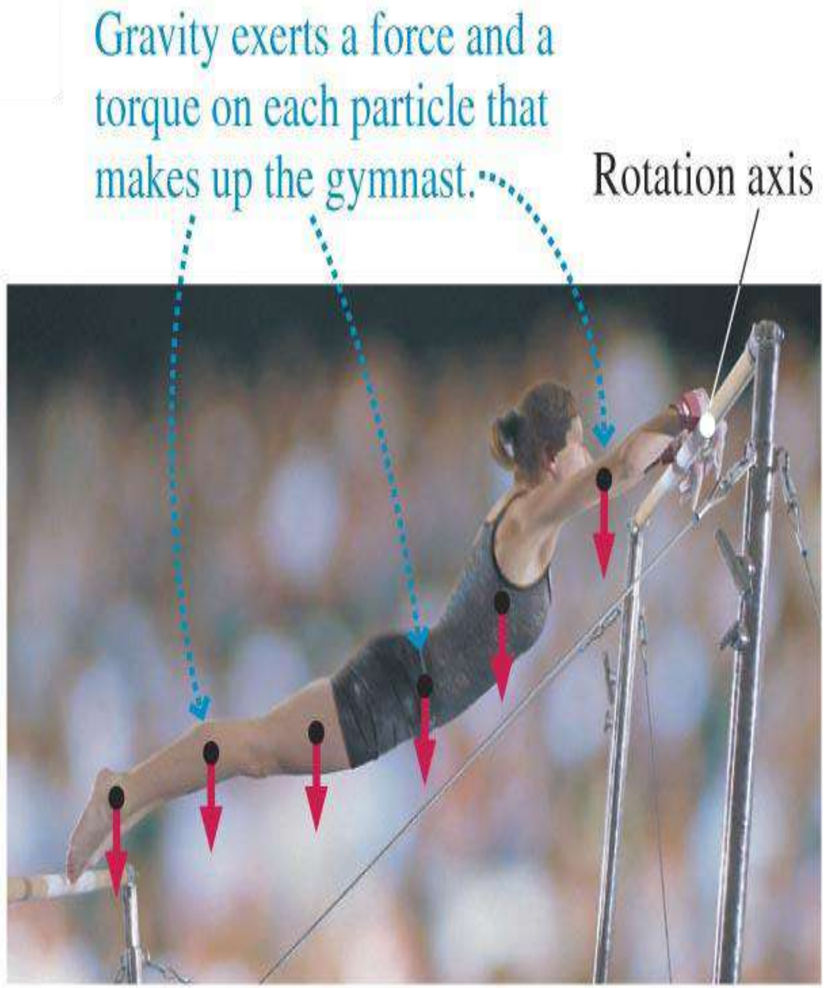
Center of Gravity



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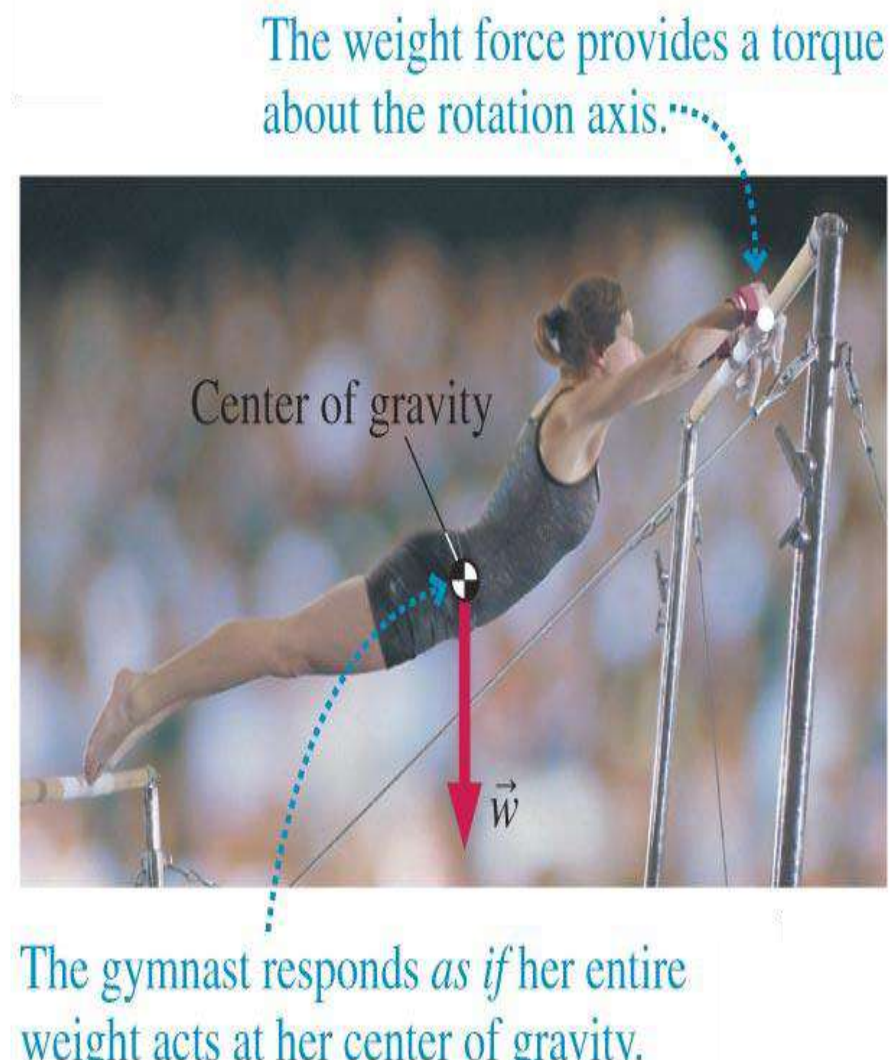


Calculating the Center-of-Gravity Position

TACTICS BOX 7.1 **Finding the center of gravity**

1. Choose an origin for your coordinate system. You can choose any convenient point as the origin.
2. Determine the coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ for the particles of masses $m_1, m_2, m_3, \ldots$, respectively.
3. The x-coordinate of the center of gravity is

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \qquad (7.8)$$

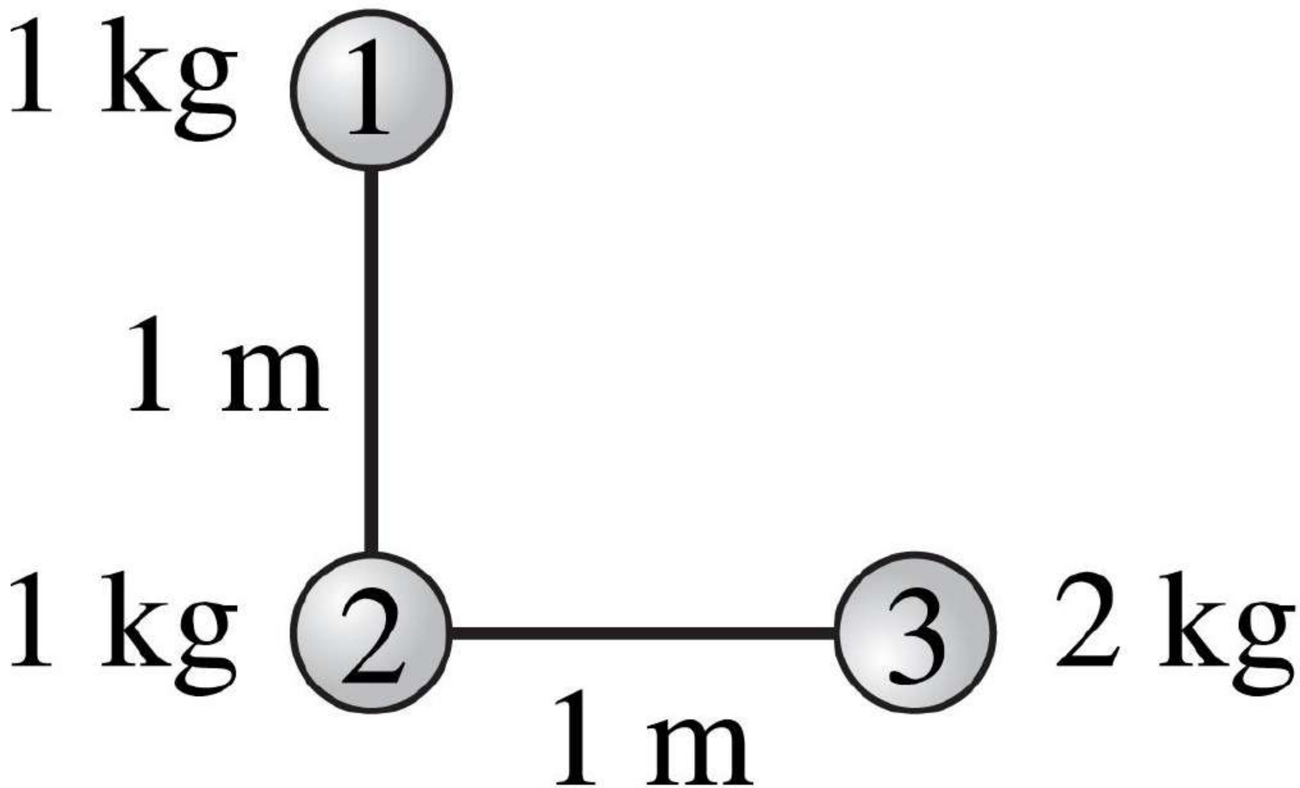
4. Similarly, the y-coordinate of the center of gravity is

$$y_{cg} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \qquad (7.9)$$

Exercises 12–15

Example Problem:

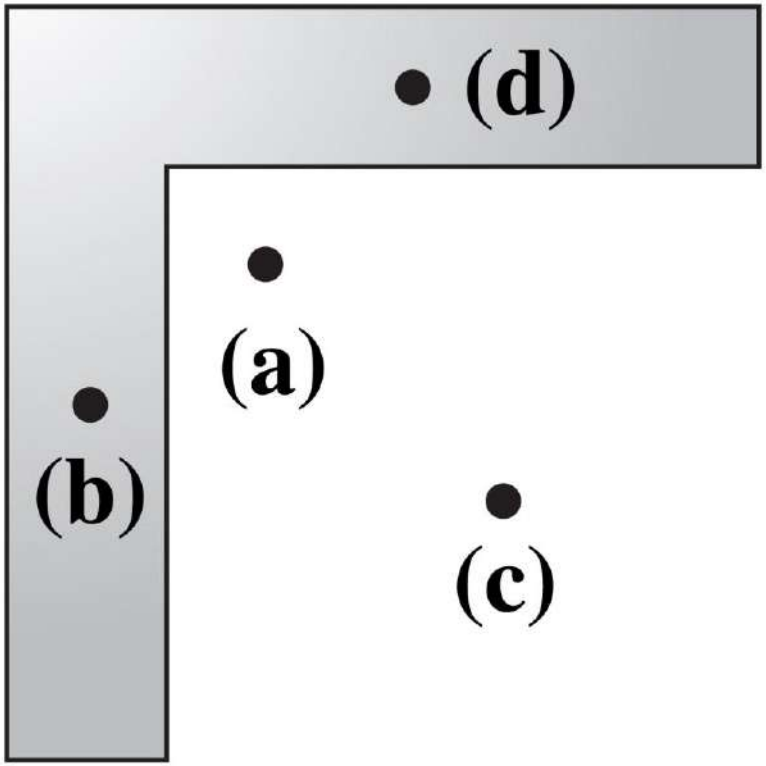
An object consists of the three balls shown, connected by massless rods. Find the *x*- and *y*-positions of the object's center of gravity.



Answer…

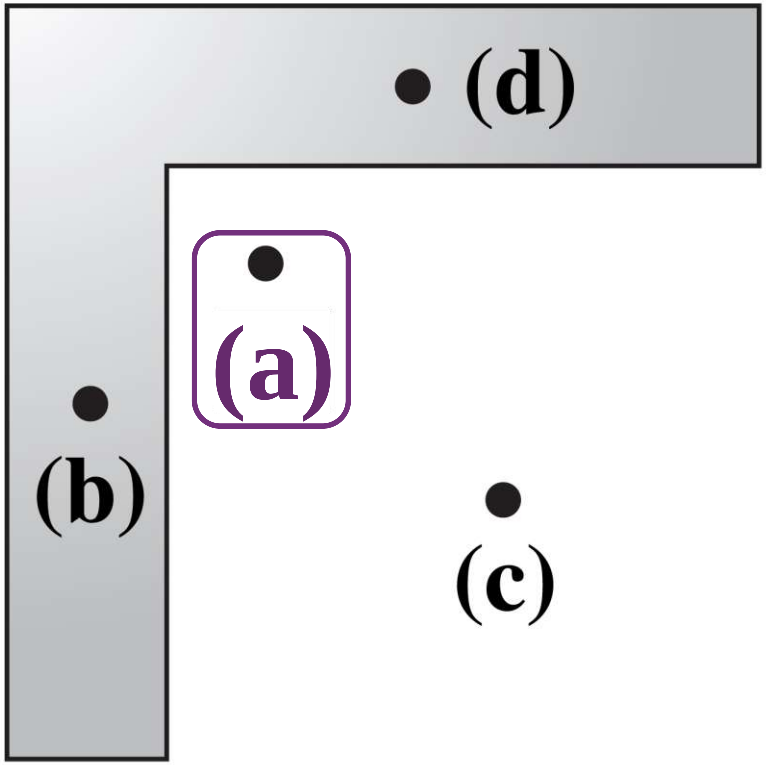
- Let mass #2 be the origin (therefore, $x_2 = 0$ m and $y_2 = 0$ m)
- $x_{cg} = [(0m)(1kg) + (1m)(2kg)] / (1kg + 1kg + 2kg) = 1/2$ m
- $y_{cg} = [(0m)(1kg) + (1m)(1kg)] / (1kg + 1kg + 2 kg) = 1/4$ m
- Answer: $x_{cg} = 1/2$ m and $y_{cg} = 1/4$ m

Which point could be the center of gravity of this L-shaped piece?



Answer

Which point could be the center of gravity of this L-shaped piece?



Newton's Second Law for Rotational Motion

You've learned Newton's second law of motion for *translational* motion: **A net force causes an object to accelerate.** In this chapter, we'll study Newton's second law for *rotational* motion: **A net torque causes an object to have an *angular* acceleration.**

To make the merry-go-round speed up, the girl has to apply a torque to it by pushing at its edge.

Looking Back ◀◀

4.6 Newton's second law

Looking Back

4.6 Newton's second law

Moment of Inertia

We have learned that *mass* is the property of an object that resists acceleration. The property of an object that resists angular acceleration is its *moment of inertia*. The moment of inertia of an object depends not only on its mass but also on how that mass is distributed. “Moment” comes from momentum… meaning motion.

By extending its tail, this cat increases its moment of inertia. This increases its resistance to angular acceleration, making it harder for it to fall.

A net torque applied to an object causes…???

A. a linear acceleration of the object.
B. the object to rotate at a constant rate.
C. the angular velocity of the object to change.
D. the moment of inertia of the object to change.

Answer

A net torque applied to an object causes…

A. a linear acceleration of the object.

B. the object to rotate at a constant rate.

C. the angular velocity of the object to change.

D. the moment of inertia of the object to change.

Another question…

Moment of inertia is…???

A. the rotational equivalent of mass.
B. the point at which all forces appear to act.
C. the time at which inertia occurs.
D. an alternative term for moment arm.

Answer

Moment of inertia is…

A. **the rotational equivalent of mass.**
B. the point at which all forces appear to act.
C. the time at which inertia occurs.
D. an alternative term for moment arm.

Question…

Which statement about an object's center of gravity is *not* true?

A. If an object is free to rotate about a pivot, the center of gravity will come to rest below the pivot.

B. The center of gravity coincides with the geometric center of the object.

C. The torque due to gravity can be calculated by considering the object's weight as acting at the center of gravity.

D. For objects small compared to the earth, the center of gravity and the center of mass are essentially the same.

Answer

Which statement about an object's center of gravity is *not* true?

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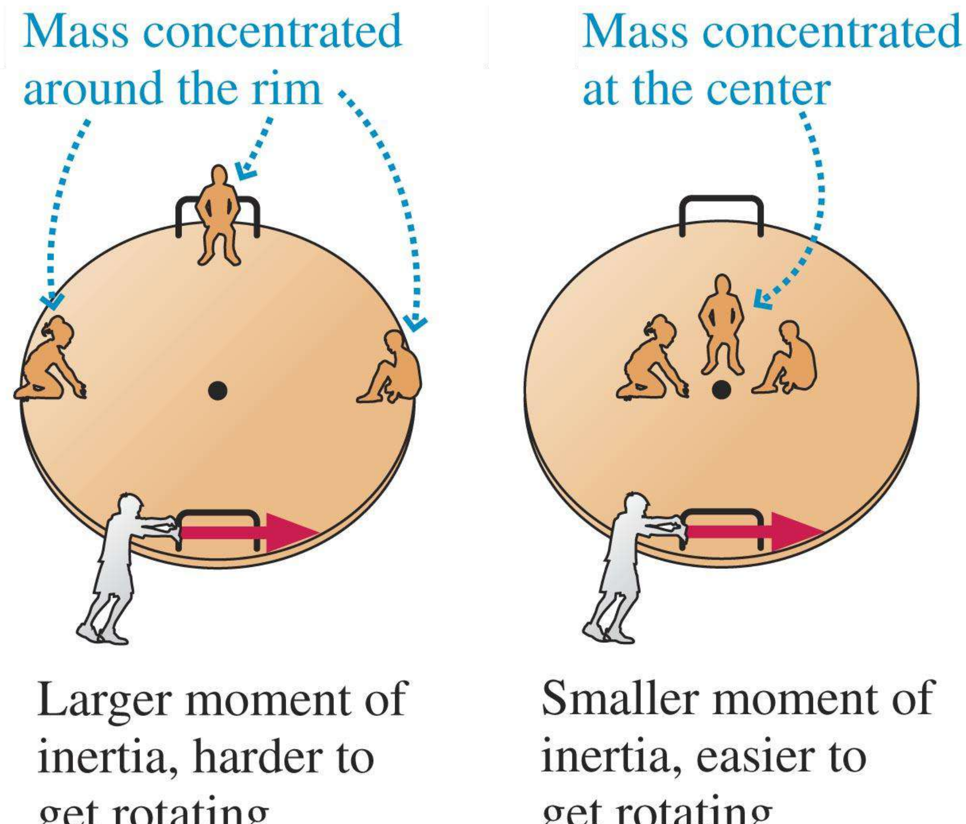
Newton’s Second Law for Rotation

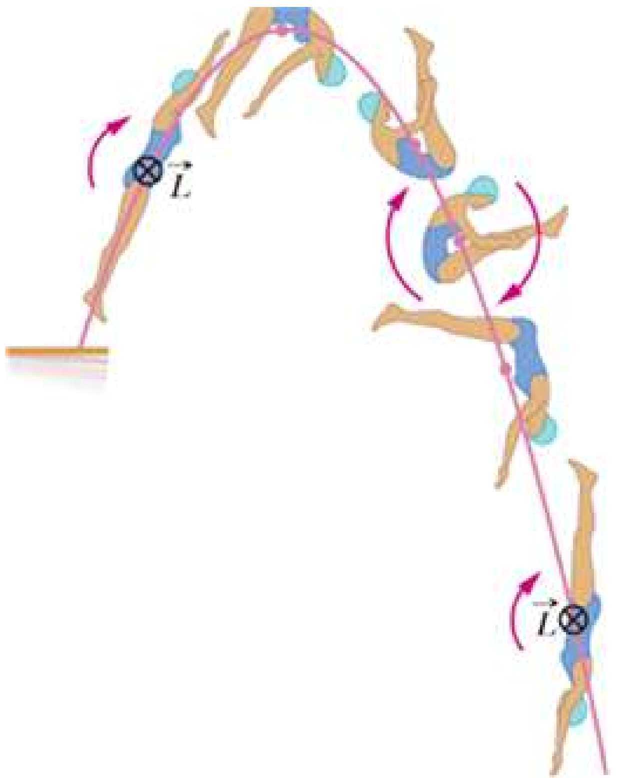
$$a = t / I$$

I = moment of inertia. Objects with larger moments of inertia are harder to get rotating.

$$I = \text{å}\, m_i r_i^2$$



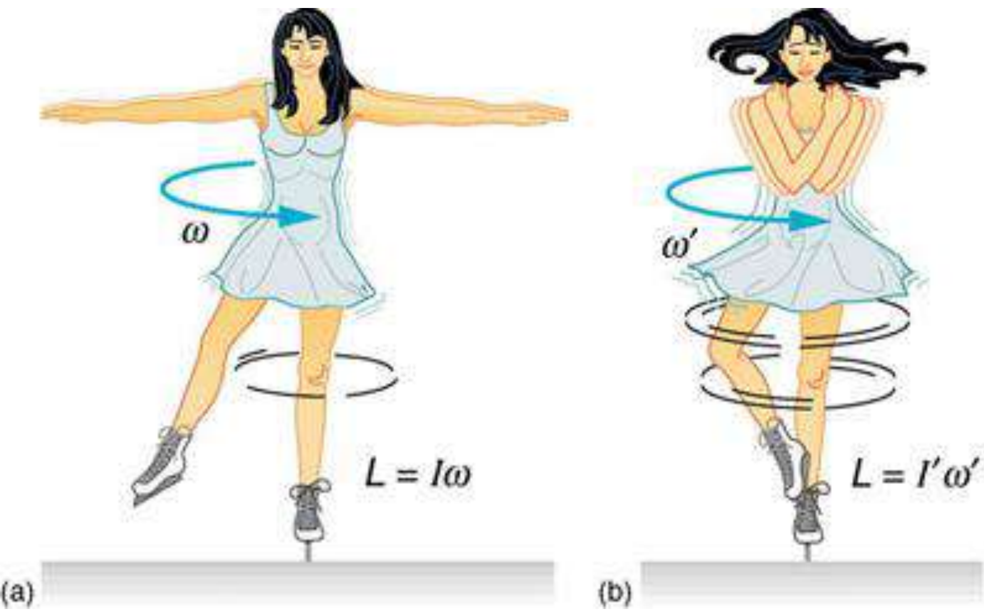
More rotational Inertia and conservation of momentum

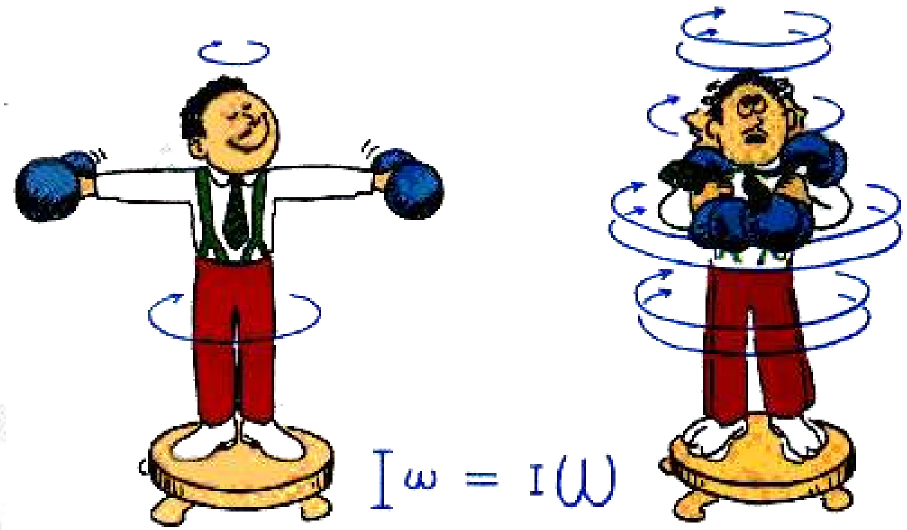


The divers rotational momentum is conserved. Where is angular velocity greatest?

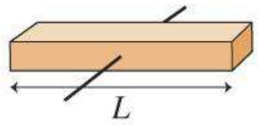
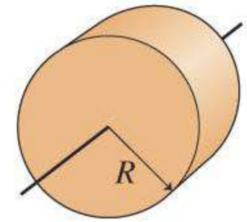
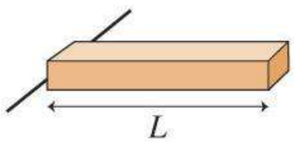
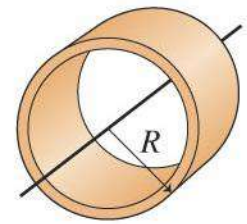
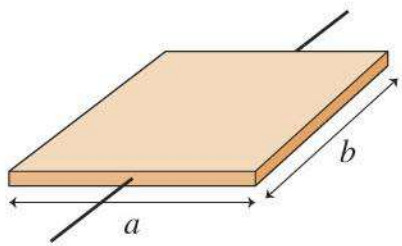
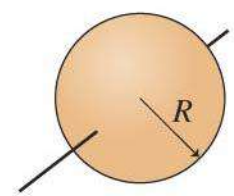
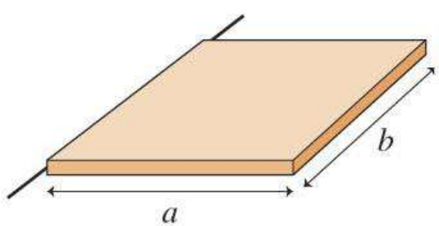
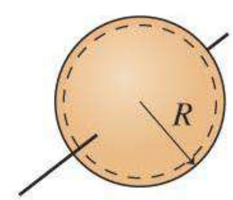
Why does the guy above have a lot of rotational inertia? Which TP has more Inertia?

Rotational Inertia (Ch. 9)





Moments of Inertia of Common Shapes

Object and axis	Picture	*I*	Object and axis	Picture	*I*
Thin rod (of any cross section), about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod (of any cross section), about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Rotational and Linear Dynamics Compared

Rotational dynamics		Linear dynamics	
Torque	τ_{net}	Force	$\vec{F}_{\text{net}}$
Moment of inertia	I	Mass	m
Angular acceleration	α	Acceleration	$\vec{a}$
Second law	$\alpha = \tau_{\text{net}}/I$	Second law	$\vec{a} = \vec{F}_{\text{net}}/m$

PROBLEM-SOLVING STRATEGY 7.1

Rotational dynamics problems

MP™

PREPARE Model the object as a simple shape. Draw a pictorial representation to clarify the situation, define coordinates and symbols, and list known information.

- Identify the axis about which the object rotates.
- Identify the forces and determine their distance from the axis.
- Calculate the torques caused by the forces, and find the signs of the torques.

SOLVE The mathematical representation is based on Newton's second law for rotational motion:

$$\tau_{\text{net}} = I\alpha \qquad \text{or} \qquad \alpha = \frac{\tau_{\text{net}}}{I}$$

- Find the moment of inertia either by direct calculation using Equation 7.14 or from Table 7.4 for common shapes of objects.
- Use rotational kinematics to find angular positions and velocities.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 25

Example Problem

The motor in a CD player exerts a torque of 7.0×10^{-4} N · m. What is the disk's angular acceleration? (A CD has a diameter of 12.0 cm and a mass of 16 g.)

- $I = ½\,MR^2 = ½\,(0.016\ \text{kg})(0.06\ \text{m})^2$
- $I = 2.88 \times 10^{-5}\ \text{kg·m}^2$
- $\alpha = \tau_{net}/I = (7.0 \times 10^{-4}\ \text{N·m}) / (2.88 \times 10^{-5}\ \text{kg·m}^2)$
- **$\alpha = 24\ \text{rad/s}^2$**

Summary

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{net}/I$, where I is the object's moment of inertia about the rotation axis.

This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{net}/m$.

Famous Quote:

- "Give me a lever long enough and I can move the world."
- $F \cdot \ell = F \cdot \ell$ or $F \cdot r = F \cdot r$
- For equilibrium, torques balance out.
- Equilibrium means no acceleration.