Ch.7 REVIEW

1. The following probability model describes the number of credits taken by a randomly selected first-year student at a large state university.

Credits	14	15	16	17
Probability	0.05	0.65	0.20	???

(a) Define the random variable.

(b)Find P(X = 17). 0.1

(c) What is the expected number of credits taken by a first-year student?

15.35 credits

(d) What is the standard deviation of number of credits taken by a first-year student?

0.73 credits

$$Var(X) = \sum (x_i - \bar{x})^2 p_i$$

$$= (14-15.35)^2(0.05) + (15-15.35)^2(0.65) + (16-15.35)^2(0.2) + (17-15.35)^2(0.1)$$

$$= .5252$$

$$\sigma_X = \sqrt{Var(X)} = \sqrt{.5252} = .73$$

2. Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

Number who actually show up	36	37	38	39	40	41
Probability	0.46	0.30	0.16	0.05	0.02	0.01

Assume that 41 tickets are sold for each flight.

(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?

(b) What is the expected number of no-shows for this flight?

(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?

ANSWER:

Question 2

Solution

Part (a):

P (everyone gets a seat) = P ($X \leq 38$) $= .46 + .30 + .16 = .92$

OR $= 1 - (.05 + .02 + .01) = .92$

Part (b):

Y = number of no shows

y	0	1	2	3	4	5
p(y)	.01	.02	.05	.16	.30	.46

$E(Y) = 0(.01) + 1(.02) + 2(.05) + 3(.16) + 4(.30) + 5(.46) = 4.1$

OR

$E(X) = 36(.46) + 37(.30) + 38(.16) + 39(.05) + 40(.02) + 41(.01) = 36.9$

$E(Y) = 41 - E(X) = 4.1$

Part (c):

$$P(X=36|\text{not all seats are filled}) = P(X = 36 \mid X < 38) = \frac{P(X=36)}{P(X<38)} = \frac{.46}{.76} = .605$$

3. A lottery offers one \$1,000 prize, one \$500 prize, and five \$100 prizes. One thousand tickets are sold at \$3 each.

(a) Find the average amount won for each ticket purchased?

$E(X) = \mu_x$

x	\$1000	\$500	\$100
$P(x)$	1/1000	1/1000	5/1000

$$(1000)\left(\frac{1}{1000}\right) + (500)\left(\frac{1}{1000}\right) + (100)\left(\frac{5}{1000}\right)$$
$$= \$2$$

(b) To make the last lotto game fair, how much would a ticket have to cost?

\$2.00 b/c then you're winning the same amount you're spending in the long run.

4. A company is considering implementing one of two quality control plans for monitoring the weights of automobile batteries that it manufactures. If the manufacturing process is working properly, the battery weights are approximately normally distributed with a specified mean and standard deviation.

   Quality control plan A calls for rejecting a battery as defective if its weight falls more than 2 standard deviations below the specified mean.

   Quality control plan B calls for rejecting a battery as defective if its weight falls more than 1.5 interquartile ranges below the lower quartile of the specified population.

   Assume the manufacturing process is under control.

   a. What proportion of batteries will be rejected by plan A ?

   b. What is the probability that at least 1 of 2 randomly selected batteries will be rejected by plan A ?

   c. What proportion of batteries will be rejected by plan B ?

ANSWER:

Question 4

Solution

1. Plan A: defective if more than two standard deviations below the mean

For Plan A

$P(\text{defective}) = P(z < -2) = .0228$ (Tables) (.02275 Calculator)

So, plan A rejects .0228 or 2.28% as defective.

OR

Students use the Empirical Rule: about 95% of the area under the normal curve lies between z = -2 and z = 2, so 5% lies outside these limits, and about 2.5% lies below z = -2.

2. For Plan A:

P(at least 1 of 2 rejected)

$= 1 - P(\text{none rejected})$

$= 1 - 2 \text{ nCr } 0\ (0.0228)^0\ (0.9772)^2$

$= 1 - (.9772)^2$

$= 1 - .9549$

$= .0451$ (.04498 Calculator)

OR

P(at least 1 of 2 rejected)

$= P(\text{exactly 1 rejected}) + P(\text{exactly 2 rejected})$

$= 2 \text{ nCr } 1\ (0.0228)\ (0.9772) + 2 \text{ nCr } 2\ (0.0228)^2\ (0.9772)^0$

$= 2\ (.0228)\ (.9772) + (0.0228)^2$

$= .0446 + .0005$

$= .0451$

OR

P(at least 1 of 2 rejected)

$= P(\text{1st rejected}) + P(\text{2nd rejected}) - P(\text{both rejected})$

$= .0228 + .0228 - (.0228)^2$

$= .0451$

Note: If P(defective) = .025 then P(at least 1 of 2 rejected) = .04938

3. Plan B: defective if more than 1.5 IQR below the lower quartile

Quartiles are at .67 std. dev. (.67449 Calculator)

IQR = 1.34 std. dev. (1.34898 Calculator)

1.5 IQR = 2.01 std. dev. (2.02347 Calculator)

Boundary for defective region is:

$Q1 - 1.5\ IQR = -.67 - 2.01 = -2.68$

For Plan B

$P(\text{defective}) = P(z < -2.68) = .0037$ (.00349 Calculator)

So, Plan B rejects .0037 or .37% as defective. (Tables) (.349% Calculator)

5. Die A has four 9's and two 0's on its faces. Die B has four 3's and two 11's on its faces. When either of these dice is rolled, each face has an equal chance of landing on top. Two players are going to play a game. The first player selects a die and rolls it. The second player rolls the remaining die. The winner is the player whose die has the higher number on top.

   a. Suppose you are the first player and you want to win the game. Which die would you select? Justify your answer.

   b. Suppose the player using die A receives 45 tokens each time he or she wins the game. How many tokens must the player using die B receive each time he or she wins in order for this to be a fair game? Explain how you found your answer.

      (A fair game is one in which the player using die A and the player using die B both end up with the same number of tokens in the long run.)

ANSWER:

Question 5

Solution

Possible Outcomes

Die A	Die B	Winner	Prob
9	3	A	$(2/3)(2/3) = 4/9$
9	11	B	$(2/3)(1/3) = 2/9$
0	3	B	$(1/3)(2/3) = 2/9$
0	11	B	$(1/3)(1/3) = 1/9$

OR

		DIE A					
DIE B		0	0	9	9	9	9
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	3	B	B	A	A	A	A
	11	B	B	B	B	B	B
	11	B	B	B	B	B	B

Winner	Prob
A	$16/36 = 4/9$
B	$20/36 = 5/9$

a. Choose die B, because the probability of winning is higher (5/9 compared to 4/9 for die A)

b. Let X be the number of tokens the player using die B should receive. For the game to be fair, we need

   $45(4/9) = X(5/9)$

   Solving this equation for X gives X = 36. Player B should receive 36 tokens.

6. TURKEY PROBLEM

The mean number of calories in three thanksgiving foods is as follows:

2 slices of turkey = 250
1 cup of gravy = 300
1 cup of mashed potatoes = 200

If you eat 3 slices of turkey, 1/2 cup of gravy, and 2 cups of mashed potatoes, what is your mean number of calories intake?

3 slices of turkey = 375
+
½ cup of gravy = 150
+
2 cups of mashed p. = 400

925
calories

7. # HOCKEY PROBLEM

Tommy and Brandon are betting a pizza on who has the fastest slap shot. After checking the speed of 40 slap shots each, the data is normally distributed.

Tommy has a mean of 86 mph with s.d. 20 mph.
Brandon has a mean of 92 mph with s.d. 10 mph.
If they each shoot one more time, what is the probability that Tommy has the faster slap shot?

T = Tommy's slapshot speed (mph)

$\mu_T = 86$ mph

$\sigma_T = 20$ mph

B = Brandon's slapshot speed (mph)

$\mu_B = 92$ mph

$\sigma_B = 10$ mph

$P(T > B) \Rightarrow P(T - B > 0)$

Combined random variable T-B:

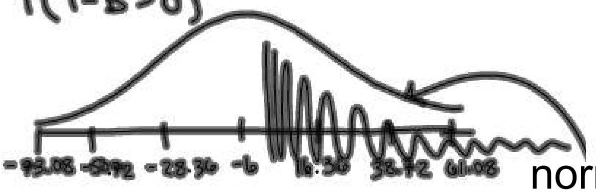
$\mu_{T-B} = \mu_T - \mu_B = 86 - 92 = -6$

$\sigma^2_{T-B} = \sigma^2_T + \sigma^2_B = (20)^2 + (10)^2 = 500$

$\sigma_{T-B} = \sqrt{500} = \underline{22.36}$

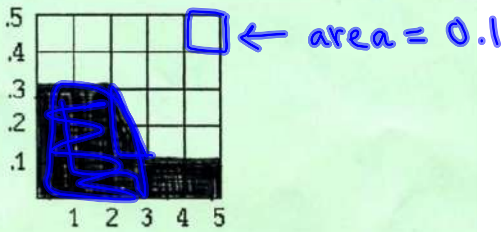
* Since T is ≈ normal & B is ≈ normal then T-B ≈ normal with $\mu_{T-B} = -6$, $\sigma_{T-B} = 22.3$

$P(T-B>0)$



normalcdf(0,1E99,-6,22.3) = 0.394

8. Here is the probability distribution function for a continuous random variable.



Determine the following probabilities:

(a) $P(0 \le X \le 3) = .8$

(b) $P(2 \le X \le 3) = .2$

(c) $P(X = 2) = 0$ (b/c this is continuous)

(d) $P(X < 2) = .6$

(e) $P(1 < X < 3) = .5$

9. Adults were surveyed on the topic of how many televisions are in your household. The table below shows the data that was collected from the 423 who participated. What is the expected number of televisions in a randomly selected household?

# of TV's	# of Adults
0	6
1	98
2	125
3	135
4	35
5	22
6	2

2.4 TV's

10. The probability model for a lottery game is shown below. If the expected value of playing this game is \$20, then what is x?

Outcome	\$0	\$5	\$15	\$x
Probability	0.70	0.12	0.10	?

\$223.75