Name: ________________

Chapter 2: Reasoning and Proof

Guided Notes

Geometry
Fall Semester

2.1 Use Inductive Reasoning

Term	Definition	Example
conjecture	An unproven statement that is based on observations.	
inductive reasoning	The process of finding a pattern for specific cases and writing a conjecture for the general case.	
counterexample	A specific example for which the conjecture is false.	

Examples:

1. Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

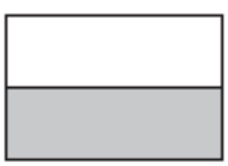
Figure 1 

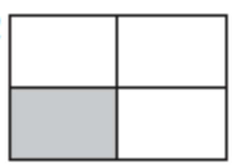
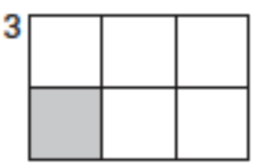
Figure 2 

Figure 3 

2. Describe the pattern in the numbers -1, -4, -16, -64, ... Then write the next three numbers in the pattern.

3. Given five noncollinear points, make a conjecture about the number of different ways to connect the points.

Number of Points	1	2	3	4	5
Picture					

4. Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of three consecutive odd numbers.

Step one: Find a pattern using groups of small numbers.

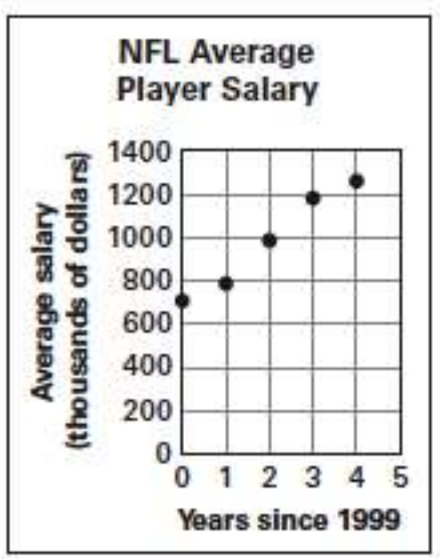
Step two: Make a conjecture.

Step three: Test your conjecture.

5. A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Student's conjecture: **The difference of any two numbers is always smaller than the larger number.**
(Hint: To find a counterexample you need to find an example that is opposite what the student is saying. Prove him/her wrong using an example.)

6. This scatter plot shows the average salary of players in the National Football League (NFL) since 1999. Make a conjecture based on the graph.



2.2 Analyze Conditional Statements

Term	Definition	Example
conditional statements ($p \rightarrow q$)	A logical statement that has two parts, a hypothesis and a conclusion.	
if-then form	A form of a conditional statement in which the "if" part contains the hypothesis and the "then" part contains the conclusion.	
hypothesis (p)	The "if" part of a conditional statement.	
conclusion (q)	The "then" part of a conditional statement.	
negation ($\sim p$)	The opposite of the original statement.	
converse ($q \rightarrow p$)	Formed by switching the hypothesis and conclusion.	
inverse ($\sim p \rightarrow \sim q$)	Formed by negating both the hypothesis and conclusion.	
contrapositive ($\sim q \rightarrow \sim p$)	Formed by writing the converse and then negating both the hypothesis and conclusion.	

equivalent statements	Two statements that are both true or both false.	
perpendicular lines	Two lines that intersect to form right angles.	
biconditional statements $(p \leftrightarrow q)$	A statement that contains the phrase "if and only if".	

Examples:

1. Rewrite the conditional statement in If-then form.

Statement: All vertebrates have a backbone.

If-then form: If ________________________________ , then ___________

_______________________.

2. Write the If-then form, the converse, the inverse, and the contrapositive of the conditional statement . . . "Olympians are athletes." Decide whether each statmenet is true or false.

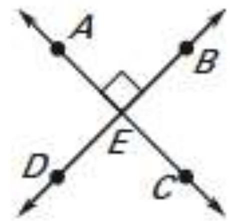
If-then form:

Converse:

Inverse:

Contrapositive:

3. Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AED$ and $\angle BEC$ are a linear pair.

4. Write the definition of parallel lines as a biconditional.

Definition: If two lines lie in the same plane and do not intersect, then they are parallel.

Converse:

Biconditional:

2.3 Apply Deductive Reasoning

Term	Definition	Example
deductive reasoning	Using facts, definitions, accepted properties, and the laws of logic to form an argument.	
Law of Detachment	If $p \rightarrow q$ is a true conditional and p is true, then q is true. Also called a direct argument.	
Law of Syllogism	If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true. Also called the chain rule.	

Examples:

1. Use the **Law of Detachment** to make a valid conclusion statement.

a). If two angles have the same measure, then they are congruent. You are given that $m\angle A = m\angle B$.

Hypothesis: ____.

Conclusion: ____.

Valid conclusion: ______________________________.

b). Jesse goes to the gym every weekday. Today is Monday.

Write as if-then statement: ____.

Hypothesis: ______________________________.

Conclusion: _______________________________.

Valid conclusion: ______________________________________.

2. If possible, use the **Law of Syllogism** to write the conditional statement that follows from the pair of true statements.

a). If Ron eats lunch today, then he will eat a sandwich. If Ron eats a sandwich, then he will drink a glass of milk.

Identify parts of first conditional statement:

Hypothesis: _________________________________.

Conclusion: _________________________________.

Identify parts of second conditional statement:

Hypothesis: _________________________________.

Conclusion: __________________________________.

New conditional statement using Law of Syllogism:

____.

b). If x^2 > 36, then x^2 > 30. If x > 6, then x^2 > 36.

1st statement: Hyp. → Concl. →

2nd statement: Hyp. → Concl. →

New conditional statement using the Law of Syllogism:

____.

c). If a triangle is equilateral, then all of its sides are congruent. If a triangle is equilateral, then all angles in the interior of the triangle are congruent.

1st statement: Hyp → Concl. →

2nd statement: Hyp. → Concl. →

New conditional statement using the Law of Syllogism:

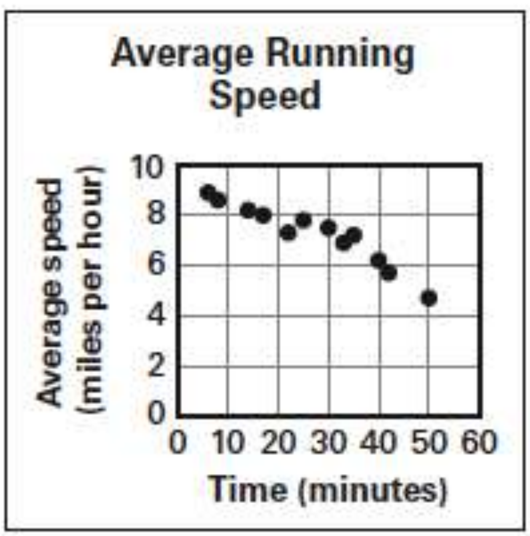
____

____.

3. Inductive or Deductive Reasoning?

Remember . . . Inductive Reasoning is based on observation and pattern. We don't always know whether our conjecture is true. Deductive Reasoning is based on fact.

Tell whether the statement is the result of inductive or deductive reasoning.



a). The runner's average speed decreases as the time spent running increases.

b). The runner's average speed is slower when running for 40 minutes than when running for 10 minutes.

Chapter 2 Extension: Symbolic Notation and Truth Tables

Term	Definition	Example
truth value		
truth table		

2.4 Use Postulates and Diagrams

Term	Definition	Example
Postulate 1	Ruler Postulate	
Postulate 2	Segment Addition Postulate	
Postulate 3	Protractor Postulate	
Postulate 4	Angle Addition Postulate	

Point, Line, and Plane Postulates

Postulate 5	Through any two points there exists exactly one line.	
Postulate 6	A line contains at least two points.	
Postulate 7	If two lines intersect, then their intersection is exactly one point.	
Postulate 8	Through any three noncollinear points there exists exactly one plane.	
Postulate 9	A plane contains at least three noncollinear points.	
Postulate 10	If two points lie in a plane, then the line containing them lies in the plane.	
Postulate 11	If two planes intersect, then their intersection is a line.	

line perpendicular to a plane	A line is ⊥ to a plane if and only if the line intersects the plane at a point that is ⊥ to every line on the plane.	

2.5 Reason Using Properties from Algebra

Algebraic Properties of Equality	
Let a, b, and c be real numbers.	
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.
Distributive Property	$a(b + c) = ab + ac$, where a, b, and c are real numbers.

Properties of Equality			
Property	**Real Numbers**	**Segments**	**Angles**
Reflexive	For any real number a, $a = a$.	For any segment AB, AB = BA.	For any angle A, $m\angle A = m\angle A$.
Symmetric	For any real numbers a and b, if $a = b$, then $b = a$.	For any segments AB and CD, if $AB = CD$, then $CD = AB$.	For any angles A and B, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive	For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	For any segments AB, CD, and EF, if $AB = CD$ and $CD = EF$, then $AB = EF$.	For any angles A, B, and C, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Examples:

1. Solve the following equation and write reasons for each step.

STEP	REASON
1. $2x+3=9-x$	1.
2.	2.
3.	3.
4.	4.

2. Solve, using the Distributive Property. Write reasons for each step.

1. $-4(6x+2)=64$	1.
2.	2.
3.	3.
4.	4.

3. A motorist travels 5 miles per hour slower than the speed limit (s) for 3.5 hours. The distance traveled (d) can be determined by the formula $d=3.5(s-5)$. Solve for s. Write reasons for each step.

1. $d=3.5(s-5)$	1.
2.	2.
3.	3.
4.	4.

4. Use properties of equality to show that $CF = AD$
Take your given statements from the diagram.

Equation	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

2.6 Prove Statements about Segments and Angles

Term	Definition	Example
proof		
two-column proof		
theorem		

Theorem 2.1 Congruence of Segments Segment congruence is reflexive, symmetric, and transitive.	
Reflexive	For any segment *AB*, $\overline{AB} \cong \overline{AB}$.
Symmetric	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
Transitive	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

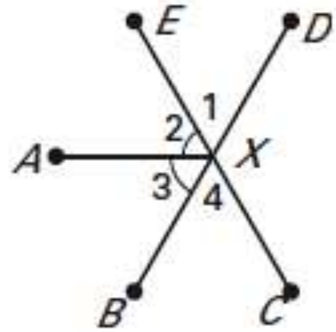
Theorem 2.2 Congruence of Angles Angle congruence is reflexive, symmetric, and transitive.	
Reflexive	For any angle *A*, $m\angle A \cong m\angle A$.
Symmetric	If $m\angle A \cong m\angle B$, then $m\angle B \cong m\angle A$.
Transitive	If $m\angle A \cong m\angle B$ and $m\angle B \cong m\angle C$, then $m\angle A \cong m\angle C$.

Midpoint Definition in Proofs:

Angle Bisector Definition in Proofs:

Congruent Angles and Segments definition in Proofs:

Examples:



1. WRITE A TWO-COLUM PROOF

Use the diagram to prove that $m\angle 1 = m\angle 4$.

Given: $m\angle 2 = m\angle 3$, $m\angle AXD = m\angle AXC$

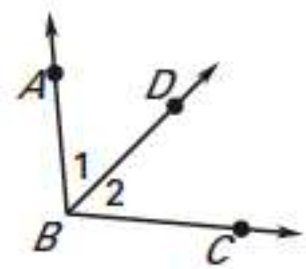
Prove: $m\angle 1 = m\angle 4$

Statements	Reasons
1. $m\angle AXC = m\angle AXD$	1.
2. $m\angle AXD = m\angle 1 + m\angle 2$	2.
3. $m\angle AXC = m\angle 3 + m\angle 4$	3.
4. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	4.
5. $m\angle 2 = m\angle 3$	5.
6. $m\angle 1 + m\angle 3 = m\angle 3 + m\angle 4$	6.
7. $m\angle 1 = m\angle 4$	7.

2. Name the property that is illustrated by the following statement.

If $\angle 5 \cong \angle 3$, then $\angle 3 \cong \angle 5$.

3. If you know that $\overrightarrow{BD}$ bisects $\angle ABC$, prove that $m\angle ABC$ is two times $m\angle 1$.



Given: $\overrightarrow{BD}$ bisects $\angle ABC$

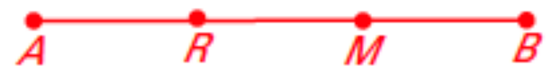
Prove: $m\angle ABC = 2 \bullet m\angle 1$

Statements	Reasons
1. $\overrightarrow{BD}$ bisects $\angle ABC$	1. ________________________
2. ____________________	2. $\angle$ bisector $\rightarrow$ $\cong\angle$'s
3. ____________________	3. $\cong\angle$'s $\rightarrow = \angle$'s
4. $m\angle 1 + m\angle 2 = m\angle ABC$	4. ________________________
5. $m\angle 1 + m\angle$ ___ $= m\angle ABC$	5. Subst. P.O.E.
6. ____________________	6. Simplifiy

4 Complete the following two-column proof.

Given: $\boldsymbol{R}$ is the midpoint of $\overline{\boldsymbol{AM}}$ and $\boldsymbol{MB} = \boldsymbol{AR}$.

Prove: $\boldsymbol{M}$ is the midpoint of $\overline{\boldsymbol{RB}}$.



Statements	Reasons
1. $\boldsymbol{R}$ is the midpoint of $\overline{\boldsymbol{AM}}$. $\boldsymbol{MB} = \boldsymbol{AR}$	1.
2. $\overline{\boldsymbol{AR}} \cong \overline{\boldsymbol{RM}}$	2.
3. $\boldsymbol{AR} = \boldsymbol{RM}$	3.
4.	4. Trans. P.O.E.
5. $\overline{\boldsymbol{MB}} \cong \overline{\boldsymbol{RM}}$	5.
6. M is the midpoint of $\overline{\boldsymbol{RB}}$	6.

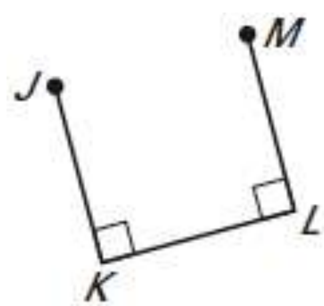
2.7 Prove Angle Pair Relationships

Term	Definition	Example
Theorem 2.3 Right Angles Congruence Theorem	All right angles are congruent.	
linear pair		
Postulate 12 Linear Pair Postulate	If two angles form a linear pair, then they are supplementary.	
vertical angles		
Theorem 2.6 Vertical Angles Congruence Theorem	Vertical angles are congruent.	

Examples:

1. Given: $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$

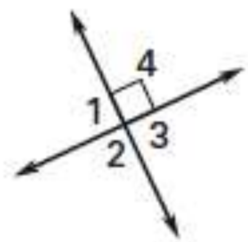
   Prove: $\angle K \cong \angle L$



Statements	**Reasons**
1. $\overline{JK} \perp \overline{KL}$, $\overline{ML} \perp \overline{KL}$	1. ________________________
2. ____________________	2. $\perp \rightarrow rt\angle$'s
3. $\angle K \cong \angle L$	3. ________________________

2. Given: $\angle 4$ **is a right angle**

Prove: $m\angle 2 = 90°$



Statements	**Reasons**
1. $\angle 4$ is a right angle	1. ____________________
2. $m\angle 4 = 90°$	2. ____________________
3. $\angle 2 \cong \angle 4$	3. ____________________
4. $m\angle 2 = m\angle 4$	4. ____________________
5. $m\angle 2 = 90°$	5. ____________________

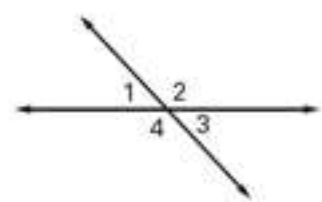
3. Use the diagram to decide if the statement is true or false.

a) If $m\angle 1 = 47°$, then $m\angle 2 = 43°$.

b) If $m\angle 1 = 47°$, then $m\angle 3 = 47°$.

c) $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$

d) $m\angle 1 + m\angle 4 = m\angle 2 + m\angle 3$



4. Find the value of the variables and the measure of each angle in the diagram.