

1. The ∠ reading from clinometer is comp. of ∠ of elevation.
2. Check students' work.
3. Check students' work. Results should be similar.
4. Possible answers: Measuring the distance between observer and object, measuring height of observer's eyes, and reading the ∠ measure from clinometer.
5. It can be used to measure height of tall objects that cannot be measured directly.

8-5 LAW OF SINES AND LAW OF COSINES, PAGES 551–558

CHECK IT OUT! PAGES 551–554

1a.
```
tan(175
    -.0874886635
```
$\tan 175° \approx -0.09$

b.
```
cos(92
    -.0348994967
```
$\cos 92° \approx -0.03$

c.
```
sin(160
     .3420201433
```
$\sin 160° \approx 0.34$

2a. $\frac{\sin N}{MP} = \frac{\sin M}{NP}$
$\frac{\sin 39°}{22} = \frac{\sin 88°}{NP}$
$NP \sin 39° = 22 \sin 88°$
$NP = \frac{22 \sin 88°}{\sin 39°}$
≈ 34.9

b. $\frac{\sin L}{JK} = \frac{\sin K}{JL}$
$\frac{\sin L}{6} = \frac{\sin 125°}{10}$
$\sin L = \frac{6 \sin 125°}{10}$
$m\angle L = \sin^{-1}\left(\frac{6 \sin 125°}{10}\right)$
$\approx 29°$

c. $\frac{\sin X}{YZ} = \frac{\sin Y}{XZ}$
$\frac{\sin X}{4.3} = \frac{\sin 50°}{7.6}$
$\sin X = \frac{4.3 \sin 50°}{7.6}$
$m\angle X = \sin^{-1}\left(\frac{4.3 \sin 50°}{7.6}\right)$
$\approx 26°$

d. $m\angle A = 180 - 67° - 44°$
$= 69°$
$\frac{\sin A}{BC} = \frac{\sin B}{AC}$
$\frac{\sin 69°}{18} = \frac{\sin 67°}{AC}$
$AC \sin 69° = 18 \sin 67°$
$AC = \frac{18 \sin 67°}{\sin 69°}$
≈ 17.7

3a. $DE^2 = DF^2 + EF^2 - 2(DF)(EF)\cos F$
$= 16^2 + 18^2 - 2(16)(18)\cos 21°$
$DE^2 \approx 42.2577$
$DE \approx 6.5$

b. $JL^2 = JK^2 + KL^2 - 2(JK)(KL)\cos K$
$8^2 = 15^2 + 10^2 - 2(15)(10)\cos K$
$64 = 325 - 300 \cos K$
$-261 = -300 \cos K$
$\cos K = \frac{261}{300}$
$m\angle K = \cos^{-1}\left(\frac{261}{300}\right) \approx 30°$

c. $YZ^2 = XY^2 + XZ^2 - 2(XY)(XZ)\cos X$
$= 10^2 + 4^2 - 2(10)(4)\cos 34°$
$YZ^2 \approx 49.6770$
$YZ \approx 7.0$

d. $PQ^2 = QR^2 + PR^2 - 2(QR)(PR)\cos R$
$9.6^2 = 10.5^2 + 5.9^2 - 2(10.5)(5.9)\cos R$
$92.16 = 145.06 - 123.9 \cos R$
$-52.9 = -123.9 \cos R$
$\cos R = \frac{52.9}{123.9}$
$m\angle R = \cos^{-1}\left(\frac{52.9}{123.9}\right) \approx 65°$

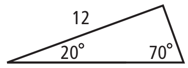
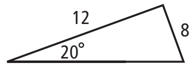
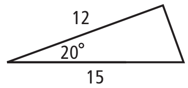
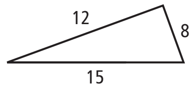
4. Step 1 Find length of cable.
$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$
$= 31^2 + 56^2 - 2(31)(56)\cos 100°$
$AC^2 = 4699.9065$
$AC = 68.6$ m

Step 2 Find angle measure between cable and ground.
$\frac{\sin A}{BC} = \frac{\sin B}{AC}$
$\frac{\sin A}{56} = \frac{\sin 100°}{68.56}$
$\sin A = \frac{56 \sin 100°}{68.56}$
$m\angle A = \sin^{-1}\left(\frac{56 \sin 100°}{68.56}\right) \approx 54°$

THINK AND DISCUSS, PAGE 554

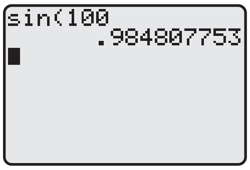
1. m$\angle A$.

2.

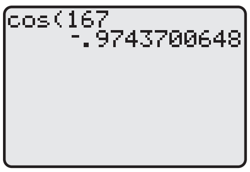
Given Information	Method	Example
Two angle measures and any side length	Law of Sines	
Two side lengths and a nonincluded angle measure	Law of Sines	
Two side lengths and the included angle measure	Law of Cosines	
Three side lengths	Law of Cosines	

EXERCISES, PAGES 555–558

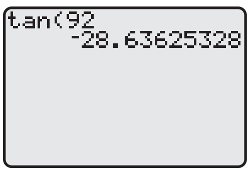
GUIDED PRACTICE, PAGE 555

1.


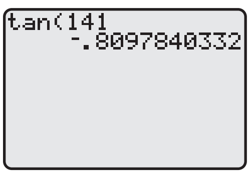
$\sin 100° \approx 0.98$

2.


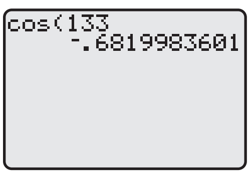
$\cos 167° \approx -0.97$

3.


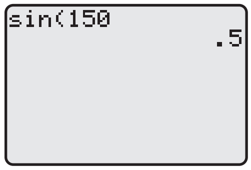
$\tan 92° \approx -28.64$

4.


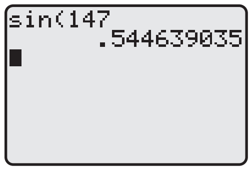
$\tan 141° \approx -0.81$

5.


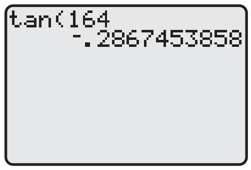
$\cos 133° \approx -0.68$

6.


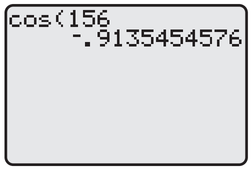
$\sin 150° = 0.5$

7.


$\sin 147° \approx 0.54$

8.


$\tan 164° \approx -0.29$

9.


$\cos 156° \approx -0.91$

10.
$$\begin{aligned}\frac{\sin R}{ST} &= \frac{\sin S}{RT}\\ \frac{\sin 36°}{15} &= \frac{\sin 70°}{RT}\\ RT\sin 36° &= 15\sin 70°\\ RT &= \frac{15\sin 70°}{\sin 36°}\\ &\approx 24.0\end{aligned}$$

11.
$$\begin{aligned}\frac{\sin B}{AC} &= \frac{\sin C}{AB}\\ \frac{\sin B}{14} &= \frac{\sin 101°}{20}\\ \sin B &= \frac{14\sin 101°}{20}\\ B &= \sin^{-1}\left(\frac{14\sin 101°}{20}\right)\\ &\approx 43°\end{aligned}$$

12.
$$\begin{aligned}\frac{\sin F}{DE} &= \frac{\sin D}{EF}\\ \frac{\sin F}{20} &= \frac{\sin 84°}{31}\\ \sin F &= \frac{20\sin 84°}{31}\\ F &= \sin^{-1}\left(\frac{20\sin 84°}{31}\right)\\ &\approx 40°\end{aligned}$$

13.
$$\begin{aligned}PR^2 &= PQ^2 + QR^2 - 2(PQ)(QR)\cos Q\\ 7^2 &= 6^2 + 10^2 - 2(6)(10)\cos Q\\ 49 &= 136 - 120\cos Q\\ -87 &= -120\cos Q\\ \cos Q &= \frac{87}{120}\\ m\angle Q &= \cos^{-1}\left(\frac{87}{120}\right) \approx 44°\end{aligned}$$

14.
$$\begin{aligned}MN^2 &= LM^2 + LN^2 - 2(LM)(LN)\cos L\\ &= 30^2 + 25^2 - 2(30)(25)\cos 77°\\ MN^2 &\approx 1187.5734\\ MN &\approx 34.5\end{aligned}$$

15. $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos C$
$= 8^2 + 11^2 - 2(8)(11)\cos 131$
$AB^2 \approx 300.4664$
$AB \approx 17.3$

16. Think: Find each $\angle$ using Law of Cosines.
$20^2 = 24^2 + 30^2 - 2(24)(30)\cos\angle 1$
$400 = 1476 - 1440\cos\angle 1$
$-1076 = -1440\cos\angle 1$
$m\angle 1 = \cos^{-1}\left(\frac{1076}{1440}\right) \approx 42°$
$24^2 = 20^2 + 30^2 - 2(20)(30)\cos\angle 2$
$576 = 1300 - 1200\cos\angle 2$
$-724 = -1200\cos\angle 2$
$m\angle 2 = \cos^{-1}\left(\frac{724}{1200}\right) \approx 53°$
$30^2 = 24^2 + 20^2 - 2(24)(20)\cos\angle 3$
$900 = 976 - 960\cos\angle 3$
$76 = -960\cos\angle 3$
$m\angle 3 = \cos^{-1}\left(\frac{76}{960}\right) \approx 85°$

PRACTICE AND PROBLEM SOLVING, PAGES 555–557

17. $\cos 95° \approx -0.09$
18. $\tan 178° \approx -0.03$
19. $\tan 118° \approx -1.88$
20. $\sin 132° \approx 0.74$
21. $\sin 98° \approx 0.99$
22. $\cos 124° \approx -0.56$
23. $\tan 139° \approx -0.87$
24. $\cos 145° \approx -0.82$
25. $\sin 128° \approx 0.79$

26. $\frac{\sin C}{6.8} = \frac{\sin 122°}{10.2}$
$m\angle C = \sin^{-1}\left(\frac{6.8\sin 122°}{10.2}\right)$
$\approx 34°$

27. $\frac{\sin 17°}{8.5} = \frac{\sin 135°}{PR}$
$PR = \frac{8.5\sin 135°}{\sin 17°}$
≈ 20.6

28. $\frac{\sin 140°}{9} = \frac{\sin 20°}{JL}$
$JL = \frac{9\sin 20°}{\sin 140°}$
≈ 4.8

29. $\frac{\sin 56°}{11.7} = \frac{\sin 47°}{EF}$
$EF = \frac{11.7\sin 47°}{\sin 56°}$
≈ 10.4

30. $\frac{\sin J}{61} = \frac{\sin 80°}{100}$
$m\angle J = \sin^{-1}\left(\frac{61\sin 80°}{100}\right)$
$\approx 37°$

31. $\frac{\sin X}{3.6} = \frac{\sin 78°}{3.9}$
$m\angle X = \sin^{-1}\left(\frac{3.6\sin 78°}{3.9}\right) \approx 65°$

32. $AB^2 = 13^2 + 5.8^2 - 2(13)(5.8)\cos 67°$
≈ 143.7177
$AB \approx 12.0$

33. $9.7^2 = 14.7^2 + 6.8^2 - 2(14.7)(6.8)\cos Z$
$94.09 = 262.33 - 199.92\cos Z$
$-168.24 = -199.92\cos Z$
$m\angle Z = \cos^{-1}\left(\frac{168.24}{199.92}\right) \approx 33°$

34. $5^2 = 12^2 + 13^2 - 2(12)(13)\cos R$
$25 = 313 - 312\cos R$
$-288 = -312\cos R$
$m\angle R = \cos^{-1}\left(\frac{288}{312}\right) \approx 23°$

35. $EF^2 = 8.4^2 + 10.6^2 - 2(8.4)(10.6)\cos 51°$
≈ 70.8506
$EF \approx 8.4$

36. $LM^2 = 10.1^2 + 12.9^2 - 2(10.1)(12.9)\cos 112°$
≈ 366.0350
$LM \approx 19.1$

37. $5^2 = 13^2 + 14^2 - 2(13)(14)\cos G$
$25 = 365 - 364\cos G$
$-340 = -364\cos G$
$m\angle G = \cos^{-1}\left(\frac{340}{364}\right) \approx 21°$

38. $AB^2 = 108^2 + 55^2 - 2(108)(55)\cos 59°$
≈ 8570.3477
$AB \approx 92.6$
$\frac{\sin B}{55} \approx \frac{\sin 59°}{92.576}$
$m\angle B \approx \sin^{-1}\left(\frac{55\sin 59°}{92.576}\right) \approx 31°$

39. $\frac{\sin B}{b} = \frac{\sin A}{a}$
$\frac{\sin 22°}{3.2} = \frac{\sin 74°}{a}$
$a = \frac{3.2\sin 74°}{\sin 22°} \approx 8.2$ cm

40. $c^2 = a^2 + b^2 - 2ab\cos C$
$= 9.5^2 + 7.1^2 - 2(9.5)(7.1)\cos 100°$
≈ 164.0851
$c \approx 12.8$ in.

41. $b^2 = a^2 + c^2 - 2ac\cos B$
$3.1^2 = 2.2^2 + 4^2 - 2(2.2)(4)\cos B$
$9.61 = 20.84 - 17.6\cos B$
$-11.23 = -17.6\cos B$
$m\angle B = \cos^{-1}\left(\frac{11.23}{17.6}\right) \approx 50°$

42. $\frac{\sin C}{c} = \frac{\sin A}{a}$
$\frac{\sin C}{8.4} = \frac{\sin 45°}{10.3}$
$m\angle C = \sin^{-1}\left(\frac{8.4\sin 45°}{10.3}\right) \approx 35°$

43. No; 3 $\angle$ measures do not uniquely determine a $\triangle$. There is not enough information to use either Law of Sines or Law of Cosines.

44. $c^2 = a^2 + b^2 - 2ab\cos C$
$= a^2 + b^2 - 2ab\cos 90°$
$= a^2 + b^2$
Law of Cosines simplifies to Pyth. Thm.

45. Let $\angle$ of turn be $\angle 1$ and let $\angle 2$ be opp. 6-km side.
$6^2 = 3^2 + 4^2 - 2(3)(4)\cos\angle 2$
$36 = 25 - 24\cos\angle 2$
$11 = -24\cos\angle 2$
$m\angle 2 = \cos^{-1}\left(-\frac{11}{24}\right)$
$m\angle 1 = 180 - m\angle 2$
$= 180 - \cos^{-1}\left(-\frac{11}{24}\right) \approx 63°$

46. Step 1 Find 3rd side length. Think: Use Law of Cosines.

$x^2 = 5^2 + 9^2 - 2(5)(9)\cos 109°$
≈ 135.3011
$x \approx 11.6$ cm

Step 2 Find perimeter.

$P \approx 5 + 9 + 11.6 \approx 25.6$ cm

47. Step 1 Find 2nd side length. Think: Use △ ∠ Sum Thm., Law of Sines.

$m\angle 3 = 180 - (93 + 24) = 63°$

$\frac{\sin 63°}{16} = \frac{\sin 24°}{x}$

$x = \frac{16 \sin 24°}{\sin 63°} \approx 7.30$ ft

Step 2 Find 3rd side length. Think: Use Law of Sines.

$\frac{\sin 63°}{16} = \frac{\sin 93°}{y}$

$y = \frac{16 \sin 93°}{\sin 63°} \approx 17.93$ ft

Step 3 Find perimeter.

$P \approx 16 + 7.30 + 17.93 \approx 41.2$ ft

48. Step 1 Find 2nd side length. Think: Use Law of Sines.

$\frac{\sin 45°}{7.3} = \frac{\sin 115°}{x}$

$x = \frac{7.3 \sin 115°}{\sin 45°} \approx 9.36$ in.

Step 2 Find 3rd side length. Think: Use △ ∠ Sum Thm., Law of Sines.

$m\angle 3 = 180 - (45 + 115) = 20°$

$\frac{\sin 45°}{7.3} = \frac{\sin 20°}{y}$

$y = \frac{7.3 \sin 20°}{\sin 45°} \approx 3.53$ in.

Step 3 Find perimeter.

$P = 7.3 + 9.36 + 3.53 \approx 20.2$

49.

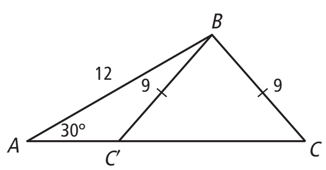


Figure shows two possible positions for C. Since $\overline{BC} \cong \overline{BC'}$, $\angle C \cong \angle BC'C$, so $\angle C$ and $\angle BC'A$ are supp.

$\frac{\sin C}{12} = \frac{\sin 30°}{9}$

$m\angle C = \sin^{-1}\left(\frac{12 \sin 30°}{9}\right) \approx 42°$

$m\angle BC'A = 180 - m\angle C$
$\approx 180 - 42 \approx 138°$

50a. Think: Use △ ∠ Sum Thm.

$m\angle F + 51 + 38 = 180$
$m\angle F + 89 = 180$
$m\angle F = 91°$

b. $\frac{\sin 91°}{18.3} = \frac{\sin 38°}{AF}$

$AF = \frac{18.3 \sin 38°}{\sin 91°}$
≈ 11 mi

$\frac{\sin 91°}{18.3} = \frac{\sin 51°}{BF}$

$BF = \frac{18.3 \sin 51°}{\sin 91°}$
≈ 14 mi

c. Dist. = speed • time

$AF = 150t_1$
$BF = 150t_2$
$BF - AF = 150(t_1 - t_2)$
$t_1 - t_2 = \frac{BF - AF}{150}$
$\approx \frac{14.2 - 11.3}{150}$
≈ 0.193 h or 1.2 min

51. Given ∠ is opp. one of given sides, so use Law of Sines.

52. Given ∠ is included by given sides, so use Law of Cosines.

53. One of given ∡ is opp. given side, so use Law of Sines.

54a. $RS = \sqrt{3^2 + 2^2} = \sqrt{13} \approx 3.6$

$ST = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10} \approx 6.3$

$RT = \sqrt{3^2 + 4^2} = 5$

b. $\angle R$, because it is opp. the longest side.

c. $ST^2 = RS^2 + RT^2 - 2(RS)(RT)\cos R$

$40 = 13 + 25 - 2\left(\sqrt{13}\right)(5)\cos R$

$2 = -10\sqrt{13}(\cos R)$

$R = \cos^{-1}\left(-\frac{2}{10\sqrt{13}}\right) \approx 93°$

55. $BC^2 = 6.46^2 + 7.14^2 - 2(6.46)(7.14)\cos 104°$
≈ 115.12197
$BC \approx 10.73$ cm
$AB^2 = 3.86^2 + 7.14^2 - 2(3.86)(7.14)\cos 138°$
≈ 106.84194
$AB \approx 10.34$ cm

$\frac{\sin ABE}{3.86} \approx \frac{\sin 138°}{10.34}$

$m\angle ABE \approx \sin^{-1}\left(\frac{3.86 \sin 138°}{10.34}\right) \approx 14.47°$

$\frac{\sin EBC}{6.46} \approx \frac{\sin 104°}{10.73}$

$m\angle EBC \approx \sin^{-1}\left(\frac{6.46 \sin 104°}{10.73}\right) \approx 35.74°$

$m\angle ABC = m\angle ABE + m\angle EBC$
$\approx 14.47 + 35.74 \approx 50°$

56. A is incorrect; possible answer: the fraction on the right side of the proportion is incorrect.
It should be $\frac{\sin 70°}{12} = \frac{\sin 85°}{x}$, as in B.

57a. $y^2 + h^2$ **b.** b^2

c. $a^2 = c^2 - 2cx + x^2 + h^2$

d. $a^2 = c^2 + b^2 - 2cx$

e. $b \cos A$ **f.** Subst.

58. No; possible answer: to use Law of Sines, you need to know at least 1 side length and ∠ measure opp. that side.

TEST PREP, PAGE 558

59. A
$AB^2 = 12^2 + 14^2 - 2(12)(14)\cos 23°$
≈ 32.71
$AB \approx 5.7$ cm
Nearest given value is 5.5 cm.

60. H

61. C
$m\angle Y = 180 - (25 + 135) = 20°$
$\dfrac{\sin 20°}{100} = \dfrac{\sin 25°}{XY}$
$XY = \dfrac{100 \sin 25°}{\sin 20°} \approx 124$ m

CHALLENGE AND EXTEND, PAGE 558

62. $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos ACB$
$(2 + 3)^2 = (2 + 4)^2 + (3 + 4)^2$
$- 2(2 + 4)(3 + 4)\cos ACB$
$25 = 85 - 84\cos ACB$
$-60 = -84\cos ACB$
$m\angle ACB = \cos^{-1}\left(\dfrac{60}{84}\right) \approx 44°$

63. Let pts. be $A(-1, 1)$, $B(1, 3)$, and $C(3, 2)$
$AB = \sqrt{2^2 + 2^2} = \sqrt{8}$; $AC = \sqrt{4^2 + 1^2} = \sqrt{17}$;
$BC = \sqrt{2^2 + 1^2} = \sqrt{5}$
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$
$5 = 8 + 17 - 2\left(\sqrt{8}\right)\left(\sqrt{17}\right)\cos A$
$-20 = -4\sqrt{34}(\cos A)$
$m\angle A = \cos^{-1}\left(\dfrac{20}{4\sqrt{34}}\right) \approx 31°$

64. Let P be position of boat after 45 min = 0.75 h.
Given information: $AB = 5$ mi, $AP = (6$ mi/h$)(0.75$ h$)$ $= 4.5$ mi, $\angle A = 180 - 32 = 148°$
$BP^2 = AB^2 + AP^2 - 2(AB)(AP)\cos A$
$= 5^2 + 4.5^2 - 2(5)(4.5)\cos 148°$
≈ 83.4122
$BP \approx 9.1$ mi

SPIRAL REVIEW, PAGE 558

65. $3n$

66. $2n + 1$

67. $2n + 2$

68. Alt. Ext. ∡ Thm.

69. Alt. Int. ∡ Thm.

70. Same-Side Int. ∡ Thm.

71. Alt. Ext. ∡ Thm.

72. $\angle 2$

73. $\angle 1$

74. $\angle 1$

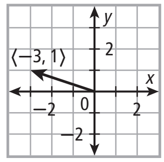
8-6 VECTORS, PAGES 559–567

CHECK IT OUT! PAGES 559–562

1a. Horiz. change along $\vec{u}$ is -3 units.
Vert. change along $\vec{u}$ is -4 units.
So component form of $\vec{u}$ is $\langle -3, -4 \rangle$.

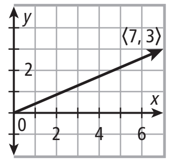
b. Horiz. change from L to M is 7 units.
Vert. change from L to M is 1 unit.
So component form of $\overrightarrow{LM}$ is $\langle 7, 1 \rangle$.

2. Step 1 Draw vector on a coord. plane. Use origin as initial pt. Then $(-3, 1)$ is terminal pt.
Step 2 Find magnitude. Use Dist. Formula.



$\left|\langle -3, 1 \rangle\right| = \sqrt{(-3 - 0)^2 + (1 - 0)^2} = \sqrt{10} \approx 3.2$

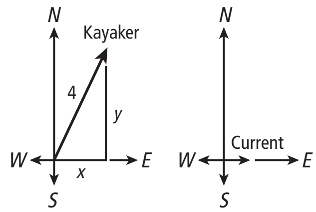
3. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction. Draw rt. $\triangle ABC$ as shown. $\angle A$ is $\angle$ formed by vector and x-axis, and $\tan A = \dfrac{3}{7}$. So
$m\angle A = \tan^{-1}\left(\dfrac{3}{7}\right) \approx 23°$



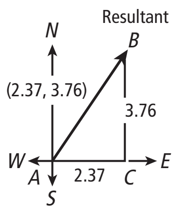
4a. $\overrightarrow{PQ} = \overrightarrow{RS}$ (same magnitude and direction)

b. $\overrightarrow{PQ} \parallel \overrightarrow{RS}$ and $\overrightarrow{XY} \parallel \overrightarrow{MN}$ (same or opp. direction)

5. Step 1 Sketch vectors for kayaker and current.



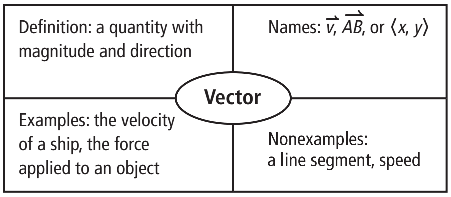
Step 2 Write vector for kayaker in component form. It has magn. 4 mi/h and makes $\angle$ of 70° with x-axis.
$\cos 70° = \dfrac{x}{4}$, so $x = 4\cos 70° \approx 1.37$
$\sin 70° = \dfrac{y}{4}$, so $y = 4\sin 70° \approx 3.76$
Kayaker's vector is $\langle 1.37, 3.76 \rangle$.
Step 3 Write vector for current in component form: $\langle 1, 0 \rangle$.
Step 4 Find and sketch resultant vector $\overrightarrow{AB}$.
Add components of kayaker's vector and current's vector.
$\langle 1.37, 3.76 \rangle + \langle 1, 0 \rangle = \langle 2.37, 3.76 \rangle$



Step 5 Find magn. and direction of resultant vector. Magn. of resultant vector is kayak's actual speed.
$\left|\langle 2.37, 3.76 \rangle\right| = \sqrt{(2.37 - 0)^2 + (3.76 - 0)^2} \approx 4.4$ mi/h
$\angle$ measure formed by resultant vector gives kayak's actual direction.
$\tan A \approx \dfrac{3.76}{2.37}$, so $m\angle A = \tan^{-1}\left(\dfrac{3.76}{2.37}\right) \approx 58°$, or N 32° E.

THINK AND DISCUSS, PAGE 563

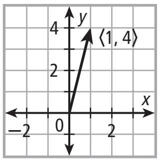
1. It does not have a direction.
2. Pyth. Thm.
3. Possible answer: Write each vector in component form and add horizontal and vertical components.
4.



EXERCISES, PAGES 563–567

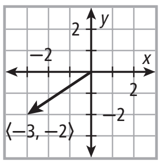
GUIDED PRACTICE, PAGES 563–564

1. equal
2. parallel
3. magnitude
4. Horiz. change from A to C is 5 units. Vert. change from A to C is 3 units. So component form of $\overrightarrow{AC}$ is $\langle 5, 3\rangle$.
5. Horiz. change from M to N is 8 units. Vert. change from M to N is −8 units. So component form of $\overrightarrow{MN}$ is $\langle 8, -8\rangle$.
6. Horiz. change from P to Q is 2 units. Vert. change from P to Q is 5 units. So component form of $\overrightarrow{PQ}$ is $\langle 2, 5\rangle$.
7. **Step 1** Draw vector on a coord. plane. Use origin as initial pt. Then (1, 4) is terminal pt.
Step 2 Find magnitude. Use Dist. Formula.



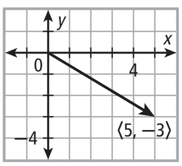
$\left|\langle 1, 4\rangle\right| = \sqrt{(1-0)^2 + (4-0)^2} = \sqrt{17} \approx 4.1$

8. **Step 1** Draw vector on a coord. plane. Use origin as initial pt. Then (−3, −2) is terminal pt.
Step 2 Find magnitude. Use Dist. Formula.



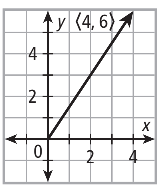
$\left|\langle -3, -2\rangle\right| = \sqrt{(-3-0)^2 + (-2-0)^2}$
$= \sqrt{13} \approx 3.6$

9. **Step 1** Draw vector on a coord. plane. Use origin as initial pt. Then (5, −3) is terminal pt.
Step 2 Find magnitude. Use Dist. Formula.

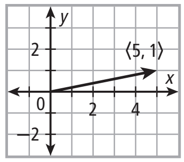


$\left|\langle 5, -3\rangle\right| = \sqrt{(5-0)^2 + (-3-0)^2} = \sqrt{34} \approx 5.8$

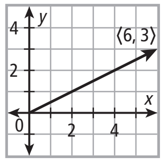
10. **Step 1** Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction. Draw rt. $\triangle ABC$ as shown. $\angle A$ is $\angle$ formed by vector and x-axis, and $\tan A = \frac{6}{4}$. So
$m\angle A = \tan^{-1}\left(\frac{6}{4}\right) \approx 56°$



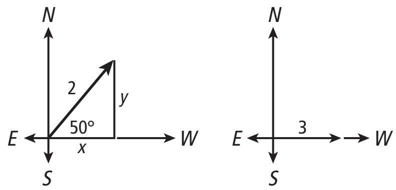
11. **Step 1** Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction. Draw rt. $\triangle ABC$ as shown. $\angle A$ is $\angle$ formed by vector and x-axis, and $\tan A = \frac{1}{5}$. So
$m\angle A = \tan^{-1}\left(\frac{1}{5}\right) \approx 11°$



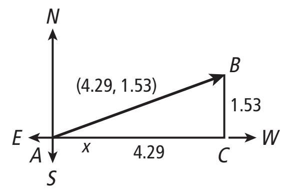
12. **Step 1** Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction. Draw rt. $\triangle ABC$ as shown. $\angle A$ is $\angle$ formed by vector and x-axis, and $\tan A = \frac{3}{6}$. So
$m\angle A = \tan^{-1}\left(\frac{3}{6}\right) \approx 27°$.



13. $\overrightarrow{CD} = \overrightarrow{EF}$ (same magn. and direction)
14. $\overrightarrow{CD} \parallel \overrightarrow{EF}$ and $\overrightarrow{AB} \parallel \overrightarrow{GH}$ (same or opp. direction)
15. $\overrightarrow{RS} = \overrightarrow{XY}$ (same magn. and direction)
16. $\overrightarrow{RS} \parallel \overrightarrow{XY}$ and $\overrightarrow{MN} \parallel \overrightarrow{PQ}$ (same or opp. direction)
17. **Step 1** Sketch vectors for 2 stages of hike.



Step 2 Write vector for 1st stage in component form. It has magn. 2 mi and makes $\angle$ of 50° with x-axis.
$\cos 50° = \frac{x}{2}$, so $x = 2\cos 50° \approx 1.29$
$\sin 50° = \frac{y}{2}$, so $y = 2\sin 50° \approx 1.53$
Vector for 1st stage is $\langle 1.29, 1.53\rangle$.
Step 3 Write vector for 2nd stage in component form: $\langle 3, 0\rangle$.
Step 4 Find and sketch resultant vector $\overrightarrow{AB}$. Add components of 1st- and 2nd-stage vectors.
$\langle 1.29, 1.53\rangle + \langle 3, 0\rangle = \langle 4.29, 1.53\rangle$



Step 5 Find magn. and direction of resultant vector. Magn. of resultant vector is straight-line dist. to campsite.
$\left|\langle 4.29, 1.53\rangle\right| = \sqrt{(4.29-0)^2 + (1.53-0)^2} \approx 4.6$ mi
$\angle$ measure formed by resultant vector gives direction of hike.
$\tan A \approx \frac{1.53}{4.29}$, so $m\angle A = \tan^{-1}\left(\frac{1.53}{4.29}\right) \approx 20°$, or N 70° E.

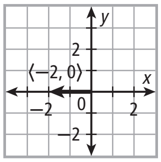
PRACTICE AND PROBLEM SOLVING, PAGES 564–566

18. $\langle 9, 2\rangle$

19. $\langle -3.5, 5.5\rangle$

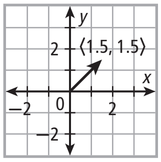
20. $\langle -4, -4\rangle$

21. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find magnitude. Since vector is horiz.,



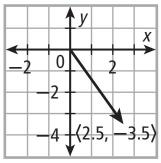
$|\langle -2, 0\rangle| = |-2| = 2.0$

22. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find magnitude. Use Pyth. Thm.



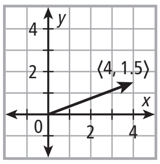
$|\langle 1.5, 1.5\rangle| = \sqrt{1.5^2 + 1.5^2} = \sqrt{4.5} \approx 2.1$

23. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find magnitude. Use Pyth. Thm.

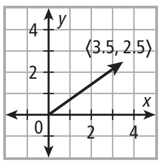


$|\langle 2.5, -3.5\rangle| = \sqrt{2.5^2 + 3.5^2} = \sqrt{18.5} \approx 4.3$

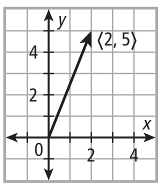
24. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction.
$m\angle A = \tan^{-1}\left(\frac{1.5}{4}\right) \approx 21°$



25. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction.
$m\angle A = \tan^{-1}\left(\frac{2.5}{3.5}\right) \approx 36°$



26. Step 1 Draw vector on a coord. plane. Use origin as initial pt.
Step 2 Find direction.
$m\angle A = \tan^{-1}\left(\frac{5}{2}\right) \approx 68°$



27. $\overrightarrow{DE} = \overrightarrow{LM}$

28. All 4 vectors are ∥.

29. $\overrightarrow{RS} = \overrightarrow{UV}$

30. $\overrightarrow{RS} \parallel \overrightarrow{UV} \parallel \overrightarrow{AB}$ and $\overrightarrow{CD} \parallel \overrightarrow{XY}$

31. Step 1 Write airplane's vector in component form.
$x = 200\cos 65° \approx 84.524$; $y = 200\sin 65° \approx 181.262$
Airplane's vector is $\langle 84.524, 181.262\rangle$.
Step 2 Write windspeed vector in component form.
$x = 40\cos 45° \approx 28.284$; $y = -40\sin 45° \approx -28.284$
Windspeed vector is $\langle 28.284, -28.284\rangle$.
Step 3 Find resultant vector $\overrightarrow{AB}$. Add components of airplane's and windspeed vectors.
$\langle 84.524, 181.262\rangle + \langle 28.284, -28.284\rangle$
$\approx \langle 112.81, 152.98\rangle$
Step 4 Find magn. and direction of resultant vector.
$|\langle 112.81, 152.98\rangle| = \sqrt{112.81^2 + 152.98^2}$
≈ 190.1 km/h
$m\angle A \approx \tan^{-1}\left(\frac{152.98}{112.81}\right) \approx 54°$, or N 36° E.

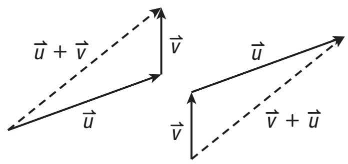
32. $\langle 1, 2\rangle + \langle 0, 6\rangle = \langle 1 + 0, 2 + 6\rangle = \langle 1, 8\rangle$

33. $\langle -3, 4\rangle + \langle 5, -2\rangle = \langle -3 + 5, 4 + (-2)\rangle = \langle 2, 2\rangle$

34. $\langle 0, 1\rangle + \langle 7, 0\rangle = \langle 0 + 7, 1 + 0\rangle = \langle 7, 1\rangle$

35. $\langle 8, 3\rangle + \langle -2, -1\rangle = \langle 8 + (-2), 3 + (-1)\rangle = \langle 6, 2\rangle$

36. Yes; possible answer: if you use head-to-tail method in both orders, you end up with a ▱ and its diag. Resultant vector is the diag. See figures below.



37a. Let $\overleftrightarrow{FG}$ be a vert. line. Use Alt. Int ∡ Thm.
$m\angle F = m\angle GFH + m\angle GFX$
$= 45 + 53 = 98°$

b. $HX^2 = 50^2 + 41^2 - 2(50)(41)\cos 98°$
≈ 4751.6097
$HX \approx 68.9$ mi/h

c. $\frac{\sin FHX}{41} \approx \frac{\sin 98°}{68.9}$
$m\angle FHX \approx \sin^{-1}\left(\frac{41\sin 98°}{68.9}\right) \approx 36°$

d. direction ≈ 45 + 36 = 81° E of N, or N 81° E

38. $\langle 15\cos 42°, 15\sin 42°\rangle \approx \langle 11.1, 10.0\rangle$

39. $\langle 7.2\cos 9°, 7.2\sin 9°\rangle \approx \langle 7.1, 1.1\rangle$

40. direction relative to x-axis = 90 − 57 = 33°
$\langle 12.1\cos 33°, 12.1\sin 33°\rangle \approx \langle 10.1, 6.6\rangle$

41. direction relative to x-axis = 90 − 22 = 68°
$\langle 5.8\cos 68°, 5.8\sin 68°\rangle \approx \langle 2.2, 5.4\rangle$

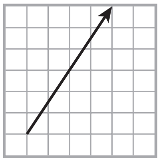
42a. $10\sin 45° \approx 7.1$ lb

b. $10\sin 75° \approx 9.7$ lb

c. Taneka; she applies more vert. force.

43a. Prob. of 1 on 1st draw is $\frac{1}{4}$; prob. of then drawing 2 is $\frac{1}{3}$. So Prob.$\left(\langle 1, 2\rangle\right) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

b. Prob.$\left(\text{vector} \parallel \text{to } \langle 1, 2\rangle\right)$
$= \text{Prob.}\left(\langle 1, 2\rangle \text{ or } \langle 2, 4\rangle\right)$
$= \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

44a. 

b. Estimates should be between 50° and 60°.

c. Answers should be close to 56°.

d. direction $= \tan^{-1}\left(\frac{6}{4}\right) \approx 56°$

e. Possible answer: Estimate is accurate only to within $\approx 5°$; measurement is accurate to within 1 or 2°; calculation is accurate to nearest degree.

45. $|\vec{u}| = |4| = 4$
direction of $\vec{u} = 0°$

46. $|\vec{v}| = |3| = 3$
direction of $\vec{v} = 90°$

47. $|\vec{w}| = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.6$
direction of $\vec{w} = \tan^{-1}\left(\frac{3}{2}\right) \approx 56°$

48. $|\vec{z}| = \sqrt{4^2 + 1^2} = \sqrt{17} \approx 4.1$
direction of $\vec{z} = \tan^{-1}\left(\frac{1}{4}\right) \approx 14°$

49. Pass pattern vectors are $\langle 0, 10\rangle$ and $\langle 10, 0\rangle$.
Resultant vector is $\langle 0, 10\rangle + \langle 10, 0\rangle = \langle 10, 10\rangle$.
Magn. of resultant is $\sqrt{10^2 + 10^2} = 10\sqrt{2}$;
Direction of resultant is $\tan^{-1}\left(\frac{10}{10}\right) = 45°$.
Jason's move is equivalent to resultant.

50.–52. Possible answers given.

50. Think: Change sign of one component only.
$\langle 3, 6\rangle$ has same magn. but different direction.
Think: Multiply both components by the same factor.
$\langle -6, 12\rangle$ has same direction but different magn.

51. $\langle -12, -5\rangle$ has same magn. but different (opp.) direction.
$\langle 24, 10\rangle$ has same direction but different magn.

52. $\langle -8, 11\rangle$ has same magn. but different (opp.) direction.
$\langle 4, -5.5\rangle$ has same direction but different magn.

53. $\vec{u} + \vec{v} = \langle 1 + 2.5, 2 + (-1)\rangle = \langle 3.5, 1\rangle$
$|\vec{u} + \vec{v}| = \sqrt{3.5^2 + 1^2} = \sqrt{13.25} \approx 3.6$
direction of $\vec{u} + \vec{v} = \tan^{-1}\left(\frac{1}{3.5}\right) \approx 16°$

54. $\vec{u} + \vec{v} = \langle -2 + 4.8, 7 + (-3.1)\rangle = \langle 2.8, 3.9\rangle$
$|\vec{u} + \vec{v}| = \sqrt{2.8^2 + 3.9^2} = \sqrt{23.05} \approx 4.8$
direction of $\vec{u} + \vec{v} = \tan^{-1}\left(\frac{3.9}{2.8}\right) \approx 54°$

55. $\vec{u} + \vec{v} = \langle 6 + (-2), 0 + 4\rangle = \langle 4, 4\rangle$
$|\vec{u} + \vec{v}| = \sqrt{4^2 + 4^2} = 4\sqrt{2} \approx 5.7$
direction of $\vec{u} + \vec{v} = \tan^{-1}\left(\frac{4}{4}\right) = 45°$

56. $\vec{u} + \vec{v} = \langle -1.2 + 5.2, 8 + (-2.1)\rangle = \langle 4, 5.9\rangle$
$|\vec{u} + \vec{v}| = \sqrt{4^2 + 5.9^2} = \sqrt{50.81} \approx 7.1$
direction of $\vec{u} + \vec{v} = \tan^{-1}\left(\frac{5.9}{4}\right) \approx 56°$

57a. $\vec{v} = \langle 1, 3\rangle$; $2\vec{v} = \langle 2, 6\rangle$

b. $|\vec{v}| = \sqrt{1^2 + 3^2} = \sqrt{10}$; $|2\vec{v}| = \sqrt{2^2 + 6^2} = 2\sqrt{10}$
$2\vec{v}$ is twice the magnitude of $\vec{v}$.

c. direction of $\vec{v} = \tan^{-1}\left(\frac{3}{1}\right) = 72°$

direction of $2\vec{v} = \tan^{-1}\left(\frac{6}{2}\right) = \tan^{-1}\left(\frac{3}{1}\right) = 72°$
Directions are the same.

d. Multiply each component by *k*.

e. $-\overrightarrow{v} = (-1)\overrightarrow{v} = (-1)\langle x, y\rangle$
$= \langle (-1)x, (-1)y\rangle = \langle -x, -y\rangle$

58. If $u > v$, resultant points due west, with magn. $u - v$.
If $v > u$, resultant points due east, with magn. $v - u$.
If $u = v$, resultant is $\langle 0, 0\rangle$.

59. A line seg. has magnitude (or length), but no direction. A ray is a part of a line that continues indefinitely in one direction. Thus it has direction and infinite magnitude. A vector has both direction and magnitude.

TEST PREP, PAGE 567

60. C
$\vec{w} = (-2)\langle 2, 1\rangle$

61. G
$\tan^{-1}\left(\frac{9}{7}\right) \approx 52°$

62. C
$\sqrt{5^2 + 11^2} = \sqrt{146} \approx 12$

63. 8.2
$|\overrightarrow{AB}| = \sqrt{(-5 + 3)^2 + (-2 - 6)^2} = \sqrt{68} \approx 8.2$

CHALLENGE AND EXTEND, PAGE 567

64. $\langle -2, 3\rangle$ is in 2nd quadrant, so direction is between 90° and 180°.
direction $= 180 + \tan^{-1}\left(\frac{3}{-2}\right) \approx 124°$

65. $\langle -4, 0\rangle$ lies along negative *x*-axis, so direction = 180°.

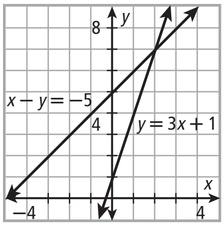
66. $\langle -5, -3\rangle$ is in 3rd quadrant, so direction is between 180° and 270°.
direction $= 180 + \tan^{-1}\left(\frac{-3}{-5}\right) \approx 211°$

67. Let $\vec{v} = \langle x, y\rangle$ be required velocity vector. Then
$\langle x, y\rangle + \langle 4, 0\rangle = \langle 10\cos 20°, 10\sin 20°\rangle$
$x + 4 = 10\cos 20°$
$x = 10\cos 20° - 4 \approx 5.40$ mi/h
$y = 10\sin 20° \approx 3.42$ mi/h
$|\vec{v}| = \sqrt{5.40^2 + 3.42^2} \approx 6.4$ mi/h
bearing $= \tan^{-1}\left(\frac{3.42}{5.40}\right) \approx 32°$, or N 58° E

68. $\vec{v} = \langle x, y\rangle = \langle 3\cos 60°, 3\cos 60°\rangle + \langle 6, 0\rangle$
$+ \langle 4\cos 40°, 4\sin 40°\rangle$
$x = 3\cos 60° + 6 + 4\cos 40° \approx 10.56$ km
$y = 3\sin 60° + 0 + 4\sin 40° \approx 5.17$ km
$|\vec{v}| = \sqrt{10.56^2 + 5.17^2} \approx 11.8$ km
bearing $= \tan^{-1}\left(\frac{5.17}{10.56}\right) = 26°$, or N 64° E

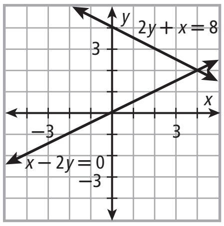
SPIRAL REVIEW, PAGE 567

69. $\begin{cases} x - y = -5 \\ y = 3x + 1 \end{cases}$
$\rightarrow \begin{cases} y = x + 5 \\ y = 3x + 1 \end{cases}$



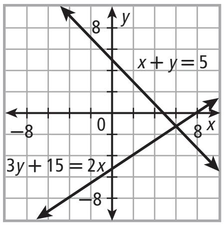
Lines intersect at (2, 7).

70. $\begin{cases} x - 2y = 0 \\ 2y + x = 8 \end{cases}$
$\rightarrow \begin{cases} y = 0.5x \\ y = -0.5x + 4 \end{cases}$



Lines intersect at (4, 2).

71. $\begin{cases} x + y = 5 \\ 3y + 15 = 2x \end{cases}$
$\rightarrow \begin{cases} y = -x + 5 \\ y = \frac{2}{3}x - 5 \end{cases}$



Lines intersect at (6, −1).

72. $NP = 3JL$
Perim. = 3(12) = 36 cm

73. Area $= (3)^2(6)$
$= 54 \text{ cm}^2$

74. $BC^2 = 3.5^2 + 4^2 - 2(3.5)(4)\cos 50°$
≈ 10.2519
$BC \approx 3.2$

75. $\frac{\sin B}{4} \approx \frac{\sin 50°}{3.20}$
$m\angle B \approx \sin^{-1}\left(\frac{4 \sin 50°}{3.20}\right) \approx 73°$

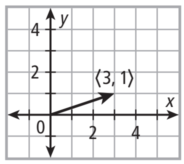
76. $m\angle C \approx 180 - (50 + 73) \approx 57°$

MULTI-STEP TEST PREP, PAGE 568

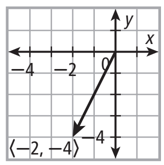
1. By Alt. Int. ∠s Thm., dist. $= \frac{1500}{\tan 14°} \approx 6016$ ft.
2. By Alt. Int. ∠s Thm., $m\angle S = 57°$
$HA^2 = 100^2 + 30^2 - 2(100)(30)\cos 57°$
≈ 7632.1658
$HA \approx 87$ mi/h
3. $\frac{\sin H}{30} \approx \frac{\sin 57°}{87.4}$
$m\angle H = \sin^{-1}\left(\frac{30 \sin 57°}{87.4}\right) \approx 73°$
So direction of $\overrightarrow{HA}$ is N 17° E.

READY TO GO ON? PAGE 569

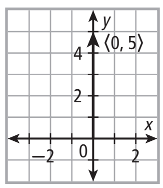
1. By Alt. Int. ∠s Thm., dist. $= \frac{1600}{\tan 34°} \approx 2372$ ft.
2. height $= 6\tan 78° \approx 28.2$ m
3. $\frac{\sin A}{14} = \frac{\sin 118°}{20}$
$A = \sin^{-1}\left(\frac{14 \sin 118°}{20}\right) \approx 38°$
4. $\frac{\sin 41°}{7} = \frac{\sin 84°}{GH}$
$GH = \frac{7 \sin 84°}{\sin 41°} \approx 10.6$
5. $m\angle X = 180 - (92 + 62) = 26°$
$\frac{\sin 26°}{8} = \frac{\sin 92°}{XZ}$
$XZ = \frac{8 \sin 92°}{\sin 26°} \approx 18.2$
6. $UV^2 = 12^2 + 9^2 - 2(12)(9)\cos 35°$
≈ 48.0632
$UV \approx 6.9$
7. $4^2 = 5^2 + 6^2 - 2(5)(6)\cos F$
$16 = 61 - 60\cos F$
$-45 = -60\cos F$
$m\angle F = \cos^{-1}\left(\frac{45}{60}\right) \approx 41°$
8. $QS^2 = 10.5^2 + 6^2 - 2(10.5)(6)\cos 39°$
≈ 48.3296
$QS \approx 7.0$
9. $|\langle 3, 1\rangle| = \sqrt{3^2 + 1^2}$
$= \sqrt{10} \approx 3.2$



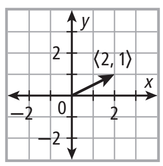
10. $|\langle -2, -4\rangle| = \sqrt{2^2 + 4^2}$
$= \sqrt{20}$
≈ 4.5



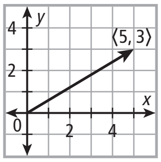
11. $|\langle 0, 5\rangle| = |5| = 5$



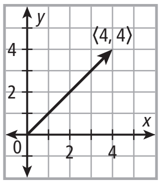
12. direction $= \tan^{-1}\left(\frac{1}{2}\right)$
$\approx 27°$



13. direction $= \tan^{-1}\left(\frac{3}{5}\right)$
$\approx 31°$



14. direction $= \tan^{-1}\left(\frac{4}{4}\right)$
$= 45°$



15. Let $\langle x, y\rangle$ be resultant vector.
$\langle x, y\rangle = \langle 6\sin 32°, 6\cos 32°\rangle + \langle 8, 0\rangle$
$x = 6\sin 32° + 8 \approx 11.18$ km
$y = 6\cos 32° \approx 5.09$ km
dist. $= \sqrt{11.18^2 + 5.09^2} \approx 12.3$ km
direction $= \tan^{-1}\left(\frac{5.09}{11.18}\right) \approx 24°$, or N 66° E

STUDY GUIDE: REVIEW, PAGES 572–575

VOCABULARY, PAGE 572

1. component form
2. equal vectors
3. geometric mean
4. angle of elevation
5. trigonometric ratio

LESSON 8-1, PAGE 572

6. $\triangle PRQ \sim \triangle RSQ \sim \triangle PSR$

7. $x^2 = \left(\frac{1}{4}\right)(100) = 25$
$x = \sqrt{25} = 5$

8. $x^2 = (3)(17) = 51$
$x = \sqrt{51}$

9. $x^2 = (5)(7) = 35$
$x = \sqrt{35}$
$y^2 = 5(5 + 7) = 60$
$y = \sqrt{60} = 2\sqrt{15}$
$z^2 = 7(5 + 7) = 84$
$z = \sqrt{84} = 2\sqrt{21}$

10. $6^2 = (x)(12)$
$36 = 12x$
$x = 3$
$y^2 = (3)(3 + 12) = 45$
$y = \sqrt{45} = 3\sqrt{5}$
$z^2 = (12)(3 + 12) = 180$
$z = \sqrt{180} = 6\sqrt{5}$

11. $\left(\sqrt{6}\right)^2 = (1)(1 + x)$
$6 = 1 + x$
$x = 5$
$y^2 = (1)(5) = 5$
$y = \sqrt{5}$
$z^2 = (5)(1 + 5) = 30$
$z = \sqrt{30}$

LESSON 8-2, PAGE 573

12. $\sin 80° = \frac{11}{UV}$
$UV = \frac{11}{\sin 80°}$
≈ 11.17 m

13. $\cos 29° = \frac{PR}{7.2}$
$PR = 7.2\cos 29°$
≈ 6.30 m

14. $\cos 33° = \frac{XY}{12.3}$
$XY = 12.3\cos 33°$
≈ 10.32 cm

15. $\tan 47° = \frac{1.4}{JL}$
$JL = \frac{1.4}{\tan 47°}$
≈ 1.31 cm

LESSON 8-3, PAGE 573

16. $m\angle C = 90 - 22 = 68°$
$AB = 5.2\cos 22° \approx 4.82$
$AC = 5.2\sin 22° \approx 1.95$

17. $m\angle H = \tan^{-1}\left(\frac{4.7}{3.5}\right) \approx 53°$
$m\angle G \approx 90 - 53 \approx 37°$
$HG = \sqrt{3.5^2 + 4.7^2} \approx 5.86$

18. $m\angle S = 90 - 50 = 40°$
$RS = \frac{32.5}{\sin 50°} \approx 42.43$
$RT = \frac{32.5}{\tan 50°} \approx 27.27$

19. $m\angle Q = \tan^{-1}\left(\frac{8.6}{9.9}\right) \approx 41°$
$m\angle N \approx 90 - 41 \approx 49°$
$QN = \sqrt{9.9^2 + 8.6^2} \approx 13.11$

LESSON 8-4, PAGE 574

20. $\angle$ of depression
21. $\angle$ of elevation
22. height $= 5.1\tan 82° \approx 36$ ft
23. horiz. dist. $= \frac{32}{\tan 4°} \approx 458$ m

LESSON 8-5, PAGES 574–575

24. $\frac{\sin Z}{4} = \frac{\sin 40°}{7}$
$m\angle Z = \sin^{-1}\left(\frac{4\sin 40°}{7}\right)$
$\approx 22°$

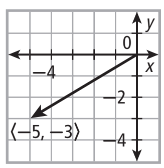
25. $\frac{\sin 23°}{16} = \frac{\sin 130°}{MN}$
$MN = \frac{16\sin 130°}{\sin 23°}$
≈ 31.4

26. $EF^2 = 14^2 + 12^2 + 2(14)(12)\cos 101°$
≈ 404.1118
$EF \approx 20.1$

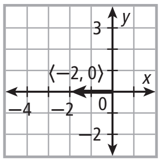
27. $10^2 = 6^2 + 12^2 - 2(6)(12)\cos Q$
$100 = 180 - 144\cos Q$
$-80 = -144\cos Q$
$m\angle Q = \cos^{-1}\left(\frac{80}{144}\right) \approx 56°$

LESSON 8-6, PAGE 575

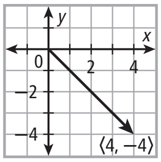
28. $\overrightarrow{AB} = \langle -2 - 5, 3 - 1\rangle = \langle -7, 2\rangle$
29. $\overrightarrow{MN} = \langle -1 - (-2), -2 - 4\rangle = \langle 1, -6\rangle$
30. $\overrightarrow{RS} = \langle -2, -5\rangle$
31. $\left|\langle -5, -3\rangle\right| = \sqrt{5^2 + 3^2}$
$= \sqrt{34} \approx 5.8$



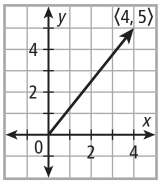
32. $|\langle -2, 0\rangle| = |2| = 2$



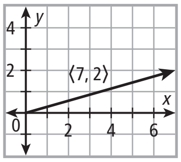
33. $|\langle 4, -4\rangle| = \sqrt{4^2 + 4^2} = \sqrt{32} \approx 5.7$



34. direction $= \tan^{-1}\left(\frac{5}{4}\right) = 51°$



35. direction $= \tan^{-1}\left(\frac{2}{7}\right) = 16°$



36. plane's vector $= \langle 600\cos 35°, 600\sin 35°\rangle$
crosswind vector $= \langle 50, 0\rangle$
resultant vector $= \langle 600\cos 35° + 50, 600\sin 35°\rangle \approx \langle 541.49, 344.15\rangle$
speed $\approx \sqrt{541.49^2 + 344.15^2} \approx 641.6$ mi/h
direction $\approx \tan^{-1}\left(\frac{344.15}{541.49}\right) \approx 32°$, or N 58° E

CHAPTER TEST, PAGE 576

1. $4^2 = (x)(8)$
$16 = 8x$
$x = 2$
$y^2 = 8(2 + 8) = 80$
$y = \sqrt{80} = 4\sqrt{5}$
$z^2 = 2(2 + 8) = 20$
$z = \sqrt{20} = 2\sqrt{5}$

2. $x^2 = (6)(12) = 72$
$x = \sqrt{72} = 6\sqrt{2}$
$y^2 = 12(6 + 12) = 216$
$y = \sqrt{216} = 6\sqrt{6}$
$z^2 = 6(6 + 12) = 108$
$z = \sqrt{108} = 6\sqrt{3}$

3. $\left(2\sqrt{30}\right)^2 = 10(10 + x)$
$120 = 100 + 10x$
$20 = 10x$
$x = 2$
$y^2 = (2)(10) = 20$
$y = \sqrt{20} = 2\sqrt{5}$
$z^2 = 2(2 + 10) = 24$
$z = \sqrt{24} = 2\sqrt{6}$

4. Let 30°-60°-90° △ have sides x, $x\sqrt{3}$, $2x$.
$\cos 60° = \frac{x}{2x} = \frac{1}{2}$

5. Let 45°-45°-90° △ have sides s, s, $s\sqrt{2}$.
$\sin 45° = \frac{s}{s\sqrt{2}} = \frac{\sqrt{2}}{2}$

6. $\tan 60° = \frac{x\sqrt{3}}{x} = \sqrt{3}$

7. $PR = 4.5\sin 18° \approx 1.39$ m

8. $AB = \frac{9}{\cos 51°} \approx 14.30$ cm

9. $FG = \frac{6.1}{\tan 34°} \approx 9.04$ in.

10. $m\angle = \tan^{-1}\left(\frac{3.5}{10}\right) \approx 19°$

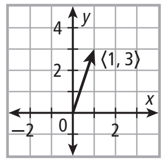
11. horiz. dist. $= \frac{910}{\tan 61°} \approx 504$ ft

12. $\frac{\sin B}{4} = \frac{\sin 85°}{9}$
$m\angle B = \sin^{-1}\left(\frac{4\sin 85°}{9}\right) \approx 26°$

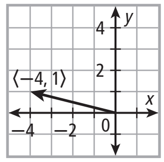
13. $\frac{\sin 35°}{11} = \frac{\sin 108°}{RS}$
$RS = \frac{11\sin 108°}{\sin 35°} \approx 18.2$

14. $7^2 = 10^2 + 15^2 - 2(10)(15)\cos M$
$49 = 325 - 300\cos M$
$-276 = -300\cos M$
$m\angle M = \cos^{-1}\left(\frac{276}{300}\right) \approx 23°$

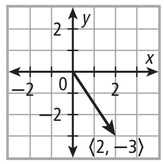
15. $|\langle 1, 3\rangle| = \sqrt{1^2 + 3^2} = \sqrt{10} \approx 3.2$



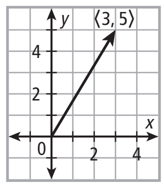
16. $|\langle -4, 1\rangle| = \sqrt{4^2 + 1^2} = \sqrt{17} \approx 4.1$



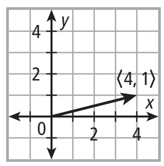
17. $|\langle 2, -3\rangle| = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.6$



18. direction $= \tan^{-1}\left(\frac{5}{3}\right) = 59°$



19. direction $= \tan^{-1}\left(\frac{1}{4}\right) = 14°$



20. boat's vector $= \langle 3.5\cos 50°, 3.5\sin 50°\rangle$
current's vector $= \langle 2, 0\rangle$
resultant vector $= \langle 3.5\cos 50° + 2, 3.5\sin 50°\rangle \approx \langle 4.25, 2.68\rangle$
speed $\approx \sqrt{4.25^2 + 2.68^2} \approx 5.0$ mi/h
direction $\approx \tan^{-1}\left(\frac{2.68}{4.25}\right) \approx 32°$, or N 58° E

COLLEGE ENTRANCE EXAM PRACTICE, PAGE 577

1. A

$$-x + 4y = 12$$
$$4y = x + 12$$
$$y = \frac{1}{4}x + 3$$

$\tan P = \text{slope} = \frac{1}{4}$

$m\angle P = \tan^{-1}\left(\frac{1}{4}\right) \approx 14°$

2. D

$\cos F = \frac{EF}{DF} = \frac{36}{39} = \frac{12}{13}$

3. C

$$\frac{\sin 61°}{8} = \frac{\sin 100°}{x}$$
$$x = \frac{8\sin 100°}{\sin 61°}$$
$$\approx 9 \text{ cm}$$

4. D

swimmer's vector $= \langle 0, 2\rangle$

current vector $= \langle 7, 0\rangle$

resultant vector $= \langle 7, 2\rangle$

speed $= \sqrt{7^2 + 2^2}$
$= \sqrt{53} \approx 7.3$ m/s

5. B

$$16^2 = 17^2 + 17^2 - 2(17)(17)\cos A$$
$$256 = 578 - 578\cos A$$
$$-322 = -578\cos A$$
$$m\angle A = \cos^{-1}\left(\frac{322}{578}\right) \approx 56.1°$$

3. Let P and Q be pts. on $\overline{AD}$ directly below B and C. Then
$AP = PB = 310$ ft, $PQ = BC = 60$ ft, and QD satisfies
$$\begin{aligned} QD^2 + QC^2 &= CD^2 \\ QD^2 + 310^2 &= 314.8^2 \\ QD^2 &= 2999.04 \\ QD &= \sqrt{2999.04} \approx 54.8 \text{ ft} \\ AD &= AP + PQ + QD \\ &\approx 310 + 60 + 54.8 \approx 425 \text{ ft} \end{aligned}$$

4. $$\begin{aligned} XY &= \frac{1}{2}(AD + BC) \\ &\approx \frac{1}{2}(424.8 + 60) \approx 242 \text{ ft} \end{aligned}$$

CHAPTER 8, PAGES 582–583

THE JOHN HANCOCK CENTER, PAGE 582

1. By Alt. Int. ∡ Thm., height and horiz. dist. x are opp. and adj. sides for 10° ∠.
$$\tan 10° = \frac{1000}{x}$$
$$x = \frac{1000}{\tan 10°} \approx 5671 \text{ ft}$$

2. $$\tan 61° = \frac{h}{818.2}$$
$$h = 818.2 \tan 61° \approx 1476 \text{ ft}$$

3. $$\tan 39° = \frac{(818.2 \tan 61°)}{x}$$
$$x = \frac{818.2 \tan 61°}{\tan 39°} \approx 1823 \text{ ft}$$

4. Shadow is longest when ∠ of elevation is smallest, on Dec 15.
$$\tan 25° = \frac{(818.2 \tan 61°)}{x}$$
$$x = \frac{818.2 \tan 61°}{\tan 25°} \approx 3165 \text{ ft}$$

ERNEST HEMINGWAY'S BIRTHPLACE , PAGE 583

1. perim. of dining room on plan ≈ 3.5 in.
$$\frac{3.5}{\text{actual length}} \approx \frac{1}{16}$$
actual length ≈ 16(3.5) ≈ 56 ft

2. area of parlor and living room on plan ≈ $1\frac{1}{8}$ in.^{2}
$$\frac{1.125}{\text{actual area}} \approx \left(\frac{1}{16}\right)^2 = \frac{1}{256}$$
actual area ≈ 256(1.125) = 288 ft^2

3. $\ell = w + 4$ and $2\ell + 2w = 40$
$$\begin{aligned} 2(w + 4) + 2w &= 40 \\ 4w + 8 &= 40 \\ 4w &= 32 \\ w &= 8 \\ \ell &= 8 + 4 = 12 \end{aligned}$$
plan dimensions are $\frac{12}{16} = \frac{3}{4}$ in. by $\frac{8}{16} = \frac{1}{2}$ in.

CHAPTER 10, PAGES 740–741

THE MELLON ARENA, PAGE 581

1. The area is a circle with a diameter of 400 ft.
area in square feet:
$A = \pi r^2 = \pi(200)^2 \approx 126{,}000$ ft^2
Area in acres:
$$A = 126{,}000 \text{ ft}^2 \cdot \frac{1 \text{ acre}}{43{,}560 \text{ ft}^2} \approx 3 \text{ acres}$$

2. $$\begin{aligned} \frac{\ell}{w} &= \frac{40}{17} \\ \ell &= \frac{40}{17}w \\ P &= 2\ell + 2w \\ 570 &= 2\left(\frac{40}{17}w\right) + 2w \\ 570(17) &= 80w + 34w \\ 9690 &= 114w \\ w &= 85 \text{ ft} \\ \ell &= \frac{40}{17}(85) = 200 \text{ ft} \end{aligned}$$
The dimensions are 200 ft by 85 ft.

3. $$\begin{aligned} \ell &= w + 130 \\ P &= 2\ell + 2w \\ 740 &= 2(w + 130) + 2w \\ 740 &= 2w + 260 + 2w \\ 480 &= 4w \\ w &= 120 \text{ ft} \\ \ell &= (120) + 130 = 250 \text{ ft} \end{aligned}$$
The dimensions are 250 ft by 120 ft.

4. **Step 1** Find the probability of sitting under the fixed sections.
area under fixed sections:
$A = \frac{2}{8}\pi(200)^2 = 10{,}000\pi$ ft^2
total area: $A = \pi(200)^2 = 40{,}000\pi$ ft^2
$$P = \frac{10{,}000\pi}{40{,}000\pi} = \frac{1}{4}$$
Step 1 Find the probability of sitting under the open sky.
area under open sky:
$A = 40{,}000\pi - 10{,}000\pi = 30{,}000\pi$ ft^2
$$P = \frac{30{,}000\pi}{40{,}000\pi} = \frac{3}{4}$$

THE U.S. MINT, PAGE 582

1. Assume the quarters are stamped out in a rectangular grid pattern, with each quarter occupying a 1-in. square. Then the number of quarters that can be stamped out of each strip equals the area of the strip in square inches.
$$\begin{aligned} \text{\# quarters} = A &= \ell w \\ &= (13 \text{ in.})\left(1500 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ft}}\right) \\ &\approx 234{,}000 \end{aligned}$$
$2(234{,}000) < 700{,}000 < 3(234{,}000)$
So, for 700,000 quarters, 3 strips are needed.

2. $V_{\text{penny}} = \pi r^2 h = \pi(9.525)^2(1.55) \approx 442$ mm^3
$$\% \text{ (copper)} = \frac{V_{\text{copper}}}{V_{\text{penny}}} \cdot 100\% \approx \frac{11}{442} \cdot 100\% \approx 2.5\%$$