Calculus Ch. 5 Reteach Sheet Name KEY

(Calculator) ***LT: I can estimate area using LRAM, MRAM, or RRAM***

1. Consider the region enclosed between the graph of $y = -x^2 + 2x$ and the x-axis for $0 \le x \le 2$. Find the $LRAM_4$, $MRAM_4$, $RRAM_4$ using 4 subintervals.

LRAM = 5/4
MRAM = 1.375
RRAM = 5/4

(Calculator) ***LT: I can evaluate an integral using NINT.***

2. Use NINT to evaluate $\int_0^5 e^{-x^2} dx$ $= .886$

(Calculator) ***LT: I can find the average value of a function.***

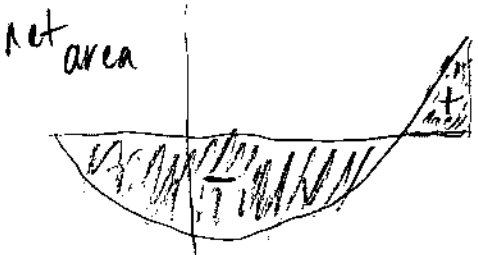
3. Find the average value of $y = -3x^2 - 1$ on the interval $[0, 1]$. At what point in the interval does the function assume its average value?

$\frac{1}{1-0}\int_0^1 -3x^2 - 1 \, dx = -2$ So... $-3x^2 - 1 = -2$ not in the interval

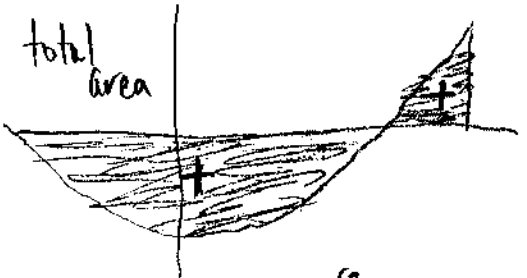
$x = -\frac{\sqrt{3}}{3}$ or $\frac{\sqrt{3}}{3}$ $\left(\frac{\sqrt{3}}{3}, -2\right)$

(Calculator) ***LT: I can evaluate an integral using NINT.***

4. Find the net area and total area of the region between $y = x^2 - 4$ and the x-axis over $[-2,3]$.



$\int_{-2}^{3} x^2 - 4 \, dx = \frac{-25}{3}$



$-\int_{-2}^{2} x^2 - 4 \, dx + \int_{2}^{3} x^2 - 4 \, dx =$

$32/3 + 7/3 = 39/3 = 13$

(Calculator) ***LT: I can approximate area under a curve using the Trap Rule and Simpson's Rule.***

5. A. Use Simpsons Rule and the Trap Rule with n=6 to approximate the value of $\int_0^3 x^3 dx$

   B. Use concavity of the function to predict whether the TRAP approximation is an overestimate or an underestimate.

   C. Find the exact value using NINT to check your answers.

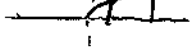
A.) $S = \frac{\frac{1}{2}}{3}\left(0 + 4\left(\frac{1}{8}\right) + 2(1) + 4\left(\frac{27}{8}\right) + 2(8) + 4\left(\frac{125}{8}\right) + 27\right)$

$S = \frac{81}{4}$

$T = \frac{\frac{1}{2}}{2}\left(0 + 2\left(\frac{1}{8}\right) + 2(1) + 2\left(\frac{27}{8}\right) + 2(8) + 2\left(\frac{125}{8}\right) + 27\right)$

$T = \frac{333}{16}$

x	y
0	0
1/2	1/8
1	1
3/2	27/8
2	8
5/2	125/8
3	27

B.)  underestimate

C) $\frac{81}{4} \approx 20.25$

(Calculator) ***LT: I can approximate area using the Trap Rule and Simpson's Rule and apply it to real life situations.***

6. Use Simpson's Rule and the Trap Rule to estimate the value of the integral $\int_3^8 f(x)dx$

$S = \frac{\frac{1}{2}}{3}\left(41 + 4(39) + 2(37) + 4(35) + 2(32) + 4(31) + 2(30) + 4(25) + 2(24) + 4(20) + 15\right)$

$S = \frac{451}{3} \approx 150.333$

$T = \frac{\frac{1}{2}}{2}\left(41 + 2(39) + 2(37) + 2(35) + 2(32) + 2(31) + 2(30) + 2(25) + 2(24) + 2(20) + 15\right)$

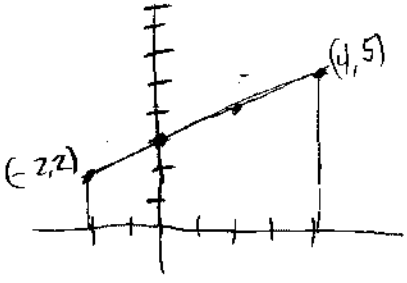
$T = \frac{301}{2} \approx 150.5$

x	y
3.0	~~41~~
3.5	~~39~~
4.0	~~37~~
4.5	~~35~~
5.0	~~32~~
5.5	~~31~~
6.0	~~30~~
6.5	~~25~~
7.0	~~24~~
7.5	20
8.0	15

(Non-Calculator) ***LT: I can evaluate an integral using graphs and area.***

7. Use the graph of the integrand and areas to evaluate the integral.

a. $\int_{-2}^{4}(\frac{x}{2}+3)dx$

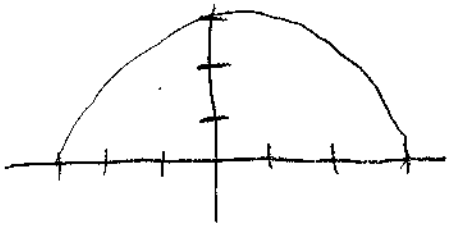


$\frac{1}{2}(b_1+b_2)h = A$

$\frac{1}{2}(2+5)6 = A$

$21 = A$

b. $\int_{-3}^{3}\sqrt{9-x^2}\,dx$



$A = \frac{\pi r^2}{2}$ half circle

$A = \frac{\pi 9}{2}$

$A = \frac{9\pi}{2}$

(Non-calculator) ***LT: I can evaluate an integral using rules for integrals.***

8. Suppose that f and h are continuous functions and that

$\int_1^{10} f(x)dx = -5$	$\int_7^{10} f(x)\,dx = 3$	$\int_7^{10} h(x)dx = 6$

Evaluate:

a. $\int_7^{10}[4f(x)-5h(x)]\,dx$ $\quad 4(3) - 5(6) = 12-30 = -18$

b. $\int_{10}^{1} f(x)dx = -\int_1^{10} f(x)dx = 5$

c. $\int_1^7 f(x)\,dx = \int_1^{10} f(x)\,dx - \int_{10}^{7} f(x)dx = -5-3 = -8$

(non-calculator) ***Lt: I can evaluate the integral using the FTC part 1.***

9. Find $\frac{dy}{dx}$ if $y = \int_0^x (z^4 - z^2)^3 dz$

$$\frac{dy}{dx} = \frac{d}{dx}\int_0^x (z^4 - z^2)^3 dz$$

$$\frac{dy}{dx} = (x^4 - x^2)^3$$

10. Find $\frac{dy}{dx}$ *if* $y = \int_{5x^2}^{2} \frac{\sqrt{1+u^2}}{u} du$

$$\frac{dy}{dx} = \frac{d}{dx} - \int_{2}^{5x^2} \frac{\sqrt{1+u^2}}{u} du \qquad \frac{d}{dx}(5x^2)$$

$$\frac{dy}{dx} = -\frac{\sqrt{1+(5x^2)^2}}{5x^2}(10x)$$

$$\frac{dy}{dx} = \frac{-2\sqrt{1+25x^4}}{x}$$

(non-calculator) ***Lt: I can evaluate the integral using the FTC part 2.***

11. Evaluate the integral using the FTC part 2.

a. $\int_{-1}^{3} (9x^2 + 5)\, dx$

$$\left[3x^3 + 5x\right]_{-1}^{3} = \left(3(3)^3 + 5(3)\right) - \left(3(-1)^3 + 5(-1)\right)$$

$$(81 + 15) - (-3 - 5)$$

$$96 - -8 = 104$$

b. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \csc^2 x\, dx$

P S S T

Sec Sec tan

CSC -CSC cot

$$\left[-\cot x\right]_{\pi/4}^{3\pi/4} = \left(-\cot\tfrac{3\pi}{4}\right) - \left(-\cot\tfrac{\pi}{4}\right)$$

$$= 1 - (-1)$$

$$= 2$$