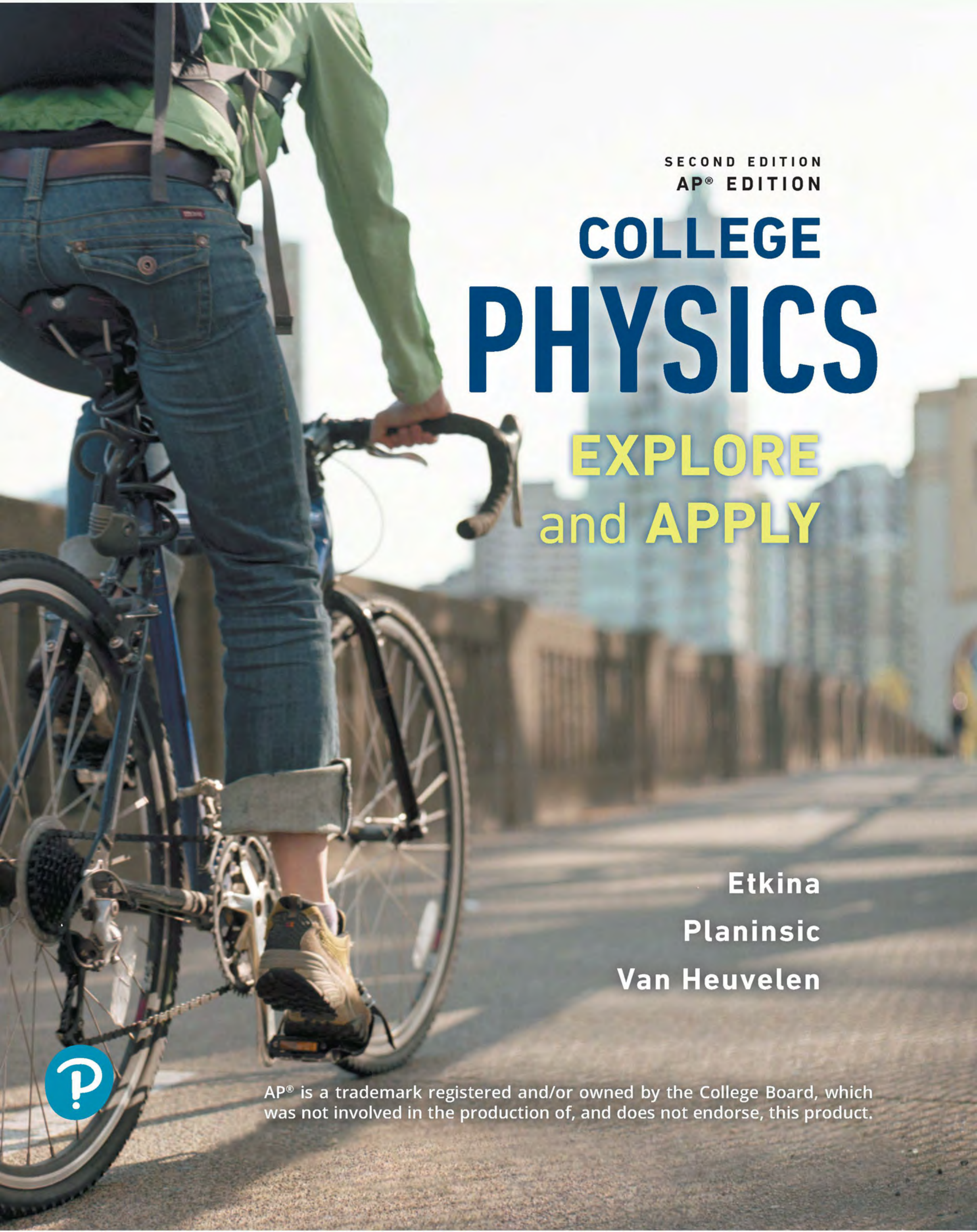

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EXPLORE and APPLY

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7

Work and Energy

- **How do pole vaulters reach a height of 5 or 6 m?**
- **Why does blood pressure increase when the walls of the aorta thicken?**
- **If our Sun were to become a black hole, how big would it be?**

Yelena Isinbayeva from Russia has held the women's pole-vaulting record of 5.06 m since 2009. The men's record is 6.16 m. How can pole vaulters reach such great heights? If you watch the event, you will see three aspects of the vault that are essential: the vaulters run very fast before inserting the pole into the vaulting box; they jump at the moment of takeoff while bending the pole as they start their vertical ascent; and they crunch their muscles at the top of the flight to give a specific body shape as they pass over the bar. All these elements contribute to the height of the vault—in this chapter, we will learn how.

BE SURE YOU KNOW HOW TO:

- Choose a system and the initial and final states of a physical process (Sections 6.2–6.4).
- Use Newton's second law and your knowledge of momentum to analyze a physical process (Sections 4.4 and 6.4).
- Use kinematics to describe motion (Section 2.9).

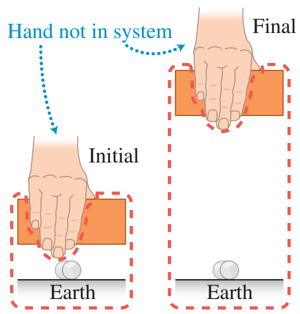
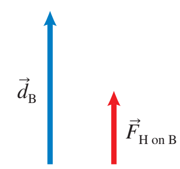
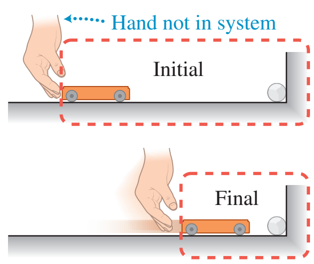
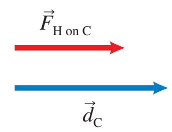
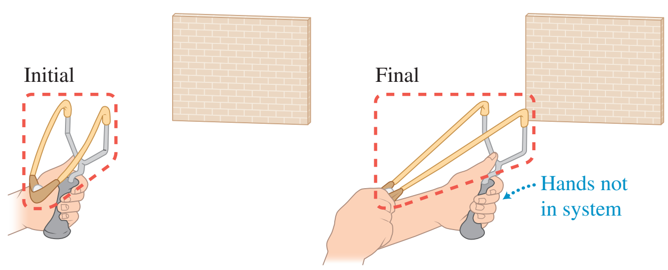
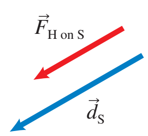
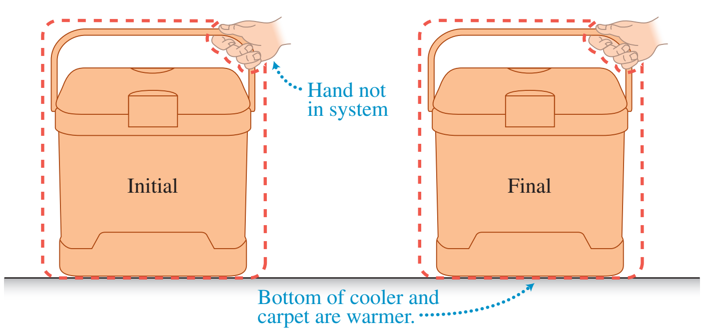
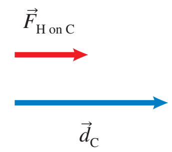
SO FAR, WE HAVE learned how to use forces and momentum to explain and analyze different types of motion. However, not all questions concerning motion can be answered using these concepts. For example, imagine a roller coaster in an amusement park. It has no engine or battery, but it goes through multiple loops and many twists and turns. The force that the track exerts on it continuously changes direction and magnitude. How is it possible to design the track so the coaster will make it through the entire ride without falling or grinding to a halt? It turns out that there are several important physical quantities, and a new principle, that are important in designing a roller coaster. We will learn about these quantities and this principle in this chapter.

7.1 Work and energy

We will begin our new approach by conducting several experiments and looking for patterns to help explain what we observe. Specifically, in Observational Experiment Table 7.1 we investigate the changes that happen to a system when an external force is exerted on it over a certain distance. In the analysis we draw vectors indicating the external force causing the displacement and the resulting displacement of the system.

OBSERVATIONAL EXPERIMENT TABLE 7.1 External forces and system changes

VIDEO OET 7.1

Observational experiment	Analysis
Experiment 1. You hold a heavy block just above a piece of chalk (the initial state) and then release the block. The chalk does not break. Now you lift the block about 30 cm above the chalk (the final state). When you release the block from the higher elevation, the block falls and smashes the chalk.  	The force you exerted and the block's displacement while being lifted were in the same direction and caused an increase in the block's elevation and in its ability to break the chalk. 
Experiment 2. You push a cart initially at rest (the initial state) until it is moving fast about two-thirds of the way across a smooth track (the final state is where you stop pushing the cart). A piece of chalk is taped to the end of the track. The fast-moving cart (no longer being pushed) collides with the piece of chalk and breaks the chalk. When you repeat the experiment with a slow-moving cart, it does not break the chalk.  	The force exerted on the cart and the cart's displacement were in the same direction and increased the cart's speed so it could break the chalk. 
Experiment 3. A piece of chalk rests in the hanging sling of a slingshot (the initial state). You then pull it back until the slingshot is fully stretched (the final state). When released from the stretched sling, the chalk flies across the room, hits the wall, and smashes.  	The force that you exerted on the sling and its displacement were in the same direction and made it possible for the stretched sling to cause the chalk to break. 
Experiment 4. A heavy cooler sits on a shag carpet (the initial state). You pull the cooler across the carpet to a position several meters from where it started (the final state).  	You exerted a force on the cooler in the direction of its displacement. After several meters of travel across the carpet, the bottom became warmer. 

(CONTINUED)

Patterns

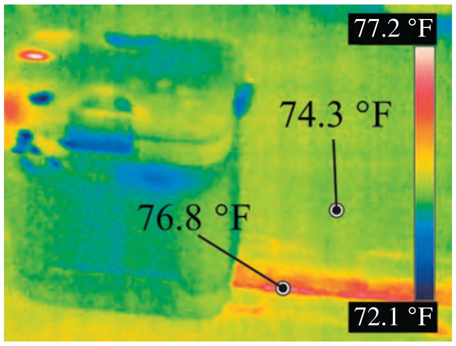
In each of these experiments, you exerted an external force $\vec{F}$ on an object in a system. The force $\vec{F}_{\text{You on Object}}$ and the object's displacement $\vec{d}_{\text{Object}}$ were in the *same direction* and caused the system to change so that it could do something that it could not do before, such as break the chalk or warm up the carpet. Specifically:

1. The *block at higher elevation* above Earth could break the chalk, but the block at low elevation could not.
2. The *fast-moving cart* could break the chalk, but the slow-moving cart could not.
3. The *stretched slingshot* could break the chalk, but the unstretched slingshot could not.
4. The *cooler and the carpet it was pulled across* became warmer than they were before the cooler was pulled across the carpet.

In the experiments in Table 7.1, the system could not break the chalk or warm the carpet in the initial state, but in the final state it could. What happened to each system to cause that change? In all cases, **an external force exerted on the system caused a displacement of an object in the system in the *same direction* as the force.** Using everyday words in a very specific way, physicists say that such a force does positive **work** on a system, and that this work done causes a change in what is called the system's **energy** (we will learn later that there can be negative work that also changes the system's energy). Energy is rather difficult to define at this point, but you can think of it loosely as a property of a system that changes when work is done on the system. Four types of energy changed in the systems in Table 7.1:

- Experiment 1: The energy of the object-Earth system associated with the elevation of the object above Earth is called **gravitational potential energy** U_g. The higher the object is above Earth, the greater the gravitational potential energy of that object-Earth system.
- Experiment 2: The energy due to an object's motion is called **kinetic energy** K. The faster the object is moving, the greater its kinetic energy.
- Experiment 3: The energy associated with an elastic object's degree of stretch is called **elastic potential energy** U_s. The greater the stretch (or compression), the greater the object's elastic potential energy.
- Experiment 4: The energy associated with both temperature and structure is called **internal energy** U_{int}. You will learn later that internal energy is the energy of motion and interaction of the microscopic particles making up the objects in the system. A component of internal energy is **thermal energy**, energy associated with the temperature of the objects in the system (see the photo). In this chapter, when we apply the concept of internal energy to people, we will also talk about muscles' **chemical energy**—this is another component of internal energy related to the interaction of particles in chemical compounds.

A photo of the carpet and cooler in Experiment 4 from Table 7.1 taken with a thermal camera. The temperature of the carpet and the cooler increases by about 2.5 °F. This shows the change in the thermal energy of both objects.



Negative and zero work

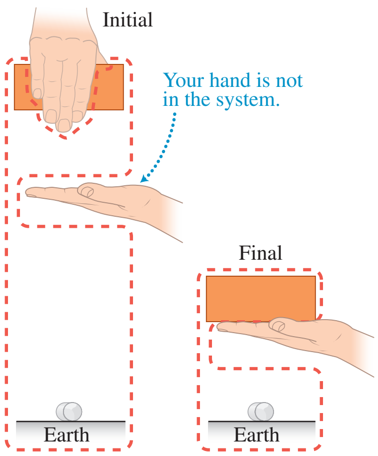
Is it possible to devise a process in which an external force causes the energy of a system to decrease or possibly causes no energy change at all? Let's try more experiments to investigate these questions. See Observational Experiment **Table 7.2** on the next page.

OBSERVATIONAL EXPERIMENT TABLE 7.2 Negative and zero work

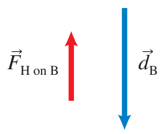
VIDEO OET 7.2

Observational experiment	Analysis

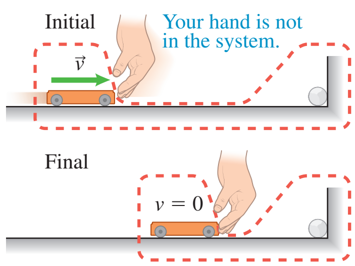
Experiment 1. A friend holds a block high above a piece of chalk (the initial state) and then releases it. Your hand catches and stops the block (the final state). The block's potential to break the chalk is greater in the initial state than in the final state.



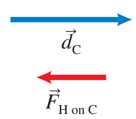
The direction of the force exerted by your hand on the block is opposite the block's displacement and reduced the system's potential to break the chalk.



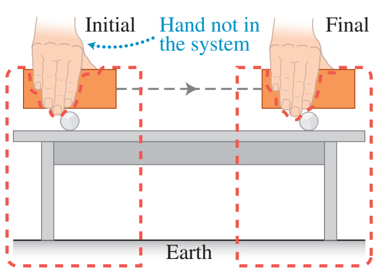
Experiment 2. A cart is moving fast (the initial state) toward a piece of chalk taped on the wall. While it is moving, you push lightly on the moving cart opposite the direction of its motion, causing it to slow down and stop (final state). The cart's potential to break the chalk is greater in the initial state than in the final state.



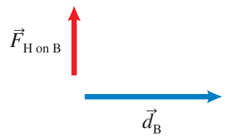
The direction of the force exerted by your hand on the cart is opposite the cart's displacement and caused the moving cart to slow down and stop, thus reducing its potential to break the chalk.



Experiment 3. Your hand is slowly moving a block to the right, keeping the block just above the tabletop. The block passes above the piece of chalk on the left (initial state) and then over the piece of chalk on the right (final state). The block is too close to the chalk in both states to break it.



The direction of the force exerted by your hand on the block is perpendicular to the block's displacement and caused no change in the block's potential to break the chalk.



Patterns

- In Experiments 1 and 2, the direction of the external force exerted on the object in the system is opposite the object's displacement, and the system's ability to break the chalk decreases.
- In Experiment 3, the direction of the external force exerted on the object in the system is perpendicular to the object's displacement, and the system's ability to break the chalk is unchanged.

We can now summarize what we have learned about work:

- When the external force is in the direction of the object's displacement, the external force does *positive work,* causing the system to gain energy.
- When the external force points opposite the object's displacement, the external force does *negative work,* causing the system's energy to decrease.
- When the external force points perpendicular to the object's displacement, the external force does *zero work* on the system, causing no change to its energy.

Defining work as a physical quantity

We now want to find a mathematical relation to define the physical quantity of work. The definition needs to be consistent with all three of the features of work listed in the previous section. Intuitively, we know that a larger force should lead to larger work, as should a larger displacement. But how do we account for the angle dependence? It turns out that there is a mathematical function that will take care of this issue—the cosine of the angle between the direction of the force vector and displacement vector (this angle is θ in **Figure 7.1**).

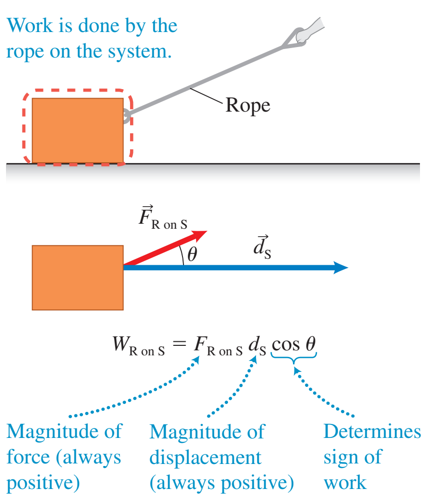


FIGURE 7.1 The definition of work.

Note that in the four experiments in Table 7.1, the force and displacement were in the same direction: $\theta = 0°$, and $\cos 0° = +1.0$. Positive work was done. In the first two experiments in Table 7.2, the force and displacement were in opposite directions: $\theta = 180°$, and $\cos 180° = -1.0$. Negative work was done. Finally, in Experiment 3 in Table 7.2, the force and displacement were perpendicular to each other: $\theta = 90°$, and $\cos 90° = 0$. Zero work was done.

Work The work done by a constant external force $\vec{F}$ exerted on an object in a system while that object undergoes a displacement $\vec{d}$ is

$$W = Fd\cos\theta \qquad (7.1)$$

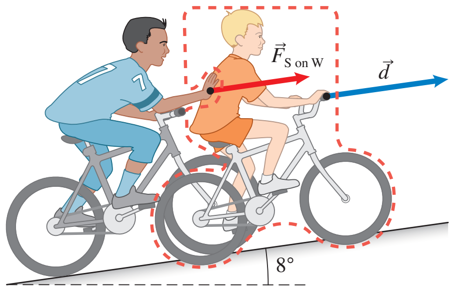
where F is the magnitude of the force in newtons (always positive), d is the magnitude of the displacement in meters (always positive), and θ is the angle between the direction of $\vec{F}$ and the direction of $\vec{d}$. The sign of $\cos\theta$ determines the sign of the work. Work is a scalar physical quantity. The unit of work is the joule (J); $1\text{ J} = 1\text{ N}\cdot\text{m}$.

The joule is named in honor of James Joule (1818–1889), one of many physicists who contributed to our understanding of work-energy relationships.

According to the definition of work, if the displacement of an object is zero, even if forces are exerted on it, no work is done. This idea might contradict your everyday experience. Imagine holding a heavy object (for example, a suitcase) in a steady position. The system is the suitcase and Earth; you are an external object. If the suitcase remains at rest, the work done by the force you exert on it is zero. But you still get tired. How is this possible? It turns out that even when you are holding something still, your muscles still contract and relax all the time, making your arm go up and down slightly (we cannot see this movement with the naked eye, but scientists use a special method called *sound myography* to detect such muscle contractions). Thus you are doing work on the suitcase by continuously lifting it a little and then dropping it, repeating the process many times. Let us now include you in the system. Careful observations show that while holding the suitcase, you breathe a little faster and small drops of perspiration appear on your face. These are the signs of increased metabolic activity—your chemical energy is converted into the kinetic and elastic potential energy of your muscles and also into thermal energy. Thus there is no contradiction between physics and everyday life.

QUANTITATIVE EXERCISE 7.1 **Pushing a bicycle uphill**

Two friends are cycling up a hill inclined at 8°—steep for bicycle riding. The stronger cyclist helps his friend up the hill by exerting a 50-N pushing force on his friend's bicycle and parallel to the hill while the friend moves a distance of 100 m up the hill. Determine the work done by the stronger cyclist on the weaker cyclist.



Represent mathematically Choose the system to be the weaker cyclist. The external force that the stronger cyclist (S) exerts on the weaker cyclist (W) $\vec{F}_{\text{S on W}}$ is parallel to the hill, as is the 100-m displacement of the weaker cyclist. The work done by the stronger cyclist on the weaker cyclist is $W = Fd\cos\theta$. The hill is inclined at 8° above the horizontal. Before reading on, decide what angle you would insert in this equation.

Solve and evaluate Note that the angle between $\vec{F}_{\text{S on W}}$ and $\vec{d}$ is 0° and not 8° because the force exerted on the cyclist is parallel to the person's displacement. Thus:

$$W_{\text{S on W}} = F_{\text{S on W}}d\cos\theta = (50\text{ N})(100\text{ m})\cos 0°$$
$$= +5000\text{ N}\cdot\text{m} = +5000\text{ J}$$

Try it yourself You pull a box 20 m up a 10° ramp. The rope is oriented 20° above the surface of the ramp. The force that the rope exerts on the box is 100 N. What is the work done by the rope on the box?

Answer

$W_{\text{R on B}} = (100\text{ N})(20\text{ m})\cos 20° = 1900\text{ J}$. Note that you use the angle between the displacement (parallel to the ramp) and the force the rope exerts on the box (20° above the ramp). You can disregard the angle of the ramp itself.

REVIEW QUESTION 7.1 Assuming that Earth's orbit around the Sun is circular, what information do you need to estimate the work done by the Sun on Earth while Earth travels 1000 km? Explain your answer.

7.2 Energy is a conserved quantity

We have found that the work done on an object in a system by an external force results in a change of one or more types of energy in the system: kinetic energy, gravitational potential energy, elastic potential energy, and internal energy. We can think of the *total energy E* of a system as the sum of all these energies in the system:

$$\text{Total energy} = E = K + U_g + U_s + U_{\text{int}} \quad (7.2)$$

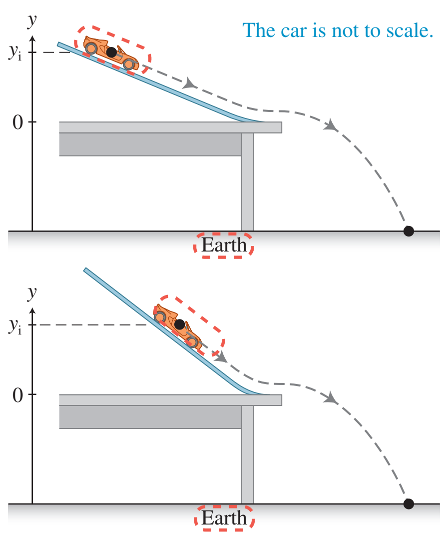
The energy of a system can be converted from one form to another. For example, the elastic potential energy of a stretched slingshot is converted into the kinetic energy of the chalk when the sling is released. Similarly, the gravitational potential energy of a separated block-Earth system is converted into kinetic energy of the block when the block falls. Here we assume that Earth does not move when the block moves toward it. What happens to the amount of energy when it is converted from one form to another? So far we have one mechanism through which the energy of the system changes—that mechanism is work. Thus it is reasonable to assume that if no work is done on the system, the energy of the system should not change; it should be *constant*. Let's test this hypothesis experimentally (see Testing Experiment Table 7.3, on the next page).

TIP Remember that an *isolated* system is a system for which we can neglect interactions with the environment. Thus, when external objects do zero work on a system, the system can be considered isolated.

TESTING EXPERIMENT TABLE 7.3 Is the energy of an isolated system constant?

VIDEO TET 7.3

Testing experiment	Prediction	Outcome
You have a toy car and a frictionless track. The bottom of the track sits near the edge of a table. You can tilt the track at different angles to vary its steepness. When the car reaches the end of the track, it flies horizontally off the table. Where should you release a car on the track so the car always lands the same distance from the table regardless of the slope of the track?	Consider car + Earth as the system. The initial state is just before we release the car and the final state is just as the car leaves the horizontal track. For the car to land on the floor at the same distance from the table's edge, the car needs to have the same horizontal velocity and hence the same kinetic energy when leaving the track. If energy is constant in the isolated system, to get the same kinetic energy at the bottom of the track, the car-Earth system must have the same initial gravitational energy when it starts. Since the gravitational potential energy depends on the separation between the car and Earth, the car should start at the same vertical elevation independent of the tilt angle of the track, $y_i = h$. Our prediction is based on the assumption that the gravitational potential energy of the system is only converted to the kinetic energy of the car and not to internal energy.	When released from the same vertical height with respect to the table, the car lands the same distance from the table.



Conclusion

The outcome of the experiment matches the prediction and thus supports our hypothesis that the energy of an isolated system is constant.

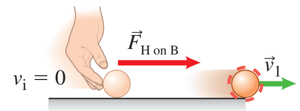
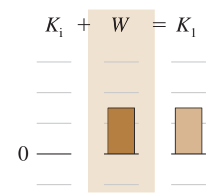
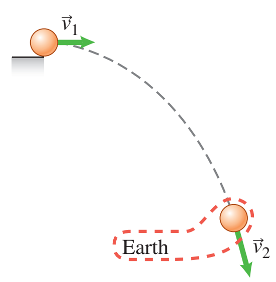
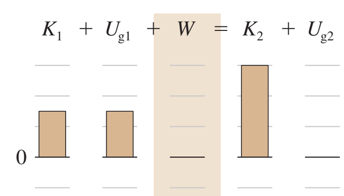
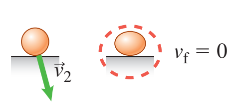
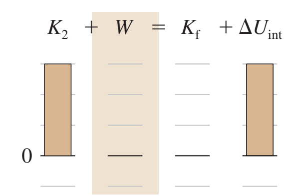
TIP It is important to note that in the above arguments we assumed that Earth is infinitely large and does not move when an object falls down or flies up.

So far we have experimental support for a qualitative hypothesis that the energy of an isolated system is constant and different processes inside the system convert energy from one form to another. We also reason that work is a mechanism through which the energy of a nonisolated system changes. This sounds a lot like our discussion of linear momentum (Chapter 6). In that chapter, we described a *conserved quantity* as one that is constant in an isolated system. We also said that when the system is not isolated and the conserved quantity changes, we can always redefine the system in such a way as to keep the quantity constant. Imagine that we did not include Earth in the system in the experiment above. We would find that the energy of the cart (our new system) changed during the experiment because it acquired kinetic energy moving down the track due to the work done by Earth on it. Thus the energy of the cart as a system was not constant. However, when we include Earth in the system, the energy remains constant.

Work-energy bar charts

We can represent work-energy processes with bar charts that are similar to the impulse-momentum bar charts we used to describe momentum. A work-energy bar chart indicates with vertical bars the relative amount of a system's different types of energy in the initial state of a process, the work done on the system by external forces during the process, and the relative amount of different types of energy in the system at the end of the process. The area for the work bar is shaded to emphasize that work is not a property of the system. The physical quantity of work characterizes the process through which energy is transferred in or out of the system due to the external forces that cause displacements of the objects inside the system. In terms of system change, work is similar to impulse in our study of momentum. In **Table 7.4** you see how bar charts can be used to represent the process when you push a clay ball off a frictionless table exerting a constant force (situation 1), the ball flies off the table (situation 2), and lands on the floor and stops (situation 3). We break the process into three parts and explain how we choose the system in each situation.

TABLE 7.4 Examples of system energy conversions

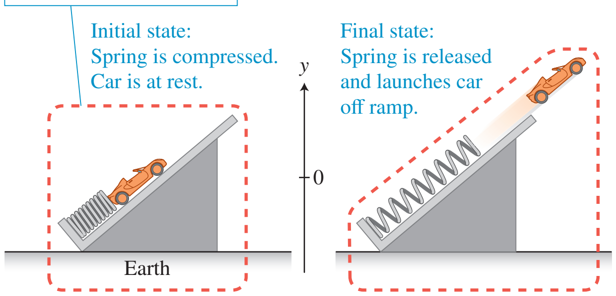
Process	Sketch of initial and final states and explanations of the system choice	Bar chart for the process
1. You push a clay ball along a frictionless table, exerting a constant force.	Initial state: The ball is at rest. Final state: The ball is moving and is just about to leave the table. System: The ball. We choose the ball as the system because there is no elevation change, thus we do not need to worry about gravitational potential energy. 	Your hand does positive work on the system. The ball gains kinetic energy. 
2. The clay ball falls down freely off the edge of the table.	Initial state: The ball is at the top of its flight. Final state: Just before the ball hits the floor (it is still moving). System: The ball and Earth. We included Earth in the system because there is elevation change and thus a change in gravitational potential energy. If we did not include Earth in the system, it would do work but there would be no gravitational potential energy in any state. 	No work is done on the system. The system's initial gravitational potential energy is converted into kinetic energy. We assume that the gravitational potential energy of the system at floor level is zero, so there is no bar for U_{gf}. 
3. The clay ball lands on the floor and stops.	Initial state: Just before the moving ball hits the floor. Final state: Just after the ball stops (and its shape changes). System: The ball and the floor. The floor stopped the ball during landing, but it did not move; thus it would be difficult to account for the effects of the floor via work. Including it in the system removes the need to account for work done by the floor. Earth is not included in the system because the elevation did not change in this situation. 	No work is done on the system. The ball's kinetic energy is converted into internal energy of the ball and the floor. 

From the examples in the table you can conclude that when choosing what objects to include in the system and what not to include, we can use the following approach: **If it is difficult to calculate the work done by that object on the system, we are usually better off including it in the system so that we describe the processes using different forms of energy.** As you proceed through the chapter you will see more examples and discussion of how to choose what to include and not to include in the system.

PHYSICS TOOL BOX 7.1 **Constructing a qualitative work-energy bar chart**

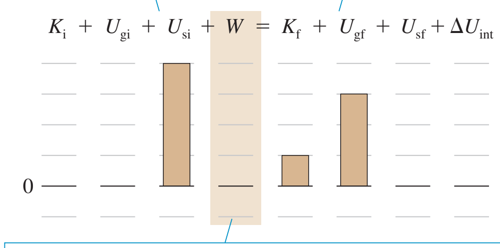
1. Choose and describe the initial and final states.

2. Choose a system.



3. Represent the energies in the initial state on the left side of the bar chart.

4. Represent the energies in the final state on the right side of the bar chart.



5. Determine the work done by external forces. (No external work is done in this process.)

TIP In Physics Tool Box 7.1, the system is isolated. No work is done on it. If we do not include Earth in the system, then Earth will do negative work on the cart. However, such a system will not have gravitational potential energy.

The generalized work-energy principle

We found experimentally that the energy of an isolated system is constant. We also defined work as a mechanism for energy transfer. Therefore, we can summarize what we have learned about work and energy with the **generalized work-energy principle**.

Generalized work-energy principle The sum of the initial energies of a system plus the work done on the system by external forces equals the sum of the final energies of the system:

$$E_i + W = E_f \qquad (7.3)$$

or

$$(K_i + U_{gi} + U_{si}) + W = (K_f + U_{gf} + U_{sf} + \Delta U_{int}) \qquad (7.3)$$

Note that we have moved $U_{int\,i}$ to the right hand side $(\Delta U_{int} = U_{int\,f} - U_{int\,i})$ since values of internal energy are rarely known, while internal energy changes are. We will also call Eq. (7.3) the *work-energy equation*.

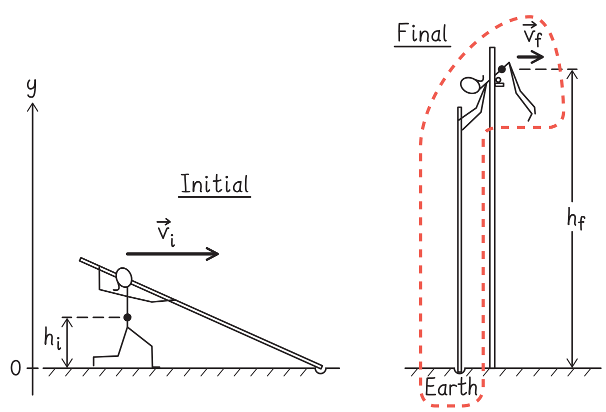
The generalized work-energy principle allows us to define the total energy of a system as the sum of the different types of energy in the system. Total energy is measured in the same units as work and changes when work is done on the system. In Eq. (7.3), kinetic energy and gravitational and elastic potential energy are forms of **mechanical energy**. Mechanical energy is associated with motion and interactions of macroscopic objects—objects that we can see with our eyes. Internal energy is associated with motion and interactions of microscopic objects—objects that we cannot see with our eyes. Unlike such physical quantities as position or force, energy is not directly observed, and there are no instruments that measure energy. We can only infer the amount of any type of energy if we can express it through measurable quantities.

The work in Eq. (7.3) can be positive or negative. Unlike velocity and acceleration, for which plus and minus signs signify direction, in the case of work the sign signifies another operation—addition or subtraction.

CONCEPTUAL EXERCISE 7.2 Pole vaulting, qualitatively

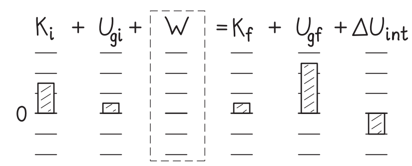
Qualitatively analyze a pole vault from an energy perspective by first drawing a sketch and then a work-energy bar chart.

Sketch and translate To analyze the process, we choose the final state to be when a pole vaulter is just clearing the bar. What should be the initial state? As we discussed at the beginning of the chapter, pole vaulters run very fast while holding an unbent pole, then insert the pole into the vaulting box. Next, using the bent pole, they lift themselves above the bar. When they have just cleared the bar, they are still holding the pole, but it is no longer bent. Thus it makes sense to choose the moment when the pole vaulter is running, just before she inserts the pole into the box, as the initial state. In both of these states the pole is not deformed, so we do not need to worry about changes in the pole's elastic energy or the pole doing work on the system. Therefore, it does not matter whether it is in the system or not. Let's choose the pole vaulter, Earth, and the pole as the system. The beauty of the energy approach is that we are only concerned with what is happening in the initial and final states, not with the processes in between. The sketch shows the pole-vaulting process with the initial and final states marked.

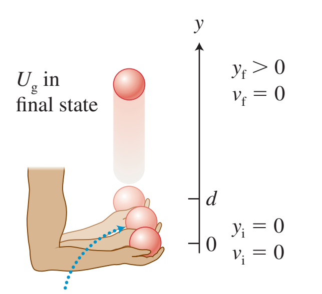


Simplify and diagram Assume that the pole vaulter can be modeled as a point-like object with her mass concentrated approximately in the middle of her body. Between the initial and final states there are no external forces exerted on the system; thus there is no work done. Using our understanding of energy, we can say that the gravitational potential energy U_{gi} of the system and the kinetic energy K_i of the vaulter in the initial state are converted to the final gravitational potential energy U_{gf} and a little bit of kinetic energy K_f (the vaulter needs to be moving horizontally a little in order not to fall straight down when no longer in contact with the pole) of the system in the final state. To draw the bars for gravitational potential energy, we assume that it is zero at ground level.

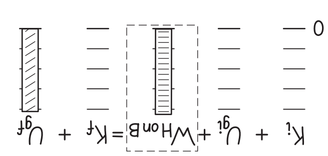
However, if we carefully watch videos of pole vaulters on the Internet, we see that right after inserting the pole into the box, they jump while bending the pole as they start their vertical ascent. They also crunch their muscles at the top of their flight to produce a specific body shape as they pass over the bar. During both of these actions, the vaulters convert some of their internal energy into the mechanical energy of the system. This means that the internal energy of the system is less in the final state than in the initial state. To represent this, we draw the bar for the internal energy *change* ΔU_{int} below the axis to indicate that the internal energy of the pole vaulter between the moment she inserts the pole in the vaulting box and when she clears the bar decreases. The pole vaulter does not cool down during the process, so where does this internal energy come from? It comes from the chemical energy stored in her body. From the bar chart, we reason that the greater the initial kinetic energy and the more internal energy the vaulter can "pump" into the vault, the greater the final gravitational potential energy, or the height of the vault. Note that we did not take into account the initial kinetic energy of the pole or the internal energy change due to its deformation (it could have gotten a little warmer).



Try it yourself You throw a ball straight upward as shown at right. The system is the ball and Earth (but not your hand). Ignore interactions with the air. Draw a work-energy bar chart starting when the ball is at rest in your hand and ending when the ball is at the very top of its flight.



Answer



See the diagram. Note that there is no kinetic energy represented in the bar chart because the ball was not moving in either the initial state or the final state. It does not matter that it was moving in the time interval between those two states.

In Table 7.4 we chose different systems to analyze different parts of a process. In general, how do we know what to include and what not to include in a system to make the analysis of a process easier? Generally, it is preferable to have a larger system so that the changes occurring can be included as energy changes within the system rather than as the work done by external forces. However, often it is best to exclude something like a motor from a system because its energy changes are complex. For instance, if you are

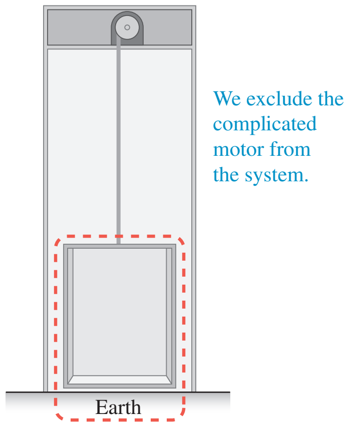


FIGURE 7.2 Energy changes in the motor pulling up on the elevator cable are very complicated.

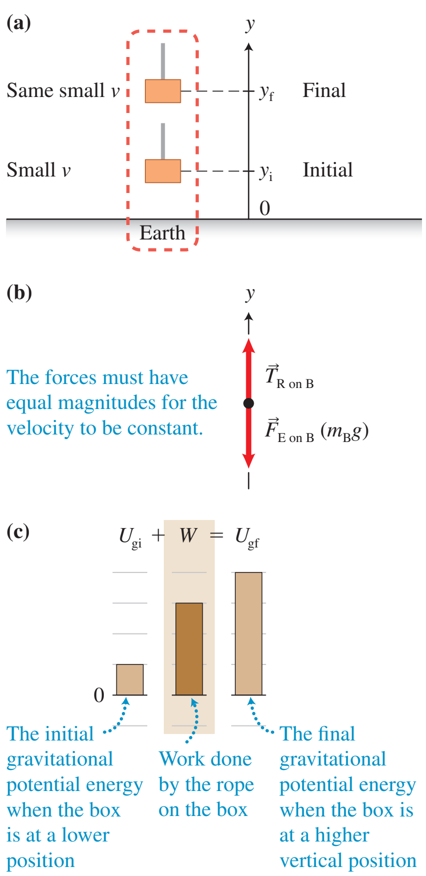


FIGURE 7.3 Lifting a box at a negligible constant speed.

studying a moving elevator (**Figure 7.2**), you might choose to exclude from the system the motor that turns the cable while lifting the elevator. The motor's cable does positive work on the system by exerting a tension force on the elevator when the elevator is moving up and negative work when the elevator is moving down. You can take into account the effect of the motor by calculating the work done by the cable on the elevator.

REVIEW QUESTION 7.2 A system can possess energy but it cannot possess work. Why?

7.3 Quantifying gravitational potential and kinetic energies

In this section we will use what we know about work, forces, and kinematics to devise mathematical expressions for gravitational potential energy and kinetic energy.

Gravitational potential energy

Imagine that a rope lifts a heavy box upward at a constant negligible velocity (**Figure 7.3a**). The rope is attached to a motor above, which is not shown in the figure. First, we choose only the box as the system and apply Newton's second law to find the magnitude of the force that the rope exerts on the box. Since the box moves up at constant velocity, the upward tension force $\vec{T}_{\text{R on B}}$ exerted by the rope on the box is equal in magnitude to the downward gravitational force $\vec{F}_{\text{E on B}}$ exerted by Earth on the box (see the force diagram in Figure 7.3b), $m_B g$, we find that the magnitude of the tension force for this process is $T_{\text{R on B}} = m_B g$.

To derive an expression for gravitational potential energy, we must change the boundaries of the system to include the box and Earth (if Earth is not included in the system, the system does not have gravitational potential energy). The origin of a vertical y-axis is the ground directly below the box with the positive direction upward. The initial state of the system is the box at position y_i moving upward at a negligible speed $v_i \approx 0$. The final state is the box at position y_f moving upward at the same negligible speed $v_f \approx 0$. According to work-energy Eq. (7.3),

$$E_i + W = E_f$$

The rope does work on the box, lifting the box from vertical position y_i to y_f:

$$W_{\text{R on B}} = T_{\text{R on B}} d \cos\theta = T_{\text{R on B}}(y_f - y_i)\cos 0° = m_B g(y_f - y_i)$$

where we substituted $T_{\text{R on B}} = m_B g$ and $\cos 0° = 1$. The kinetic energy did not change. Substituting this expression for work into the work-energy equation, we get

$$U_{gi} + m_B g(y_f - y_i) = U_{gf}$$

Figure 7.3c represents this information with an energy bar chart. We now have an expression for the change in the gravitational potential energy of the system: $U_{gf} - U_{gi} = m_B g y_f - m_B g y_i$. This suggests the following definition for the gravitational potential energy of a system.

Gravitational potential energy The gravitational potential energy of an object-Earth system is

$$U_g = mgy \qquad (7.4)$$

where m is the mass of the object, $g = 9.8\ \text{N/kg}$, and y is the position of the object with respect to the **zero level of gravitational potential energy** (the origin of the coordinate system is our choice). The units of gravitational potential energy are $\text{kg(N/kg)m} = \text{N} \cdot \text{m} = \text{J}$ (joule), the same unit used to measure work and the same unit for every type of energy.

TIP Three things are important here:

1. The value of U_g depends on our choice of zero for the y-coordinate. According to Eq. (7.4), this is where U_g is zero. However, the *change* in U_g does not depend on the choice of the zero level.
2. The derivation of Eq. (7.4) assumed that g is constant. We will look at nonconstant g later in the chapter.
3. If Earth is not included in the system, the system does not have any U_g, but Earth can do work on the system.

Kinetic energy

Next, we analyze a simple thought experiment to determine an expression for the kinetic energy of a system that consists of a single object. Imagine that your hand exerts a force $\vec{F}_{\text{H on C}}$ on a cart of mass m while pushing it toward the right a displacement $\vec{d}$ on a horizontal frictionless surface (**Figure 7.4a**). A bar chart for this process is shown in Figure 7.4b. There is no change in gravitational potential energy. The kinetic energy changes from the initial state to the final state because of the work done by the external force exerted by your hand on the cart:

$$K_i + W_{\text{H on C}} = K_f$$

or

$$W_{\text{H on C}} = K_f - K_i$$

We know that the work in this case equals $F_{\text{H on C}}d\cos 0° = F_{\text{H on C}}d$. Thus,

$$F_{\text{H on C}}d = K_f - K_i$$

This does not look like a promising result—the kinetic energy change on the right equals quantities on the left side that do not depend on the mass or speed of the cart. However, we can use dynamics and kinematics to get a result that does depend on these properties of the cart. The horizontal component form of Newton's second law is

$$m_C a_C = F_{\text{H on C}}$$

We can rearrange a kinematics equation $(v_f^2 = v_i^2 + 2ad)$ to get an expression for the displacement of the cart in terms of its initial and final speeds and its acceleration:

$$d = \frac{v_f^2 - v_i^2}{2a_C}$$

Now, we insert these expressions for force and displacement into the left side of the equation $F_{\text{H on C}}d = K_f - K_i$:

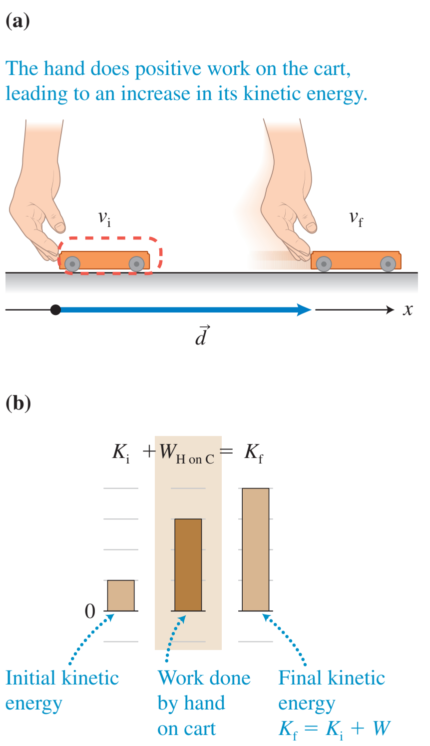
$$F_{\text{H on C}}d = (m_C a_C)\left(\frac{v_f^2 - v_i^2}{2a_C}\right) = m_C\left(\frac{v_f^2}{2} - \frac{v_i^2}{2}\right) = \tfrac{1}{2}m_C v_f^2 - \tfrac{1}{2}m_C v_i^2$$

We can now insert this result into the equation $F_{\text{H on C}}d = K_f - K_i$:

$$\tfrac{1}{2}m_C v_f^2 - \tfrac{1}{2}m_C v_i^2 = K_f - K_i$$

It appears that $\frac{1}{2}m_C v^2$ is an expression for the kinetic energy of the cart.

FIGURE 7.4 The work done by the hand causes the cart's kinetic energy to increase.



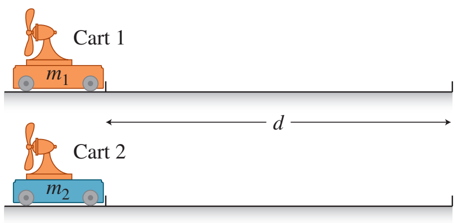
Kinetic energy The kinetic energy of an object is

$$K = \tfrac{1}{2}mv^2 \qquad (7.5)$$

where m is the object's mass and v is its speed relative to the chosen coordinate system. Notice that kinetic energy is always positive.

TIP Remember that speed is relative—it depends on the observer. A person on a Boeing 747 airplane would consider the kinetic energy of a 70-kg passenger next to her as zero, while a person on the ground would find the kinetic energy of the same person to be about 2.5 million joules.

FIGURE 7.5 Two fan carts of different masses with identical fans.



To check whether the unit of kinetic energy is the joule (J), we insert the units into Eq. (7.5): $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right)\text{m} = \text{N}\cdot\text{m} = \text{J}$.

It is important to understand the differences between force, acceleration, work, impulse, momentum, and kinetic energy. To illustrate this, we will perform the following thought experiment. Imagine two carts on a smooth surface that have identical fans attached but different masses. The fans rotate and push away the air around the carts; according to Newton's third law, the air pushes on the carts in the opposite direction, making them accelerate. Both carts start from rest, and we let them move the *same distance* on the floor (see **Figure 7.5**). **Table 7.5** compares the above physical quantities for the two carts. Notice that some quantities are the same for both carts, some are larger for cart 1, and some are larger for cart 2. Your task is to think about the explanations for the statements in the table.

TABLE 7.5 Physical quantities describing the motion of two fan carts

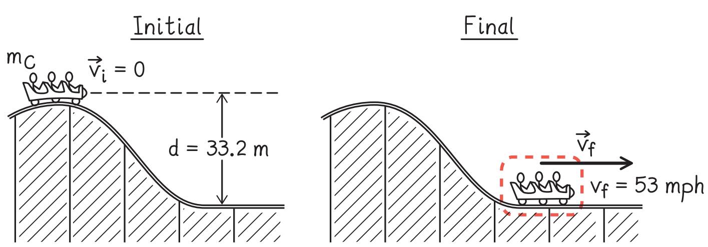
Quantity	Cart 1 (mass m_1)	Cart 2 (mass $m_2 = 2m_1$)
Force exerted by the air on the fan cart	$\vec{F}_{\text{A on 1}} = \vec{F}$	$\vec{F}_{\text{A on 2}} = \vec{F}_{\text{A on 1}} = \vec{F}$ Same as cart 1
Acceleration of the cart	$\vec{a}_1 = \frac{\vec{F}}{m_1}$	$\vec{a}_2 = \frac{\vec{F}}{m_2} = \frac{\vec{F}}{2m_1} = \frac{1}{2}\vec{a}_1$ Less than cart 1
Time interval to travel distance d	$\Delta t_1 = \sqrt{\frac{2d}{a_1}}$	$\Delta t_2 = \sqrt{\frac{2d}{a_2}} = \sqrt{\frac{4d}{a_1}} = \sqrt{2}\Delta t_1 > \Delta t_1$ Greater than cart 1
Work done by the fan on the cart	$W_{\text{A on 1}} = Fd\cos 0°$	$W_{\text{A on 2}} = Fd\cos 0° = W_{\text{A on 1}}$ Same as cart 1
Change of kinetic energy of the cart after moving distance d	$\Delta K_1 = \frac{m_1 v_{1\text{f}}^2}{2} = Fd$	$\Delta K_2 = \frac{m_2 v_{2\text{f}}^2}{2} = Fd = \Delta K_1$ Same as cart 1
Speed of the cart after moving distance d	$v_{1\text{f}} = \sqrt{\frac{2Fd}{m_1}}$	$v_{2\text{f}} = \frac{v_{1\text{f}}}{\sqrt{2}}$ Less than cart 1
Change of momentum of the cart after moving distance d	$m_1 v_{1\text{f}}$	$m_2 v_{2\text{f}} = 2m_1 \frac{v_{1\text{f}}}{\sqrt{2}} = \sqrt{2} m_1 v_{1\text{f}}$ Greater than cart 1
Impulse exerted on the cart after moving distance d	$\vec{J} = \vec{F}\Delta t_1 = m_1\vec{v}_{1\text{f}}$	$\vec{J} = \vec{F}\Delta t_2 = m_2\vec{v}_{2\text{f}}$ Greater than cart 1

You might be surprised that the heavier, slower cart has a greater momentum gain than the lighter, faster cart but the same gain in kinetic energy. But if you think about the work done by the air on each cart (equal to force times distance because the cosine of the angle between the force and displacement is 1), it is the same; thus the change in kinetic energy must be the same.

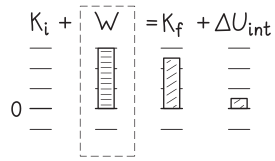
EXAMPLE 7.3 Mystic Timbers roller coaster

The Mystic Timbers roller coaster in Ohio has a drop of 33.2 m. The maximum speed of the coaster at the bottom of this drop is 53 mph. Can we say that all work done by Earth on the roller coaster is converted to the coaster's kinetic energy during the ride?

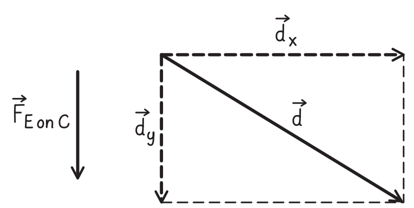
Sketch and translate We first draw a sketch of the roller coaster and mark the givens, assuming that the coaster starts from rest at the top of the track. We choose the initial state to be just before the roller coaster starts to drop and the final state to be at the very bottom of the drop. Because the problem asks about the work done by Earth, Earth should not be in the system—we include only the coaster and the track in the system.



Simplify and diagram A work-energy bar chart representing the process is shown at right. Because Earth is not in the system, Earth does work on the coaster, but the system does not have gravitational potential energy in any state. In the initial state, the system has no kinetic energy. In the final state, it has kinetic energy and possibly a change in internal energy (the coaster wheels and the track could warm up), but we are not sure.



Because the force that Earth exerts on the coaster points down, it only does work on the coaster due to the coaster's *vertical* displacement. This is because the work done by Earth on the coaster during the horizontal displacement of the coaster is zero, and the total displacement of the coaster can be decomposed into vertical and horizontal displacement components, as shown below.



Represent mathematically We use the bar chart to write the work-energy equation:

$$0 + W = K_f + \Delta U_{int}$$

Because we do not know the mass of the coaster, we cannot calculate the work done by Earth on the coaster or the coaster's final kinetic energy. However, we can check whether all work goes into the kinetic energy increase. If it does, the final speed of the coaster calculated using this assumption should be equal to the reported final speed of the coaster.

$$W_{E\text{ on }C} = F_{E\text{ on }C}d\cos\theta = F_{E\text{ on }C}d\cos 0^\circ = F_{E\text{ on }C}d = m_C g d$$

$$\text{and}\quad K_f = \tfrac{1}{2}m_C v_f^2$$

Inserting these into the work-energy equation with the assumption that $\Delta U_{int} = 0$ gives

$$0 + m_C g d = \tfrac{1}{2}m_C v_f^2 \;\Rightarrow\; v_f = \sqrt{2gd}$$

Solve and evaluate $v_f = \sqrt{2 \times (9.8\text{ m/s}^2) \times (33.2\text{ m})} = 25.5\text{ m/s}.$

The reported final speed of the coaster is

$$53\text{ mph} = \frac{(53\text{ miles/h}) \times (1600\text{ m/mile})}{3600\text{ s/h}} = 23.6\text{ m/s}$$

This reported final speed is about 7% $\left(\frac{25.5 - 23.6}{25.5} = 0.07\right)$ less than it would be if we assumed that all of the work done by the gravitational force went into the increase of the kinetic energy of the roller coaster. Thus some of this work went into an increase of the coaster and track's internal energy (and maybe some to the air around it).

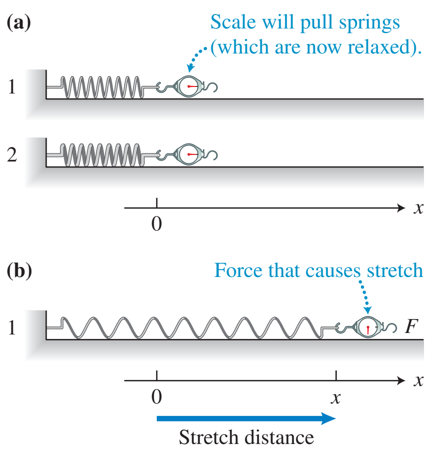
Try it yourself Change the system in this example to include Earth. Does the result change? How does the work-energy bar chart change?

Answer The roller coaster speed calculated for the new system is the same. On the bar chart there is no work done, but the system instead has an initial gravitational potential energy bar of the same height as the previous work done bar. The kinetic energy and change in internal energy bars remain the same.

REVIEW QUESTION 7.3 When we use the work-energy equation, how do we incorporate the force that Earth exerts on an object?

7.4 Quantifying elastic potential energy

FIGURE 7.6 Measuring spring stretch x caused by a scale exerting force F on a spring.



Our next goal is to construct a mathematical expression for the elastic potential energy stored by an elastic object when it has been stretched or compressed. We have a special problem in deriving this expression. Previously, we investigated work done by constant forces. However, when you stretch or compress an elastic spring-like object, you have to pull or push harder the more the object is stretched or compressed. The force is not constant. How does the force you exert to stretch a spring-like object change as the object stretches?

Hooke's law

To answer this question, we use two springs of the same length: a thinner and less stiff spring 1 and a thicker and stiffer spring 2 (**Figure 7.6a**). The springs are attached at the left end to a rigid object and placed on a smooth surface. We use a scale to pull on the right end of each spring, exerting a force whose magnitude F can be measured by the scale (Figure 7.6b). We record F and the distance x that each spring stretches from its unstretched position (see **Table 7.6**).

TABLE 7.6 Result of pulling on springs while exerting an increasing force

Force F exerted by the scale on the spring	Spring 1 stretch distance x	Spring 2 stretch distance x
0.00 N	0.000 m	0.000 m
1.00 N	0.050 m	0.030 m
2.00 N	0.100 m	0.060 m
3.00 N	0.150 m	0.090 m
4.00 N	0.200 m	0.120 m

FIGURE 7.7 A graph of the stretch of springs 1 and 2 when the same force is exerted on each spring.

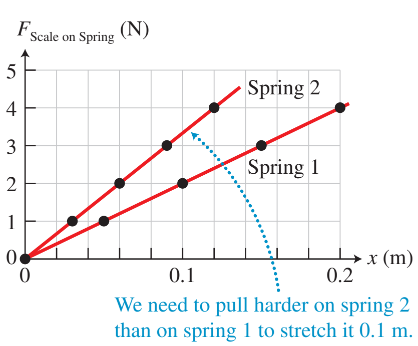


Figure 7.7 shows a graph of the Table 7.6 data. We use stretch distance x as an independent variable and the magnitude of the force $\vec{F}_{\text{Scale on Spring}}$ as a dependent variable. The magnitude of the force exerted by the scale on each spring is proportional to the distance that each spring stretches.

$$F_{\text{Scale on Spring}} = kx$$

The coefficient of proportionality k (the slope of the F-versus-x graph) is called the **spring constant**. The slope for the stiffer spring 2 is larger (33 N/m) than the slope for spring 1 (20 N/m). In other words, to stretch spring 1 by 1.0 m we would have to exert only a 20-N force, but we would need a 33-N force for spring 2.

Often we are interested not in the force that something exerts on the spring, but in the force that the spring exerts on something else, $\vec{F}_{\text{Spring on Scale}}$. Using Newton's third law we have $\vec{F}_{\text{Spring on Scale}} = -\vec{F}_{\text{Scale on Spring}}$. The x-component of this force is

$$F_{\text{Spring on Scale } x} = -kx$$

Note that if an object stretches the spring to the right in the positive direction, the spring pulls back on the object in the opposite negative direction (**Figure 7.8a**; in this case, the object is your finger). If the object compresses the spring to the left in the negative direction, the spring pushes back on the object in the opposite positive direction (Figure 7.8b). These observations are the basis for a relation first developed by Robert Hooke (1635–1703), called **Hooke's law**.

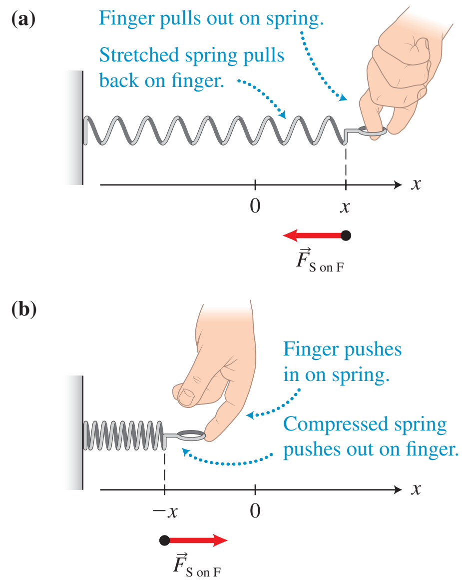
Elastic force (Hooke's law) If any object causes a spring to stretch or compress, the spring exerts an elastic force on that object. If the object stretches the spring along the *x*-direction, the *x*-component of the force the spring exerts on the object is

$$F_{\text{S on O}\,x} = -kx \qquad (7.6)$$

The spring constant *k* is measured in newtons per meter and is a measure of the stiffness of the spring (or any elastic object); *x* is the distance that the object has been stretched/compressed (not the total length of the object). The elastic force exerted by the spring on the object points in a direction opposite to the direction it was stretched (or compressed)—hence the negative sign in front of *kx*. The object in turn exerts a force on the spring:

$$F_{\text{O on S}\,x} = +kx$$

FIGURE 7.8 Hooke's law.



Elastic potential energy

Our goal now is to develop an expression for the elastic potential energy of an elastic stretched or compressed object (such as a stretched spring). Consider the constant slopes of the lines in the graph shown in Figure 7.7. While stretching the spring with your hand from zero stretch $(x = 0)$ to some arbitrary stretch distance *x*, the magnitude of the force your hand exerts on the spring changes in a linear fashion from zero when unstretched to *kx* when stretched.

To calculate the work done on the spring by such a variable force, we can replace this variable force with the average force $(F_{\text{H on S}})_{\text{average}}$:

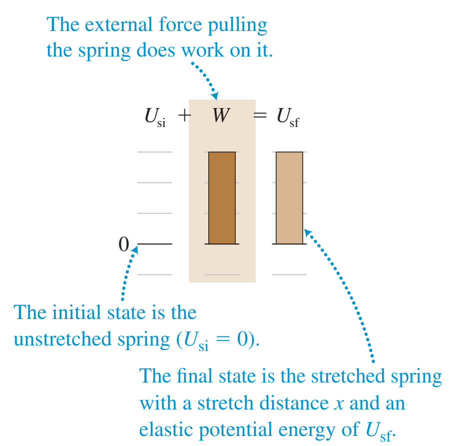
$$(F_{\text{H on S}})_{\text{average}} = \frac{0 + kx}{2}$$

The force your hand exerts on the spring is in the same direction as the direction in which the spring stretches. Thus the work done by this force on the spring to stretch it a distance *x* is

$$W = (F_{\text{H on S}})_{\text{average}}\,x = (\tfrac{1}{2}kx)x = \tfrac{1}{2}kx^2$$

This work equals the change in the spring's elastic potential energy (see **Figure 7.9** for the bar chart). Assuming that the elastic potential energy of the unstretched spring is zero, the work we calculated above equals the final elastic potential energy of the stretched spring.

FIGURE 7.9 Bar chart showing that external work increases the elastic potential energy of the spring (the system).



Elastic potential energy The elastic potential energy of a spring-like object with a spring constant *k* that has been stretched or compressed a distance *x* from its undisturbed position is

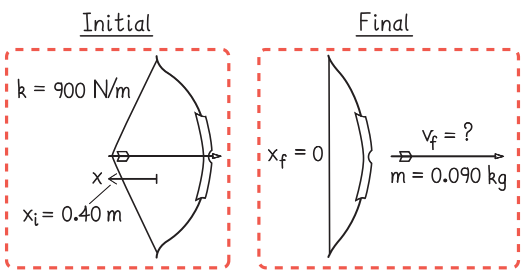
$$U_s = \tfrac{1}{2}kx^2 \qquad (7.7)$$

Just like any other type of energy, the unit of elastic potential energy is the joule (J).

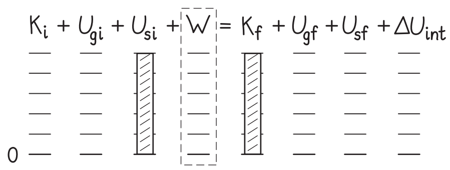
EXAMPLE 7.4 **Shooting an arrow**

You load an arrow (mass $= 0.090$ kg) into a bow and pull the bowstring back 0.40 m. The bow has a spring constant $k = 900$ N/m. Determine the arrow's speed as it leaves the bow.

Sketch and translate We sketch the process, as shown below. The system is the bow and arrow. In the initial state, the bowstring is pulled back 0.40 m. In the final state, the string has just relaxed and the arrow has left the string.



Simplify and diagram Since the arrow moves horizontally, we do not need to keep track of gravitational potential energy. The initial elastic potential energy of the bow is converted into the final kinetic energy of the arrow. We represent the process with the bar chart.



Represent mathematically Use the bar chart to help apply the work-energy equation:

$$E_i + W = E_f$$
$$\Rightarrow K_i + U_{si} + 0 = K_f + U_{sf}$$
$$\Rightarrow 0 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 + 0$$

Multiply both sides of the above by 2, divide by m, and take the square root to get

$$v = \sqrt{\frac{k}{m}}x$$

Solve and evaluate

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{900\ \text{N/m}}{0.090\ \text{kg}}}(0.40\ \text{m}) = 40\ \text{m/s}$$

This is reasonable for the speed of an arrow fired from a bow. The units of $\sqrt{\frac{k}{m}}x$ are equivalent to $\frac{\text{m}}{\text{s}}$:

$$\sqrt{\frac{\text{N}}{\text{m}\cdot\text{kg}}}(\text{m}) = \sqrt{\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{m}\cdot\text{kg}}}(\text{m}) = \frac{\text{m}}{\text{s}}$$

Try it yourself If the same arrow were shot vertically, how high would it go?

Answer 82 m.

REVIEW QUESTION 7.4 If the magnitude of the force exerted by a spring on an object is kx, why is it that the work done to stretch the spring a distance x is not equal to $kx \cdot x = kx^2$?

7.5 Friction and energy conversion

In nearly every mechanical process, objects exert friction forces on each other. Sometimes the effect of friction is negligible (for example, in an air hockey game), but most often friction is important (for example, when a driver applies the brakes to avoid a collision). Our next goal is to investigate how we can incorporate friction into work and energy concepts. Let's analyze a car skidding to avoid an accident.

Can a friction force do work?

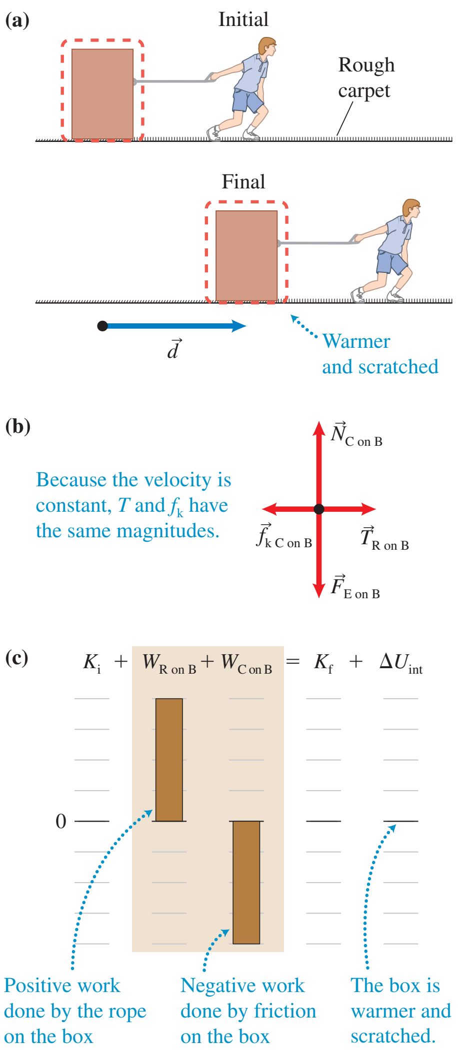
Imagine that you pull a rope attached to a box (the system) so that the box moves at constant velocity on a rough carpet (**Figure 7.10a**). A force diagram for the box is shown in Figure 7.10b.

The bar chart (Figure 7.10c) shows that the force exerted by the rope does positive work on the box system; the friction force does negative work. These horizontal forces have exactly the same magnitude (the box is moving at constant velocity). Thus, the sum of the work done by those forces on the system is zero and the energy of the system should not change.

$$T_{\text{R on B}}d\cos 0° + f_{\text{k C on B}}d\cos 180° = T_{\text{R on B}}d + (-f_{\text{k C on B}}d) = 0$$

However, if you touch the bottom of the box at its final position, you find that it is warmer than before you started pulling and the box has scratches on its bottom. Again, the internal energy of the box increased. We get $0 = \Delta U_{\text{int}}$, where ΔU_{int} is greater than zero. The zero work done on the box does *not* equal the positive increase in internal energy. This is a contradiction of the work-energy principle. How can we resolve this?

FIGURE 7.10 Pulling a box across a rough carpet.



The effect of friction as a change in internal energy

The key to resolving this problem is to change the system. The new system will include both surfaces that are in contact, for example, the box and the carpet. The friction force is then an internal force and therefore does no work on the system. But there is a change in the internal energy of the system caused by friction between the two surfaces.

When the rope is pulling the box across the rough carpet at constant velocity, we know that if the rope pulls horizontally on the box, the magnitude of the force that it exerts on the box $T_{\text{R on B}}$ must equal the magnitude of the friction force that the carpet exerts on the box $f_{\text{k C on B}}$. But now the box and carpet are both in the system—so the force exerted by the rope is the only external force. Thus the work done on the box-carpet system is

$$W = T_{\text{R on B}}d\cos 0°$$

Substituting $T_{\text{R on B}} = f_{\text{k C on B}}$ and $\cos 0° = 1.0$ into the above, we get

$$W = +f_{\text{k C on B}}d$$

The only system energy change is its internal energy ΔU_{int}. The work-energy equation for pulling the box across the surface is

$$W = \Delta U_{\text{int}}$$

After inserting the expression for the work done on the box and rearranging, we get

$$\Delta U_{\text{int}} = +f_k d$$

We have constructed an expression for the change in internal energy of a system caused by the friction force that the two contacting surfaces in the system exert on each other when one object moves a distance d across the other.

Increase in the system's internal energy due to friction

$$\Delta U_{\text{int}} = +f_k d \qquad (7.8)$$

where f_k is the magnitude of the average friction force exerted by the surface on the object moving relative to the surface, and d is the distance that the object moves across that surface. The increase in internal energy is shared between the moving object and the surface.

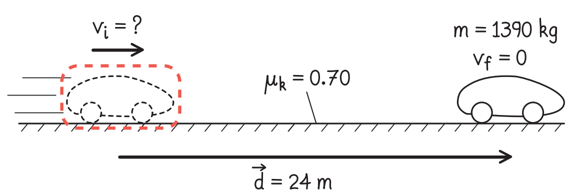
Including friction in the work-energy equation as an increase in the system's internal energy produces the same result as calculating the work done by friction force. In this new approach, there is an increase in internal energy $\Delta U_{\text{int}} = +f_k d$ in a system that includes the two surfaces rubbing against each other (this expression goes on the right side of the work-energy equation). When we include the two surfaces in the system, we can see why the moving box gets warmer and why there might be structural changes.

If we include only the box in the system, tracking the interactions between the system and the environment becomes very complicated because mass and thermal energy are transferred between the system and the environment. Therefore, in this book we prefer to include both surfaces in the system and consider the increase in internal energy caused by friction.

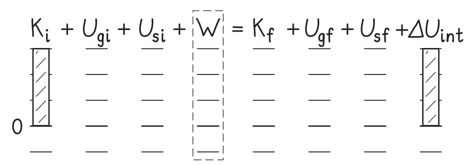
EXAMPLE 7.5 **Skidding to a stop**

You are driving your car when another car crosses the road at an intersection in front of you. To avoid a collision, you apply the brakes, leaving 24-m skid marks on the road while stopping. A police officer observes the near collision and gives you a speeding ticket, claiming that you were exceeding the 35 mi/h speed limit. She estimates your car's mass as 1390 kg and the coefficient of kinetic friction μ_k between your tires and this particular road as about 0.70. Do you deserve the speeding ticket?

Sketch and translate We sketch the process. We choose your car and the road surface as the system. We need to decide if your car was traveling faster than 35 mi/h at the instant you applied the brakes.



Simplify and diagram Assume that the process occurs on a horizontal level road and neglect interactions with the air. The initial state is just before the brakes are applied. The final state is just after your car has come to rest. We draw the energy bar chart to represent the process. In the initial state, the system has kinetic energy. In the final state, the system has no kinetic energy and has increased internal energy due to friction.



Represent mathematically Convert the bar chart into an equation:

$$K_i + 0 = U_{\text{int f}}$$

$$\Rightarrow \tfrac{1}{2}m_C v_i^2 = f_{\text{k R on C}}d = (\mu_k N_{\text{R on C}})d$$

The magnitude of the upward normal force $N_{\text{R on C}}$ that the road exerts on the car equals the magnitude of the downward gravitational force $F_{\text{E on C}} = m_C g$ that Earth exerts on the car: $N_{\text{R on C}} = m_C g$. Thus the above becomes

$$\tfrac{1}{2}m_C v_i^2 = \mu_k m_C g d$$

Solve and evaluate Rearranging the above and canceling the car mass, we get

$$v_i = \sqrt{2\mu_k g d} = \sqrt{2(0.70)(9.8\ \text{N/kg})(24\ \text{m})}$$

$$= (18.1\ \text{m/s})\left(\frac{1\ \text{mi/h}}{0.447\ \text{m/s}}\right) = 41\ \text{mi/h}$$

It looks like you deserve the ticket. We ignored the resistive drag force that air exerts on the car, which could be significant for a car traveling at 41 mi/h. Thus, air resistance helps the car's speed decrease, which means you actually were traveling faster than 41 mi/h.

Try it yourself Imagine the same situation as above, only you are driving a 2000-kg minivan. Will the answer for the initial speed increase or decrease?

Answer The speed does not change.

REVIEW QUESTION 7.5 Why, when friction cannot be neglected, is it useful to include both surfaces in the system when analyzing processes using the energy approach?

7.6 Skills for analyzing processes using the work-energy principle

The general strategy for solving problems with the work-energy principle is described on the left side of the table in Example 7.6 and illustrated on the right side for a specific process.

PROBLEM-SOLVING STRATEGY 7.1 Applying the work-energy principle

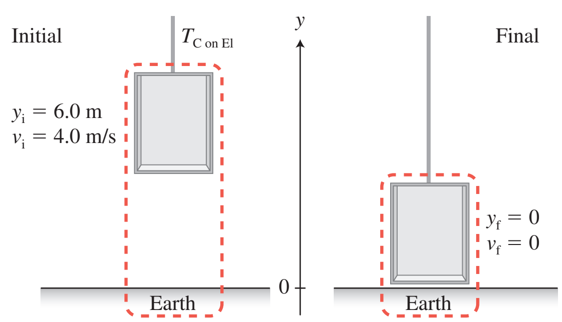
EXAMPLE 7.6 An elevator slows to a stop

A 1000-kg elevator is moving downward. While moving down at 4.0 m/s, its speed decreases steadily until it stops in 6.0 m. Determine the magnitude of the tension force that the cable exerts on the elevator ($T_{\text{C on El}}$) while it is stopping.

Sketch and translate

- Sketch the initial and final states of the process, labeling known and unknown information.
- Include the object of reference and the coordinate system.
- Choose the system of interest. Sometimes you might need to go back and redefine your system after you draw a bar chart if you find that the chosen system is not convenient.

- The elevator and Earth are in the system. We exclude the cable—its effect will be included as the work done by the cable on the elevator.

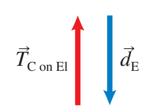


- The observer is on the ground; the coordinate system has a vertical axis pointing up with the zero at the bottom of the shaft.

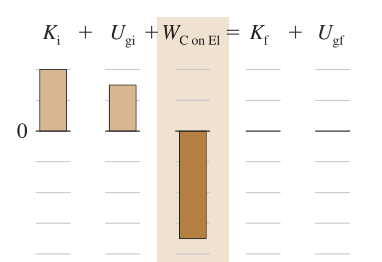
Simplify and diagram

- What simplifications can you make to the objects, interactions, and processes?
- Decide which energy types are changing.
- Are external objects doing work?
- Use the initial-final sketch to help draw a work-energy bar chart. Include work bars (if needed) and initial and final energy bars for the types of energy that are changing. Specify the zero level of gravitational potential energy.

- We assume that the cable exerts a constant force on the elevator and that the elevator can be considered a point-like object since all of its points move the same way (no deformation).
- We will keep track of kinetic energy (since the elevator's speed changes) and gravitational potential energy (since the elevator's vertical position changes).
- The tension force exerted by the cable on the elevator does negative work (the tension force points up, and the displacement of the elevator points down).



- The zero gravitational potential energy is at the lowest position of the elevator. In its initial state the system has kinetic energy and gravitational potential energy. In its final state the system has no energy.



Represent mathematically

- Convert the bar chart into a mathematical description of the process. Each bar in the chart will appear as a single term in the equation.

$$E_i + W = E_f$$

$$\tfrac{1}{2}mv_i^2 + mgy_i + T_{\text{C on El}}(y_i - 0)\cos 180° = 0$$

$$\tfrac{1}{2}mv_i^2 + mgy_i - T_{\text{C on El}}\,y_i = 0$$

(CONTINUED)

Solve and evaluate

- Solve for the unknown and evaluate the result.
- Does it have the correct units? Is its magnitude reasonable? Do the limiting cases make sense?

$$T_{\text{C on El}} = mg + \frac{mv_i^2}{2y_i}$$

$$= (1000\ \text{kg})(9.8\ \text{N/kg}) + \frac{(1000\ \text{kg})(4.0\ \text{m/s})^2}{2(6.0\ \text{m})}$$

$$= 11{,}000\ \text{N}$$

- The result has the correct units. The force that the cable exerts is more than the 9800-N force that Earth exerts. This is reasonable since the elevator slows down while moving down; thus the sum of the forces exerted on it should point up.
- Limiting case: If the elevator slowed down over a much longer distance (y_f = very large number instead of 6.0 m), then the force would be closer to 9800 N.

Try it yourself Solve the same problem using Newton's second law and kinematics.

Answer 11,000 N.

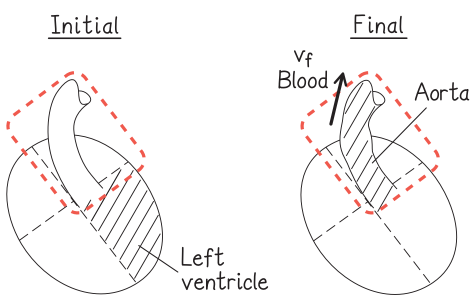
The "Try it yourself" question above demonstrates that we can obtain the same result for Example 7.6 using Newton's second law and kinematics. However, the energy approach is often easier and quicker.

We have used the work-energy approach to examine several real-world phenomena. Let's use it to examine how the body pumps blood.

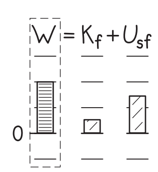
BIO CONCEPTUAL EXERCISE 7.7 Stretching the aorta

Every time your heart beats, the left ventricle pumps about 80 cm^3 of blood into the aorta, the largest artery in the human body. This pumping action occurs during a very short time interval, about 0.13 s. The elastic aorta walls stretch to accommodate the extra volume of blood. During the next 0.4 s or so, the walls of the aorta contract, applying pressure on the blood and moving it out of the aorta into the rest of the circulatory system. Represent this process with a qualitative work-energy bar chart.

Sketch and translate We sketch the process, as shown below. Choose the system to be the aorta and the 80 cm^3 of blood that is being pumped. The left ventricle is not in the system. Choose the initial state to be just before the left ventricle contracts. Choose the final state to be when the blood is in the stretched aorta before moving out into the rest of the circulatory system.

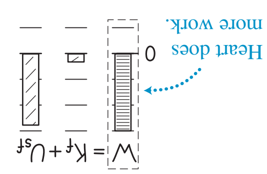


Simplify and diagram We keep track of the kinetic energy (the blood speed changes) and the elastic potential energy (the aorta wall stretches). We ignore the slight increase in the vertical elevation of the blood. The left ventricle is not in the system and does positive work pushing the blood upward in the direction of the blood's displacement. The work-energy bar chart below left represents this process for an aorta with flexible walls.



Try it yourself Modify the work-energy bar chart for a person with hardened and thickened artery walls.

Answer If the person has stiff, thick arteries (a condition called atherosclerosis), more energy than normal is required to stretch the aorta walls. This process is represented by the bar chart to the right. In this case, the blood pressure is higher than it would be for a healthy cardiovascular system because the heart has to do more work to stretch the walls while pushing blood into the aorta.



In Conceptual Exercise 7.2 we discussed the physics of pole vaulting qualitatively; here we will solve a quantitative problem to figure out the contributions of different types of energy to the height of the jump. Make sure you review and continually consult Exercise 7.2, as both the sketch and the energy bar charts in that exercise are the same as in this problem.

EXAMPLE 7.8 Pole vaulting, quantitatively

65-kg, 1.74-m-tall Yelena Isinbayeva from Russia holds the women's pole-vaulting record of 5.06 m. Estimate the amount of internal energy that she converts into mechanical energy during the vault if before take-off she is running at a speed of 8.2 m/s, assuming that the change in the internal energy of the pole is negligible.

Sketch and translate See the sketch of the situation in Exercise 7.2. We choose the initial state as Yelena is running just before she inserts the pole into the vaulting box and the final state when she just clears the bar. The givens are $m = 65$ kg, $H_Y = 1.74$ m, $h_f = 5.06$ m, and $v_i = 8.2$ m/s. Yelena, Earth, and the pole are the system.

Simplify and diagram Again, we assume that Yelena can be modeled as a point-like object with her mass concentrated approximately in the middle of her body, which is moving at 8.2 m/s about $h_i = \frac{1}{2}H_Y \approx 0.9$ m above the ground. We also assume that she needs to be moving at a small but nonzero horizontal speed when she clears the bar, about 0.1 m/s ($v_f = 0.1$ m/s). In the final state, there should be some internal energy decrease because she converts this energy into mechanical energy when jumping as she starts her vertical ascent and at the top of the flight when she is crunching her muscles to give a specific body shape as she passes over the bar. We also assume that no other external forces are doing work on the system.

Represent mathematically We use the bar chart (see Exercise 7.2) to write the work-energy equation:

$$K_i + U_{gi} = K_f + U_{gf} + \Delta U_{int}$$
$$\Rightarrow \Delta U_{int} = K_i + U_{gi} - K_f - U_{gf} = (K_i - K_f) + (U_{gi} - U_{gf})$$
$$= \frac{m}{2}(v_i^2 - v_f^2) - mg(h_f - h_i)$$

Solve and evaluate We can now substitute the givens into the final equation:

$$\Delta U_{int} = \frac{(65.0 \text{ kg})}{2}\left[(8.2 \text{ m/s})^2 - (0.1 \text{ m/s})^2\right]$$
$$- (65.0 \text{ kg}) \times (9.8 \text{ m/s}^2)(5.06 \text{ m} - 0.90 \text{ m})$$
$$= 2184.98 \text{ kg} \cdot \text{m}^2/\text{s}^2 - 2649.92 \text{ kg} \cdot \text{m}^2/\text{s}^2$$
$$= -464.95 \text{ J} \approx -465 \text{ J}$$

The negative sign shows that the internal energy *decreases*—in agreement with the bar chart. We could now adjust the length of the bars: The bar for ΔU_{int} should be about 20% of the height of the bar for K_i. The internal energy decrease is significant but reasonable—it is still a small part of the kinetic energy of the pole vaulter. This is the energy that Yelena converted to the mechanical energy to reach the desired height. However, she probably converted some more of her internal energy into other forms of energy because we ignored the increase in the internal energy of the pole as well as the air.

Try it yourself The men's record for pole vaulting is 6.16 m. Estimate how much internal energy the vaulter converts to mechanical energy assuming his mass is 69 kg, his height is 1.77 m, and he runs 100 m in 11.4 s (8.77 m/s). Specify all assumptions you made.

Answer About 930 J, assuming that all the vaulter's mass is concentrated at a point at the middle of the vaulter's height, and that the vaulter is not moving at the top of the flight.

Pole vaulting may not be part of your daily life, but there are plenty of everyday processes that we can analyze using a work-energy approach, for example, walking up the stairs. Let's choose the system to be you and Earth. The initial state is you on the first floor and the final state is you on the 10th floor. The gravitational potential energy of the system definitely increases. In addition, on the 10th floor you are sweating and the soles of your shoes and the surface of the stairs warm up a little due to the repeated deformation of the materials; thus the thermal energy (part of the internal energy) also increases. We have two increases; where does the energy for these increases come from? You might think that the stairs do work on you. However, it is not clear what the displacement is when you are in contact with each stair—different parts of your body move differently, so it is not clear what work the staircase does on you. Therefore, it is best to redefine the system to include the entire staircase.

The increases in gravitational potential and thermal energy of the you-Earth-staircase system came from the chemical energy of your muscles. The bar chart in

Figure 7.11a shows the average energy conversion across several stairs. We show the bar for the initial U_{gi} and the taller bar for the final U_{gf}, and we split the bar for internal energy change ΔU_{int} into two bars: chemical energy ΔU_{ch} and thermal energy ΔU_{th}. The thermal energy increases, but the chemical energy decreases. This is why you need to eat lunch to get back the expended energy! Note that if we did not include Earth in the system, the system would not have any gravitational potential energy, but Earth would do negative work. Therefore, some energy will need to increase to compensate for this negative work (see Figure 7.11b). In this situation, we see that including or excluding Earth from the system does not make any difference to the analysis, but including the staircase in the system simplifies the analysis.

FIGURE 7.11 Work-energy bar charts describing the process of walking up the stairs for different systems. (a) You, Earth and the staircase are in the system. (b) Earth is not in the system.

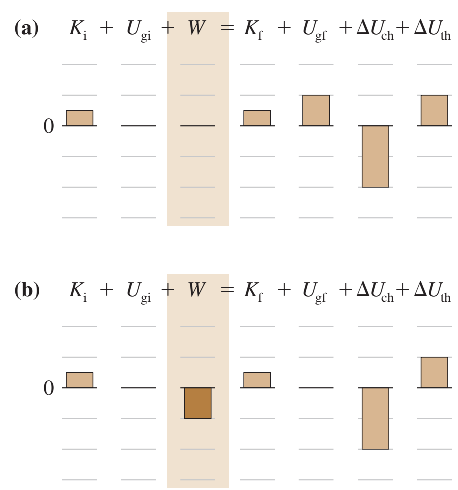
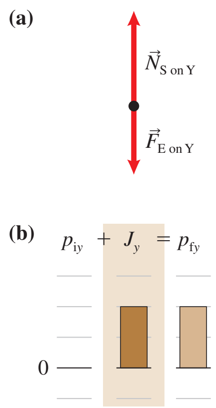


FIGURE 7.12 Force diagram and impulse-momentum bar chart for you pushing upward off a stair.



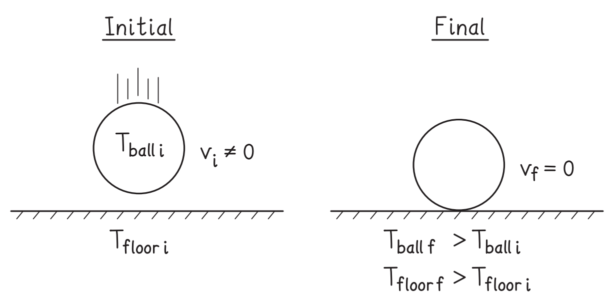
The work-energy approach describes the process, but does not explain it. During every step, you start at rest and then accelerate upward. If you are the system, then what are the objects responsible for your acceleration? You interact with Earth and the stair. The force exerted by the stair on you ($\vec{N}_{S\ on\ Y}$) should be larger than the force exerted by Earth ($\vec{F}_{E\ on\ Y}$) for you to accelerate upward (see the force diagram in **Figure 7.12a**).

Although the sum of the forces exerted by the stair and Earth on you does zero work (as there is no displacement of your feet while they are in contact with the floor), the sum of these two forces exerts a nonzero impulse on you due to the nonzero time interval of contact. Figure 7.12b shows the impulse-momentum bar chart for the process of each step. The initial state is right before you start your step, and the final state is when you are moving upward, but before you reach the next stair.

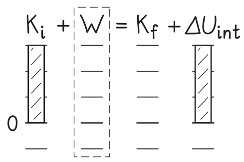
CONCEPTUAL EXERCISE 7.9 Dropping a steel ball

You drop a steel ball on the floor and take a picture of the floor with a thermal camera. The camera shows an increase in the temperature of the floor of about 4 °F; the ball gets a little hotter, too. Choose an appropriate system to analyze the energy conversions for the process that starts when the ball is about to hit the floor and ends when the ball is at rest on the floor.

Sketch and translate The initial and final states are provided in the givens. We show on the sketch a ball that is moving downward at speed v_i and then at rest (speed $v_f = 0$) on the floor. We also show the increased temperatures of the floor and the ball $\Delta T_{floor} > 0$; $\Delta T_{ball} > 0$. Because the floor did not move during the collision, it could not do any work on the ball. Therefore, we will include it in the system. Earth is irrelevant here because the distance between the ball and the floor does not change between the initial and final states.



Simplify and diagram The bar chart for the process is shown below. The left side shows the initial kinetic energy; the right side shows the change in internal energy. We assume that all this change is due to the temperature change of the ball and the floor.



Try it yourself Imagine that the ball lands on a trampoline instead of the floor. Analyze the process using the ball and Earth as the system. The initial state is when the ball touches the trampoline, and the final state is when the trampoline is stretched downward and the ball stops momentarily.

Answer After the ball first touches the trampoline, the trampoline stretches downward and exerts an increasing upward force on the ball in the direction opposite to the ball's downward motion. The trampoline therefore does negative work on the ball. This negative work decreases the kinetic energy of the ball. There is also a small decrease in the gravitational potential energy as the ball moves down with the trampoline.

REVIEW QUESTION 7.6 What would change in the solution to the problem in Example 7.8 if we did not include Earth in the system? Would the answer be different? Explain.

7.7 Collisions

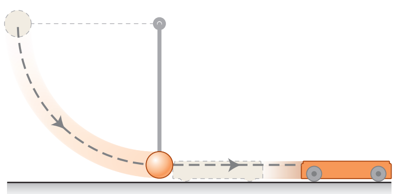
A collision is a process that occurs when two (or more) objects are in direct contact with each other for a short time interval, such as when a baseball is hit by a bat (**Figure 7.13**). The ball compresses during the first half of the collision, then decompresses during the second half of the collision. We have already used impulse and momentum principles to analyze collisions (in Chapter 6). We learned that the forces that the two colliding objects exert on each other during the collision are complicated, nonconstant, and exerted for a very brief time interval—roughly 1 ms in the case of the baseball-bat collision. Can we learn anything new by analyzing collisions using the work-energy principle?

FIGURE 7.13 A baseball is compressed while being hit by a bat.

Analyzing collisions using momentum and energy principles

Observational Experiment **Table 7.7**, on the next page, shows three different observational experiments involving collisions. In each case a 1.0-kg object attached to the end of a string (the bob of a pendulum) swings down and hits a 4.0-kg wheeled cart at the lowest point of its swing (**Figure 7.14**). In each experiment, the pendulum bob and cart start at the same initial positions, but the compositions of the pendulum bob and cart are varied. After each collision, the cart moves at a nearly constant speed because of the smoothness of the surface on which it rolls.

FIGURE 7.14 A pendulum bob hits a cart, causing it to move forward.



OBSERVATIONAL EXPERIMENT TABLE 7.7 Analyzing momentum and energy during collisions

VIDEO OET 7.7

Observational experiment	Analysis
Experiment 1. A 1.0-kg metal bob swings and hits a 4.0-kg metal cart. Their velocity components just before and just after the collision are shown below: $m_1 = 1.0\text{ kg}; v_{1ix} = 10\text{ m/s}$ $v_{1fx} = -6.0\text{ m/s}$ $m_2 = 4.0\text{ kg}; v_{2ix} = 0\text{ m/s}$ $v_{2fx} = 4.0\text{ m/s}$	**Momentum:** Before collision: $(1.0\text{ kg})(+10\text{ m/s}) + (4.0\text{ kg})\,0 = +10\text{ kg}\cdot\text{m/s}$ After collision: $(1.0\text{ kg})(-6.0\text{ m/s}) + (4.0\text{ kg})(+4.0\text{ m/s}) = +10\text{ kg}\cdot\text{m/s}$ **Kinetic energy:** Before collision: $(1/2)(1.0\text{ kg})(+10\text{ m/s})^2 + (1/2)(4.0\text{ kg})\,0^2 = 50\text{ J}$ After collision: $(1/2)(1.0\text{ kg})(-6.0\text{ m/s})^2 + (1/2)(4.0\text{ kg})(+4.0\text{ m/s})^2 = 50\text{ J}$
Experiment 2. A 1.0-kg sand-filled balloon swings and hits a 4.0-kg flimsy cardboard cart. After the collision, the damaged cart moves across the table at constant speed. The side of the balloon that hit the cart is flattened in the collision. Their velocity components just before and just after the collision are shown below: $m_1 = 1.0\text{ kg}; v_{1ix} = 10\text{ m/s}$ $v_{1fx} = -4.2\text{ m/s}$ $m_2 = 4.0\text{ kg}; v_{2ix} = 0\text{ m/s}$ $v_{2fx} = 3.55\text{ m/s}$	**Momentum:** Before collision: $(1.0\text{ kg})(+10\text{ m/s}) + (4.0\text{ kg})\,0 = +10\text{ kg}\cdot\text{m/s}$ After collision: $(1.0\text{ kg})(-4.2\text{ m/s}) + (4.0\text{ kg})(+3.55\text{ m/s}) = +10\text{ kg}\cdot\text{m/s}$ **Kinetic energy:** Before collision: $(1/2)(1.0\text{ kg})(+10\text{ m/s})^2 + (1/2)(4.0\text{ kg})\,0^2 = 50\text{ J}$ After collision: $(1/2)(1.0\text{ kg})(-4.2\text{ m/s})^2 + (1/2)(4.0\text{ kg})(+3.55\text{ m/s})^2 = 34\text{ J}$
Experiment 3. A 1.0-kg sand-filled balloon covered with Velcro swings down and sticks to a 4.0-kg Velcro-covered cardboard cart. The string holding the balloon is cut by a razor blade immediately after the balloon contacts the cart. The damaged cart and flattened balloon move off together across the table. Their velocity components just before and just after the collision are shown below: $m_1 = 1.0\text{ kg}; v_{1ix} = 10\text{ m/s}$ $v_{1fx} = 2.0\text{ m/s}$ $m_2 = 4.0\text{ kg}; v_{2ix} = 0\text{ m/s}$ $v_{2fx} = 2.0\text{ m/s}$	**Momentum:** Before collision: $(1.0\text{ kg})(+10\text{ m/s}) + (4.0\text{ kg})\,0 = +10\text{ kg}\cdot\text{m/s}$ After collision: $(1.0\text{ kg})(+2.0\text{ m/s}) + (4.0\text{ kg})(+2.0\text{ m/s}) = +10\text{ kg}\cdot\text{m/s}$ **Kinetic energy:** Before collision: $(1/2)(1.0\text{ kg})(+10\text{ m/s})^2 + (1/2)(4.0\text{ kg})\,0^2 = 50\text{ J}$ After collision: $(1/2)(1.0\text{ kg})(+2.0\text{ m/s})^2 + (1/2)(4.0\text{ kg})(+2.0\text{ m/s})^2 = 10\text{ J}$

Patterns

Two important patterns emerge from the data collected from these different collisions.

- The momentum of the system is constant in all three experiments.
- The kinetic energy of the system is constant when no damage is done to the objects during the collision (Experiment 1 but not in Experiments 2 and 3).

We can understand the first pattern in Table 7.7 using our knowledge of impulse-momentum. The *x*-component of the net force exerted on the system in all three cases was zero; hence the *x*-component of momentum should be constant.

What about the second pattern? In Experiment 1, the objects in the system were very rigid, but in Experiments 2 and 3, they were more fragile and as a result were deformed during the collision. Using the data, we can determine the amount of kinetic energy that was converted to internal energy. But it is impossible to predict this amount ahead of time. Unfortunately, this means that in collisions where any deformation of the objects occurs, we cannot make predictions about the amount of kinetic energy converted to internal energy. However, we now know that even in collisions where the objects become damaged, the momentum of the system still remains constant. So even though the work-energy equation is less useful in these types of collisions, the impulse-momentum equation is still very useful.

Types of collisions

The experiments in Table 7.7 are examples of the three general collision categories: elastic collisions (Experiment 1), inelastic collisions (Experiment 2), and totally inelastic collisions (Experiment 3). **Table 7.8** summarizes these three types of collisions.

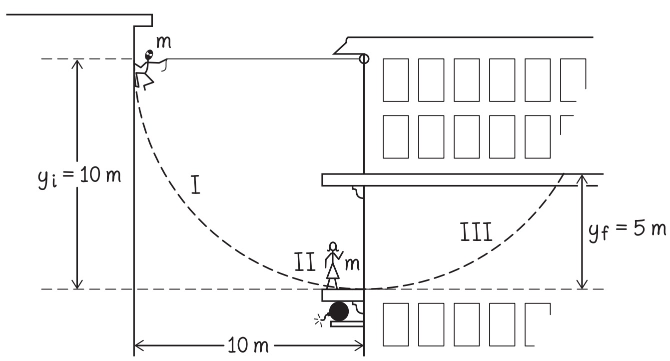
TABLE 7.8 Types of collisions

Elastic collisions	Inelastic collisions	Totally inelastic collisions
Both the momentum and kinetic energy of the system are constant. The internal energy of the system does not change. The colliding objects never stick together. Examples: There are no perfectly elastic collisions in nature, although collisions between very rigid objects (such as billiard balls) come close. Collisions between atoms or subatomic particles are almost exactly elastic.	The momentum of the system is constant but the kinetic energy is not. The colliding objects do not stick together. Internal energy increases during the collisions. Examples: A volleyball bouncing off your arms, or you jumping on a trampoline.	These are inelastic collisions in which the colliding objects stick together. Typically, a large fraction of the kinetic energy of the system is converted into internal energy in this type of collision. Examples: You catching a football, or a car collision where the cars stick together.

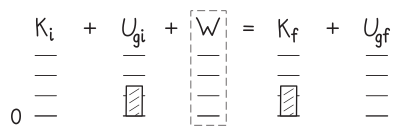
EXAMPLE 7.10 Spiderman saves Mary Jane

Spiderman is climbing up a building on one side of a 10-m-wide street when he spots Mary Jane 10 m below, standing on a platform on the other side of the street. The platform is about to explode. Spiderman swings down using his web and grabs Mary Jane, hoping to swing up with her to an open terrace, 5 m above the platform on the same building—where they both would be safe. Will they make it?

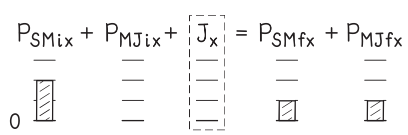
Sketch and translate Let's break the process into three parts: (I) Spiderman's swing from the first building down to Mary Jane (before they join together); (II) their brief impact, an inelastic collision (because they are traveling together after the impact); and (III) their swing together until they stop on the terrace. The givens are in the sketch below. We assume that the mass of Spiderman is the same as that of Mary Jane.



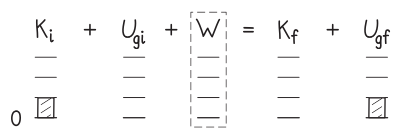
Simplify and diagram Consider Spiderman and Mary Jane to be point-like objects. For Part I, choose Spiderman, the web, and Earth as the system. During Spiderman's downward swing, the gravitational potential energy of the system is converted to his kinetic energy just before colliding with Mary Jane (see the work-energy bar chart, top right). We can use this to determine his speed just before impact. This way, we do not need to worry about the impulse exerted by the gravitational force on Spiderman to find his momentum at the bottom of the swing. The direction of the force exerted by Earth on Spiderman continually changes relative to the direction of his motion as he swings down, making calculations of impulse very difficult.



For Part II, the inelastic collision of Spiderman with Mary Jane, there will probably be an increase in internal energy. We can't use an energy analysis to decide how much—we don't have a way to independently determine internal energy change. Thus, we use only momentum conservation and impulse-momentum bar charts to determine Mary Jane and Spiderman's speed immediately after the collision (see the impulse-momentum bar chart below—the system is Spiderman and Mary Jane, who we have assumed have equal masses).



For Part III, knowing their speed after the collision, we can now use energy conservation to determine the maximum height they reach after the collision (see the work-energy bar chart below). Again, momentum conservation is very difficult because it involves the impulse caused by the gravitational force, which continually changes direction relative to the changing direction of motion as they swing upward.



Represent mathematically Use the energy bar charts from Part I to find that

$$mgy_i = \frac{1}{2}mv_{SM}^2$$

where $y_i = 10$ m is Spiderman's original height, and v_{SM} is his unknown speed just before grabbing Mary Jane. Dividing both sides of the equation by Spiderman's (nonzero) mass and rearranging, we get

$$v_{SM} = \sqrt{2gy_i}$$

We now use the outcome of the Part II momentum conservation collision process to determine their joint speed $v_{SM\text{-}MJ}$ immediately after the collision:

$$mv_{SM} = (m + m)v_{SM-MJ} \quad \text{or} \quad v_{SM-MJ} = \frac{1}{2}v_{SM}$$

For Part III, the height to which they rise after the collision is determined using energy conservation:

$$\frac{1}{2}(m + m)v_{SM-MJ}^2 = (m + m)gy_f \quad \text{or} \quad y_f = \frac{v_{SM-MJ}^2}{2g}$$

Solve and evaluate We can work backward to find the maximum height they will reach:

$$y_f = \frac{v_{SM-MJ}^2}{2g} = \frac{[(1/2)v_{SM}]^2}{2g} = \frac{v_{SM}^2}{4 \times 2g} = \frac{y_i}{4}$$

Therefore, they will rise to only a quarter of Jane's original height, or $(10\text{ m})/4 = 2.5$ m—too low to reach the terrace that is 5 m up on the other side of the street. The units are correct. Why is the answer so small? It turns out that half the initial gravitational potential energy is converted to internal energy during the collision.

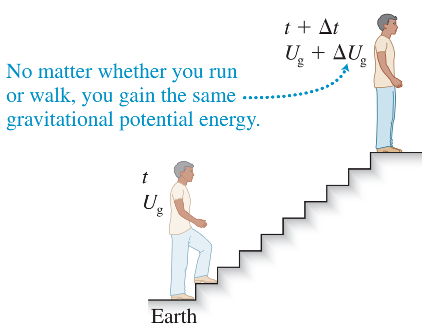
Try it yourself Suppose Mary Jane's mass is half Spiderman's mass. Would there be more, less, or the same mechanical energy converted into internal energy during the collision?

Answer Using the same process as above except with Mary Jane's mass $m/2$ instead of m, we find that the final height of Spiderman and Mary Jane is $4/9$ of the initial height, or about 4.4 m. Therefore, there is less mechanical energy converted into internal energy during this collision.

REVIEW QUESTION 7.7 Imagine that a collision occurs. You measure the masses of the two objects before the collision and measure the velocities of the objects both before and after the collision. Describe how you could use these data to determine which type of collision had occurred.

7.8 Power

FIGURE 7.15 Walking or running up stairs.



Why is it harder for a person to run up a flight of stairs than to walk up the stairs if the change in gravitational potential energy of the system person-Earth is the same? The *amount* of internal energy converted into gravitational energy is the same in both cases (as shown in **Figure 7.15**), but the *rate* of that conversion is not. When you run upstairs, you convert the energy at a faster rate. The rate at which the conversion occurs is called the **power**.

Power The power of a process is the amount of some type of energy converted into a different type divided by the time interval Δt in which the process occurred:

$$\text{Power} = P = \left|\frac{\Delta E}{\Delta t}\right| \tag{7.9}$$

If the process involves external forces doing work, then power can also be defined as the magnitude of the work W done on or by the system divided by the time interval Δt needed for that work to be done:

$$\text{Power} = P = \left|\frac{W}{\Delta t}\right| \tag{7.10}$$

The SI unit of power is the watt (W). 1 watt is 1 joule/second $(1\text{ W} = 1\text{ J/s})$.

A cyclist in good shape pedaling at moderate speed will convert about 400–500 J of internal chemical energy each second (400–500 W) into kinetic, gravitational potential, and thermal energies.

Power is sometimes expressed in *horsepower* (hp): 1 hp = 746 W. Horsepower is most often used to describe the power rating of engines or other machines. A 50-hp gasoline engine (typical in cars) converts the internal energy of the fuel into other forms of energy at a rate of $50 \times 746\text{ W} = 37{,}300\text{ W}$, or 37,300 J/s.

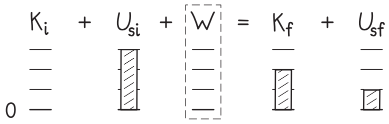
EXAMPLE 7.11 Car power

Miguel and Sierra are investigating a new toy car. The 350-g car is powered by a spring-based engine that delivers approximately constant power. They measure the time dependence of the speed of the car using a motion detector; their data are shown in the table at right. Estimate the power of the spring-based engine during the motion represented in the table. Indicate any assumptions that you made.

t (s)	v (m/s)
0.00	0
0.25	2.4
0.50	3.4
0.75	4.1
1.00	4.8

Sketch and translate We choose the car as the system. The initial state is at time $t = 0$ when the car starts moving from rest, and the final state is the moment at some later time t when the car is moving with the speed v.

Simplify and diagram Inside the car's engine, the elastic potential energy of the spring is converted into the kinetic energy of the car. There are two external forces that are exerted on the car, but neither of them does any work on the car. The force exerted by Earth on the car is perpendicular to the displacement of the car, and the point where the ground exerts a force on the car has zero displacement with respect to the car (the part of the car that is in contact with the ground is the tire—if it rolls without slipping, the point of contact is stationary). Ignoring air resistance and any friction in bearings, the work-energy bar chart for the process is shown at right.



Represent mathematically We can express the power of the conversion of elastic potential energy into kinetic energy using the definition of power [see Eq. (7.9)]:

$$P = \left|\frac{\Delta E}{\Delta t}\right| = \frac{\frac{1}{2}mv^2 - 0}{t - 0} = \frac{mv^2}{2t}$$

Solve and evaluate We know the mass of the car (0.350 kg), and we have five data points for t and the corresponding values of v. We want to estimate the unknown power P. Could we just pick one pair of data (t, v) from the table and calculate P using the equation above? We could, but we would discard valuable information that is hidden in the other data points that we did not use. Note that each measurement in the table contains random uncertainties, inherent to all measurements. The more data we use, the more precise our result will be. How can we use all data in the table?

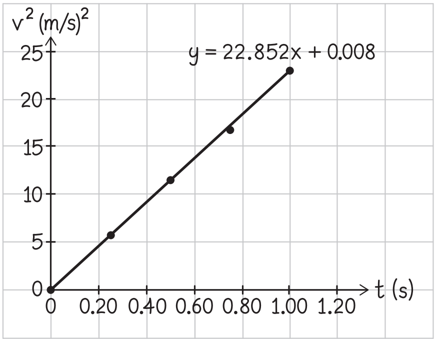
First, we rearrange the previous equation to express v as a function of t, as suggested in the table. We obtain

$$v = \sqrt{\frac{2Pt}{m}}$$

This equation suggests that the speed of the car powered by a constant-power engine is directly proportional to the square root of time. Since we are much more familiar with linear functions and have the tools to analyze them, we square both sides of this equation to obtain a linear dependence on time t:

$$v^2 = \frac{2P}{m}t$$

Next, we draw a graph by plotting the t values on the horizontal axis and the v^2 values on the vertical axis. The best-fit line through the points has a slope equal to $22.9\text{ m}^2/\text{s}^3$.



Since the slope of the line is given by $\frac{2P}{m}$, we can finally calculate the power:

$$P = \frac{(22.9\text{ m}^2/\text{s}^3)(0.35\text{ kg})}{2} = 4.0\,\frac{\text{m}\cdot\text{kg}\cdot\text{m/s}^2}{\text{s}} = 4.0\text{ W}$$

The units are correct. The qualitative dependence also makes sense: a heavier car with the same (t, v) data would need a more powerful engine. The actual power is probably larger than 4 W since we neglected air resistance and friction in the bearings.

Try it yourself A 1400-kg car is traveling on a level road at a constant speed of 27 m/s (60 mph). The drag force exerted by the air on the car and the rolling friction force exerted by the road on the car tires add to give a net force of 680 N pointing opposite the direction of motion of the car. Determine the rate of work done by the drag and rolling friction forces. Express your results in watts and in horsepower.

Answer $P = 1.8 \times 10^4\text{ W} = 24\text{ hp}$.

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REVIEW QUESTION 7.8 Toyota says that the power of their Prius car is 121 hp. What does this number mean?

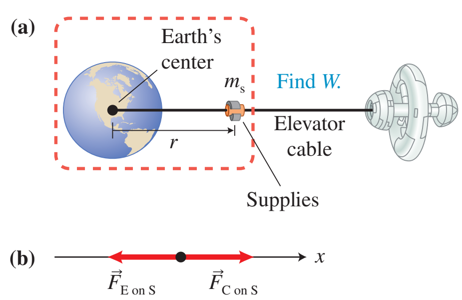
7.9 Improving our model of gravitational potential energy

So far, we have assumed that the gravitational force exerted by Earth on an object is constant ($F_{\text{E on O}} = mg$). Using this equation we devised the expression for the gravitational potential energy of an object-Earth system as $U_g = mgy$ (with respect to a chosen zero level). This expression is only valid when an object is close to Earth's surface. We know from our study of gravitation (at the end of Chapter 5) that the gravitational force exerted by planetary objects on moons and satellites and by the Sun on the planets changes with the distance between the objects according to Newton's law of gravitation ($F_{1 \text{ on } 2} = Gm_1m_2/r^2$). How would the expression for the gravitational potential energy change when we take into account that the gravitational force varies with distance?

Gravitational potential energy for large separations between objects

Imagine that a "space elevator" has been built to transport supplies from the surface of Earth to the International Space Station (ISS). The elevator moves at constant velocity, except for the very brief acceleration and deceleration at the beginning and end of the trip. How much work must be done to lift the supplies from the surface to the ISS?

FIGURE 7.16 A cable lifts supplies to the International Space Station via a space elevator. (a) Determine the work required to lift the supplies. (b) The cable exerts the force in the direction of motion. In this force diagram, the supplies are the system.



The initial state is the moment just as the supplies leave the surface. The final state is the moment just as they arrive at the ISS (**Figure 7.16a**). We choose Earth and the supplies as the system. The force that the elevator cable exerts on the supplies is an external force that does positive work on the system. We keep track of gravitational potential energy only; it is the only type of energy that changes between the initial and final states. Since the supplies are moving at constant velocity, the force exerted by the elevator cable on the supplies is equal in magnitude to the gravitational force exerted by Earth on the supplies (Figure 7.16b). Earth is the object of reference, and the origin of the coordinate system is at the center of Earth. The process can be described mathematically as follows:

$$U_{gi} + W = U_{gf} \Rightarrow W = U_{gf} - U_{gi}$$

At this point it would be tempting to say that the work done by the elevator cable on the system is $W = Fd\cos\theta$, where F is the constant force that the cable exerts on the supplies. However, as the supplies reach higher and higher altitudes, the force exerted by the elevator cable decreases—Earth exerts a weaker and weaker force on the supplies.

Determining the work done by a variable force requires a complex mathematical procedure. The outcome of this procedure is

$$W = \left(-G\frac{m_E m_S}{R_E + h_{ISS}}\right) - \left(-G\frac{m_E m_S}{R_E}\right)$$

where m_E is the mass of Earth, R_E is the radius of Earth, m_S is the mass of the supplies, h_{ISS} is the altitude of the ISS above Earth's surface, and $R_E + h_{ISS}$ is the distance of the ISS from the center of Earth. Before you move on, check whether this complicated equation makes sense (for example, check the units).

The work is written as the difference in two quantities; the latter describes the initial state and the former the final state. If we compare the above result with $W = U_{gf} - U_{gi}$, we see that each term is an expression for the gravitational potential energy of the Earth-object system for a particular object's distance from the center of Earth.

Gravitational potential energy of a system consisting of Earth and any object

$$U_g = -G\frac{m_E m_O}{r_{E\text{-}O}} \qquad (7.11)$$

where m_E is the mass of Earth $(5.97 \times 10^{24}\ \text{kg})$, m_O is the mass of the object, $r_{E\text{-}O}$ is the distance from the center of Earth to the center of the object, and $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$ is Newton's gravitational constant.

We can use Eq. (7.11) to find the gravitational potential energy of the system of any two spherical or point-like objects if we know the masses of the objects and the distance between their centers.

Note that the cable did positive work on the system while pulling the supplies away from Earth. When the object (the supplies in this case) is infinitely far away, the gravitational potential energy is zero. The only way to add positive energy to a system and have it become zero is if it started with negative energy, for example, $-5 + 5 = 0$. Thus, for the case of zero energy at infinity, the gravitational potential energy is a negative number when the object is closer to Earth. We can represent the process of pulling an object from the surface of Earth to infinity using a work-energy bar chart (**Figure 7.17**). The initial state is when the object is near Earth, and the final state is when it is infinitely far away.

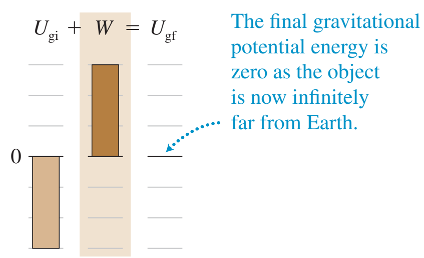


FIGURE 7.17 A bar chart representing the work needed to take an object from near Earth to infinitely far away. The system is the supplies and Earth.

Now we can determine the amount of work needed to raise 1000 kg of supplies to the International Space Station, which is 3.50×10^5 m above Earth's surface.

$$\begin{aligned} W &= \left(-G\frac{m_E m_S}{R_E + h_{ISS}}\right) - \left(-G\frac{m_E m_S}{R_E}\right) \\ &= -Gm_E m_S\left(\frac{1}{R_E + h_{ISS}} - \frac{1}{R_E}\right) \\ &= -(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})(1000\ \text{kg}) \\ &\quad \times \left(\frac{1}{6.37 \times 10^6\ \text{m} + 3.50 \times 10^5\ \text{m}} - \frac{1}{6.37 \times 10^6\ \text{m}}\right) \\ &= 3.26 \times 10^9\ \text{J} \end{aligned}$$

Let's compare this with what would have been calculated had we used our original expression for gravitational potential energy. Choose the zero level at the surface of Earth.

$$\begin{aligned} W = m_S g y_f - m_S g y_i &= m_S g y_f - 0 \\ &= (1000\ \text{kg})(9.8\ \text{N/kg})(3.50 \times 10^5\ \text{m}) = 3.43 \times 10^9\ \text{J} \end{aligned}$$

This differs by only about 5% from the more accurate result. Remember that $U_g = mgy$ is reasonable when the distance above the surface of Earth is a small fraction of the radius of Earth. The altitude of the ISS (350 km) is a small fraction of the radius of Earth (6371 km), so $U_g = mgy$ is still reasonably accurate.

Escape speed

The best Olympic high jumpers can leap over bars that are about 2.5 m (8 ft) above Earth's surface. Let's estimate a jumper's speed when leaving the ground in order to attain that height. We choose the jumper and Earth as the system and the zero level of gravitational potential energy at ground level. The kinetic energy of the jumper as he leaves the ground is converted into the gravitational potential energy of the system.

$$\frac{1}{2}mv^2 = mgy$$

$$\Rightarrow v = \sqrt{2gy} = \sqrt{2(9.8\ \text{N/kg})(2.5\ \text{m})} = 7.0\ \text{m/s}$$

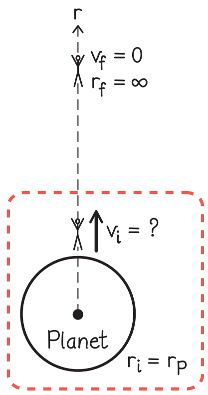
The gravitational coefficient of objects near the Moon's surface is $g_M = \frac{Gm_M}{R_M^2} = 1.6\ \text{N/kg}$. Using the equation above, we find the height of the jump to be 15.3 m. That's about 50 feet!

Is it possible to jump entirely off a celestial body—that is, jump up and never come down? What is the minimum speed you would need in order to do this? This minimum speed is called **escape speed**.

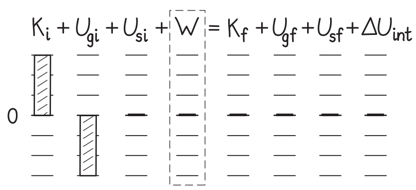
EXAMPLE 7.12 Escape speed

What vertical speed must a jumper have in order to leave the surface of a planet and never come back down?

Sketch and translate First, we draw a sketch of the process. The initial state is the instant after the jumper's feet leave the surface. The final state is when the jumper has traveled far enough away from the planet to no longer feel the effects of its gravity (at $r = \infty$). Choose the system to be the jumper and the planet.



Simplify and diagram We represent the process with the bar chart. In the initial state, the system has both kinetic and gravitational potential energy. In the final state, both the kinetic energy and the gravitational potential energy are zero.



Represent mathematically Using the generalized work-energy equation and the bar chart:

$$E_i + W = E_f$$
$$\Rightarrow K_i + U_{gi} + 0 = K_f + U_{gf}$$
$$\Rightarrow \tfrac{1}{2} m_J v^2 + \left(-G\frac{m_P m_J}{r_P}\right) + 0 = 0 + 0$$

where m_P is the mass of the planet, m_J is the mass of the jumper, r_P is the radius of the planet, and $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ is the gravitational constant.

Solve and evaluate Solving for the escape speed of the jumper,

$$v = \sqrt{\frac{2Gm_P}{r_P}} \tag{7.12}$$

We can use the above equation to determine the escape speed for any celestial body. For example, the escape speed for the Moon is

$$v = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{1.74 \times 10^6\ \text{m}}}$$
$$= 2370\ \text{m/s}$$

The escape speed for Earth is

$$v = \sqrt{\frac{2Gm_E}{R_E}}$$
$$= \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})}{6.37 \times 10^6\ \text{m}}}$$
$$= 11{,}200\ \text{m/s} = 11.2\ \text{km/s}$$

Try it yourself What is the escape speed of a particle near the surface of the Sun? The mass of the Sun is 2.0×10^{30} kg and its radius is 700,000 km.

Answer 620 km/s.

Black holes

TIP Notice that the escape speed does not depend on the mass of the escaping object—a tiny speck of dust and a huge boulder would need the same initial speed to leave Earth. Why is that?

Equation (7.12) for the escape speed suggests something amazing. If the mass of a star were large enough and/or its radius small enough, the escape speed could be made arbitrarily large. What if the star's escape speed were greater than light speed ($c = 3.00 \times 10^8\ \text{m/s}$)? What would this star look like? Light leaving the star's surface would not be moving fast enough to escape the star. The star would be completely dark.

Let's imagine what would happen if the Sun started shrinking so that its material were compressed into a smaller volume. How small would the Sun have to be for its escape speed to be greater than light speed? Use Eq. (7.12) with $v = c$ to answer this question.

$$v = c = \sqrt{\frac{2Gm_S}{r_S}} \tag{7.13}$$

or

$$r_S = \frac{2Gm_S}{c^2} = \frac{2(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(1.99 \times 10^{30}\,\mathrm{kg})}{(3.00 \times 10^8\,\mathrm{m/s})^2}$$
$$= 2.95 \times 10^3\ \mathrm{m} \approx 3\ \mathrm{km}$$

So, if the Sun collapsed to smaller than 3 km in radius, it would become invisible for the outside world. Could this happen? We will return to this question in later chapters.

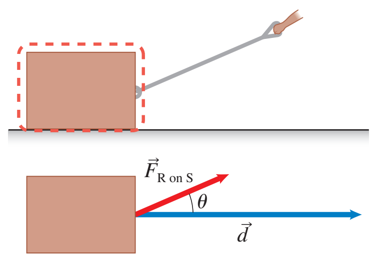
Equation (7.13) was first constructed by the brilliant astronomer Pierre-Simon Laplace (1749–1827), who used classical mechanics to predict the presence of what he called "dark stars"—now more commonly called black holes.

We've been talking about objects whose escape speed is larger than light speed. Physicists once thought that since light has zero mass, the gravitational force exerted on it would always be zero. At the beginning of the 20th century, Albert Einstein's theory of general relativity improved greatly on Newton's law of universal gravitation. According to general relativity, light is affected by gravity. Amazingly, the theory predicts the same size for a black hole that is provided by the Newtonian theory.

REVIEW QUESTION 7.9 In this section you read that the gravitational potential energy of two large bodies (for example, the Sun and Earth) is negative. Why is this true?

Summary

Work (W) is a way to change the energy of a system. Work is done on a system when an external object exerts a force of magnitude F on an object in the system as it undergoes a displacement of magnitude d. The work depends on the angle θ between the directions of $\vec{F}$ and $\vec{d}$. It is a scalar quantity. (Section 7.1)



$$W = Fd\cos\theta \qquad \text{Eq. (7.1)}$$

Gravitational potential energy (U_g) is the energy that a system has due to the relative separation of two objects with mass. It is a scalar quantity. U_g depends on the gravitational interaction of the objects. A single object cannot have gravitational potential energy.

$$U_g = mgy \qquad \text{Eq. (7.4)}$$

(near Earth's surface; zero level is at the surface)

$$U_g = -G\frac{m_A m_B}{r_{AB}} \qquad \text{Eq. (7.11)}$$

(general expression; zero level is at infinity)

Kinetic energy (K) is the energy of an object of mass m moving at speed v. It is a scalar quantity.

$$K = \tfrac{1}{2}mv^2 \qquad \text{Eq. (7.5)}$$

Elastic potential energy (U_s) is the energy of a stretched or compressed elastic object (e.g., coils of a spring or a stretched bow string).

$$U_s = \tfrac{1}{2}kx^2 \qquad \text{Eq. (7.7)}$$

Internal energy (U_{int}) is the energy of motion and interaction of the microscopic particles making up the objects in the system. (Section 7.1)

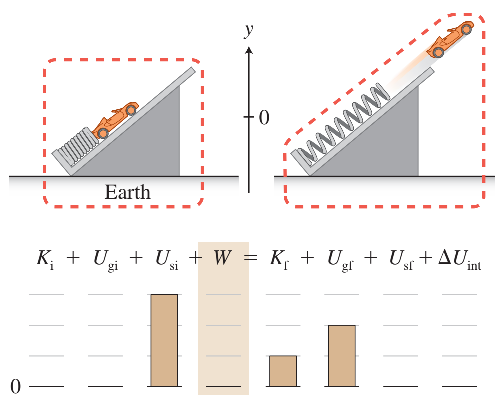
$$\Delta U_{int} = +f_k d \qquad \text{Eq. (7.8)}$$

(conversion of mechanical energy to internal due to friction)

Total energy (E) is the sum of all the energies of the system. (Section 7.2)

$$E = K + U_g + U_s + U_{int} + \cdots \qquad \text{Eq. (7.2)}$$

Work-energy principle The energy of a system changes when external forces do work on it. Internal forces do not change the energy of the system. When there are no external forces doing work on the system, the system's energy is constant. (Section 7.2)



$$E_i + W = E_f \qquad \text{Eq. (7.3)}$$

$$(K_i + U_{gi} + U_{si}) + W = (K_f + U_{gf} + U_{sf} + \Delta U_{int})$$

Collisions

- **Elastic:** momentum and kinetic energy of the system are constant—no changes in internal energy.
- **Inelastic:** momentum is constant but kinetic energy is not—internal energy increases and kinetic energy decreases.
- **Totally inelastic:** an inelastic collision in which the colliding objects stick together. (Section 7.7)

All collisions:

$$\Sigma\vec{p}_i = \Sigma\vec{p}_f$$

Elastic collisions only:

$$\Sigma K_i = \Sigma K_f$$

For other collisions,

$$\Delta U_{int} > 0$$

Power (P) is the rate of energy conversion, or rate of work done on a system during a process. (Section 7.8)

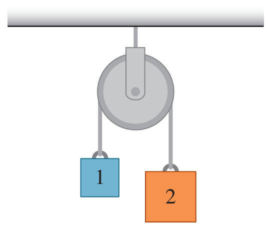
$$P = \left|\frac{\Delta E}{\Delta t}\right| \text{ or } P = \left|\frac{W}{\Delta t}\right| \qquad \text{Eq. (7.9) or (7.10)}$$

Questions

Multiple Choice Questions

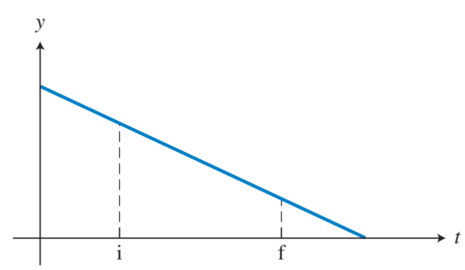
1. In which of the following is positive work done by a person on a suitcase?
   (a) The person holds a heavy suitcase.
   (b) The person lifts a heavy suitcase.
   (c) The person stands on a moving walkway carrying a heavy suitcase.
   (d) All of the above
   (e) None of the above
2. Which answer best represents the system's change in energy for the following process? The system includes Earth, two carts, and a compressed spring between the carts. The spring is released, and in the final state, one cart is moving up a frictionless ramp until it stops. The other cart is moving in the opposite direction on a horizontal frictionless track.
   (a) Kinetic energy to gravitational potential energy
   (b) Elastic potential energy to gravitational potential energy
   (c) Elastic potential energy to kinetic energy and gravitational potential energy
   (d) Elastic potential energy to kinetic energy
3. An Atwood machine is shown in Figure Q7.3. As the blocks are released and block 1 moves downward, the energy of the block 1-Earth system
   (a) increases.
   (b) decreases.
   (c) stays constant.
   (d) It's impossible to say without including block 2 in the system.

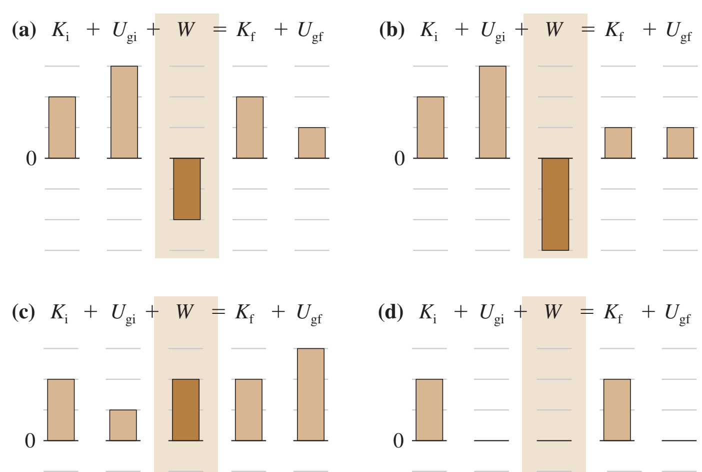
FIGURE Q7.3



4. Below you see several statements analyzing the process described in the previous question. Match the energy analysis with the system choice for which the analysis is correct.
   I. The total energy of the system decreases.
   II. The total energy of the system increases.
   III. The total energy of the system stays constant.
   Systems:
   (a) Block 2 and Earth (b) Block 1 and Earth
   (c) Both blocks, the string, and Earth (d) Both blocks and the string
5. Three processes are described below. Choose one process in which there is work done on the system. The spring, Earth, and the cart are part of the system.
   (a) A relaxed spring rests upright on a tabletop. You slowly compress the spring. You then release the spring and it flies up several meters to its highest point.
   (b) A cart at the top of a smooth inclined surface coasts at increasing speed to the bottom (ignore friction).
   (c) A cart at the top of a smooth inclined surface slides at increasing speed to the bottom where it runs into and compresses a spring (ignore friction).
6. Choose which statement describes a process in which an external force does negative work on the system. The person is not part of the system.
   (a) A person slowly lifts a box from the floor to a tabletop.
   (b) A person slowly lowers a box from a tabletop to the floor.
   (c) A person carries a bag of groceries horizontally from one location to another.
   (d) A person holds a heavy suitcase.
7. Which example(s) below involve(s) zero physics work? Choose all that apply.
   (a) A person holds a child.
   (b) A person pushes a car stuck in the snow but the car does not move.
   (c) A rope supports a swinging chandelier.
   (d) A person uses a self-propelled lawn mower on a level lawn.
   (e) A person pulls a sled uphill.
8. Estimate the change in gravitational potential energy when you rise from bed to a standing position.
   (a) No change (0 J) (b) About 250 J
   (c) About 2500 J (d) About 25 J
9. What does it mean if object 1 does +10 J of work on object 2?
   (a) Object 1 exerts a 10-N force on object 2 in the direction of its 1-m displacement.
   (b) Object 1 exerts a 1-N force on object 2 in the direction of its 10-m displacement.
   (c) Object 1 exerts a 10-N force on object 2 at a 60° angle relative to its 2-m displacement.
   (d) All of the above
   (e) None of the above
10. You pull on a spring, which obeys Hooke's law, in two steps. In step 1, you extend it by distance x_s. In step 2, you further extend it by the same distance x_s. Choose the answer that correctly compares the elastic potential energy of the spring after the first step (U_{s1}) with that after the second step (U_{s2}).
    (a) $U_{s2} = 2U_{s1}$ because the force exerted on the spring at the end of step 2 is two times larger than at the end of step 1.
    (b) $U_{s2} = 2U_{s1}$ because the extension of the spring at the end of step 2 is two times larger than the extension of the spring at the end of step 1.
    (c) $U_{s2} > 2U_{s1}$ because the more the spring is stretched, the larger the force that is needed to extend the spring for a unit distance.
    (d) $U_{s2} > 2U_{s1}$ because the average force exerted on the spring in step 2 is more than two times larger than the average force exerted on the spring in step 1.
11. The graph in Figure Q7.11 shows the time dependence of the vertical displacement of a lead ball with marked initial and final states. Choose all the work-energy bar charts (a) to (d) that can represent this process (multiple answers may be correct). Note that the y-axis can point either up or down.

FIGURE Q7.11

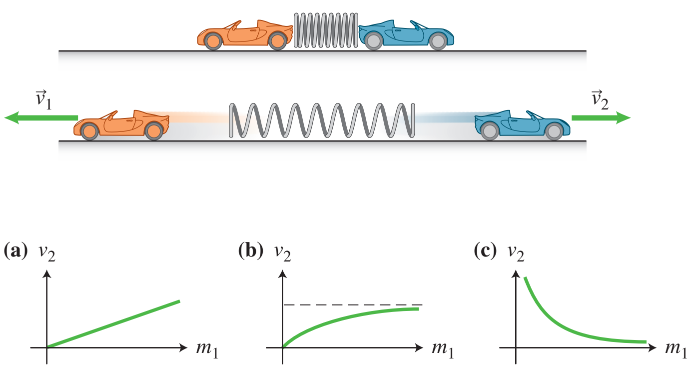




12. A 1400-kg car is traveling on a level road at a constant speed of 60 mi/h. The drag force exerted by the air on the car and the rolling friction force exerted by the road on the car tires add to give a net force of 680 N pointing opposite to the direction of the car's motion. What is the rate of work done by the air and the road on the car?
    (a) 820 kW (b) 370 kW
    (c) 41 kW (d) 18 kW

13. You place two toy cars on a horizontal table and connect them with a light compressed spring as shown in **Figure Q7.13**. The spring tries to push the cars apart, but they are tied together by a thread. When the thread is burned, the spring pushes the cars apart. You decide to investigate how the final speed of car 2 depends on the mass of car 1. You run several experiments changing m_1 and measuring v_2 while keeping the compression of the spring and the mass of car 2 constant. Which of the v_2-versus-m_1 graphs (a) to (c) do you expect to obtain? Evaluate the graphs by analyzing limiting cases.

FIGURE Q7.13



14. Two clay balls are moving toward each other. The balls have equal kinetic energies ($K_1 = K_2$), but ball 1, which is coming from the left, is moving faster than ball 2 ($v_1 > v_2$). Which answer correctly explains how the balls will move after a totally inelastic collision?
   (a) The balls will not move because $K_1 = K_2$ before the collision.
   (b) The balls will move to the left because $p_2 > p_1$ before the collision.
   (c) The balls will move to the right because $v_1 > v_2$ before the collision.
   (d) We cannot predict how the balls will move after the collision.

Conceptual Questions

15. Is energy a physical phenomenon, a model, or a physical quantity? Explain your answer.
16. Your friend thinks that the escape speed should be greater for more massive objects than for less massive objects. Provide an argument for his opinion. Then provide a counterargument for why the escape speed is independent of the mass of the object.
17. Suggest how you can measure the following quantities: the power of a motor, the kinetic energy of a moving car, and the elastic potential energy of a stretched spring.
18. How can satellites stay in orbit without any jet propulsion system? Explain using work-energy ideas.
19. Why does the Moon have no atmosphere, but Earth does?
20. What will happen to Earth if our Sun becomes a black hole?
21. In the equation $U_g = mgy$, the gravitational potential energy is directly proportional to the distance of the object from a planet. In the equation $U_g = -G\frac{m_P m}{r}$, it is inversely proportional. How can you reconcile those two equations?
22. You push a small cart by exerting a constant force F along a table. After you push the cart a distance s, the cart falls off the table and lands on the floor a distance D_1 away from the table edge. You repeat the same experiment (same F, same s) using a heavier cart and obtain a new distance D_2. Choose the answer that correctly compares the distances D_1 and D_2 and gives the best explanation of the outcome.
   (a) $D_1 = D_2$ because the same work was done on both carts before they flew off the table.
   (b) $D_1 = D_2$ because the carts had the same momentum before they flew off the table.
   (c) $D_1 > D_2$ because the carts had the same kinetic energy before they flew off the table, and consequently the speed of the lighter cart had been greater.
   (d) $D_1 > D_2$ because the kinetic energy of cart 1 had been greater than that of cart 2 before they flew off the table.

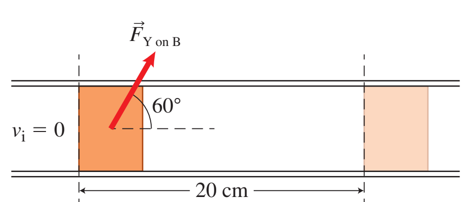
Problems

Below, BIO indicates a problem with a biological or medical focus. Problems labeled EST ask you to estimate the answer to a quantitative problem rather than derive a specific answer. Asterisks indicate the level of difficulty of the problem. Problems with no * are considered to be the least difficult. A single * marks moderately difficult problems. Two ** indicate more difficult problems.

7.1 Work and energy

1. Jay fills a wagon with sand (about 20 kg) and pulls it with a rope 30 m along the beach. He holds the rope 25° above the horizontal. The rope exerts a 20-N tension force on the wagon. How much work does the rope do on the wagon?
2. You have a 15-kg suitcase and (a) slowly lift it 0.80 m upward, (b) hold it at rest to test whether you will be able to move the suitcase without help in the airport, and then (c) lower it 0.80 m. What work did you do in each case? What assumptions did you make to solve this problem?
3. * You use a rope to slowly pull a sled and its passenger 50 m up a 20° incline, exerting a 150-N force on the rope. The system is the sled. (a) How much work will you do if you pull parallel to the incline? (b) How much work will you do if you exert the same magnitude force while slowly lowering the sled back down the incline and pulling parallel to the incline? (c) How much work did Earth do on the sled for the trip in part (b)?
4. A rope attached to a truck pulls a 180-kg motorcycle at 9.0 m/s. The rope exerts a 400-N force on the motorcycle at an angle of 15° above the horizontal. (a) What is the work that the rope does in pulling the motorcycle 300 m? (b) How will your answer change if the speed is 12 m/s? (c) How will your answer change if the truck accelerates?
5. You lift a 25-kg child 0.80 m, slowly carry him 10 m to the playroom, and finally set him back down 0.80 m onto the playroom floor. What work do you do on the child for each part of the trip and for the whole trip? List your assumptions.
6. A truck runs into a pile of sand, moving 0.80 m as it slows to a stop. The magnitude of the work that the sand does on the truck is 6.0×10^5 J. (a) Determine the magnitude of the average force that the sand exerts on the truck. (b) Did the sand do positive or negative work? (c) How does the average force change if the stopping distance is doubled? Indicate any assumptions you made.
7. A 0.50-kg block is placed in a straight gutter that allows the block to move only along the gutter. The friction between the gutter walls and the block is negligible. You are exerting a constant force of 3.0 N on the block at an angle of 60° with respect to the gutter (**Figure P7.7** shows the top view). The block is initially at rest. Determine (a) the work done on the block if the distance it moved is 20 cm and (b) the final speed of the block.

FIGURE P7.7



8. ** You are pushing a block of mass m at constant speed v over distance s up a smooth incline, which makes an angle θ with the horizontal. (a) Derive an expression for the work done by Earth on the block. (b) How would the answer to part (a) change if there is friction between the block and the incline? (c) How would the answer to part (a) change if you are pushing the block so that it is speeding up but there is no friction between the block and the incline?
9. ** It is a windy day. You are moving a 20-kg box at a constant speed over 10 m by pulling a rope that is attached to the box. The rope makes an angle of 25° with the horizontal. The coefficient of friction between the box and the ground is 0.4. The force exerted by the wind on the box is 15.0 N. Determine the work done by you on the box if (a) the wind is blowing toward you and (b) if the wind is blowing on your back.

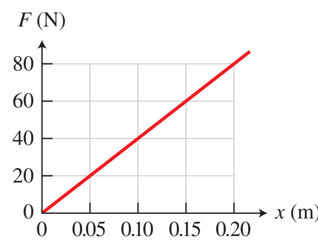
7.2 and 7.3 Energy is a conserved quantity and Quantifying gravitational potential and kinetic energies

10. A 5.0-kg rabbit and a 12-kg Irish setter have the same kinetic energy. If the setter is running at speed 4.0 m/s, how fast is the rabbit running?
11. * **EST** Estimate your average kinetic energy when walking to physics class. What assumptions did you make?
12. * A pickup truck (2268 kg) and a compact car (1100 kg) have the same momentum. (a) What is the ratio of their kinetic energies? (b) If the same horizontal net force were exerted on both vehicles, pushing them from rest over the same distance, what is the ratio of their final kinetic energies?
13. * When does the kinetic energy of a car change more: when the car accelerates from 0 to 10 m/s or from 30 m/s to 40 m/s? Explain.
14. * When exiting the highway, a 1100-kg car is traveling at 22 m/s. The car's kinetic energy decreases by 1.4×10^5 J. The exit's speed limit is 35 mi/h. Did the driver reduce the car's speed enough? Explain.
15. * You are on vacation in San Francisco and decide to take a cable car to see the city. A 5200-kg cable car goes 360 m up a hill inclined 12° above the horizontal. The system is the car and Earth. Determine the change in the total energy of the system when the car moves from the bottom to the top. Indicate any assumptions that you made.
16. * **Flea jump** A 5.4×10^{-7}-kg flea pushes off a surface by extending its rear legs for a distance of slightly more than 2.0 mm, consequently jumping to a height of 40 cm. What physical quantities can you determine using this information? Make a list and determine three of them.
17. * **Roller coaster ride** A roller coaster car drops a maximum vertical distance of 35.4 m. (a) Determine the maximum speed of the car at the bottom of that drop. (b) Describe any assumptions you made. (c) Will a car with twice the mass have more or less speed at the bottom? Explain.
18. * **BIO** **EST** **Heart pumps blood** The heart does about 1 J of work while pumping blood into the aorta during each heartbeat. (a) Estimate the work done by the heart in pumping blood during a lifetime. (b) If all of that work was used to lift a person, to what height could an average person be lifted? Indicate any assumptions you used for each part of the problem.
19. * **Wind energy** Air circulates across Earth in regular patterns. A tropical air current called the Hadley cell carries about 2×10^{11} kg of air per second past a cross section of Earth's atmosphere while moving toward the equator. The average air speed is about 1.5 m/s. (a) What is the kinetic energy of the air that passes the cross section each second? (b) About 1×10^{20} J of energy was consumed in the United States in 2005. What is the ratio of the kinetic energy of the air that passes toward the equator each second and the energy consumed in the United States each second?
20. * **BIO** **Bone break** The tibia bone in the lower leg of an adult human will break if the compressive force on it exceeds about 4×10^5 N (we assume that the ankle is pushing up). Suppose that a student of mass 60 kg steps off a chair that is 0.40 m above the floor. If landing stiff-legged on the surface below, what minimum stopping distance does he need to avoid breaking his tibias? Indicate any assumptions you made in your answer to this question.
21. * **BIO** **EST** **Climbing Mt. Everest** In 1953 Sir Edmund Hillary and Tenzing Norgay made the first successful ascent of Mt. Everest. How many slices of bread did each climber have to eat to compensate for the increase of the gravitational potential energy of the system climbers-Earth? (One piece of bread releases about 1.0×10^6 J of energy in the body.) Indicate all of the assumptions used. Note: The body is an inefficient energy converter—see the reading passage at the end of this section.

7.4 Quantifying elastic potential energy

22. A door spring is difficult to stretch. (a) What maximum force do you need to exert on a relaxed spring with a 1.2×10^4-N/m spring constant to stretch it 6.0 cm from its equilibrium position? (b) How much does the elastic potential energy of the spring change? (c) Determine its change in elastic potential energy as it returns from the 6.0-cm stretch position to a 3.0-cm stretch position. (d) Determine its elastic potential energy change as it moves from the 3.0-cm stretch position back to its equilibrium position.
23. * A moving car has 40,000 J of kinetic energy while moving at a speed of 7.0 m/s. A spring-loaded automobile bumper compresses 0.30 m when the car hits a wall and stops. What can you learn about the bumper's spring using this information? Answer quantitatively and list the assumptions that you made.
24. * The force required to stretch a slingshot by different amounts is shown in the graph in Figure P7.24. (a) What is the spring constant of the sling? (b) How much work does a child need to do to stretch the sling 15 cm from equilibrium?

FIGURE P7.24



7.5 Friction and energy conversion

25. Jim is driving a 2268-kg pickup truck at 20 m/s and releases his foot from the accelerator pedal. The car eventually stops due to an effective friction force that the road, air, and other things exert on the car. The friction force has an average magnitude of 800 N. Determine the stopping distance of the truck.
26. * A car skids 18 m on a level road while trying to stop before hitting a stopped car in front of it. The two cars barely touch. The coefficient of kinetic friction between the first car and the road is 0.80. A policewoman gives the driver a ticket for exceeding the 35 mi/h speed limit. Can you defend the driver in court? Explain.
27. ** A waterslide of length l has a vertical drop of h. Abby's mass is m. An average friction force of magnitude f opposes her motion. She starts down the slide at initial speed v_i. Use work-energy ideas to develop an expression for her speed at the bottom of the slide. Then evaluate your result using unit analysis and limiting case analysis.
28. ** You are pulling a crate on a rug, exerting a constant force on the crate $\vec{F}_{\text{Y on C}}$ at an angle θ above the horizontal. The crate moves at constant speed. Represent this process using a motion diagram, a force diagram, a momentum bar chart, and an energy bar chart. Specify your choice of system for each representation. Make a list of physical quantities you can determine using this information.
29. * Your friend Devin has to solve the following problem: "You have a spring with spring constant k. You compress it by distance x and use it to shoot a steel ball of mass m into a sponge of mass M. After the collision, the ball and the sponge move a distance s along a rough surface and stop (see Figure P7.29). The coefficient of friction between the sponge and the surface is μ. Derive an expression that shows how the distance s depends on relevant physical quantities."

FIGURE P7.29

Devin derived the following equation:

$$s = \frac{kmx^2}{2(m + M)^2 g\mu}$$

Without deriving it, evaluate the equation that Devin came up with. Is it reasonable? How do you know?

7.6 Skills for analyzing processes using the work-energy principle

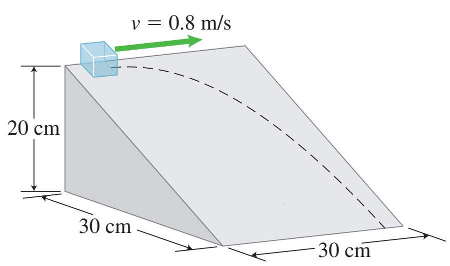
30. In a popular new hockey game, the players use small launchers with springs to move the 0.0030-kg puck. Each spring has a 120-N/m spring constant and can be compressed up to 0.020 m. Determine the maximum speed of the puck. First, represent the process with a work-energy bar chart and then solve the problem.

31. The top of a descending ski slope is 50 m higher than the bottom of the slope. A 60-kg skier starts from rest and skis straight to the bottom of the slope. (a) Represent the process with work-energy bar charts. Indicate the system, the initial state, and the final state. (b) Determine the speed of a skier at the bottom of the slope, assuming the friction is negligible. (c) Would a heavier skier have a larger, smaller, or the same speed at the bottom of the slope? Explain.

32. * If 20% of the gravitational potential energy change of the skier in the previous problem is converted into internal energy (due to friction and air drag), how fast is the 60-kg skier traveling at the bottom of the slope? Again, represent the process with work-energy bar charts indicating the system, the initial state, and the final state.

33. You exert a quick push on a 20-g ice cube horizontally along the top of a smooth incline so that it starts moving at a speed of 0.8 m/s, as shown in **Figure P7.33**. After you stop pushing it, the cube slides down the incline as shown in the figure. Calculate the speed of the ice cube when it reaches the bottom of the incline.

FIGURE P7.33



34. A driver loses control of a car, drives off an embankment, and lands in a canyon 6.0 m below. What was the car's speed just before touching the ground if it was traveling on the level surface at 12 m/s before the driver lost control?

35. * You are pulling a box so it moves at increasing speed. Compare the work you need to do to accelerate it from 0 m/s to speed v to the work needed to accelerate it from speed v to the speed of $2v$. Discuss whether your answer makes sense. How many different situations do you need to consider?

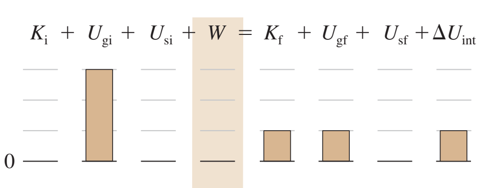
36. ** A cable lowers a 1200-kg elevator so that the elevator's speed increases from zero to 4.0 m/s in a vertical distance of 6.0 m. What is the force that the cable exerts on the elevator while lowering it? Specify the system, its initial and final states, and any assumptions you made. Then change the system and solve the problem again. Do the answers match?

37. ** EST **Hit by a hailstone** A 0.030-kg hailstone the size of a golf ball (4.3 cm in diameter) is falling at about 16 m/s when it reaches Earth's surface. Estimate the force that the hailstone exerts on your head—a head-on collision. Indicate any assumptions used in your estimate. Note that the cheekbone will break if something exerts a 900-N or larger force on the bone for more than 6 ms. Is this hailstone likely to break a bone?

38. * BIO **Froghopper jump** Froghoppers may be the insect jumping champs. These 6-mm-long bugs can spring 70 cm into the air, about the same distance as the flea. But the froghopper is 60 times more massive than a 12-mg flea. The froghopper pushes off for about 4 mm. What average force does it exert on the surface? Compare this to the gravitational force that Earth exerts on the bug.

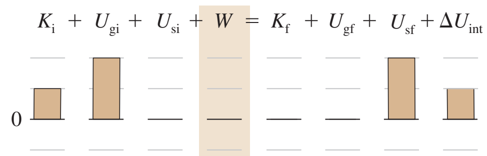
39. * **Bar chart Jeopardy 1** Describe in words and with a sketch a process that is consistent with the qualitative work-energy bar chart shown in **Figure P7.39**. Then apply in symbols the generalized work-energy principle for that process.

FIGURE P7.39



40. * **Bar chart Jeopardy 2** Describe in words and with a sketch a process that is consistent with the qualitative work-energy bar chart shown in **Figure P7.40**. Then apply in symbols the generalized work-energy principle for that process.

FIGURE P7.40



41. * **Equation Jeopardy 1** Construct a qualitative work-energy bar chart for a process that is consistent with the equation below. Then describe in words and with a sketch a process that is consistent with both the equation and the bar chart.

$$(1/2)(400\ \text{N/m})(0.20\ \text{m})^2 = (1/2)(0.50\ \text{kg})v^2 + (0.50\ \text{kg})(9.8\ \text{m/s}^2)(0.80\ \text{m})$$

42. * **Equation Jeopardy 2** Construct a qualitative work-energy bar chart for a process that is consistent with the equation below. Then describe in words and with a sketch a process that is consistent with both the equation and the bar chart.

$$(120\ \text{kg})(9.8\ \text{m/s}^2)(100\ \text{m})\sin 53° = (1/2)(120\ \text{kg})(20\ \text{m/s})^2 + f_k(100\ \text{m})$$

43. * **Evaluation 1** Your friend provides a solution to the following problem. Evaluate his solution. Constructively identify any mistakes he made and correct the solution. Explain possible reasons for the mistakes.

The problem: A 400-kg motorcycle, including the driver, travels up a 10-m-long ramp inclined 30° above the paved horizontal surface holding the ramp. The cycle leaves the ramp at speed 20 m/s. Determine the cycle's speed just before it lands on the paved surface.

Your friend's solution:

$$(1/2)(400\ \text{kg})(20\ \text{m/s}) = (400\ \text{kg})(9.8\ \text{m/s}^2)(10\ \text{m}) + (1/2)(400\ \text{kg})v^2$$
$$v = -13.2\ \text{m/s}$$

44. * **Evaluation 2** Your friend provides a solution to the following problem. Evaluate her solution. Constructively identify any mistakes she made and correct the solution. Explain possible reasons for the mistakes.

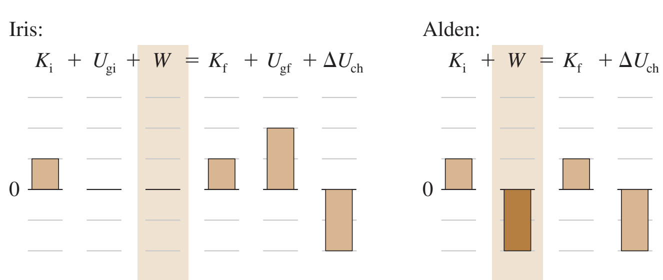
The problem: Jim (mass 50 kg) steps off a ledge that is 2.0 m above a platform that sits on top of a relaxed spring of force constant 8000 N/m. How far will the spring compress while stopping Jim?

Your friend's solution:

$$(50\ \text{kg})(9.8\ \text{m/s}^2)(2.0\ \text{m}) = (1/2)(8000\ \text{N/m})x$$
$$x = 0.25\ \text{m}$$

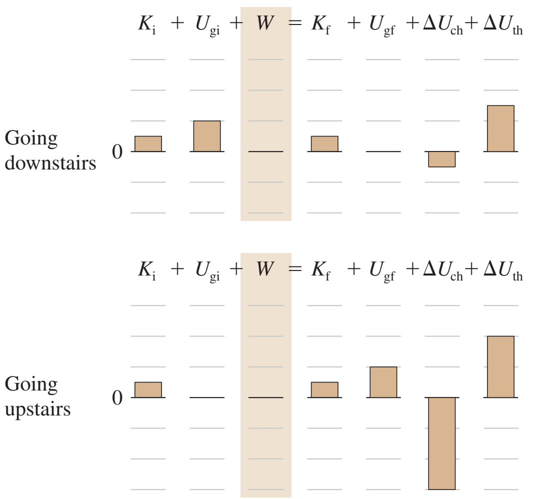
45. A crab climbs up a vertical rock with a constant speed. Students Iris and Alden represent this process using the work-energy bar charts in **Figure P7.45**. They choose the same initial and final states but different systems. On both bar charts ΔU_{ch} represents change in chemical energy. The students also assumed that the temperature of the crab did not change. Describe the system and the initial and final states that each student has chosen.

FIGURE P7.45



46. * Work-energy bar charts for a person going downstairs and upstairs are shown in **Figure P7.46**. The bar charts show the average energy conversion across several steps. The system is the person, Earth, and the stairs. (a) What are the initial and final states in each case? (b) Describe and explain the similarities in the bar charts. (c) Describe the differences in the bar charts. How do these differences explain why we are less tired going downstairs? (d) How do you explain that when we walk upstairs $|\Delta U_{th}| < |\Delta U_{ch}|$, but when we walk downstairs $|\Delta U_{th}| > |\Delta U_{ch}|$? (Hint: Think about what happens to our shoes and the stairs when we go downstairs.)

FIGURE P7.46



7.7 Collisions

47. A 4.0-kg block moving at 2.0 m/s west on a frictionless surface collides totally inelastically with a second 1.0-kg block traveling east at 3.0 m/s. (a) Determine the final velocity of the blocks. (b) Determine the kinetic energy of each block before and after the collision. (c) What happens to the difference in kinetic energies in this process?

48. * A 1060-kg car moving west at 16 m/s collides with and locks onto a 1830-kg stationary car. (a) Determine the velocity of the cars just after the collision. (b) After the collision, the road exerts a 1.2×10^4-N friction force on the car tires. How far do the cars skid before stopping? Specify the system and the initial and final states of the process.

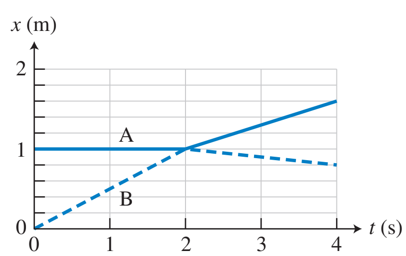
49. * You fire an 80-g arrow so that it is moving at 80 m/s when it hits and embeds in a 10-kg block resting on ice. (a) What is the velocity of the block and arrow just after the collision? (b) How far will the block slide on the ice before stopping? A 7.2-N friction force opposes its motion. Specify the system and the initial and final states for (a) and (b).

50. * You fire a 50-g arrow that moves at an unknown speed. It hits and embeds in a 350-g block that slides on an air track. At the end, the block runs into and compresses a 4000-N/m spring 0.10 m. How fast was the arrow traveling? Indicate the assumptions that you made and discuss how they affect the result.

51. * To confirm the results of Problem 7.50, you try a new experiment. The 50-g arrow is launched in an identical manner so that it hits and embeds in a 3.50-kg block. The block hangs from strings. After the arrow joins the block, they swing up so that they are 0.50 m higher than the block's starting point. How fast was the arrow moving before it joined the block?

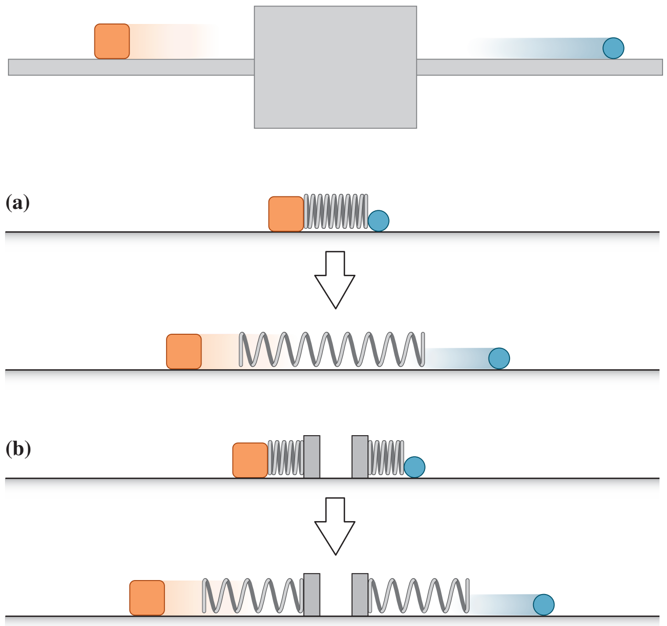
52. * Somebody tells you that **Figure P7.52** shows a displacement-versus-time graph of two carts on a linear track before and after a collision, when friction and air drag are negligible. The mass of cart A is 2.0 kg and mass of cart B is 1.0 kg. (a) Is the graph in agreement with the laws of physics? Explain. (b) If yes, is the collision elastic, inelastic, or totally inelastic? Explain. You will need a ruler to solve this problem.

FIGURE P7.52



53. ** Some students are investigating an unknown device that is shooting objects in opposite directions, as shown in **Figure P7.53**. The objects can have different masses and are therefore ejected from the device with different speeds. The objects are always launched in pairs. The students propose two different explanations for the object-launching mechanism.

FIGURE P7.53



Mechanism 1: Two objects are touching opposite ends of a very light spring that is initially compressed always for the same amount (Figure P7.53a). When the spring is released, it pushes on the objects and makes them fly in opposite directions. The spring is not attached to anything.

Mechanism 2: Each object is touching its own light spring that is initially compressed always for the same amount. The springs are identical and are compressed by the same distance (Figure P7.53b). When the springs are released, they push on the objects and cause them to fly in opposite directions.

The students also measure the masses and speeds of the pairs of the objects that are launched together (see the table below). Can you reject one or both mechanisms using the data in the table? Are there any events in the table that are in agreement with both mechanisms? Explain.

Event #	m_1 (g)	v_1 (m/s)	m_2 (g)	v_2 (m/s)
1	1.0	3.0	2.0	1.5
2	1.0	2.0	1.0	2.0
3	2.0	1.5	3.0	1.0
4	2.0	2.0	1.0	4.0
5	3.0	1.0	2.0	1.5

7.8 Power

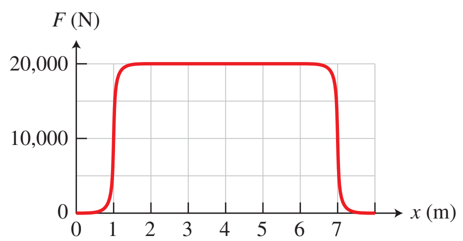
54. A roofing shingle elevator is lifting a 10.0-kg package to the top of a building at constant speed. The angle between the elevator track and the horizontal is 75°. The power of the elevator is 700 W. What is the vertical component of the speed with which the package is moving up? Indicate any assumptions that you made.

55. (a) What is the power involved in lifting a 1.0-kg object 1.0 m in 1.0 s? (b) While lifting a 10-kg object 1.0 m in 0.50 s? (c) While lifting the 10-kg object 2.0 m in 1.0 s? (d) While lifting a 20-kg object 1.0 m in 1.0 s?

56. * A fire engine must lift 30 kg of water a vertical distance of 20 m each second. What is the amount of power needed for the water pump for this fire hose?

57. * **BIO** **Internal energy change while biking** You set your stationary bike on a high 80-N friction-like resistive force and cycle for 30 min at a speed of 8.0 m/s. Your body is 10% efficient at converting chemical energy in your body into mechanical work. (a) What is your internal chemical energy change? (b) How long must you bike to convert 3.0×10^5 J of chemical potential while staying at this speed? (This amount of energy equals the energy released by the body after eating three slices of bread.)
58. * **Climbing Mt. Mitchell** An 82-kg hiker climbs to the summit of Mount Mitchell in western North Carolina. During one 2.0-h period, the climber's vertical elevation increases 540 m. Determine (a) the change in gravitational potential energy of the climber-Earth system and (b) the power of the process needed to increase the gravitational energy.
59. * **BIO** **EST** **Sears stair climb** The fastest time for the Sears Tower (now Willis Tower) stair climb (103 flights, or 2232 steps) is about 13 min. (a) Estimate the mechanical power in watts for a top climber. Indicate any assumptions you made. (b) If the body is 20% efficient at converting chemical energy into mechanical energy, approximately how many joules and kilocalories of chemical energy does the body expend during the stair climb? Note: 1 food calorie = 1 kilocalorie = 4186 J.
60. * **BIO** **EST** **Exercising so you can eat ice cream** You curl a 5.5-kg (12 lb) dumbbell that is hanging straight down in your hand up to your shoulder. (a) Estimate the work that your hand does in lifting the dumbbell. (b) Estimate the average mechanical power of the lifting process. Indicate any assumptions used in making the estimate. (c) Assuming the efficiency described at the end of Problem 7.59, how many times would you have to lift the dumbbell in order to burn enough calories to use up the energy absorbed by eating a 300-food-calorie dish of ice cream? (Problem 7.59 provides the joule equivalent of a food calorie.) List the assumptions that you made.
61. ** **BIO** **Salmon move upstream** In the past, salmon would swim more than 1130 km (700 mi) to spawn at the headwaters of the Salmon River in central Idaho. The trip took about 22 days, and the fish consumed energy at a rate of 2.0 W for each kilogram of body mass. (a) What is the total energy used by a 3.0-kg salmon while making this 22-day trip? (b) About 80% of this energy is released by burning fat and the other 20% by burning protein. How many grams of fat are burned? One gram of fat releases 3.8×10^4 J of energy. (c) If the salmon is about 15% fat at the beginning of the trip, how many grams of fat does it have at the end of the trip?
62. * **EST** Estimate the maximum horsepower of the process of raising your body mass as fast as possible up a flight of 20 stair steps. Justify any numbers used in your estimate. The only energy change you should consider is the change in gravitational potential energy of the system you-Earth.
63. * A 1600-kg car smashes into a shed and stops. The force that the shed exerts on the car as a function of a position at the car's center is shown in **Figure P7.63**. How fast was the car traveling just before hitting the shed?

FIGURE P7.63



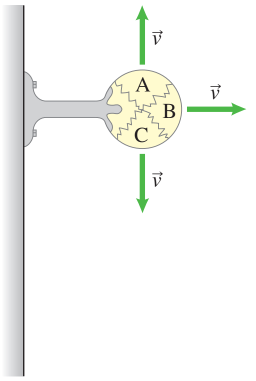
7.9 Improving our model of gravitational potential energy

64. At what distance from Earth is the gravitational potential energy of a spaceship-Earth system reduced to half the energy of the system before the launch?
65. * **Possible escape of different air molecule types** (a) Determine the ratio of escape speeds from Earth for a hydrogen molecule (H_2) and for an oxygen molecule (O_2). The mass of the oxygen is approximately 16 times that of the hydrogen. (b) In the atmosphere, the average random kinetic energy of hydrogen molecules and oxygen molecules is the same. Determine the ratio of the average speeds of the hydrogen and the oxygen molecules. (c) Based on these two results, give one reason why our atmosphere lacks hydrogen but retains oxygen.
66. Determine the escape speed for a rocket to leave Earth's Moon.
67. Determine the escape speed for a rocket to leave the solar system.
68. If the Sun were to become a black hole, how much would it increase the gravitational potential energy of the Sun-Earth system?
69. * A satellite moves in elliptical orbit around Earth, which is one of the foci of the elliptical orbit. (a) The satellite is moving faster when it is closer to Earth. Explain why. (b) If the satellite moves faster when it is closer to Earth, is the energy of the satellite-Earth system constant? Explain.
70. * Determine the maximum radius Earth's Moon would have to have in order for it to be a black hole.

General Problems

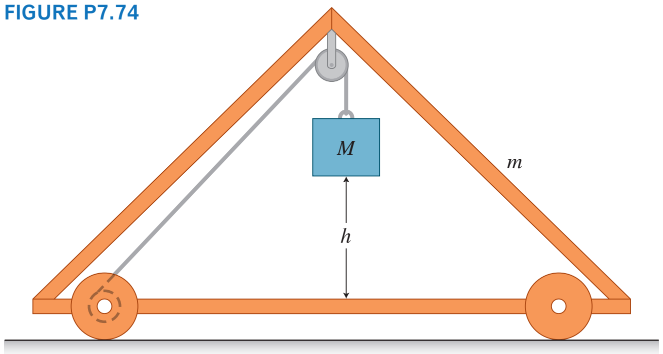
71. You throw a clay ball vertically upward. The ball hits the ceiling and sticks to it. Let the initial state be just after the ball leaves your hand and the final state just after the ball sticks to the ceiling. Choose a system and represent the process with a work-energy bar chart. Indicate any assumptions that you made.
72. * A spherical street lamp accidentally explodes. Three equal pieces A, B, and C fly off the lamp holder with equal speeds but in different directions, as shown in **Figure P7.72**. (a) Compare the speeds with which each piece hits the ground. (b) Compare qualitatively the times needed for each piece to reach the ground. Indicate any assumptions that you made.

FIGURE P7.72



73. Assume that in the previous problem each piece of the lamp has mass m. Develop expressions for the magnitude of (a) the total kinetic energy and (b) the total momentum of the three pieces together just after the explosion.
74. * **EST** A "gravity force car" is powered by the force exerted by Earth on an object of mass M. The object is tied to a rope that is turning the axle of the rear wheels, as shown in **Figure P7.74**. The mass of the car (excluding the hanging object) is m. As the car accelerates, the object descends a distance h. (a) Ignoring air resistance and friction in the pulley and wheels, derive an expression for the final speed of the car. (b) Based on your answer to part (a), what would you suggest to the designers of such a car if the goal is to achieve the greatest final speed? (c) Estimate the maximum final speed of a tabletop version of a gravity force car (think of reasonable values for the variables in the equation that you derived).[1]

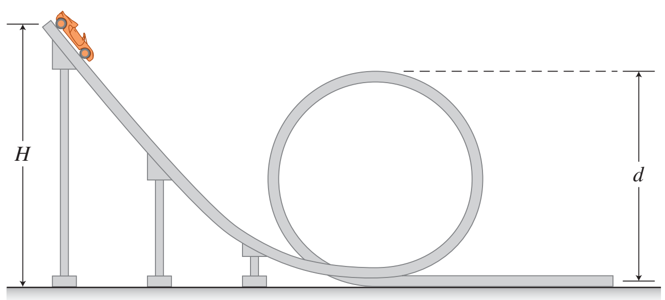
FIGURE P7.74



[1]This problem was inspired by Gordon Aubrecht, who sadly passed away in 2016.

75. * **Loop the loop** You are given a loop raceway for your Hot Wheels cars (as shown in **Figure P7.75**). While playing with the cars, you and your friends notice that you need to release the car from at least a height H of 1.30 diameters of the loop above the ground for the car not to fall off the track at the top of the circular loop. Two of your friends have different explanations for the observed pattern:

FIGURE P7.75



Jordan: If there were no friction forces exerted on the car, the minimum height H would be equal to the loop diameter d because the mechanical energy of the car-Earth system is constant in the process.

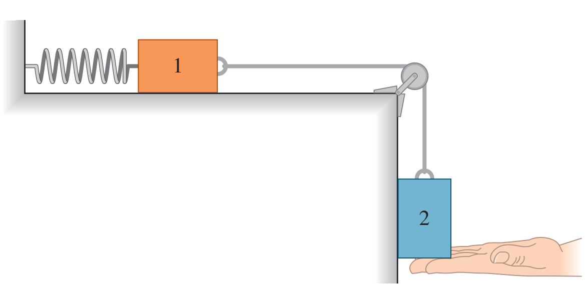
Leila: The minimum height H is larger than the loop diameter d even if the friction forces are negligible. The car's speed at the top of the loop should be large enough that the force exerted by Earth on the car alone keeps the car moving in a circle determined by the loop.

Analyze the problem and decide which of your friends is correct.

76. ** **Atwood machine** Two blocks of masses m_1 and m_2 hang at the ends of a string that passes over a very light pulley (see Figure Q7.3). Friction in the pulley can be ignored. The blocks are initially at rest at the same height (the initial state). After they are released, the heavier block m_2 moves through a distance d (the final state). Represent the process with work-energy bar charts choosing as a system (a) block 1 and Earth, (b) block 1, block 2, and Earth, and (c) block 1 and block 2. (d) Derive an expression for the final speed of the blocks in terms of m_1, m_2, and d, and evaluate the expression using unit analysis and extreme case analysis.

77. ** Two blocks of masses m_1 and m_2 are connected with a string that passes over a very light pulley (see **Figure P7.77**). Friction in the pulley can be ignored. Block 1 is resting on a rough table and block 2 is hanging over the edge. The coefficient of friction between the block 1 and the table is μ (assume static and kinetic friction have the same value). Block 1 is also connected to a spring with a constant k. In the initial state, the spring is relaxed as a person is holding block 2, but the string is still taut. When block 2 is released, it moves down for a distance d until it stops (the final state). (a) Explain why block 2 eventually stops moving. (b) Choose a system and represent the process with a work-energy bar chart. (c) Derive the expression for distance d in terms of m_1, m_2, μ, and k, and evaluate the expression using extreme case analysis and unit analysis.

FIGURE P7.77



78. ** BIO EST **Impact extinction** 65 million years ago over 50% of all species became extinct, ending the reign of dinosaurs and opening the way for mammals to become the dominant land vertebrates. A theory for this extinction, with considerable supporting evidence, is that a 10-km-wide 1.8×10^{15}-kg asteroid traveling at speed 11 km/s crashed into Earth. Use this information and any other information or assumptions of your choosing to (a) estimate the change in velocity of Earth due to the impact; (b) estimate the average force that Earth exerted on the asteroid while stopping it; and (c) estimate the internal energy produced by the collision (a bar chart for the process might help). By comparison, the atomic bombs dropped on Japan during World War II were each equivalent to 15,000 tons of TNT (1 ton of TNT releases 4.2×10^9 J of energy).

79. ** Newton's cradle is a toy that consists of several metal balls touching each other and suspended on strings (**Figure P7.79**). When you pull one ball to the side and let it strike the next ball, only one ball swings out on the other side. When you use two balls to hit the others, two balls swing out. Can you account for this effect using your knowledge about elastic collisions?

FIGURE P7.79

80. * Pose a problem involving work-energy ideas with real numbers. Then solve the problem choosing two different systems and discuss whether the answers were different.

81. ** **Six Flags roller coaster** A loop-the-loop on the Six Flags Shockwave roller coaster has a 10-m radius (**Figure P7.81**). The car is moving at 24 m/s at the bottom of the loop. Determine the force exerted by the seat of the car on an 80-kg rider when passing inverted at the top of the loop.

FIGURE P7.81

82. ** **Designing a ride** You are asked to help design a new type of loop-the-loop ride. Instead of rolling down a long hill to generate the speed to go around the loop, the 300-kg cart starts at rest (with two passengers) on a track at the same level as the bottom of the 10-m radius loop. The cart is pressed against a compressed spring that, when released, launches the cart along the track around the loop. Choose a spring of the appropriate spring constant to launch the cart so that the downward force exerted by the track on the cart as it passes the top of the loop is 0.2 times the force that Earth exerts on the cart. The spring is initially compressed 6.0 m.

Reading Passage Problems

BIO **Metabolic rate** Energy for our activities is provided by the chemical energy of the foods we eat. The absolute value of the rate of conversion of this chemical energy into other forms of energy $(\Delta E/\Delta t)$ is called the metabolic rate. The metabolic rate depends on many factors—a person's weight, physical activity, the efficiency of bodily processes, and the fat-muscle ratio. **Table 7.9** lists the metabolic rates of people under several different conditions and in several different units of measure: 1 kcal = 1000 calories = 4186 J. Dieticians call a kcal simply a Cal. A piece of bread provides about 70 kcal of metabolic energy.

In 1 hour of heavy exercise a 68-kg person metabolizes 600 kcal − 90 kcal = 510 kcal more energy than when at rest. Typically, reducing kilocalorie intake by 3500 kcal (either by burning it in exercise or not consuming it in the first place) results in a loss of 0.45 kg of body mass (the mass is lost through exhaling carbon dioxide—the product of metabolism).

TABLE 7.9 Energy usage rate during various activities

Type of activity	$\Delta E/\Delta t$ (watts)	$\Delta E/\Delta t$ (kcal/h)	$\Delta E/\Delta t$ (kcal/day)
45-kg person at rest	80	70	1600
68-kg person at rest	100	90	2100
90-kg person at rest	120	110	2600
68-kg person walking 3 mph	280	240	5800
68-kg person moderate exercise	470	400	10,000
68-kg person heavy exercise	700	600	14,000

83. Why is the metabolic rate different for different people?
(a) They have different masses.
(b) They have different body function efficiencies.
(c) They have different levels of physical activity.
(d) All of the above

84. A 50-kg mountain climber moves 30 m up a vertical slope. If the muscles in her body convert chemical energy into gravitational potential energy with an efficiency of no more than 5%, what is the chemical energy used to climb the slope?
(a) 7 kcal (b) 3000 J
(c) 70 kcal (d) 300,000 J

85. If 10% of a 50-kg rock climber's total energy expenditure goes into the gravitational energy change when climbing a 100-m vertical slope, what is the climber's average metabolic rate during the climb if it takes her 10 min to complete the climb?
(a) 8 W (b) 80 W (c) 500 W
(d) 700 W (e) 800 W

86. **EST** A 68-kg person wishes to lose 4.5 kg in 2 months. Estimate the time that this person should spend in moderate exercise each day to achieve this goal (without altering her food consumption).
(a) 0.4 h (b) 0.9 h (c) 1.4 h
(d) 1.9 h (e) 2.4 h

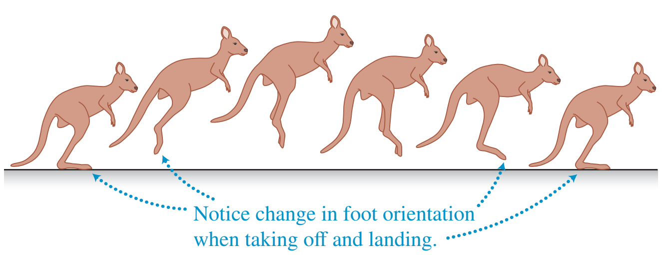
87. **EST** A 68-kg person walks at 5 km per hour for 1 hour a day for 1 year. Estimate the *extra* number of kilocalories of energy used because of the walking.
(a) 40,000 kcal (b) 47,000 kcal
(c) 88,000 kcal (d) 150,000 kcal

88. Suppose that a 90-kg person walks for 1 hour each day for a year, expending 50,000 extra kilocalories of metabolic energy (in addition to his normal resting metabolic energy use). What approximately is the person's mass at the end of the year, assuming his food consumption does not change?
(a) 57 kg (b) 61 kg (c) 64 kg
(d) 66 kg (e) 67 kg

BIO Kangaroo hopping Hopping is an efficient method of locomotion for the kangaroo (see **Figure 7.18**). When the kangaroo is in the air, the Earth-kangaroo system has a combination of gravitational potential energy and kinetic energy. When the kangaroo lands, its Achilles tendons and the attached muscles stretch—a form of elastic potential energy. This elastic potential energy is used along with additional muscle tension to launch the kangaroo off the ground for the next hop. In the red kangaroo, more than 50% of the total energy used during each hop is recovered elastic potential energy. This is so efficient that the kangaroo's metabolic rate actually decreases slightly as its hopping speed increases from 8 km/h to 25 km/h.

The horizontal and vertical force components exerted by a firm surface on a kangaroo's feet while it hops are shown in **Figure 7.19a**. The vertical force $N_{S\text{ on }K\,y}$ (Figure 7.19b) varies: when the kangaroo is not touching the surface S, the force is zero; when it is pushing off, the force is about three times the gravitational force that Earth exerts on the kangaroo. The surface exerts a backward horizontal force ($F_{S\text{ on }K\,x}$) on the kangaroo's foot while it lands and a forward horizontal force as it pushes off for the next hop (Figure 7.19c), similar to what happens to a human foot when landing in front of the body and when pushing off for another step when behind the body.

FIGURE 7.18 A kangaroo hopping.



89. Why is hopping an energy-efficient mode of transportation for a kangaroo?
(a) There is less resistance since there is less contact with the ground.
(b) The elastic energy stored in muscles and tendons when landing is returned to help with the next hop.
(c) The kangaroo has long feet that cushion the landing.
(d) The kangaroo's long feet help launch the kangaroo.

90. Why does the horizontal force exerted by the ground on the kangaroo change direction as the kangaroo lands and then hops forward?
(a) The backward force when it lands prevents it from slipping, and the forward force when taking off helps propel it forward.
(b) One horizontal force is needed to help stop the kangaroo's fall and the other to help launch its upward vertical hop.
(c) Both forces oppose the kangaroo's motion, but one looks like it is forward because the kangaroo is moving fast.
(d) The kangaroo is not an inertial reference frame, and the forward force is not real.
(e) All of the above

91. Which answer below is closest to the vertical impulse that the ground exerts on the kangaroo while it takes off?
(a) 0 (b) $+50\ \text{N}\cdot\text{s}$ (c) $+150\ \text{N}\cdot\text{s}$
(d) $-50\ \text{N}\cdot\text{s}$ (e) $-150\ \text{N}\cdot\text{s}$

92. Which answer below is closest to the vertical impulse due to the gravitational force exerted on the kangaroo by Earth during the short time interval while it takes off?
(a) 0 (b) $+50\ \text{N}\cdot\text{s}$ (c) $+150\ \text{N}\cdot\text{s}$
(d) $-50\ \text{N}\cdot\text{s}$ (e) $-150\ \text{N}\cdot\text{s}$

93. Suppose the net vertical impulse on the 50-kg kangaroo due to all external forces was $+100\ \text{N}\cdot\text{s}$. Which answer below is closest to its vertical component of velocity when it leaves the ground? Assume that the kangaroo is initially at rest.
(a) $+2.0$ m/s (b) $+3.0$ m/s (c) $+4.0$ m/s
(d) $+8.0$ m/s (e) $+10$ m/s

94. Which answer below is closest to the vertical height above the ground that the kangaroo reaches if it leaves the ground traveling with a vertical component of velocity of 2.5 m/s?
(a) 0.2 m (b) 0.3 m (c) 0.4 m
(d) 0.6 m (e) 0.8 m

FIGURE 7.19 The kangaroo stores elastic potential energy in its muscle and tendon when landing and uses some of that energy to make the next hop.