Name: _______________

Chapter 5:
Relationships within Triangles

Guided Notes

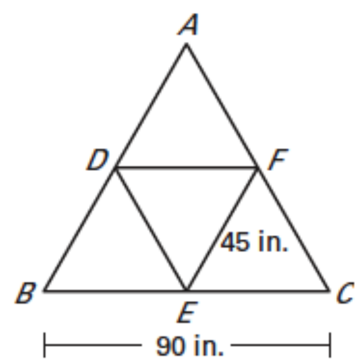
Geometry
Fall Semester

5.1 Midsegment Theorem and Coordinate Proof

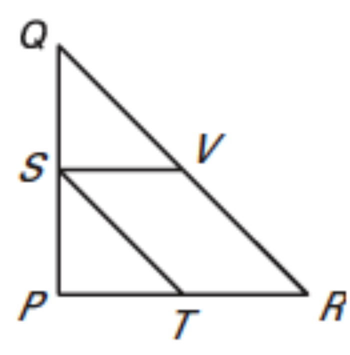
Term	Definition	Example
midsegment of a triangle		
Theorem 5.1 Midsegment Theorem	**The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.**	
coordinate proof		

Examples:

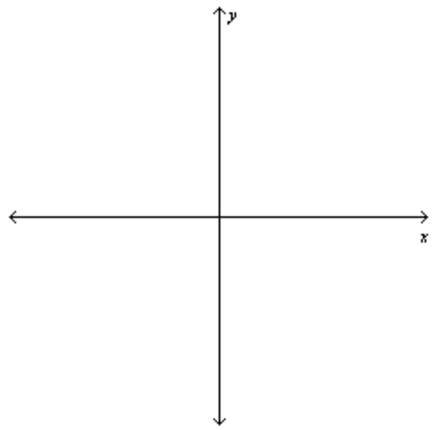
1. In the diagram, $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$. Find DF and AB.



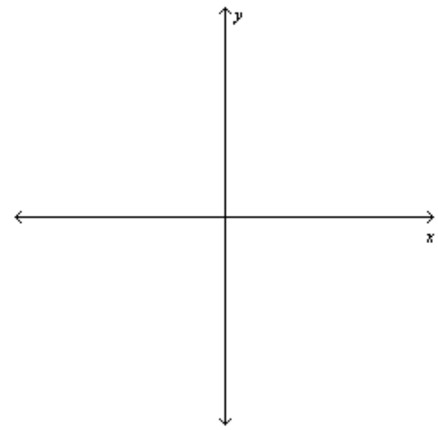
2. In the diagram at the right, $QS = SP$ and $PT = TR$. Show that $\overline{QR} \parallel \overline{ST}$.



3. Place a square and an acute triangle in a coordinate plane in a way that's convenient for finding side lengths. Then assign coordinates.



Square



Acute Triangle

4. Find the length (using distance formula), and midpoint (using midpoint formula) of a diagonal of the square from #3 above.

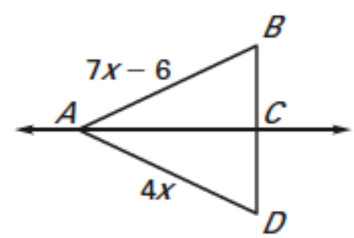
5.2 Use Perpendicular Bisectors

Term	Definition	Example
perpendicular bisector		
equidistant		
Theorem 5.2 Perpendicular Bisector Theorem	In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.	
Theorem 5.3 Converse of the Perpendicular Bisector Theorem	In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.	
concurrent		
point of concurrency		
perpendicular bisector of a triangle		

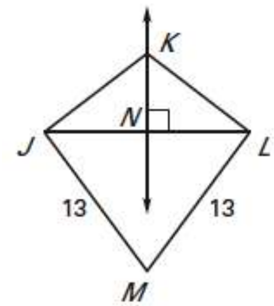
Theorem 5.4 Concurrency of Perpendicular Bisectors of a Triangle	**The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.**	
circumcenter		**1. Acute Triangle** **2. Right Triangle** **3. Obtuse Triangle**
circumscribed		

Examples:

1. $\overleftrightarrow{AC}$ is the perpendicular bisector of $\overline{BD}$. Find AD.



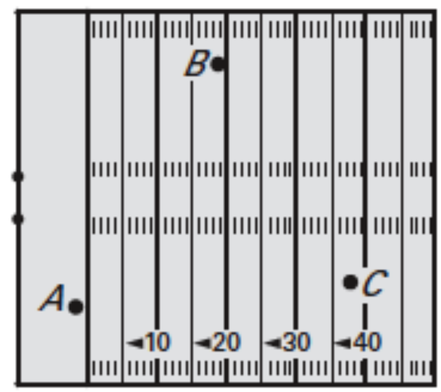
2. In the diagram, $\overleftrightarrow{KN}$ is the perpendicular bisector of $\overline{JL}$.



a) What segment lengths in the diagram are equal?

b) Is M on $\overleftrightarrow{KN}$?

3. Three of your friends are playing catch. You want to join and position yourself so that you are the same distance from each of your friends. Find a location for you to stand.

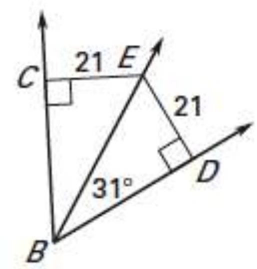


5.3 Use Angle Bisectors of Triangles

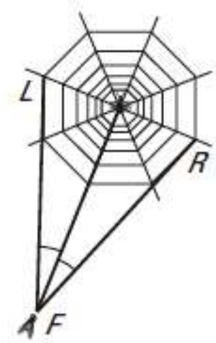
Term	Definition	Example
angle bisector		
distance from a point to a line		
Theorem 5.5 Angle Bisector Theorem	If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.	
Theorem 5.6 Converse of the Angle Bisector Theorem	If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.	
angle bisector of a triangle		
Theorem 5.7 Concurrency of Angle Bisectors of a Triangle	The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.	
incenter		
inscribed		

Examples:

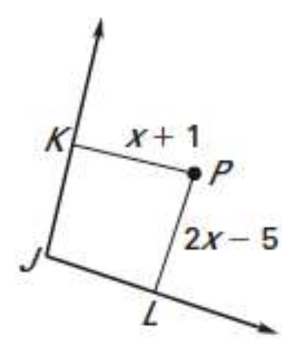
1. Find the measure of $\angle CBE$.



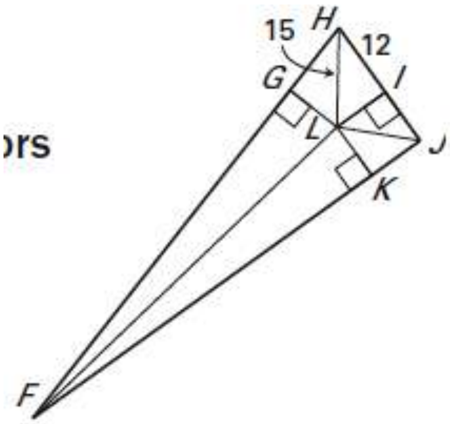
2. A spider's position on its web relative to an approaching fly and the opposite sides of the web form congruent angles as shown. Will the spider have to move farther to reach a fly toward the right edge or the left edge?



3. For what value of x does P lie on the bisector of $\angle J$?



4. In the diagram, L is the incenter of $\triangle FHJ$. Find LK.



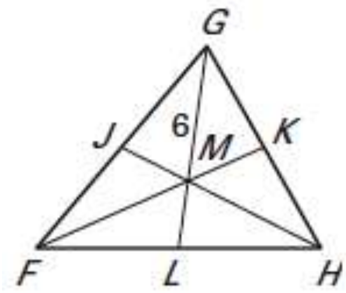
5.4 Use Medians and Altitudes

Term	Definition	Example
median of a triangle		
centroid		
Theorem 5.8 Concurrency of Medians of a Triangle	The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude of a triangle		
Theorem 5.9 Concurrency of Altitudes of a Triangle	The lines containing the altitudes of a triangle are concurrent.	
orthocenter		1. Acute Triangle 2. Right Triangle 3. Obtuse Triangle

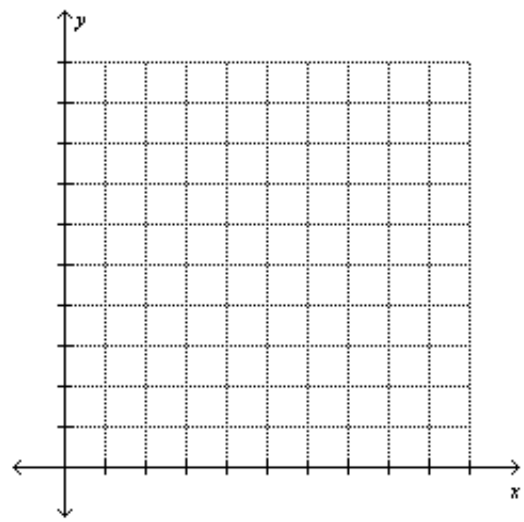
isosceles triangles	**The perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment.**	
equilateral triangle	**The perpendicular bisector, angle bisector, median, and altitude from any angle to the opposite side are all the same segment.**	

Examples:

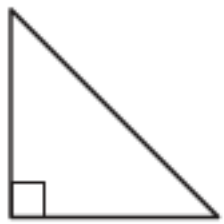
1. In ΔFGH, M is the centroid and $GM = 6$. Find ML and GL.

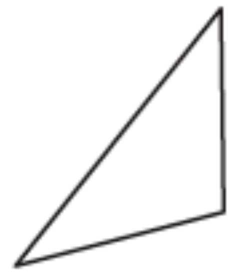


2. The vertices of ΔJKL are $J(1,2), K(4,6)$ and $L(7,4)$. Find the coordinates of the centroid P in ΔJKL.

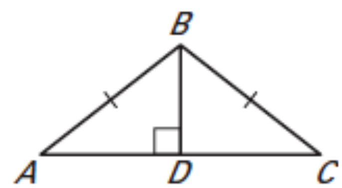


3. Find the orthocenter P of the triangle.

a) 

b) 

4. Prove that the altitude to the base of an isosceles triangle is a median.

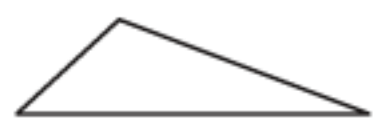


5.5 Use Inequalities in a Triangle

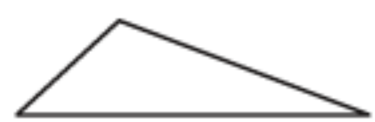
Term	Definition	Example
Theorem 5.10	**If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.**	
Theorem 5.11	**If one angle of a triangle is longer than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.**	
Theorem 5.12 Triangle Inequality Theorem	**The sum of the lengths of any two sides of a triangle is greater than the length of the third side.** **Some sets of line segments cannot be used to form a triangle, because their lengths do not satisfy the inequality.** **When you know two side lengths of a triangle, you can find the range of lengths for the third side.**	

Examples:

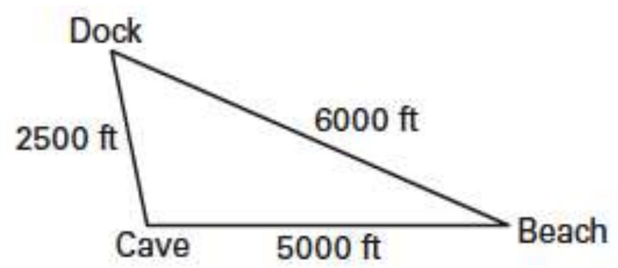
1. a) Mark the largest angle and the longest side.



   b) Mark the smallest angle and shortest side.

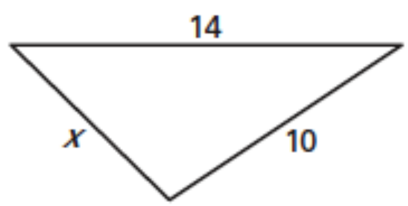


2. A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown in the diagram. One of the angles in the path is about $55°$ and one is about $24°$. What is the angle measure of the path made at the cave?

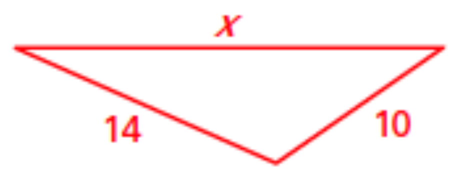


3. A triangle has one side length of 14 and another length of 10. Describe the possible lengths of the third side.

Small values of x
(what if x is the shortest side?)



Large values of x
(what if x is the longest side?)



5.6 Inequalities in Two Triangles and Indirect Proof

Term	Definition	Example
Theorem 5.13 Hinge Theorem (SAS Inequality)	If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.	
Theorem 5.14 Converse of the Hinge Theorem (SSS Inequality)	If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is larger than the included angle of the second triangle.	
indirect reasoning		
indirect proof (proof by contradiction)		

How to Write an Indirect Proof

Step 1	*Identify* the statement you want to prove. *Assume* temporarily that this statement is false by assuming that its opposite is true.
Step 2	*Reason* logically until you reach a contradiction.
Step 3	*Point out* that the desired conclusion must be true because the contradiction proves the temporary assumption false.