Solutions Key

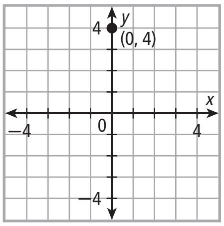
Extending Transformational Geometry

ARE YOU READY? PAGE 821

1. E **2.** C

3. A **4.** D

5.



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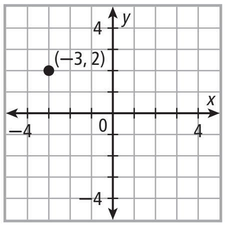

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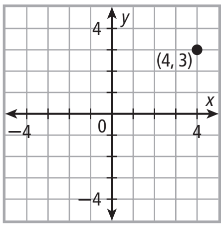

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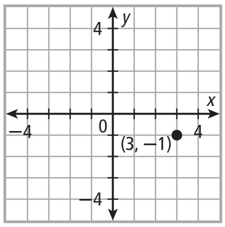

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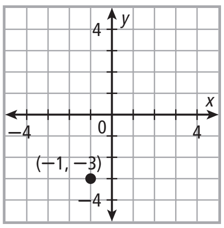
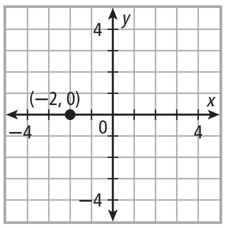


10.



11. no

12. Yes; by Alt. Int. ∠s Thm., $\angle DGE \cong \angle FEG$ and $\angle DEG \cong \angle FGE$. By Reflex. Prop. of ≅, $\overline{GE} \cong \overline{GE}$. So by ASA, $\triangle DEG \cong \triangle FGE$.

13. Yes; $\frac{JM}{JK} = \frac{10}{4} = \frac{5}{2}$, $\frac{JN}{JL} = \frac{7.5}{3} = \frac{5}{2}$. so $\frac{JM}{JK} = \frac{JN}{JL}$; $\angle J \cong \angle J$ by Reflex. Prop. of ≅; $\triangle JKL \sim \triangle JMN$ by SAS ~.

14. Yes; corr. ∠s are ≅, corr. sides are proportional.
$\frac{PQ}{UV} = \frac{QR}{VW} = \frac{RS}{WX} = \frac{SP}{XU} = 2$

15. $(n)\text{m(int. }\angle) = (n - 2)180$
$8\text{m(int. }\angle) = 6(180)$
$\text{m(int. }\angle) = \frac{6}{8}(180) = 135°$

16. sum of internal ∠ measures $= (5 - 2)180 = 540°$

17. $(n)\text{m(ext. }\angle) = 360$
$6\text{m(ext. }\angle) = 360$
$\text{m(ext. }\angle) = 60°$

18. $90 + 5x = (6 - 2)180 = 720$
$5x = 630$
$x = 126$

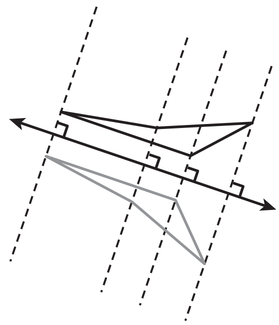
12-1 REFLECTIONS, PAGES 824–830

CHECK IT OUT! PAGES 824–826

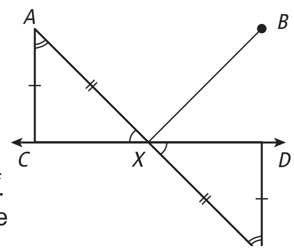
1a. No; the image does not appear to be flipped.

b. Yes; the image appears to be flipped across a line.

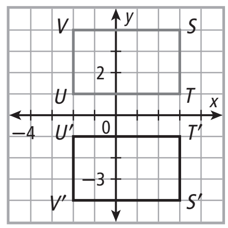
2. **Step 1** Through each vertex draw a line ⊥ to the line of reflection.
Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.
Step 3 Connect the images of the vertices.



3. If *A* and *B* were the same distance from the river, would have $\triangle ACX \cong \triangle BDX$ (see diagram). So $\overline{AX} = \overline{XB'}$ (CPCTC) and therefore $\overline{AX} = \overline{BX}$. So $\overline{AX}$ and $\overline{BX}$ would be congruent.



4. The reflection of (x, y) is $(x, -y)$.
$S(3, 4) \rightarrow S'(3, -4)$
$T(3, 1) \rightarrow T'(3, -1)$
$U(-2, 1) \rightarrow U'(-2, -1)$
$V(-2, 4) \rightarrow V'(-2, -4)$



THINK AND DISCUSS, PAGE 826

1. Possible answer: *ABA′C* is a kite, because $\overline{AB} \cong \overline{A'B}$ and $\overline{AC} \cong \overline{A'C}$. So there are exactly two pairs of ≅ adjacent sides.

2. ℓ is the ⊥ bisector of $\overline{AA'}$.

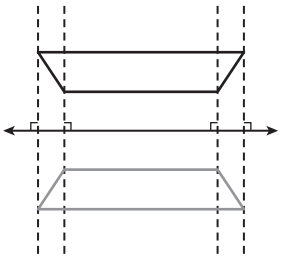
3.

Line of Reflection	Image of (*a*, *b*)	Example
x-axis	(*a*, −*b*)	(1, 2) → (1, −2)
y-axis	(−*a*, *b*)	(1, 2) → (−1, 2)
y = *x*	(*b*, *a*)	(1, 2) → (2, 1)

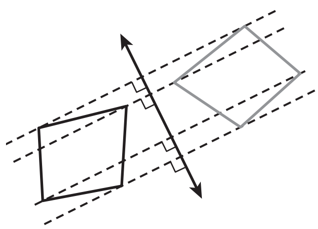
EXERCISES, PAGES 827–830

GUIDED PRACTICE, PAGE 827

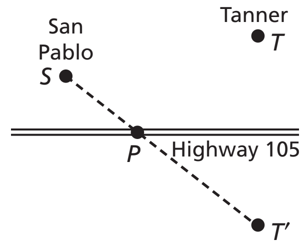
1. They are congruent.
2. Yes; the image appears to be flipped across a line.
3. No; the image does not appear to be flipped.
4. Yes; the image appears to be flipped across a line.
5. No; the image does not appear to be flipped.
6. **Step 1** Through each vertex draw a line ⊥ to the line of reflection.
   Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.
   Step 3 Connect the images of the vertices.



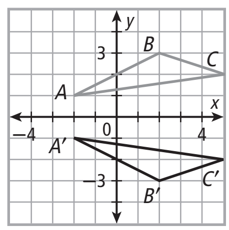
7. **Step 1** Through each vertex draw a line ⊥ to the line of reflection.
   Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.
   Step 3 Connect the images of the vertices.



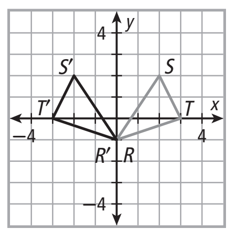
8. **1 Understand the Problem**
   The problem asks you to draw a diagram, locating point. *P* on a highway so that $\overline{SP} + \overline{PT}$ has the least possible value.
   2 Make a Plan
   Let *T′* be the reflection of *T* across the highway. For any point *P* on the highway, $\overline{PT'} \cong \overline{PT}$. So $\overline{SP} + \overline{PT} = \overline{SP} + \overline{PT'}$. $\overline{SP} + \overline{PT'}$ is the least when *S*, *P*, and *T′* are collinear.
   3 Solve
   Reflect *T* across the highway to locate *T′*. Draw $\overline{ST'}$ and locate *P* at the intersection of $\overline{ST'}$ and the highway.
   4 Look Back
   To verify the answer, choose several possible locations for *P* and measure the total length of the access roads for each location.



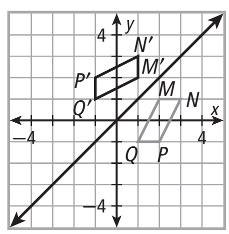
9. The reflection of (*x*, *y*) is (*x*, −*y*).
   A(−2, 1) → *A′*(−2, −1)
   B(2, 3) → *B′*(2, −3)
   C(5, 2) → *C′*(5, −2)



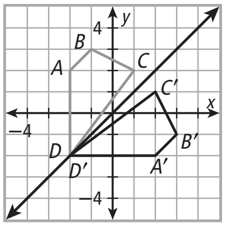
10. The reflection of (*x*, *y*) is (−*x*, *y*).
   R(0, −1) → *R′*(0, −1)
   S(2, 2) → *S′*(−2, 2)
   T(3, 0) → *T′*(−3, 0)



11. The reflection of (*x*, *y*) is (*y*, *x*).
   M(2, 1) → *M′*(1, 2)
   N(3, 1) → *N′*(1, 3)
   P(2, −1) → *P′*(−1, 2)
   Q(1, −1) → *Q′*(−1, 1)



12. The reflection of (x, y) is (y, x).
$A(-2, 2) \rightarrow A'(2, -2)$
$B(-1, 3) \rightarrow B'(3, -1)$
$C(1, 2) \rightarrow C'(2, 1)$
$D(-2, -2) \rightarrow D'(-2, -2)$



PRACTICE AND PROBLEM SOLVING, PAGES 827–829

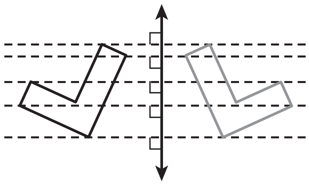
13. No; the image does not appear to be flipped.

14. Yes; the image appears to be flipped across a line.

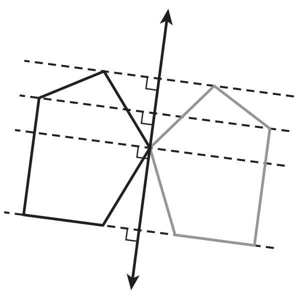
15. Yes; the image appears to be flipped across a line.

16. No; the image does not appear to be flipped.

17. **Step 1** Through each vertex draw a line ⊥ to the line of reflection.
Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.
Step 3 Connect the images of the vertices.

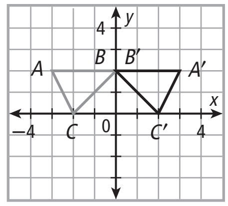


18. **Step 1** Through each vertex draw a line ⊥ to the line of reflection.
Step 2 Measure the distance from each vertex to the line of reflection. Locate image of each vertex on the opposite side of the line of reflection and the same distance from it.
Step 3 Connect the images of the vertices.

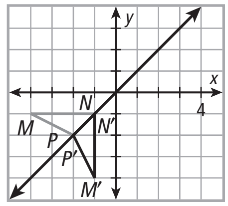


19. **1 Understand the Problem**
The problem asks you to draw a diagram, locating point X on the side rail so that $\overline{AX}$ and $\overline{XC}$ make the same ∠ with the side rail.
2 Make a Plan
Let C' be the reflection of C across the side rail. For any point X on side rail, the ∡ of $\overline{AX}$ and $\overline{XC'}$ with the side rail are ≅ when A, X, and C' are collinear.
3 Solve
Reflect C across the side rail to locate C'. Draw $\overline{AC'}$ and locate P at the intersection of $\overline{AC'}$ and the side rail.
4 Look Back
To verify the answer, choose several possible locations for X and measure the ∡ for each location.

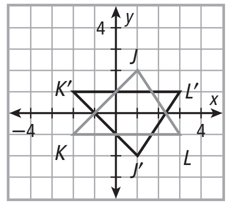
20. The reflection of (x, y) is ($-x$, y).
$A(-3, 2) \rightarrow A'(3, 2)$
$B(0, 2) \rightarrow B'(0, 2)$
$C(-2, 0) \rightarrow C'(2, 0)$



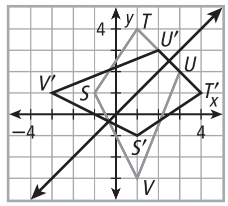
21. The reflection of (x, y) is (y, x).
$M(-4, -1) \rightarrow M'(-1, -4)$
$N(-1, -1) \rightarrow N'(-1, -1)$
$P(-2, -2) \rightarrow P'(-2, -2)$



22. The reflection of (x, y) is (x, $-y$).
$J(1, 2) \rightarrow J'(1, -2)$
$K(-2, -1) \rightarrow K'(-2, 1)$
$L(3, -1) \rightarrow L'(3, 1)$



23. The reflection of (x, y) is (y, x).
$S(-1, 1) \rightarrow S'(1, -1)$
$T(1, 4) \rightarrow T'(4, 1)$
$U(3, 2) \rightarrow U'(2, 3)$
$V(1, -3) \rightarrow V'(-3, 1)$



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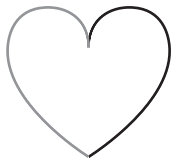
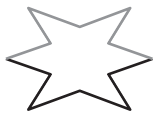


25.

26.



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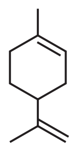
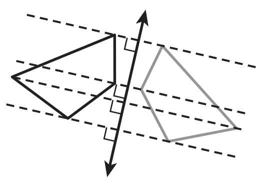


S-(−)-limonene

28.



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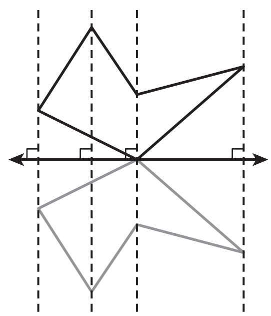

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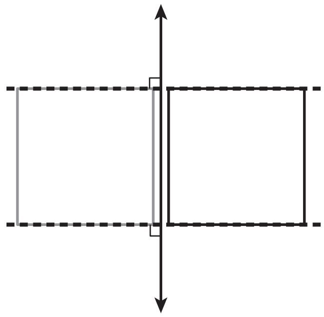


31. (5, 2) → (5, −2)

32. (−3, −7) → (3, −7)

33. (0, 12) → (0, −12)

34. (−3, −6) → (−6, −3)

35. (0, −5) → (−5, 0)

36. (4, 4) → (4, 4)

37a. No; $\overline{TH}$ passes through (4, 3.25) which lies above *E*(4, 3).

b. *H*(5, 4) → *H*′(7, 4)

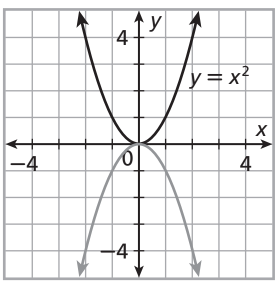
c. slope of $\overline{TH'} = \frac{4-1}{7-1} = 0.5$

equation of $\overline{TH'}$:
$y - 1 = 0.5(x - 1)$ or $y = 0.5x + 0.5$
equation of $\overline{BC}$: $x = 6$
At the intersection, $x = 6$ and
$y = 0.5(6) + 0.5 = 3.5$.
The intersection is at (6, 3.5).

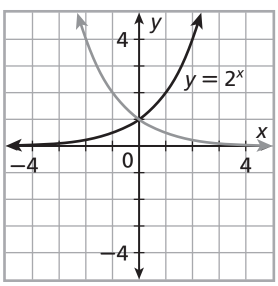
38. The figures are formed by the numerals 1, 2, … 7 and their reflections. The figure formed by numeral 8 is:

39. The line of reflection is the line $y = x$. It is the only possible line of reflection because it must be the ⊥ bisector of the segment connecting the given points.; there is only one such ⊥ bisector.

40.



41.



42. The points on line ℓ remain fixed. Each of these pts. is its own image under a reflection across line ℓ.

43–45. Check students' constructions.

TEST PREP, PAGES 829–830

46. A
(−2, 4) → (−2, −4)

47. J
P → *S*, *M* → *D*, *J* → *G*, and *N* → *W*, so
PMJN → *SDGW*

48. C
(−3, 4) → (−(−3), 4) = (3, 4)

CHALLENGE AND EXTEND, PAGE 830

49. $y \to 3 + (3 - y) = 6 - y$
(4, 2) → (4, 6 − 2) = (4, 4)

50. $x \to 2(1) - x = 2 - x$
(−3, 2) → (2 − (−3), 2) = (5, 2)

51. $y \to x + 2$ *and* $x \to y - 2$
(3, 1) → (1 − 2, 3 + 2) = (−1, 5)

52. Draw $\overline{AA'}$ and $\overline{BB'}$; let *C* be the point. where $\overline{AA'}$ intersects ℓ and let *D* be the point where $\overline{BB'}$ intersects ℓ. By definition of reflection, ℓ is the ⊥ bisector of $\overline{AA'}$ and $\overline{BB'}$.
Therefore, ∠*ACD* and ∠*A′CD* are right. ∠s, $\overline{AC} \cong \overline{A'C}$ by definition of bisector, and $\overline{CD} \cong \overline{CD}$ by Reflex. Prop. of ≅. By SAS, △*ACD* ≅ △*A′CD*. By CPCTC, ∠*CDA* ≅ ∠*CDA′*. ∠*ADB* is comp. to ∠*CDA*, and ∠*A′DB′* is comp. to ∠*CDA′*. So ∠*ADB* ≅ ∠*A′DB′*. $\overline{AD} \cong \overline{A'D}$ by CPCTC, and $\overline{BD} \cong \overline{B'D}$ by def. of bisector. Therefore, △*ADB* ≅ △*A′DB′* by SAS. By CPCTC, $\overline{AB} \cong \overline{A'B'}$.

53. Use the fact that the reflection of a segment is ≅ to its preimage and the definition of ≅ segs.

54. Draw $\overline{AC}$ and $\overline{A'C'}$. Use the fact that the reflection of a segment is ≅ to its preimage to prove △*ABC* ≅ △*A′B′C′* by SSS. By CPCTC ∠*ABC* ≅ ∠*A′B′C′*. So m∠*ABC* = m∠*A′B′C′* by definition of ≅ ∠s.

55. Use the fact that the reflection of a segment is ≅ to its preimage to prove that △*ABC* ≅ △*A′B′C′* by SSS.

56. Since *C* is between *A* and *B*, *AC* + *BC* = *AB*. Use the fact that the reflection of a segment is ≅ to its preimage to prove that *A′C′* + *B′C′* = *A′B′*. Then use the definition of betweenness to prove that *C′* is between *A′* and *B′*.

57. Since *A*, *B*, and *C* are collinear, one point. is between other two. Case 1: If *C* is between *A* and *B*, then *AC* + *BC* = *AB*. Use the fact that the reflection of a segment is ≅ to its preimage to prove that *A′C′* + *B′C′* = *A′B′*. Then *C′* is between *A′* and *B′*. So *A′*, *B′*, and *C′* are collinear. Prove the other two cases similarly.

SPIRAL REVIEW, PAGE 830

58. $P = \left(\frac{4}{12}\right)\left(\frac{4}{12}\right) = \frac{16}{144} = \frac{1}{9}$

59. $P = \left(\frac{10}{12}\right)\left(\frac{10}{12}\right) = \frac{100}{144} = \frac{25}{36}$

60. $P = \left(\frac{6}{12}\right)\left(\frac{4}{12}\right) = \frac{24}{144} = \frac{1}{6}$

61. dimensions of scale drawing: $\frac{1 \text{ cm}}{30 \text{ m}}(60 \text{ m})$ by $\frac{1 \text{ cm}}{30 \text{ m}}(105 \text{ m})$ or 2 cm by 3.5 cm
$P = 2(2) + 2(3.5) = 11$ cm

62. dimensions of scale drawing: $\frac{1.5 \text{ cm}}{15 \text{ m}}(60 \text{ m})$ by $\frac{1.5 \text{ cm}}{15 \text{ m}}(105 \text{ m})$ or 6 cm by 10.5 cm
$P = 2(6) + 2(10.5) = 33$ cm

63. dimensions of scale drawing: $\frac{1 \text{ cm}}{25 \text{ m}}(60 \text{ m})$ by $\frac{1 \text{ cm}}{25 \text{ m}}(105 \text{ m})$ or 2.4 cm by 4.2 cm
$P = 2(2.4) + 2(4.2) = 13.2$ cm

64. $BC^2 + AB^2 = AC^2$
$BC^2 + 2^2 = (\sqrt{7})^2$
$BC^2 = 3$
$BC = \sqrt{3} \approx 1.73$

65. $\cos A = \frac{AB}{AC} = \frac{2}{\sqrt{7}}$
$m\angle A = \cos^{-1}\left(\frac{2}{\sqrt{7}}\right) \approx 41°$

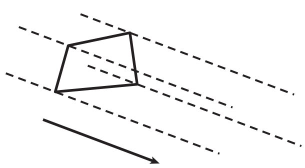
66. $m\angle C + m\angle A = 90$
$m\angle C + 41 \approx 90$
$m\angle C \approx 49°$

12-2 TRANSLATIONS, PAGES 831–837

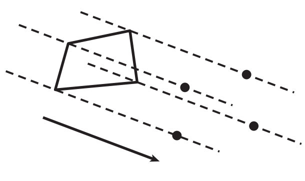
CHECK IT OUT! PAGES 831–833

1a. Yes; all the points have moved the same distance in the same direction.
b. No; not all the points have moved the same distance.

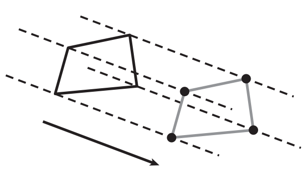
2. **Step 1** Draw a line ∥ to the vector through each vertex of the quadrilateral.



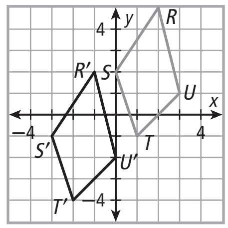
Step 2 Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the ∥ lines.



Step 3 Connect the images of the vertices.



3. The image of (x, y) is $(x - 3, y - 3)$.
$R(2, 5) \rightarrow R'(2 - 3, 5 - 3) = R'(-1, 2)$
$S(0, 2) \rightarrow S'(0 - 3, 2 - 3) = S'(-3, -1)$
$T(1, -1) \rightarrow T'(1 - 3, -1 - 3) = T'(-2, -4)$
$U(3, 1) \rightarrow U'(3 - 3, 1 - 3) = U'(0, -2)$
Graph the image and the preimage.



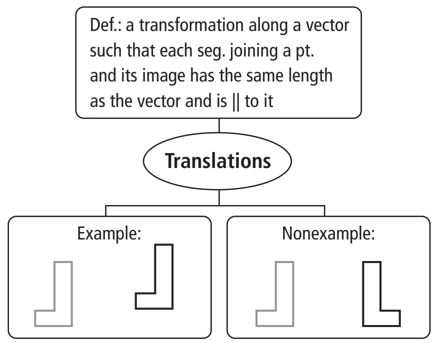
4. The drummer's starting coordinates are (0, 0).
The second position is (0 + 16, 0) = (16, 0).
The final position is (16, 0 − 24) = (16, −24).

THINK AND DISCUSS, PAGE 833

1. Possible answer: $\vec{v} \parallel \overline{AA'}$ and $|\vec{v}| = AA'$

2. Possible answer: $\overline{AA'}$ and $\overline{BB'}$ are both ⊥ to the translation vector, so they are ∥ to each other. They are ≅ because their lengths equal to the length of the translation vector. So $AA'B'B$ is a quadrilateral, because the opposite sides $\overline{AA'}$ and $\overline{BB'}$ are ∥ and ≅.

3.



EXERCISES, PAGES 834–837

GUIDED PRACTICE, PAGE 834

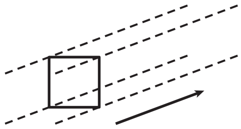
1. No; not all the points have moved the same distance.

2. No; not all the points have moved the same distance.

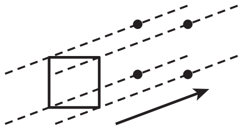
3. Yes; all the points have moved the same distance in the same direction.

4. No; not all the points have moved the same distance.

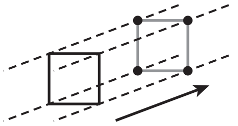
5. **Step 1** Draw a line ∥ to the vector through each vertex of the square.



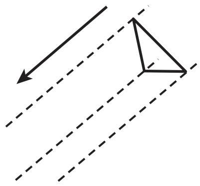
Step 2 Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the ∥ lines.



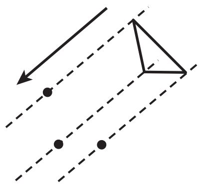
Step 3 Connect the images of the vertices.



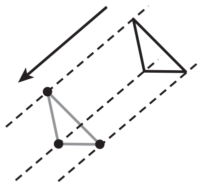
6. **Step 1** Draw a line ∥ to the vector through each vertex of the triangle.



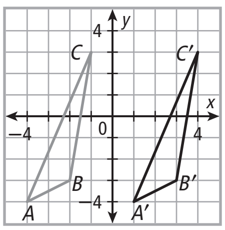
Step 2 Measure the length of vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the ∥ lines.



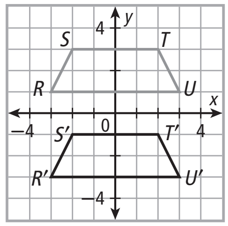
Step 3 Connect the images of the vertices.



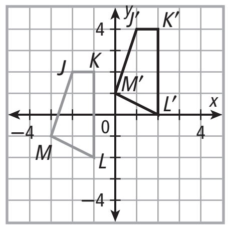
7. The image of (x, y) is $(x + 5, y)$.
$A(-4, -4) \rightarrow A'(-4 + 5, -4) = A'(1, -4)$
$B(-2, -3) \rightarrow B'(-2 + 5, -3) = S'(3, -3)$
$C(-1, 3) \rightarrow C'(-1 + 5, 3) = C'(4, 3)$
Graph image and preimage.



8. The image of (x, y) is $(x, y - 4)$.
$R(-3, 1) \rightarrow R'(-3, 1 - 4) = R'(-3, -3)$
$S(-2, 3) \rightarrow S'(-2, 3 - 4) = S'(-2, -1)$
$T(2, 3) \rightarrow T'(2, 3 - 4) = T'(2, -1)$
$U(3, 1) \rightarrow U'(3, 1 - 4) = U'(3, -3)$
Graph image and preimage.



9. The image of (x, y) is $(x + 3, y + 2)$.
$J(-2, 2) \rightarrow J'(-2 + 3, 2 + 2) = J'(1, 4)$
$K(-1, 2) \rightarrow K'(-1 + 3, 2 + 2) = K'(2, 4)$
$L(-1, -2) \rightarrow L'(-1 + 3, -2 + 2) = L'(2, 0)$
$M(-3, -1) \rightarrow M'(-3 + 3, -1 + 2) = U'(0, 1)$
Graph image and preimage.



10. The second polygon has coordinates
$(1, 5 - 4) = (1, 1)$, $(2, 3 - 4) = (2, -1)$,
$(1, 1 - 4) = (1, -3)$, and $(0, 3 - 4) = (0, -1)$.
The third polygon has coordinates
$(1, 1 - 4) = (1, -3)$, $(2, -1 - 4) = (2, -5)$,
$(1, -3 - 4) = (1, -7)$, and $(0, -1 - 4) = (0, -5)$.
The fourth polygon has coordinates
$(1, -3 - 4) = (1, -7)$, $(2, -5 - 4) = (2, -9)$,
$(1, -7 - 4) = (1, -11)$, and $(0, -5 - 4) = (0, -9)$.

PRACTICE AND PROBLEM SOLVING, PAGES 834–836

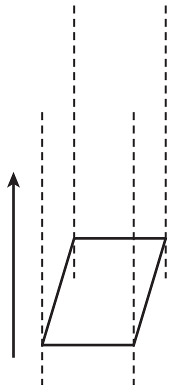
11. Yes; all the points have moved the same distance in the same direction.

12. No; not all points have moved the same distance.

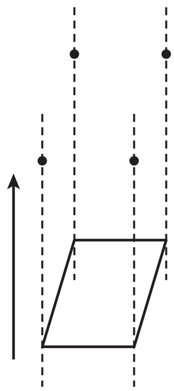
13. No; not all the points have moved the same distance.

14. Yes; all the points have moved the same distance in the same direction.

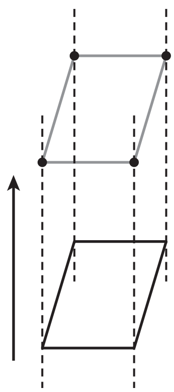
15. **Step 1** Draw a line || to the vector through each vertex of the square.



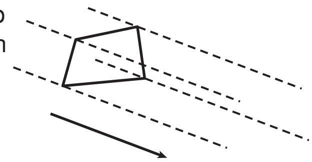
Step 2 Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the || lines.



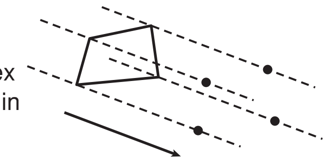
Step 3 Connect the images of the vertices.



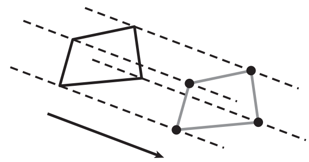
16. **Step 1** Draw a line || to the vector through each vertex of the square.



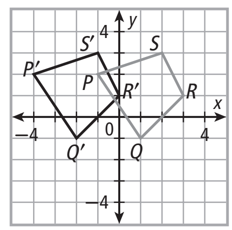
Step 2 Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the || lines.



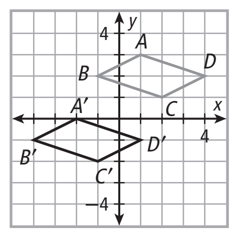
Step 3 Connect the images of the vertices.



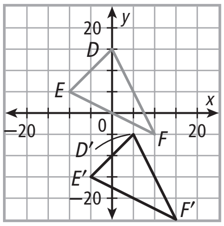
17. The image of (x, y) is $(x - 3, y)$.
$P(-1, 2) \rightarrow P'(-1 - 3, 2) = P'(-4, 2)$
$Q(1, -1) \rightarrow Q'(1 - 3, -1) = Q'(-2, -1)$
$R(3, 1) \rightarrow R'(3 - 3, 1) = R'(0, 1)$
$S(2, 3) \rightarrow S'(2 - 3, 3) = S'(-1, 3)$
Graph the image and the preimage.



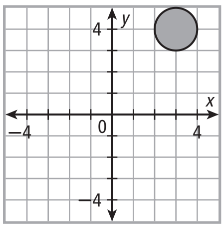
18. The image of (x, y) is $(x - 3, y - 3)$.
$A(1, 3) \rightarrow A'(1 - 3, 3 - 3) = A'(-2, 0)$
$B(-1, 2) \rightarrow B'(-1 - 3, 2 - 3) = B'(-4, -1)$
$C(2, 1) \rightarrow C'(2 - 3, 1 - 3) = C'(-1, -2)$
$D(4, 2) \rightarrow D'(4 - 3, 2 - 3) = D'(1, -1)$
Graph the image and the preimage.



19. The image of (x, y) is $(x + 5, y - 20)$.
$D(0, 15) \rightarrow D'(0 + 5, 15 - 20) = D'(5, -5)$
$E(-10, 5) \rightarrow E'(-10 + 5, 5 - 20) = E'(-5, -15)$
$F(10, -5) \rightarrow F'(10 + 5, -5 - 20) = F'(15, -25)$
Graph the image and the preimage.

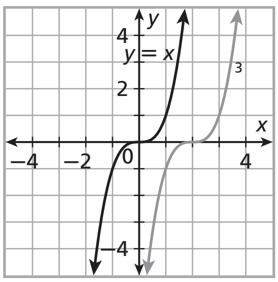


20a. The ladybug starts at $(-2.5, -1.5)$.
The second position is at
$(-2.5 + 1, -1.5 + 1) = (-1.5, -0.5)$.
The third position is at $(-1.5 + 2, -0.5 + 2)$
$= (0.5, 1.5)$.
The final position is at $(0.5 + 3, 1.5 + 3)$
$= (3.5, 4.5)$.

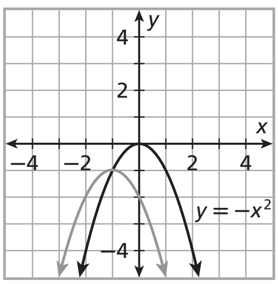


b. $\langle 1, 1\rangle + \langle 2, 2\rangle + \langle 3, 3\rangle = \langle 6, 6\rangle$

21.



22.



23a. Possible images are $(3 - 3, 2) = (0, 2)$,
$(3 - 1, 2 - 4) = (2, -2)$, $(3 + 3, 2 - 2) = (6, 0)$,
and $(3 + 2, 2 + 3) = (5, 5)$.
The points in the fourth quadrant: $(2, -2)$; $P = \frac{1}{4}$

b. Points on an axis: $(0, 2)$ and $(6, 0)$. $P = \frac{2}{4} = \frac{1}{2}$

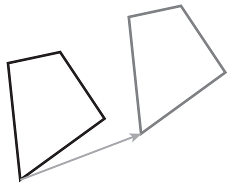
c. Points at origin: none. $P = \frac{0}{4} = 0$

24a. ball travels from (1, 2) to (3, 3)
vector is $\langle 3 - 1, 3 - 2\rangle = \langle 2, 1\rangle$

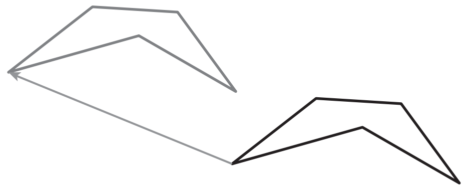
b. ball travels from (3, 3) to (7, 1)
vector is $\langle 7 - 3, 1 - 3\rangle = \langle 4, -2\rangle$

c. sum of vectors is $\langle 2 + 4, 1 + (-2)\rangle = \langle 6, -1\rangle$
ball travels from (1, 2) to (7, 1)
vector is $\langle 7 - 1, 1 - 2\rangle = \langle 6, -1\rangle$

25.



26.



27. No; there are no fixed points because, by definition of a translation, every point must move by the same distance.

28. First use the adjustable ||s to draw a line through the given pt. that is || to the given vector. Then use a ruler to measure a distance along this line that is equal to the magnitude of the vector. The only additional tool needed to do this construction is the ruler.

29. vector is $\langle 1 - (-3), 2 - 2\rangle = \langle 4, 0\rangle$;
$(-3, 2) \rightarrow (1, 2)$

30. vector is $\langle -3 - 1, 2 - 2\rangle = \langle -4, 0\rangle$;
$(1, 2) \rightarrow (-3, 2)$

31. vector is $\langle 0 - 3, -3 - (-1)\rangle = \langle -3, -2\rangle$;
$(3, -1) \rightarrow (0, -3)$

32. vector is $\langle 1 - (-4), 2 - (-3)\rangle = \langle 5, 5\rangle$;
$(-4, -3) \rightarrow (1, 2)$

33. vector is $\langle 0 - 3), 0 - (-1)\rangle = \langle -3, 1\rangle$;
$(3, -1) \rightarrow (0, 0)$

34. The overlap is a rectangle with
$\ell = 8 - \frac{2}{3}(8) = 2\frac{2}{3}$ in. and
$w = 3 - \frac{2}{3}(3) = 1$ in.
$A = \ell w = \left(2\frac{2}{3}\right)(1) = 2\frac{2}{3}$ in.^{2}

35. The distance between P and its image is equal to the magnitude of the translation vector $\langle a, b\rangle$. By Distance Formula,
the magnitude of this vector is $\sqrt{a^2 + b^2}$.

36–38. Check students' constructions.

TEST PREP, PAGES 836–837

39. A
$P(1, 3) \rightarrow P'(1 - 3, 3 + 5) = P'(-2, 8)$

40. G
$A(-6, -2) \rightarrow B(-4, -4) = B(-6 + 2, -2 -2)$
$(3, -1) \rightarrow (3 + 2, -1 - 2) = (5, -3)$

41. C
$Q(3, -1) \rightarrow P(1, 3) = P(3 - 2, -1 + 4)$
vector is $\langle -2, 4 \rangle$

CHALLENGE AND EXTEND, PAGE 837

42. vector $\langle a, b \rangle$ satisfies $b = 2a$ and $\sqrt{a^2 + b^2} = \sqrt{5}$.
$a^2 + (2a)^2 = 5$
$a^2 + 4a^2 = 5$
$5a^2 = 5$
$a^2 = 1$
$a = \pm 1$
$b = 2(\pm 1) = \pm 2$
$M(1, 2) \rightarrow M'(1 - 1, 2 - 2) = M'(0, 0)$
or $\rightarrow M'(2, 4) = M'(2, 4)$

43a. the vector $\overrightarrow{PQ}$

b. $\overrightarrow{PQ} = \vec{u} + \vec{v} + \vec{w}$
$= \langle 2, 0, 0 \rangle + \langle 2, 0, 0 \rangle + \langle 2, 0, 0 \rangle$
$= \langle 2, 2, 2 \rangle$
$\left|\overrightarrow{PQ}\right| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{12} \approx 3.46$ cm

44. Draw $\overline{AA'}$ and $\overline{BB'}$. By def. of translation, $\overline{AA'} \cong \overline{BB'}$ and $\overline{AA'} \parallel \overline{BB'}$. Opposite sides $\overline{AA'}$ and $\overline{BB'}$ are ≅ and ∥. So $AA'B'B$ is a parallelogram. Opposite sides of a quadrilateral are ≅. So $\overline{AB} \cong \overline{A'B'}$.

45. Use fact that the translation of a segment is ≅ to its preimage and def. of ≅ segs.

46. Draw $\overline{AC}$ and $\overline{A'C'}$. Use the fact that the translation of a segment is ≅ to its preimage to prove $\triangle ABC \cong \triangle A'B'C'$ by SSS. By CPCTC, $\angle ABC \cong \angle A'B'C'$. So $m\angle ABC = m\angle A'B'C'$ by def. of ≅ ∠s.

47. Use the fact that the translation of a segment is ≅ to its preimage to prove $\triangle ABC \cong \triangle A'B'C'$ by SSS.

48. Since C is between A and B, $AC + BC = AB$. Use the fact that the translation of a segment is ≅ to its preimage to prove $A'C' + B'C' = A'B'$. Then use def. of betweenness to prove that C' is between A' and B'.

49. Since A, B, and C are collinear, one point is between other two. Case 1: If C is between A and B, $AC + BC = AB$. Use the fact that the translation of a segment is ≅ to its preimage to prove $A'C' + B'C' = A'B'$. Then C' is between A' and B'. So A', B', and C' are collinear. Prove the other two cases similarly.

SPIRAL REVIEW, PAGE 837

50. $\begin{cases} -5x - 2y = 17 & (1) \\ 6x - 2y = -5 & (2) \end{cases}$
(2) − (1):
$6x - (-5x) = -5 - 17$
$11x = -22$
$x = -2$
Substitute in (2):
$6(-2) - 2y = -5$
$-12 - 2y = -5$
$-2y = 7$
$y = -3.5$
Solution is: $(-2, -3.5)$ or $\left(-2, -\frac{7}{2}\right)$

51. $\begin{cases} 2x - 3y = -7 & (1) \\ 6x + 5y = 49 & (2) \end{cases}$
(2) − 3(1):
$6x - 3(2x) + 5y - 3(-3y) = 49 - 3(-7)$
$6x - 6x + 5y + 9y = 49 + 21$
$14y = 70$
$y = 5$
Substitute in (1):
$2x - 3(5) = -7$
$2x - 15 = -7$
$2x = 8$
$x = 4$
Solution is: (4, 5)

52. $\begin{cases} 4x + 4y = -1 & (1) \\ 12x - 8y = -8 & (2) \end{cases}$
2(1) + (2):
$2(4x) + 12x + 2(4y) - 8y = 2(-1) - 8$
$8x + 12x + 8y - 8y = -2 - 8$
$20x = -10$
$x = -0.5$
Substitute in (1):
$4(-0.5) + 4y = -1$
$-2 + 4y = -1$
$4y = 1$
$y = 0.25$
Solution is: $(-0.5, 0.25)$ or $\left(-\frac{1}{2}, \frac{1}{4}\right)$

53. Think: All ∠s in the diagram are right. ∠s.
$x + 15y = 90$ (1)
$3x + 9y = 90$ (2)
3(1) − (2):
$3x - 3x + 3(15y) - 9y = 3(90) - 90$
$36y = 180$
$y = 5$
Substitute in (1):
$x + 15(5) = 90$
$x + 75 = 90$
$x = 15$

54. Think: $4x°$ is a right ∠.
$4x = 90$
$x = 22.5$
Think: $y°$ and $2x°$ ∠s are comp.
$y + 2x = 90$
$y + 2(22.5) = 90$
$y + 45 = 90$
$y = 45$

55. General point $(x, y) \rightarrow (x, -y)$
$M(-2, 0) \rightarrow M'(-2, 0)$
$N(-3, 2) \rightarrow N'(-3, -2)$
$P(0, 4) \rightarrow P'(0, -4)$

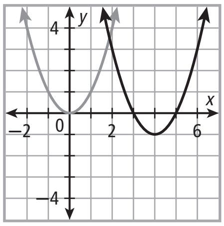
56. General point $(x, y) \rightarrow (-x, y)$
$M(-2, 0) \rightarrow M'(2, 0)$
$N(-3, 2) \rightarrow N'(3, 2)$
$P(0, 4) \rightarrow P'(0, 4)$

57. General point $(x, y) \rightarrow (y, x)$
$M(-2, 0) \rightarrow M'(0, -2)$
$N(-3, 2) \rightarrow N'(2, -3)$
$P(0, 4) \rightarrow P'(4, 0)$

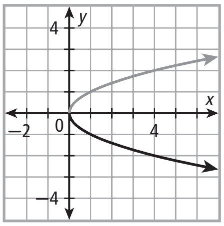
CONNECTING GEOMETRY TO ALGEBRA: TRANSFORMATIONS OF FUNCTIONS, PAGE 838

TRY THIS, PAGE 838

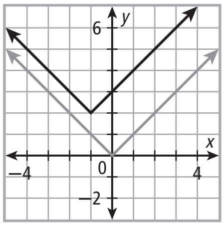
1. function rule:
   $y = (x - 4)^2 - 1$
   graph:



2. function rule:
   $y = -\sqrt{x}$
   graph:



3. function rule:
   $y = |x + 1| + 2$
   graph:



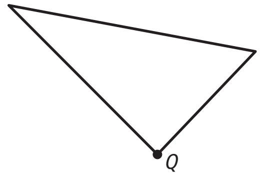
12-3 ROTATIONS, PAGES 839–845

CHECK IT OUT! PAGES 839–841

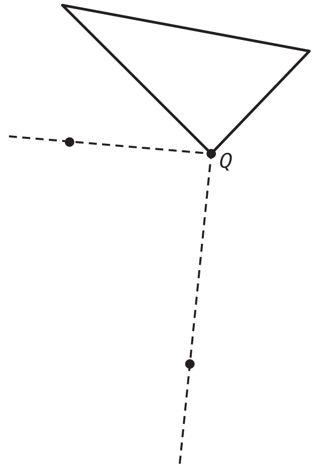
1a. No; the figure appears to be translated, not turned.

b. Yes; the figure appears to be turned around a point.

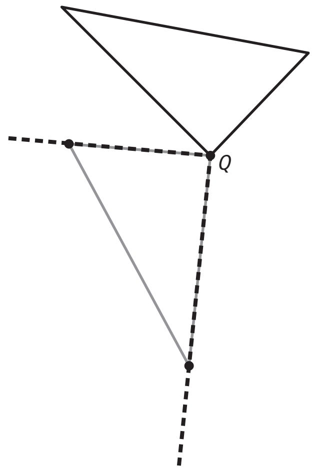
2. **Step 1** Draw a segment from each vertex to point *Q*.



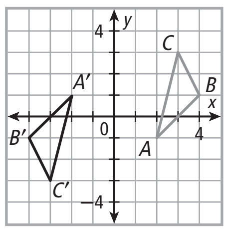
Step 2 Construct an $\angle \cong$ to $\angle X$ onto each segment. Measure the distance from each vertex to point *Q* and mark off this distance on the corresponding ray to locate the image of each vertex.



Step 3 Connect the images of the vertices.

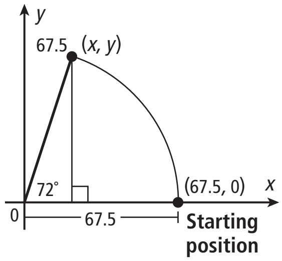


3. The rotation of (x, y) is $(-x, -y)$.
$A(2, -1) \rightarrow A'(2, -1)$
$B(4, 1) \rightarrow B'(4, 1)$
$C(3, 3) \rightarrow C'(3, 3)$



4. Step 1 Find the ∠ of rotation. 6 min is $\frac{6}{30} = \frac{1}{5}$ of a complete rotation, or $\frac{1}{5}(360) = 72°$.

Step 2 Draw a right △ to represent the car's location (x, y) after a rotation of 72° about the origin.



Step 3 Use the cosine ratio to find the *x*-coordinate

$\cos 72° = \frac{x}{67.5}$

$x = 67.5 \cos 72° \approx 20.9$

Step 4 Use the sine ratio to find the *y*-coordinate

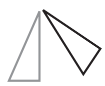
$\sin 72° = \frac{y}{67.5}$

$y = 67.5 \sin 72° \approx 64.2$

The car's location after 6 min is approximately (20.9, 64.2).

THINK AND DISCUSS, PAGE 841

1. The image is in the same position as the preimage.
2. $\overline{AP} \cong \overline{A'P}$
3.

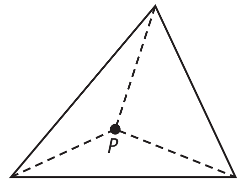
	Reflection	Translation	Rotation
Definition	A transformation across a line (the line of reflection) such that the line of reflection is the ⊥ bisector of each seg. joining a point and its image	A transformation along a vector such that each seg. joining a point and its image has the same length as the vector and is ∥ to it	A transformation about a pt. *P* such that each pt. and its image are the same dist. from *P* and all of the ∡ with vertex *P* formed by a pt. and its image are ≅
Example			

EXERCISES, PAGES 842–845

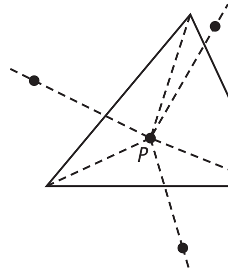
GUIDED PRACTICE, PAGE 842

1. Yes; the figure appears to be turned around a point.
2. No; the figure appears to be flipped, not turned.
3. No; the figure appears to be translated, not turned.
4. Yes; the figure appears to be turned around a point.

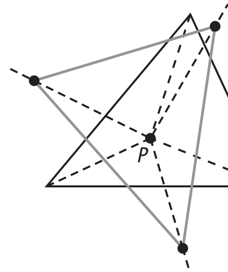
5. Step 1 Draw a segment from each vertex to point *P*.



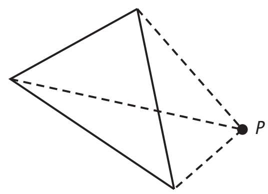
Step 2 Construct an ∠ ≅ to ∠*A* onto each segment. Measure distance from each vertex to point *P* and mark off this distance on the corresponding ray to locate image of each vertex.



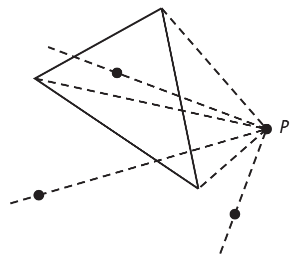
Step 3 Connect the images of the vertices.



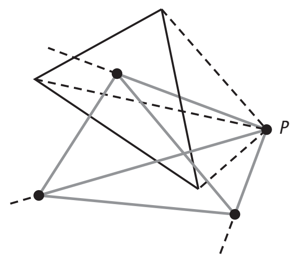
6. Step 1 Draw a segment from each vertex to point *P*.



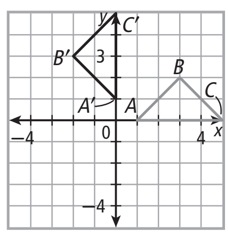
Step 2 Construct an ∠ ≅ to ∠*A* onto each segment. Measure the distance from each vertex to point *P* and mark off this distance on the corresponding ray to locate image of each vertex.



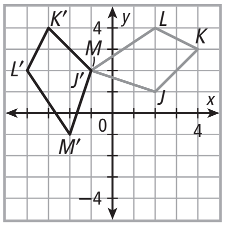
Step 3 Connect the images of the vertices.



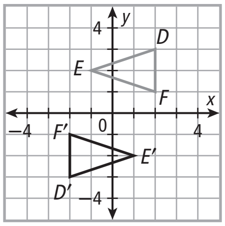
7. The rotation of (x, y) is $(-y, x)$.
$A(1, 0) \rightarrow A'(0, 1)$
$B(3, 2) \rightarrow B'(-2, 3)$
$C(5, 0) \rightarrow C'(0, 5)$



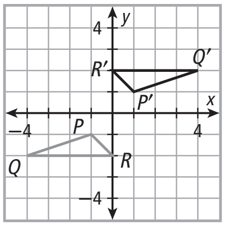
8. The rotation of (x, y) is $(-y, x)$.
$J(2, 1) \rightarrow J'(-1, 2)$
$K(4, 3) \rightarrow K'(-3, 4)$
$L(2, 4) \rightarrow L'(-4, 2)$
$M(-1, 2) \rightarrow M'(-2, -1)$



9. The rotation of (x, y) is $(-x, -y)$.
$D(2, 3) \rightarrow D'(-2, -3)$
$E(-1, 2) \rightarrow E'(1, -2)$
$F(2, 1) \rightarrow F'(-2, -1)$

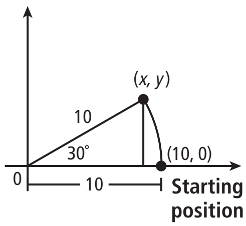


10. The rotation of (x, y) is $(-x, -y)$.
$P(-1, -1) \rightarrow P'(1, 1)$
$Q(-4, -2) \rightarrow Q'(4, 2)$
$R(0, -2) \rightarrow R'(0, 2)$



11. **Step 1** Draw a right △ to represent the car's location (x, y) after a rotation of 30° about the origin.
Step 2 Use the cosine ratio to find the x-coordinate

$$\cos 30° = \frac{x}{10}$$
$$x = 10 \cos 30° \approx 8.7$$



Step 3 Use the sine ratio to find the y-coordinate

$$\sin 30° = \frac{y}{10}$$
$$y = 10 \sin 30° = 5$$

The car's final position is approximately (8.7, 5).

PRACTICE AND PROBLEM SOLVING, PAGES 842–844

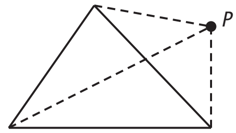
12. No; the figure appears to be flipped and translated, not turned.

13. Yes; the figure appears to be turned around a point.

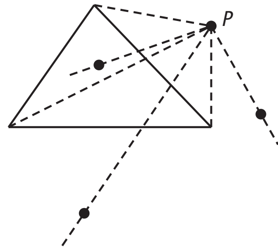
14. Yes; the figure appears to be turned around a point.

15. No; the figure appears to be enlarged, not turned.

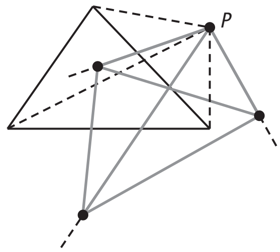
16. **Step 1** Draw a segment from each vertex to point P.



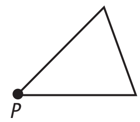
Step 2 Construct an $\angle \cong$ to $\angle A$ onto each segment. Measure the distance from each vertex to point P and mark off this distance on the corresponding ray to locate the image of each vertex.



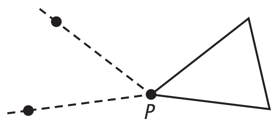
Step 3 Connect the images of the vertices.



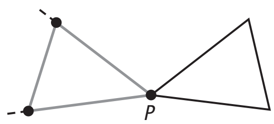
17. **Step 1** Draw a segment from each vertex to point P.



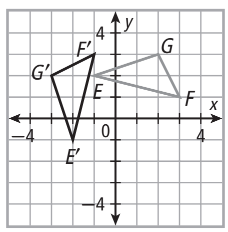
Step 2 Construct an $\angle \cong$ to $\angle A$ onto each segment. Measure the distance from each vertex to point P and mark off this distance on the corresponding ray to locate the image of each vertex.



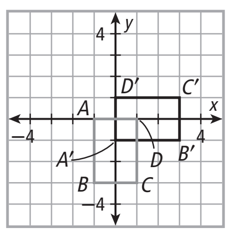
Step 3 Connect the images of the vertices.



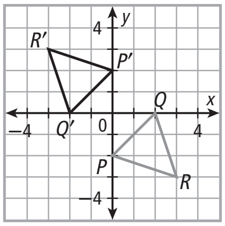
18. The rotation of (x, y) is $(-y, x)$.
$E(-1, 2) \rightarrow E'(-2, -1)$
$F(3, 1) \rightarrow F'(-1, 3)$
$G(2, 3) \rightarrow G'(-3, 2)$



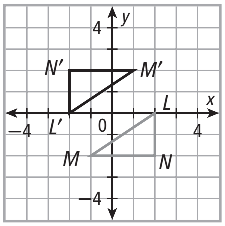
19. The rotation of (x, y) is $(-y, x)$.
$A(-1, 0) \rightarrow A'(0, -1)$
$B(-1, -3) \rightarrow B'(3, -1)$
$C(1, -3) \rightarrow C'(3, 1)$
$D(1, 0) \rightarrow D'(0, 1)$



20. The rotation of (x, y) is $(-x, -y)$.
$P(0, 2) \rightarrow P'(0, -2)$
$Q(2, 0) \rightarrow Q'(-2, 0)$
$R(3, -3) \rightarrow R'(-3, 3)$

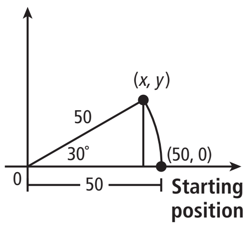


21. The rotation of (x, y) is $(-x, -y)$.
$L(2, 0) \rightarrow L'(-2, 0)$
$M(-1, -2) \rightarrow M'(1, 2)$
$N(2, -2) \rightarrow N'(-2, 2)$



22. Step 1 Find the ∠ of rotation. 6 min is $\frac{6}{72} = \frac{1}{12}$ of a complete rotation, or $\frac{1}{12}(360) = 30°$.

Step 1 Draw a right △ to represent the table's location (x, y) after a rotation of 30° about the origin.

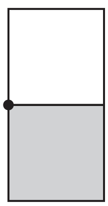


Step 2 Use the cosine ratio to find the x-coordinate
$\cos 30° = \frac{x}{50}$
$x = 50 \cos 30° \approx 43.3$

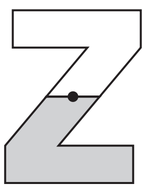
Step 3 Use the sine ratio to find the y-coordinate
$\sin 30° = \frac{y}{50}$
$y = 50 \sin 30° = 25$

The table's final position is approximately (43.3, 25).

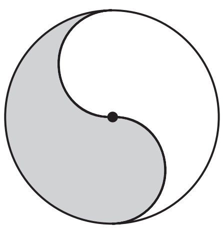
23.



24.



25.



26a. $\tan \theta = \frac{2}{3}$
$\theta = \tan^{-1}\left(\frac{2}{3}\right) \approx 34°$

b. The x-coordinate of the image is 0.
The y-coordinate of the image is
$OQ = \sqrt{2^2 + 3^2} = \sqrt{13} \approx 3.6$
The location of the image is approximately (0, 3.6).

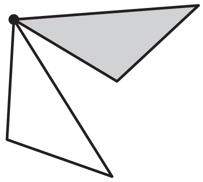
27. T **28.** M

29. $\overline{ST}$ **30.** $\overline{NP}$

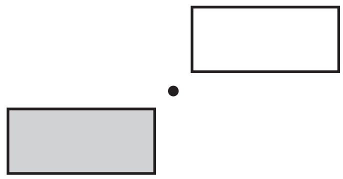
31a. $\frac{4}{20}(360) = 72°$

b. $A(7, 3) = A(4 + 3, 3 + 0)$
$A'(4 + 3\cos 72°, 3 + 3\sin 72°) \approx A'(4.9, 5.9)$

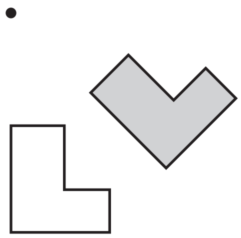
32.



33.



34.



35a. Possible answer: 90° **b.** $\frac{x}{24} = \frac{90}{360} = \frac{1}{4}$
$x = \frac{1}{4}(24) = 6$ h

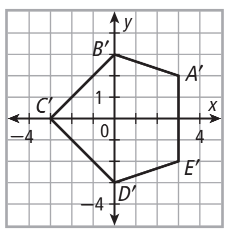
36a. Possible answer: 45°. Check students' estimates.

b. Draw $\overline{AP}$ and $\overline{A'P}$ and use the protractor to measure $\angle A'PA$.

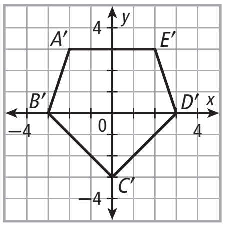
c. Check students' constructions; possible answer: 50°.

37. No; although all points are rotated around the center of rotation by the same ∠, points that are farther from the center of rotation move a greater distance than points that are closer to the center of rotation.

38. The rotation of (x, y) is $(-y, x)$.
$A(2, -3) \rightarrow A'(3, 2)$
$B(3, 0) \rightarrow B'(0, 3)$
$C(0, 3) \rightarrow C'(-3, 0)$
$D(-3, 0) \rightarrow D'(0, -3)$
$E(-2, -3) \rightarrow E'(3, -2)$



39. The rotation of (x, y) is $(-x, -y)$.
$A(2, -3) \to A'(-2, 3)$
$B(3, 0) \to B'(-3, 0)$
$C(0, 3) \to C'(0, -3)$
$D(-3, 0) \to D'(3, 0)$
$E(-2, -3) \to E'(2, 3)$



40. The image of $ABCDE$ under a rotation of 180° is not the same as its reflection across the x–axis because the images of the specific points are in different locations. For example, $A(2, -3) \to A'(-2, 3)$ is under a 180° rotation, but $\to A'(2, 3)$ is under a reflection in the x–axis.

41. Check students' constructions.

TEST PREP, PAGE 845

42. C
$(x, y) \to (-y, x)$
$(-2, 5) \to (-5, -2)$

43. H
$\frac{1}{3}(360) = 120°$

44. 180
$(-3, 4) \to (3, -4) = (-(-3), -(4))$
$(x, y) \to (-x, -y)$: rotation of 180°

CHALLENGE AND EXTEND, PAGE 845

45. Gear B has 8 teeth. So one complete counterclockwise rotation of gear B will move gear A by 8 teeth clockwise. Gear A has 18 teeth. So 8 teeth is $\frac{4}{9}$ of a complete rotation, or $\frac{4}{9}(360) = 160°$ clockwise.

46. Draw auxiliary segs. $\overline{AP}$, $\overline{BP}$, $\overline{A'P}$, and $\overline{B'P}$. By the def. of rotation, $\overline{AP} \cong \overline{A'P}$ and $\overline{BP} \cong \overline{B'P}$. Also by the def. of rotation, $\angle A'PA \cong \angle B'PB$. By the Common $\angle$ Thm., $\angle B'PA' \cong \angle BPA$. Thus $\triangle B'PA' \cong \triangle BPA$ by SAS, and $\overline{AB} \cong \overline{A'B'}$ by CPCTC.

47. Use the fact that the rotation of a segment is $\cong$ to its preimage and the def. of $\cong$ segs.

48. Draw auxiliary segs. $\overline{AC}$ and $\overline{A'C'}$. Use the fact that the rotation of a segment is $\cong$ to its preimage to prove $\triangle ABC \cong \triangle A'B'C'$ by SSS. By CPCTC, $\angle ABC \cong \angle A'B'C'$. So $m\angle ABC = m\angle A'B'C'$ by the def. of $\cong$ ∠s.

49. Use the fact that the rotation of a segment is $\cong$ to its preimage to prove $\triangle ABC \cong \triangle A'B'C'$ by SSS.

50. Since C is between A and B, $AC + BC = AB$. Use the fact that the rotation of a segment is $\cong$ to its preimage to prove $A'C' + B'C' = A'B'$. Then use the def. of betweenness to prove C' is between A' and B'.

51. Since A, B, and C are collinear, one point is between other two. Case 1: If C is between A and B, then $AC + BC = AB$. Use the fact that the rotation of a segment is $\cong$ to its preimage to prove $A'C' + B'C' = A'B'$. Then C' is between A' and B'. So A', B', and C' are collinear. Prove the other two cases similarly.

SPIRAL REVIEW, PAGE 845

52. $3 = x^2 - 4x + 7$
$0 = x^2 - 4x + 4$
$0 = (x - 2)^2$
$x = 2$

53. $3 = 2x^2 - 5x - 9$
$0 = 2x^2 - 5x - 12$
$0 = (2x + 3)(x - 4)$
$x = -\frac{3}{2}$ or 4

54. $3 = x^2 - 2$
$5 = x^2$
$x = \pm\sqrt{5}$

55. $QRYX$ is an isosceles trapazoid.
$m\angle XYR = 180 - m\angle QRY$
$= 180 - m\angle XQR$
$= 180 - 86 = 94°$

56. $\frac{QR - PS}{XY - PS} = \frac{PQ}{PX} = 2$
$QR - PS = 2(XY - PS)$
$QR - PS = 2XY - 2PS$
$QR = 2XY - PS$
$= 2(4.2) - (4)$
$= 8.4 - 4 = 4.4$

57. $A(1, 3) \to D(5, -6) = D(1 + 4, 3 - 9)$
translation vector: $\langle 4, -9 \rangle$

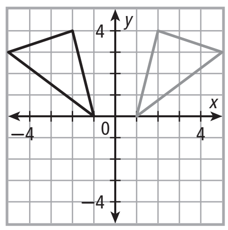
58. $D(5, -6) \to B(5, 0) = B(5 + 0, -6 + 6)$
translation vector: $\langle 0, 6 \rangle$

59. $C(-3, -2) \to (0, 0) = (-3, +3, -2, +2)$
translation vector: $\langle 3, 2 \rangle$

12-3 TECHNOLOGY LAB: EXPLORE TRANSFORMATIONS WITH MATRICES, PAGES 846–847

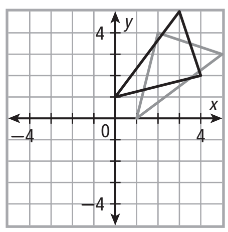
ACTIVITY 1, TRY THIS, PAGE 846

1. $[A] \cdot [B] = \begin{bmatrix} -1 & -2 & -5 \\ 0 & 4 & 3 \end{bmatrix}$



a reflection across the y-axis

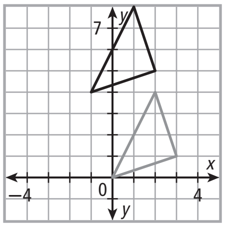
2. $[A] \cdot [B] = \begin{bmatrix} 0 & 4 & 3 \\ 1 & 2 & 5 \end{bmatrix}$



a reflection across the line $y = x$

ACTIVITY 2, TRY THIS, PAGE 847

3. $[A] + [B] = \begin{bmatrix} -1 & 2 & 1 \\ 4 & 5 & 8 \end{bmatrix}$

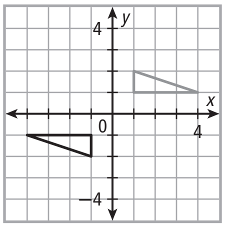


a translation 1 unit left and 4 units up

4. Let $[A] = \begin{bmatrix} a & a & a \\ b & b & b \end{bmatrix}$. Add $[A] + [B]$ and use the solution matrix to graph the image of the △.

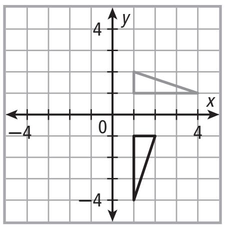
ACTIVITY 3, TRY THIS, PAGE 847

5. $[A] \cdot [B] = \begin{bmatrix} -1 & -4 & -1 \\ -1 & -1 & -2 \end{bmatrix}$



a 180° rotation about the origin

6. $[A] \cdot [B] = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & -1 \end{bmatrix}$

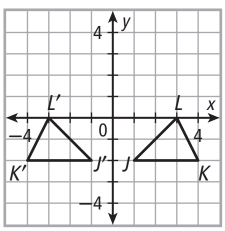


a 270° rotation about the origin

12-4 COMPOSITIONS OF TRANSFORMATIONS, PAGES 848–853

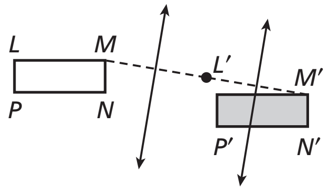
CHECK IT OUT! PAGES 849–850

1. **Step 1** The reflection image of (x, y) is $(x, -y)$.
   $J(1, -2) \rightarrow J'(1, 2)$
   $K(4, -2) \rightarrow K'(4, 2)$
   $L(3, 0) \rightarrow L'(3, 0)$
   Step 2 The rotation image of (x, y) is $(-x, -y)$.
   $J'(1, 2) \rightarrow J''(-1, 2)$
   $K'(4, 2) \rightarrow K''(-4, 2)$
   $L'(3, 0) \rightarrow L''(-3, 0)$
   Step 3 Graph the preimage and the image.



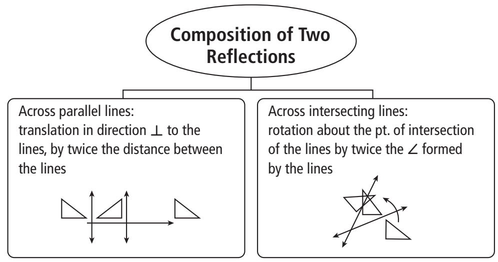
2. By Thm. 12-4-2, the composition of the two reflections across ∥ lines is equivalent to a translation ⊥ to these lines, n and p. By Thm. 12-4-2, the distance of translation is
   $2(3) = 6$ in.

3. **Step 1** Draw $\overline{MM'}$ and locate its midpoint X.
   Step 2 Draw the ⊥ bisectors of $\overline{MX}$ and $\overline{XM'}$.



THINK AND DISCUSS, PAGE 850

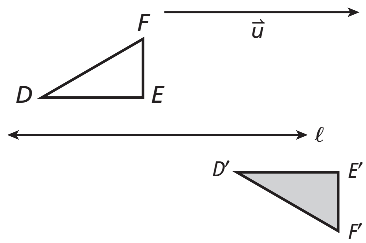
1. Theorem 12-4-1; a composition of two isometries is an isometry.
2. $\vec{v} \parallel \ell$; possible answer; Translate the preimage along the vector then reflect the image across the line.
3.



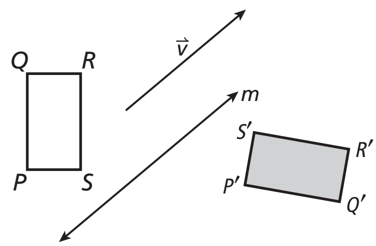
EXERCISES, PAGES 851–853

GUIDED PRACTICE, PAGE 851

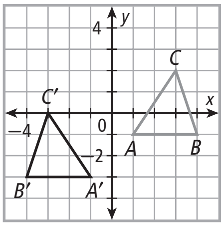
1. Draw a figure and translate it along a vector. Then reflect the image across a line.
2. **Step 1** Translate △DEF along $\vec{u}$.
   Step 2 Reflect the image across line ℓ.



3. **Step 1** Reflect rectangle $PQRS$ across line m.
   Step 2 Translate the image along $\vec{v}$.

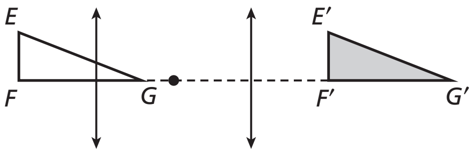


4. **Step 1** The reflection image of (x, y) is $(-x, y)$.
$A(1, -1) \rightarrow A'(-1, -1)$
$B(4, -1) \rightarrow B'(-4, -1)$
$C(3, 2) \rightarrow C'(-3, 2)$
Step 2 The translation image of (x, y) is $(x, y - 2)$.
$A'(-1, -1) \rightarrow A''(-1, -3)$
$B'(-4, -1) \rightarrow B''(-4, -3)$
$C'(-3, 2) \rightarrow C''(-3, 0)$
Step 3 Graph the preimage and the image.

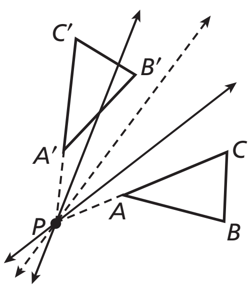


5. By Thm. 12-4-2, the composition of the two reflections across the intersecting lines is equivalent to a rotation about the point of intersection. By Thm. 12-4-2, the ∠ of rotation is 2(50) = 100°.

6. **Step 1** Draw $\overline{FF'}$ and locate its midpoint *X*.
Step 2 Draw the ⊥ bisectors of $\overline{FX}$ and $\overline{XF'}$.

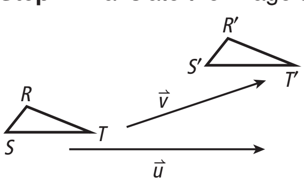


7. **Step 1** Draw ∠*APA′*. Draw its ∠ bisector $\overrightarrow{PX}$.
Step 2 Draw ∠ bisectors of ∠*APX* and ∠*A′PX*.

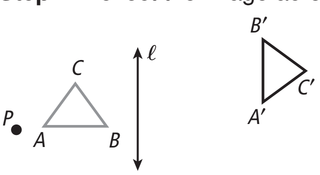


PRACTICE AND PROBLEM SOLVING, PAGES 851–853

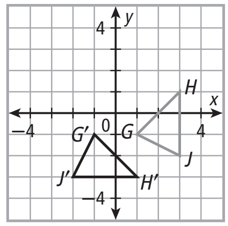
8. **Step 1** Translate △*RST* along $\vec{u}$.
Step 2 Translate the image along $\vec{v}$.



9. **Step 1** Rotate △*ABC* 90° around point *P*.
Step 2 Reflect the image across the line ℓ.

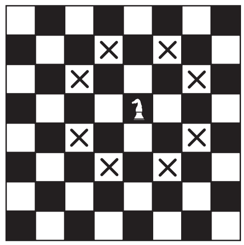


10. **Step 1** The reflection image of (x, y) is (y, x).
$G(1, -1) \rightarrow G'(-1, 1)$
$H(3, 1) \rightarrow H'(1, 3)$
$J(3, -2) \rightarrow J'(-2, 3)$
Step 2 The reflection image of (x, y) is $(x, -y)$.
$G'(-1, 1) \rightarrow G''(-1, -1)$
$H'(1, 3) \rightarrow H''(1, -3)$
$J'(-2, 3) \rightarrow J''(-2, -3)$
Step 3 Graph the preimage and the image.

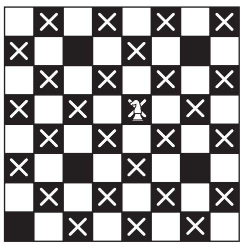


11a. The move is a horizontal or vertical translation by 2 spaces followed by a vertical or horizontal translation by 1 space.

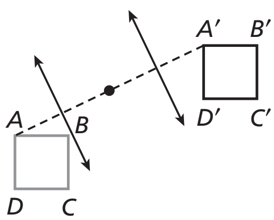
b.



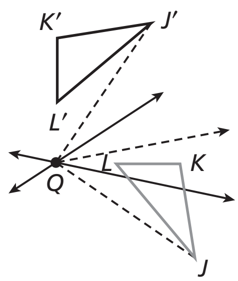
c.



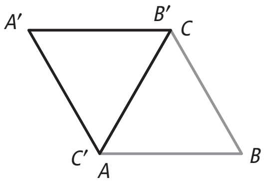
12. **Step 1** Draw $\overline{BB'}$ and locate its midpoint *X*.
Step 2 Draw the ⊥ bisectors of $\overline{BX}$ and $\overline{XB'}$.



13. **Step 1** Draw ∠*JQJ′*. Draw its ∠ bisector $\overrightarrow{QX}$.
Step 2 Draw ∠ bisectors of ∠*JQX* and ∠*J′QX*.



14. Solution A is incorrect because the endpts. are not written in the same order as they are written for the preimage.

15. 

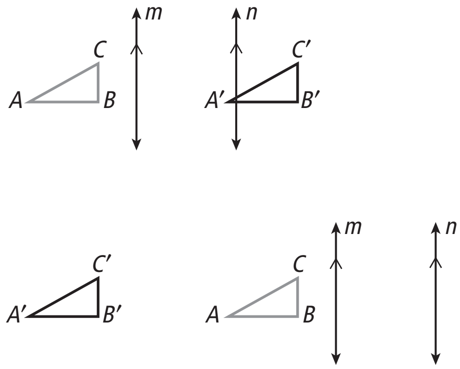
16. sometimes (if reflection lines intersect)

17. never (by def. of isometry)

18. always (Thm. 12-4-1)

19. always (Th. 12-4-3)

20. Yes; the order matters, as shown in the figures.



21. $R(-3, -2) \to (-3, 2) \to R'(2, 2)$
$S(-1, -2) \to (-1, 2) \to S'(4, 2)$
$T(-1, 0) \to (-1, 0) \to T'(2, 0)$
$(x, y) \to (x, -y); (x, y) \to (x + 5, y)$
The line of reflection is the x-axis, and the translation vector is $\langle 5, 0\rangle$.

22a. $\langle 1, 3\rangle + \langle 3, 1\rangle$

b. Possible answer: $\langle 3, -3\rangle + \langle 4, 4\rangle + \langle -3, 3\rangle$

TEST PREP, PAGE 853

23. A
$(x, y) \to (-x, y); (x, y) \to (-y, x)$
$A(2, 1) \to (-2, 1) \to (-1, -2)$

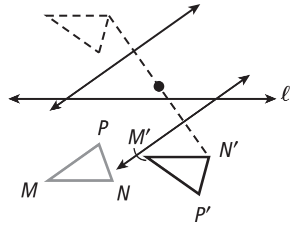
24. G
The rotation maps $\triangle ABC$ into 3rd quadrant; reflection maps image into 4th quadrant.

25. C
Add the two translation vectors to get the vector for the new translation.

CHALLENGE AND EXTEND, PAGE 853

26. $A(3, 1) = A(-1 + 4, 2 + (-1))$
$\to (-1 - (-1), 2 + 4)) = (0, 6);$
$(0, 6) = (0, 5 + 1) \to A'(0, 5 - 1) = A'(0, 4)$

27. Possible answer: Reflect $\triangle MNP$ across a horizontal line ℓ. $\triangle M'N'P'$ is a translation of this image. This means there are two || lines such that the composition of the reflections across these lines is equivalent to the translation. These lines can be found as shown in the figure.



28. First reflection: $(x, y) \to (y - 1, x + 1)$,
second reflection: $(x, y) \to (y - 3, x + 3)$,
composition: $(x, y) \to (y - 1, x + 1)$
$\to ((x + 1) - 3, (y - 1) + 3) = (x - 2, y + 2)$
translation vector: $\langle -2, 2\rangle$

SPIRAL REVIEW, PAGE 853

29. yes (no x-coordinate is repeated)

30. no (-3 would be mapped to -1 and to 1)

31. $5(EJ) = 4(8)$
$5EJ = 32$
$EJ = 6.4$

32. $5(5 + CD) = 4(4 + 12)$
$25 + 5CD = 64$
$5CD = 39$
$CD = 7.8$

33. $FH^2 = 4(4 + 16)$
$FH^2 = 64$
$FH = 8$

34. rotation sends $(x, y) \to (-y, x)$
$F(2, 3) \to F'(-3, 2)$

35. rotation sends $(x, y) \to (-x, -y)$
$N(-1, -3) \to N'(1, 3)$

36. rotation sends $(x, y) \to (-y, x)$
$Q(-2, 0) \to Q'(0, -2)$

12A MULTI-STEP TEST PREP, PAGE 854

1. translation vector: $\vec{v} = \langle 5 - 1, 1 - 3\rangle = \langle 4, -2\rangle$
$|\vec{v}| = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5} \approx 4.5$ m

2. (2, 4); First reflect H across $\overline{DC}$. Image is $H'(5, 7)$. Then draw $\overline{TH'}$ and find this segment's intersection with $\overline{DC}$.

3. path is $\langle 1, 1\rangle + \langle 3, -3\rangle$
distance is $\sqrt{1^2 + 1^2} + \sqrt{3^2 + 3^2} = \sqrt{2} + \sqrt{8}$
$= \sqrt{2} + 3\sqrt{2}$
$= 4\sqrt{2} \approx 5.7$ m

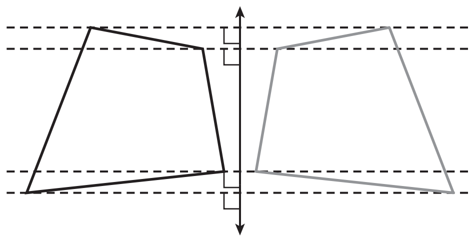
4. The turntable rotates $\frac{2}{16}(360) = 45°$ in 2 s.
Coordinates of pillar:
$(4, 2) = (3 + 1, 2) \to \left(3 + \frac{\sqrt{2}}{2}, 2 + \frac{\sqrt{2}}{2}\right) \approx (3.7, 2.7)$

12A READY TO GO ON? PAGE 855

1. Yes; the image appears to be flipped across a horizontal line.
2. No; the image does not appear to be flipped.
3. **Step 1** Through each vertex draw a line ⊥ to the the line of reflection.

   Step 2 Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.

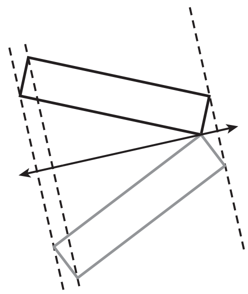
   Step 3 Connect the images of the vertices.



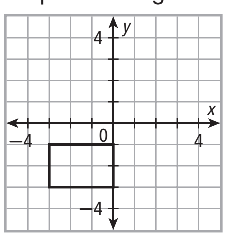
4. **Step 1** Through each vertex draw a line ⊥ to line of reflection.

   Step 2 Measure distance from each vertex to line of reflection. Locate image of each vertex on opposite side of line of reflection and same distance from it.

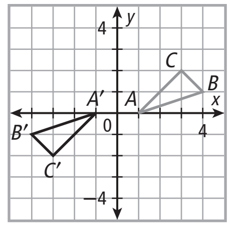
   Step 3 Connect images of vertices.



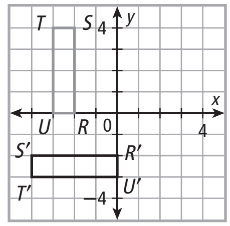
5. No; not all points have moved the same distance.
6. Yes; all points have moved the same distance in the same direction.
7. The image of (x, y) is $(x - 4, y - 3)$.
   $(1, 0) \to (1 - 4, 0 - 3) = (-3, -3)$
   $(4, 0) \to (4 - 4, 0 - 3) = (0, -3)$
   $(4, 2) \to (4 - 4, 2 - 3) = (0, -1)$
   $(1, 2) \to (1 - 4, 2 - 3) = (-3, -1)$
   Graph the image.



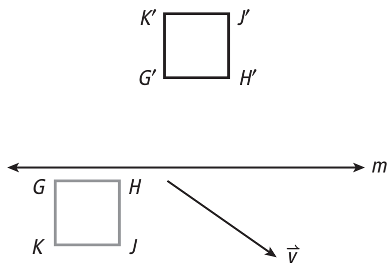
8. Yes; the figure appears to be turned around a point.
9. Yes; the figure appears to be turned around a point.
10. The rotation of (x, y) is $(-x, -y)$.
    $P(0, 2) \to P'(0, -2)$
    $Q(2, 0) \to Q'(-2, 0)$
    $R(3, -3) \to R'(-3, 3)$



11. The rotation of (x, y) is $(-y, x)$.
    $A(-1, 0) \to A'(0, -1)$
    $B(-1, -3) \to B'(3, -1)$
    $C(1, -3) \to C'(3, 1)$
    $D(1, 0) \to D'(0, 1)$



12.

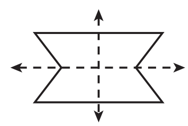


13. $A(1, 0) \to (-1, 0) \to A'(-1, 0)$
    $B(1, 3) \to (-1, 3) \to B'(-1, -3)$
    $C(2, 3) \to (-2, 3) \to C'(-1, -3)$
    $(x, y) \to (-x, y) \to (-x, -y)$
    a rotation by 180° about the origin

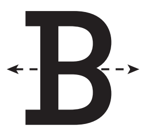
12-5 SYMMETRY, PAGES 856–862

CHECK IT OUT! PAGES 856–858

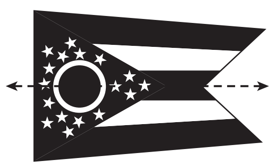
1a. yes; two lines of symmetry



b. yes; one line of symmetry



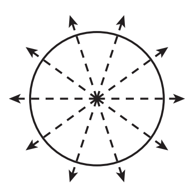
c. yes; one line of symmetry



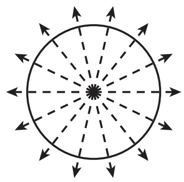
2a. yes; 120°; order: 3 **b.** yes; 180°; order: 2

c. no rotational symmetry

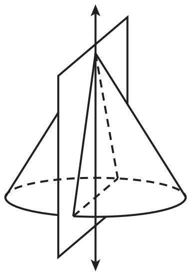
3a. line symmetry and rotational symmetry; ∠ of rotational symmetry: 72°; order: 5



b. line symmetry and rotational symmetry; ∠ of rotational symmetry: 51.4°; order: 7



4a. both; plane symmetry and symmetry about an axis



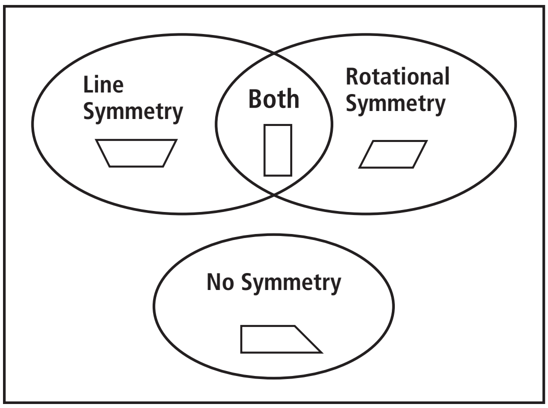
b. neither

THINK AND DISCUSS, PAGE 858

1. First, fold the paper in half. Cut out a design that goes up the fold, then unfold paper.

2. The ∠ of rotational symmetry is $\frac{360°}{n}$.

3.



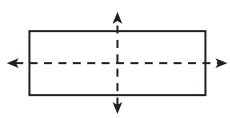
EXERCISES, PAGES 859–862

GUIDED PRACTICE, PAGE 859

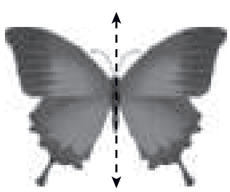
1. The line of symmetry is ⊥ bisector of base.

2. line symmetry

3. yes; two lines of symmetry



4. yes; one line of symmetry



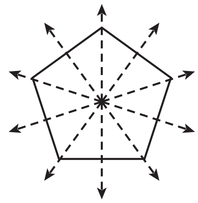
5. no line symmetry

6. yes; 180°; order: 2

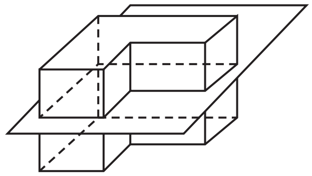
7. no rotational symmetry

8. yes; 120°; order: 3

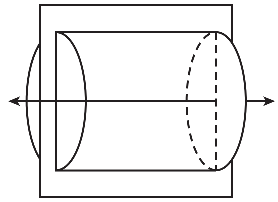
9. The ∠ of rotational symmetry is 72°; order: 5.



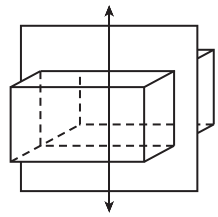
10. plane symmetry



11. both; plane symmetry and symmetry about an axis

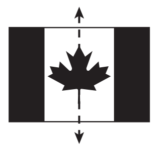


12. both; plane symmetry and symmetry about an axis

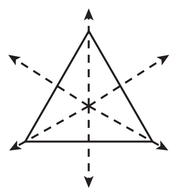


PRACTICE AND PROBLEM SOLVING, PAGES 859–861

13. yes; one line of symmetry



14. yes; three lines of symmetry



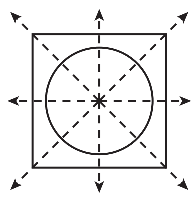
15. no line of symmetry

16. yes; 60°; order: 6

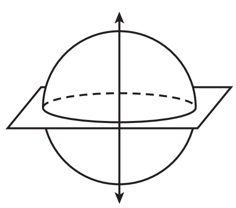
17. yes; 72°; order: 5

18. yes; 72°, order: 5.

19. The ∠ of rotational symmetry is 90°; order: 4.

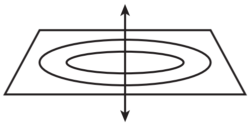


20. both; plane symmetry and symmetry about an axis



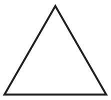
21. neither

22. both; plane symmetry and symmetry about an axis

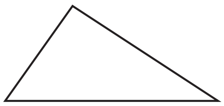


23. isosceles

24. equilateral



25. scalene



26. (area for $x < 0$) = (area for $x > 0$)

(area for $x < 0$) + (area for $x > 0$) = (area under whole curve)

2(area for $x > 0$) = 1

(area for $x > 0$) = 0.5

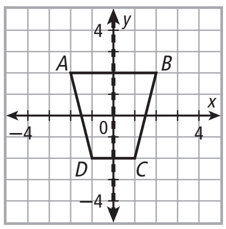
27. (area for $x > 3$) ≈ 0

28. $P = \dfrac{(\text{area for } -1 < x < 1)}{(\text{area under whole curve})}$

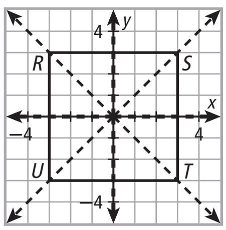
$= \dfrac{2(\text{area for } 0 < x < 1)}{1}$

$= 2(0.34) = 0.68$

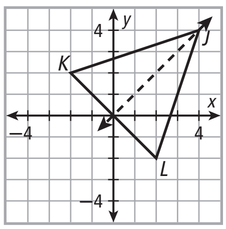
29. line symmetry



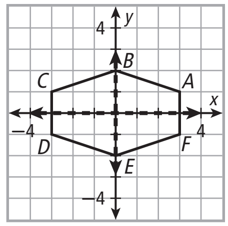
30. line symmetry and rotational symmetry; 90°; order: 4



31. line symmetry

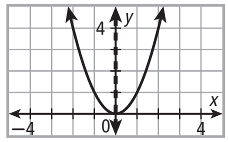


32. line symmetry and rotational symmetry; 180°; order: 2

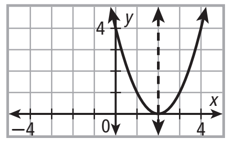


33. rotational symmetry of order 4

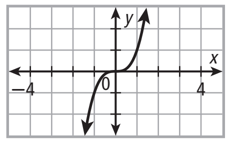
34. line symmetry; $x = 0$



35. line symmetry; $x = 2$



36. rotational symmetry; 180°; order: 2

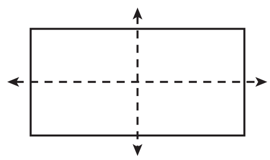


37a. No; there is no line that divides the woodcut into two identical reflected halves.

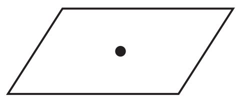
b. yes; 180°; order: 2

c. Yes; if color is not taken into account, the ∠ of rotational symmetry is 90°, and the order is 4.

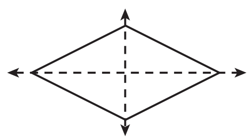
38. rectangle



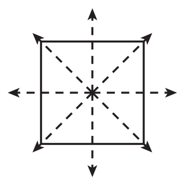
39. parallelogram



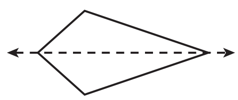
40. rhombus



41. square



42. kite



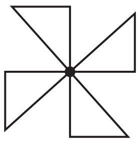
43. The ∠ of rotational symmetry is $\frac{1}{24}(360) = 15°$.

44. All rectangles have two lines of symmetry, one through midpoints of each opposite pair of sides. The only other possible lines of symmetry are through opposite vertices, which means that adjacent sides are ≅. Therefore, the rectangle must be a square.

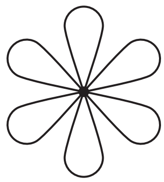
45. rotational symmetry with an ∠ of 120°, order 3

46. The figures all have a rotational symmetry of order 2.

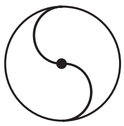
47.



48.



49.



50. If the ∠ of rotational symmetry is x °, then the order of rotational symmetry is $\frac{360}{x}$. If the order of rotational symmetry is x, then the ∠ of rotational symmetry is $\left(\frac{360}{x}\right)^{\circ}$.

TEST PREP, PAGE 862

51. A

52. J

53. C

54. H

CHALLENGE AND EXTEND, PAGE 862

55. If the polygon has n sides, then the ∠ of rotational symmetry is $\frac{360}{n}$.

$$\frac{360}{n} = 5$$
$$n = \frac{360}{5} = 72$$

56. If n is even, the lines of symmetry pass through each pair of opposite vertices and each pair of opposite midpoints of sides.

There are $\frac{n}{2} + \frac{n}{2} = n$ lines of symmetry.

If n is odd, the lines of symmetry pass through a vertex and the midpoint opposite it. So there are n lines of symmetry.

57. The line of symmetry is vertical and passes through the vertex of parabola, (−4, 0). So the line of symmetry is $x = -4$.

58. The line of symmetry is vertical and passes through vertex of the graph, (2, 0). So the line of symmetry is $x = 2$.

59. The line of symmetry is vertical and passes through the vertex of the parabola, (0, 5). So the line of symmetry is $x = 0$.

60. 13; a line through the centers of each pair of opposite faces (3 lines), a line through the midpoints of each pair of opposite edges (6 lines), and a line through each pair of opposite vertices (4 lines).

61. 8; a line through each vertex and the center of the opposite face (4 lines) and a line through the midpoints of each pair of opposite edges (4 lines).

62. 13; a line through the centers of each pair of opposite faces (4 lines), a line through the midpoints of each pair of opposite edges (6 lines), and a line through each pair of opposite vertices (3 lines).

SPIRAL REVIEW, PAGE 862

63. $\frac{E}{197.12} = \frac{20}{16}$

$$E = \frac{20}{16}(197.12) = \$246.40$$

64.
$$S = \pi r^2 + \pi r\ell$$
$$61\pi = \pi(5)^2 + \pi(5)\ell$$
$$61\pi = 25\pi + 5\pi\ell$$
$$36\pi = 5\pi\ell$$
$$36 = 5\ell$$
$$\ell = 7.2 \text{ in.}$$

65. Step 1 Find the base edge length.
$$S = B + L$$
$$65.25 = s^2 + 45$$
$$20.25 = s^2$$
$$s = 4.5 \text{ cm}$$
Step 2 Find the slant height.
$$L = \tfrac{1}{2}P\ell$$
$$45 = \tfrac{1}{2}(18)\ell = 9\ell$$
$$\ell = 5 \text{ cm}$$

66. Step 1 Find the base area.
$$b = \tfrac{1}{3}P = 8\sqrt{3} \text{ m}$$
$$h = \left(4\sqrt{3}\right)\sqrt{3} = 12 \text{ m}$$
$$B = \tfrac{1}{2}bh = \tfrac{1}{2}\left(8\sqrt{3}\right)(12) = 48\sqrt{3} \text{ m}^2$$
Step 2 Find the slant height.
$$S = B + L$$
$$120\sqrt{3} = 48\sqrt{3} + \tfrac{1}{2}P\ell$$
$$72\sqrt{3} = \tfrac{1}{2}\left(24\sqrt{3}\right)\ell$$
$$72 = 12\ell$$
$$\ell = 6 \text{ m}$$

67. $P(-1, 4) \rightarrow (4, -1) \rightarrow P'(4 + 2, -1 - 4) = P'(6, -5)$

68. $P(-1, 4) \rightarrow (-4, -1) \rightarrow P'(4, -1)$

69. $P(-1, 4) \rightarrow (-1 + 1, 4) = (0, 4) \rightarrow P'(0, -4)$

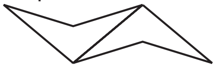
12-6 TESSELLATIONS, PAGES 863–869

CHECK IT OUT! PAGES 863–865

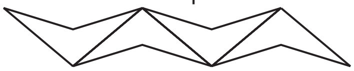
1a. translation symmetry

b. translation symmetry and glide reflection symmetry

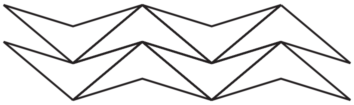
2. **Step 1** Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



Step 3 Translate the row of quadrilaterals to make a tessellation.

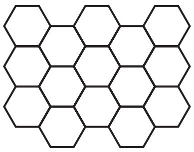


3a. Only hexagons are used. The tessellation is regular.

b. Different vertices have different arrangements of polygons. The tessellation is neither regular nor semiregular.

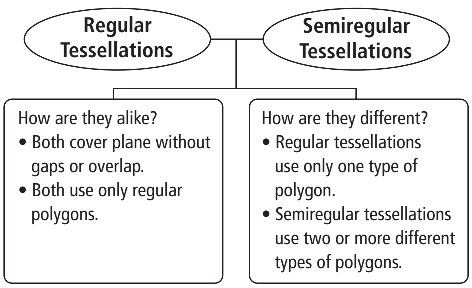
c. Two hexagons and two △s meet at each vertex. The tessellation is semiregular.

4a. Yes; three hexagons meet at each vertex. 120° + 120° + 120° = 360°



b. No; no combination of 90° and 120° ∠s is equal to 360°.

THINK AND DISCUSS, PAGE 866

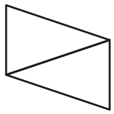
1. In a pattern that has glide reflection symmetry, you go from one figure to the next by a composition of a translation and a reflection.
2. It is not possible. No matter how circles are arranged to cover a plane, there will always be overlaps or gaps between them.
3. 

EXERCISES, PAGES 866–869

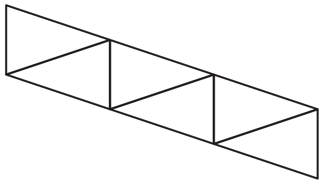
GUIDED PRACTICE, PAGE 866

1. Possible answer:

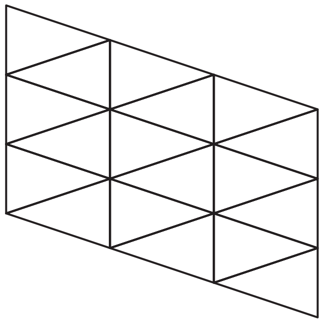
2. Possible answers: checkerboard, hexagonal floor tiles, honeycomb
3. translation symmetry, and glide reflection symmetry
4. translation symmetry, and glide reflection symmetry
5. translation symmetry, and glide reflection symmetry
6. **Step 1** Rotate △ 180° about the midpoint of one side.



Step 2 Translate the resulting pair of △s to make a row of △s.

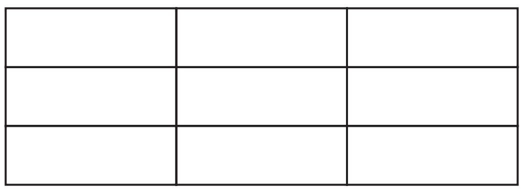


Step 3 Translate the row of △s to make a tessellation.

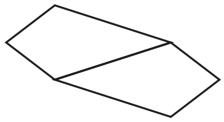


7. **Step 1** Translate the rectangle to make a row of rectangles.

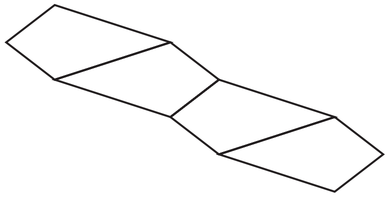
Step 2 Translate the row of rectangles to make a tessellation.



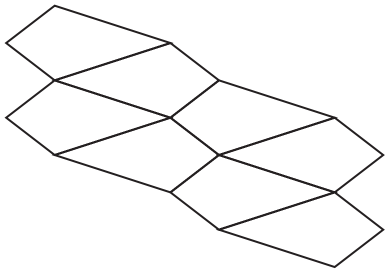
8. **Step 1** Rotate the kite 180° about the midpoint of one side.



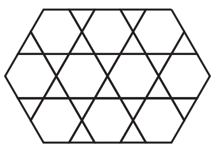
Step 2 Translate the resulting pair of kites to make a row of kites.



Step 3 Translate the row of kites to make a tessellation.



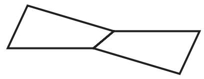
9. Only hexagons are used. The tessellation is regular.
10. Irregular quadrilaterals (rectangles) are used. The tessellation is neither regular nor semiregular.
11. Two hexagons and two △s meet at each vertex. The tessellation is semiregular.
12. No; each ∠ of the octagon measures 135°, and 135 is not a divisor of 360.
13. Yes; possible answer: two hexagons and two △s meet at each vertex. $120° + 120° + 60° + 60° = 360°$



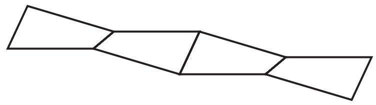
14. No; no combination of 90° and 72° ∠s is = to 360°.

PRACTICE AND PROBLEM SOLVING, PAGES 867–868

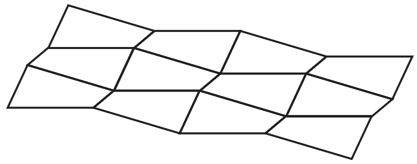
15. translation symmetry
16. translation symmetry and glide reflection symmetry
17. translation symmetry
18. **Step 1** Rotate quadrilateral 180° about the midpoint of one side.



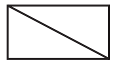
Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals



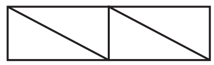
Step 3 Translate the row of quadrilaterals to make a tessellation.



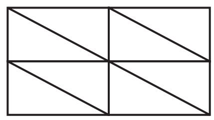
19. **Step 1** Rotate the △ 180° about the midpoint of one side.



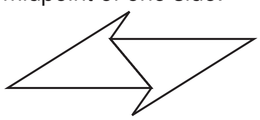
Step 2 Translate the resulting pair of △s to make a row of △s.



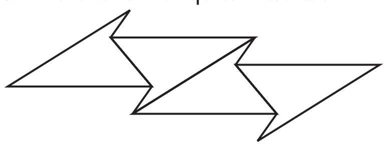
Step 3 Translate the row of △s to make a tessellation.



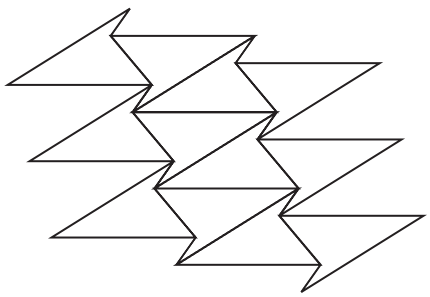
20. **Step 1** Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



Step 3 Translate the row of quadrilaterals to make a tessellation.



21. Different vertices have different arrangements of polygons. The tessellation is neither regular nor semiregular.

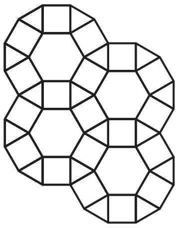
22. Two hexagons, two squares, and one △ meet at each vertex. The tessellation is semiregular.

23. Irregular pentagons are used. The tessellation is neither regular nor semiregular.

24. No; each ∠ of the heptagon measures $\frac{5}{7}(180) \approx 128.6°$, which is not a divisor of 360.

25. No; no combination of 60° and 135° ∡ is = to 360°.

26. Yes; two hexagons, two squares, and one △ meet at each vertex.
$2(120°) + 2(90°) + 60° = 360°$



27. translation and glide reflection symmetry

28. translation

29. translation, glide reflection, rotation

30. translation, glide reflection, rotation

31. always

32. sometimes

33. always

34. never

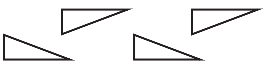
35. never

36a. equilateral △

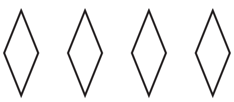
36b. parallelogram

37.

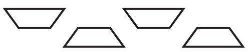
38.



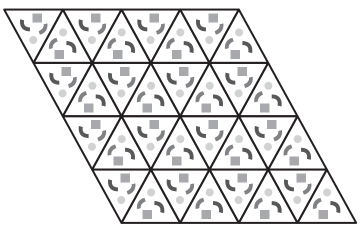
39.



40.



41.

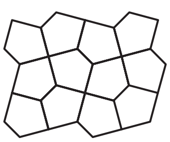


The tessellation has translation symmetry, reflection symmetry, and order 3 rotation symmetry.

42. No, because the interior ∠ measures of a regular pentagon and a regular hexagon are 120° and 108°, and there is no possible arrangement of both these ∠ measures that adds to 360°.

43. CH_2 — CH
with CH_3 attached below CH

44.



45. Equilateral ∡, squares, and regular hexagons are the only regular polygons with interior ∠ measures that divide evenly into 360°.

TEST PREP, PAGE 869

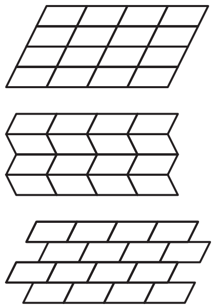
46. B

47. H

48. D
$2(90°) + 3(60°) = 360°$
Two squares and three ∡ fit around vertex.

CHALLENGE AND EXTEND, PAGE 869

49. Possible answer:



50. No; you cannot fit cylinders together in any way without creating gaps or overlaps.

51. Each prism is made up of quadrilaterals and since any quadrilateral can be used to tessellate the plane, the prism will tessellate three-dimensional space.

52. Each prism is made up of quadrilateral and since any quadrilateral can be used to tessellate the plane, the prism will tessellate three-dimensional space.

SPIRAL REVIEW, PAGE 869

53. Let r be the tax rate.

$$8.00(0.85)(1 + r) + 2.69 = 10.00$$
$$6.8(1 + r) = 7.71$$
$$1 + r = 1.075$$
$$r = 0.075 = 7.5\%$$

54. Let m be the minutes before 7:00 AM
Louis' jogging time is $\frac{5 \text{ mi}}{6 \text{ mi/h}}(60 \text{ min/h}) = 50$ min
Andrea's jogging time is $\frac{3.9 \text{ mi}}{6.5 \text{ mi/h}}(60 \text{ min/h}) = 36$ min
$50 = 36 + m$
$m = 14$
Louis should leave home 14 min before 7:00 AM, or at 6:46 AM

55. $(x - (-2))^2 + (y - 3)^2 = \left(\sqrt{5}\right)^2$
$(x + 2)^2 + (y - 3)^2 = 5$

56. $x^2 + y^2 = r^2$
$3^2 + 4^2 = r^2$
$25 = r^2$
$x^2 + y^2 = 25$

57. $(x - 5)^2 + (y + 3)^2 = r^2$
$(1 - 5)^2 + (-1 + 3)^2 = r^2$
$20 = r^2$
$(x - 5)^2 + (y + 3)^2 = 20$

58. no rotational symmetry

59. ∠ of rotational symmetry: 72°; order: 5

60. ∠ of rotational symmetry: 180°; order: 2

12-6 GEOMETRY LAB: USE TRANSFORMATIONS TO EXTEND TESSELLATIONS, PAGES 870–871

ACTIVITY 1, TRY THIS, PAGE 870

1–3. Check students' work.

ACTIVITY 2, TRY THIS, PAGE 871

4. Check students' work.

5. Possible answer: This tessellation has rotational and translation symmetry, but the tessellation from Activity 1 has only translation symmetry.

6. Check students' work.

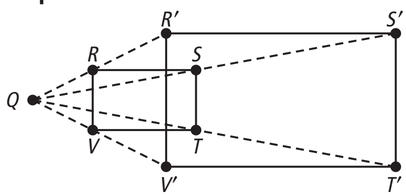
12-7 DILATIONS, PAGES 872–879

CHECK IT OUT! PAGES 872–874

1a. No; the figures are not similar.

1b. Yes; the figures are similar and the image is not turned or flipped.

2. **Step 1** Draw a ray through Q and each vertex.
Step 2 On each ray, mark three times the distance from Q to each vertex.
Step 3 Connect the vertices of the image.



3. Scale factor is 4. So a 1 in. by 1 in. square on the photo represents a 4 in. by 4 in. square on the painting.
Find the dimensions of the painting.
$s = 4(10) = 40$ in.
Find the area of the painting.
$A = s^2 = (40)^2 = 1600$ in.^{2}

4. The dilation of (x, y) is $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.

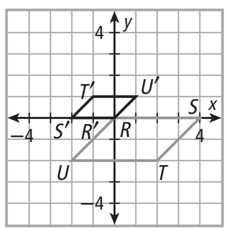
$R(0, 0) \rightarrow R'\left(-\frac{1}{2}(0), -\frac{1}{2}(0)\right) = R'(0, 0)$

$S(4, 0) \rightarrow S'\left(-\frac{1}{2}(4), -\frac{1}{2}(0)\right) = S'(-2, 0)$

$T(2, -2) \rightarrow T'\left(-\frac{1}{2}(2), -\frac{1}{2}(-2)\right) = T'(-1, 1)$

$U(-2, -2) \rightarrow U'\left(-\frac{1}{2}(-2), -\frac{1}{2}(-2)\right) = U'(1, 1)$

Graph the preimage and the image.

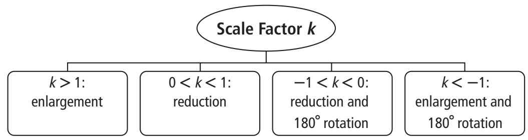


THINK AND DISCUSS, PAGE 874

1. Measure the length of one side of the image and the length of the corresponding side of the preimage. Form the ratio of these two lengths to find the scale factor.

2. dilation by a scale factor of $-k$

3.



EXERCISES, PAGES 875–879

GUIDED PRACTICE, PAGE 875

1. The center is the origin; the scale factor is 3.

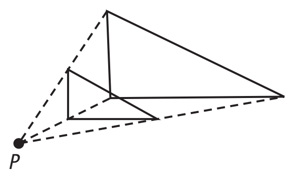
2. Yes; the figures are similar and the image is not turned or flipped.

3. Yes; the figures are similar and the image is not turned or flipped.

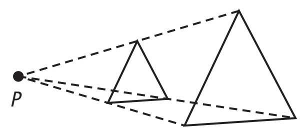
4. No; the figures are not similar.

5. Yes; the figures are similar and the image is not turned or flipped.

6. **Step 1** Draw a ray through P and each vertex.
Step 2 On each ray, mark two times the distance from P to each vertex.
Step 3 Connect vertices of the image.

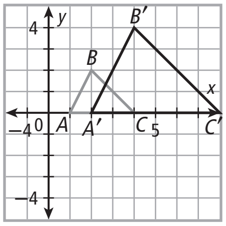


7. **Step 1** Draw a ray through P and each vertex.
Step 2 On each ray, mark $\frac{1}{2}$ the distance from P to each vertex.
Step 3 Connect the vertices of the image.



8. The scale factor of the dilation from the room to the blueprint is $\frac{1}{50}$.
Therefore, the scale factor of the dilation from the blueprint to the room is 50. So a 1 in. by 1 in. square on the blueprint represents a 50 in. by 50 in. square in the room.
Find the dimensions of the room.
$\ell = 50(8) = 400$ in.
$w = 50(6) = 300$ in.
Find perimeter of room.
$A = 2\ell + 2w$
$= 2(400) + 2(300) = 1400$ in.
$= 116$ ft 8 in.

9. The dilation of (x, y) is $(2x, 2y)$.
$A(1, 0) \rightarrow A'(2(1), 2(0)) = A'(2, 0)$
$B(2, 2) \rightarrow B'(2(2), 2(2)) = B'(4, 4)$
$C(4, 0) \rightarrow C'(2(4), 2(0)) = C'(8, 0)$
Graph the preimage and the image.



10. Dilation of (x, y) is $\left(\frac{1}{2}x, \frac{1}{2}y\right)$.

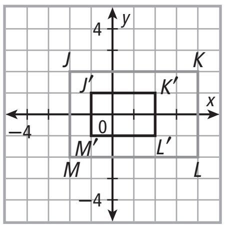
$J(-2, 2) \rightarrow J'\left(\frac{1}{2}(-2), \frac{1}{2}(2)\right) = J'(-1, 1)$

$K(4, 2) \rightarrow K'\left(\frac{1}{2}(4), \frac{1}{2}(2)\right) = K'(2, 1)$

$L(4, -2) \rightarrow L'\left(\frac{1}{2}(4), \frac{1}{2}(-2)\right) = L'(2, -1)$

$M(-2, -2) \rightarrow M'\left(\frac{1}{2}(-2), \frac{1}{2}(-2)\right) = M'(-1, -1)$

Graph the preimage and the image.



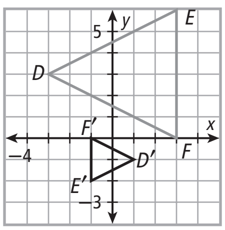
11. The dilation of (x, y) is $\left(-\frac{1}{3}x, -\frac{1}{3}y\right)$.

$D(-3, 3) \rightarrow D'\left(-\frac{1}{3}(-3), -\frac{1}{3}(3)\right) = D'(1, -1)$

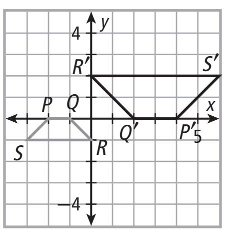
$E(3, 6) \rightarrow E'\left(-\frac{1}{3}(3), -\frac{1}{3}(6)\right) = E'(-1, -2)$

$F(3, 0) \rightarrow F'\left(-\frac{1}{3}(3), -\frac{1}{3}(0)\right) = F'(-1, 0)$

Graph the preimage and the image.



12. The dilation of (x, y) is $(-2x, -2y)$.
$P(-2, 0) \rightarrow P'(-2(-2), -2(0)) = P'(4, 0)$
$Q(-1, 0) \rightarrow Q'(-2(-1), -2(0)) = Q'(2, 0)$
$R(0, -1) \rightarrow R'(-2(0), -2(-1)) = R'(0, 2)$
$S(-3, -1) \rightarrow S'(-2(-3), -2(-1)) = S'(6, 2)$
Graph the preimage and the image.



PRACTICE AND PROBLEM SOLVING, PAGES 875–878

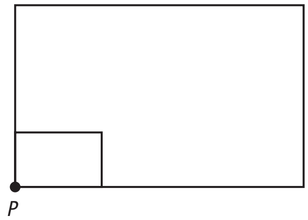
13. Yes; the figures are similar and the image is not turned or flipped.

14. Yes; the figures are similar and the image is not turned or flipped.

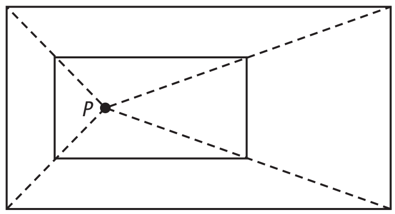
15. No; the image is flipped.

16. Yes; the figures are similar and the image is not turned or flipped.

17. **Step 1** Draw a ray through P and each vertex.
Step 2 On each ray, mark 3 times the distance from P to each vertex.
Step 3 Connect the vertices of the image.



18. **Step 1** Draw a ray through P and each vertex.
Step 2 On each ray, mark one half the distance from P to each vertex.
Step 3 Connect the vertices of the image.



19. Scale factor is 1.5, so a 1 cm by 1 cm square on photo represents a 1.5 cm by 1.5 cm square on enlargement.
Find dimensions of enlargement.
$\ell = 1.5(6) = 9$ cm
$w = 1.5(8) = 12$ cm
# of tiles = area of enlargement
$= \ell w$
$= (9)(12) = 108$ tiles

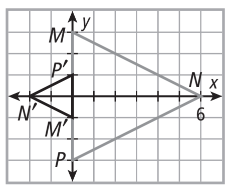
20. The dilation of (x, y) is $\left(-\frac{1}{3}x, -\frac{1}{3}y\right)$.

$M(0, 3) \rightarrow M'\left(-\frac{1}{3}(0), -\frac{1}{3}(3)\right) = M'(0, -1)$

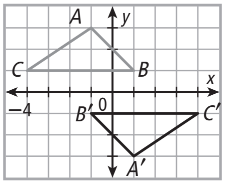
$N(6, 0) \rightarrow N'\left(-\frac{1}{3}(6), -\frac{1}{3}(0)\right) = N'(-2, 0)$

$P(0, -3) \rightarrow P'\left(-\frac{1}{3}(0), -\frac{1}{3}(-3)\right) = P'(0, 1)$

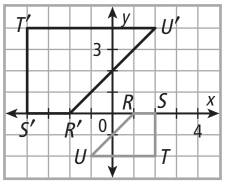
Graph the preimage and the image.



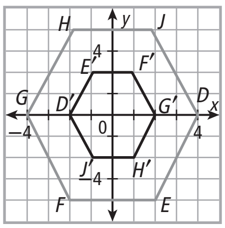
21. The dilation of (x, y) is $(-x, -y)$.
$A(-1, 3) \rightarrow A'(-(-1), -(3)) = A'(1, -3)$
$B(1, 1) \rightarrow B'(-(1), -(1)) = B'(-1, -1)$
$C(-4, 1) \rightarrow C'(-(-4), -(1)) = C'(4, -1)$
Graph the preimage and the image.



22. The dilation of (x, y) is $(-2x, -2y)$.
$R(1, 0) \rightarrow R'(-2(1), -2(0)) = R'(-2, 0)$
$S(2, 0) \rightarrow S'(-2(2), -2(0)) = S'(-4, 0)$
$T(2, -2) \rightarrow T'(-2(2), -2(-2)) = T'(-4, 4)$
$U(-1, -2) \rightarrow U'(-2(-1), -2(-2)) = U'(2, 4)$
Graph the preimage and the image.



23. The dilation of (x, y) is $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.
$D(4, 0) \rightarrow D'\left(-\frac{1}{2}(4), -\frac{1}{2}(0)\right) = D'(-2, 0)$
$E(2, -4) \rightarrow E'\left(-\frac{1}{2}(2), -\frac{1}{2}(-4)\right) = E'(-1, 2)$
$F(-2, -4) \rightarrow F'\left(-\frac{1}{2}(-2), -\frac{1}{2}(-4)\right) = F'(1, 2)$
$G(-4, 0) \rightarrow G'\left(-\frac{1}{2}(-4), -\frac{1}{2}(0)\right) = G'(2, 0)$
$H(-2, 4) \rightarrow H'\left(-\frac{1}{2}(-2), -\frac{1}{2}(-4)\right) = H'(1, -2)$
$J(2, 4) \rightarrow J'\left(-\frac{1}{2}(2), -\frac{1}{2}(4)\right) = J'(-1, -2)$
Graph the preimage and the image.

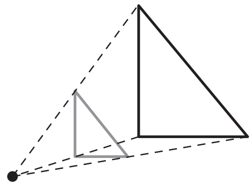


24. $\triangle FGH \sim \triangle KLM$

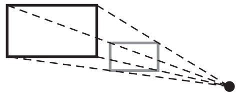
25. $ABCDE \sim MNPQR$

26. Find the the dimensions of the image.
$\ell = 4(5) = 20$ cm
$w = 4(2) = 8$ cm
$h = 4(3) = 12$ cm
Find the surface area and the volume.
$S = L + B$
$= (2\ell + 2w)h + 2\ell w$
$= (2(20) + 2(8))(12) + 2(20)(8)$
$= 56(12) + 40(8) = 992 \text{ cm}^2$
$V = Bh$
$= \ell wh$
$= (20)(8)(12) = 1920 \text{ cm}^3$

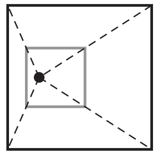
27.



28.



29.



30a. Let the scale factor be k.
poster length = $k \cdot$ original length
$82.8 = k(27.6)$
$k = 3$

b. $A = \ell w$
$= 82.8(3 \cdot 19.9)$
$= 4943.16 \text{ cm}^2$

31. Solution B is incorrect. The area of the rectangle $A'B'C'D'$ is $(2.5)^2 \cdot 6 = 37.5$.

32a. $8 = k(6)$
$k = 1\frac{1}{3}$

b. before dilation:
$A = \pi(3)^2 = 9\pi \approx 28.3 \text{ mm}^2$
after dilation:
$A = \pi(4)^2 = 16\pi \approx 50.3 \text{ mm}^2$

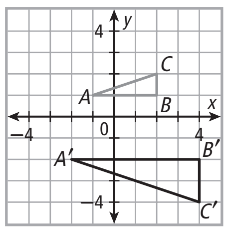
c. light increases by a factor m, where
$50.3 \approx (1 + m)(28.3)$
$1.78 \approx 1 + m$
$m = 0.78 = 78\%$

33a. Possible answer: 2.5. Check students' estimates.

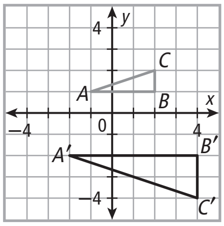
b. Measure $\overline{AB}$ and $\overline{A'B'}$, then calculate $\frac{A'B'}{AB}$.

c. $\frac{16}{6.5} \approx 2.5$

34a. The dilation of (x, y) is $(2x, 2y)$.
$A(-1, 1) \rightarrow A'(-2, 2) \rightarrow A''(-2, -2)$
$B(2, 1) \rightarrow B'(4, 2) \rightarrow B''(4, -2)$
$C(2, 2) \rightarrow C'(4, 4) \rightarrow C''(4, -4)$



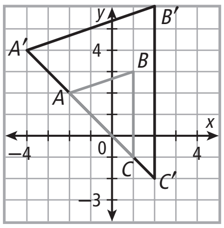
b. The dilation of (x, y) is $(2x, 2y)$.
$A(-1, 1) \to A'(-1, -1) \to A''(-2, -2)$
$B(2, 1) \to B'(2, -1) \to B''(4, -2)$
$C(2, 2) \to C'(2, -2) \to C''(4, -4)$



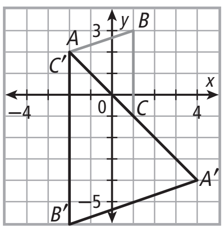
c. The images are the same. The order of transformations does not matter.

35. $\text{scale factor} = \dfrac{-0.25 \text{ in.}}{870{,}000 \text{ mi}}$
$= \dfrac{-0.25 \text{ in.}}{(870{,}000 \text{ mi})(5280 \text{ ft/mi})(12 \text{ in./ft})}$
$\approx -4.5 \times 10^{-12}$

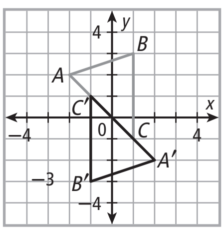
36. $A'(k(-2), k(2)) = A'(-4, 4)$
$k = 2$
$B(1, 3) \to B'(2, 6)$
$C(1, -1) \to C'(2, -2)$



37. $C'(k(1), k(-1)) = C'(-2, 2)$
$k = -2$
$A(-2, 2) \to A'(4, -4)$
$B(1, 3) \to B'(-2, -6)$



38. $B'(k(1), k(3)) = B'(-1, -3)$
$k = -1$
$A(-2, 2) \to A'(2, -2)$
$C(1, -1) \to C'(-1, 1)$



39. For $k = 1$, the image and the preimage are the same figure. For $k = -1$, the dilation is equivalent to a 180° rotation. A rotation is an isometry. So the image is ≅ to the preimage.

40. A dilation is equivalent to a 180° rotation when the scale factor is −1. In this case, the image has the same size as the preimage. So the only effect of transformation is the rotation by 180°.

41. Yes; first dilation multiplies all the linear measures of the preimage by m. The second dilation multiplies all the linear measures of the image by n. The overall effect is to multiply all the linear measures by mn, which is equivalent to a single dilation with the scale factor mn.

42–45. Check students' constructions.

TEST PREP, PAGE 878

46. B
$D = D(0, 2)$, so $D'(0, 2k) = D'(0, -2)$; $k = -1$

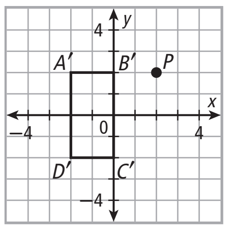
47. H
original dimensions: $\ell = 4$, $w = 2$
dilated dimensions: $\ell = 2.5(4) = 10$, $w = 2.5(2) = 5$
$P = 2\ell + 2w = 2(10) + 2(5) = 30$

48. 4.2
$(k(-2), k(3)) = (-8.4, 12.6)$
$-2k = -8.4$
$k = 4.2$

49. No; the dimensions of the enlargement are $1.5(6) = 9$ cm by $1.5(8) = 12$ cm; $A = 9(12) = 108 \text{ cm}^2$

CHALLENGE AND EXTEND, PAGE 879

50a. $A(0, 2) = A(2 + (-2), 2 + 0)$
$\to A'(2 + 2(-2), 2 + 2(0)) = A'(-2, 2)$
$B(1, 2) = B(2 + (-1), 2 + 0)$
$\to B'(2 + 2(-1), 2 + 2(0)) = B'(0, 2)$
$C(1, 0) = C(2 + (-1), 2 + (-2))$
$\to C'(2 + 2(-1), 2 + 2(-2)) = C'(0, -2)$
$D(0, 0) = D(2 + (-2), 2 + (-2))$
$\to D'(2 + 2(-2), 2 + 2(-2)) = D'(-2, -2)$



b. The transformation is the composition of the dilation centered at the origin with the scale factor two followed by the translation along the vector $\langle -2, -2 \rangle$.

c. The transformation is the composition of the dilation centered at the origin with the scale factor k followed by the translation along the vector $\langle -a, -b \rangle$.

51. The dilation sends $(x, y) \to (3x, 3y)$.
For (x, y) on line ℓ,
$y = -x + 2$
$(3y) = -(3x) + 6$
So the image point $(3x, 3y)$ lies on the line $y = -x + 6$; therefore this line is the image of ℓ after dilation.

SPIRAL REVIEW, PAGE 879

52. The data fit the equation $y = 1.6x - 4$.
For \$68 in tips,
$68 = 1.6x - 4$
$72 = 1.6x$
$x = 45$
Jerry would need to serve 45 customers.

53. $P = JK + KL + LM + JM$
$= \sqrt{3^2 + 4^2} + (7 - 0) + \sqrt{3^2 + 4^2} + (4 - (-3))$
$= 5 + 7 + 5 + 7 = 24$ units
$JKLM$ is a ▱ with $b = JM = 7$ and $h = 2 - (-2) = 4$.
$A = bh = 7(4) = 28$ units^2

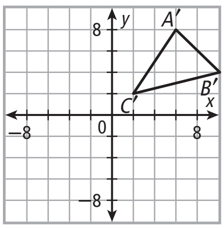
54. $P = DE + EF + DF$
$= \sqrt{4^2 + 2^2} + \sqrt{2^2 + 6^2} + \sqrt{2^2 + 4^2}$
$= 2\sqrt{5} + 2\sqrt{10} + 2\sqrt{5}$
$= \left(4\sqrt{5} + 2\sqrt{10}\right)$ units
$\overline{DE}$ and $\overline{DF}$ are $\perp$, so $\triangle DEF$ is a right $\triangle$ with
$b = DE = 2\sqrt{5}$ and $h = DF = 2\sqrt{5}$.
$A = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{5}\right)\left(2\sqrt{5}\right) = 10$ units^2

55. Yes; for example, you can fit two squares and two right △s around a vertex.

56. No; the internal ∠s are $\frac{7(180)}{9} = 140°$ and 60°, and no combination of these ∠s is = to 360°.

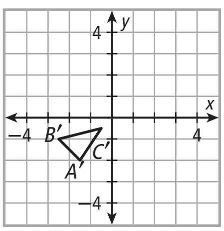
USING TECHNOLOGY, PAGE 879

2. $\begin{bmatrix} 6 & 10 & 2 \\ 8 & 4 & 2 \end{bmatrix}$



3. $\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$

4. $[A] \cdot [B] = \begin{bmatrix} -1.5 & -2.5 & -0.5 \\ -2 & -1 & -0.5 \end{bmatrix}$



12B MULTI-STEP TEST PREP, PAGE 880

1. A: none; B: ||; C: none; D: intersecting; E: none

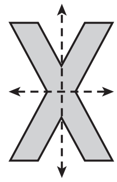
2. A: no; B: no; C: yes, 60°, order: 6; D: yes, 120°, order: 3; E: no

3. A: yes, ▱; B: yes, parallelogram; C: no; D: yes, equilateral △; E: no

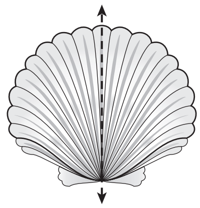
4. $21{,}098.88 = k^2 \cdot (13.2)(11.1) = k^2 \cdot 146.52$
$k^2 = 144$
$k = 12$

12B READY TO GO ON? PAGE 881

1. yes



2. yes



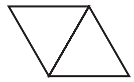
3. no

4. yes; 180°; order: 2

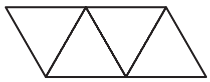
5. yes; 30°; order: 12

6. yes; 72°; order: 5

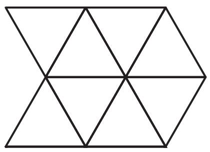
7. **Step 1** Rotate the △ 180° about the midpoint of one side.



Step 2 Translate the resulting pair of △s to make a row of △s.

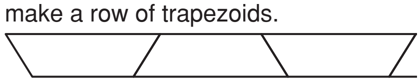


Step 3 Translate the row of △s to make a tessellation.

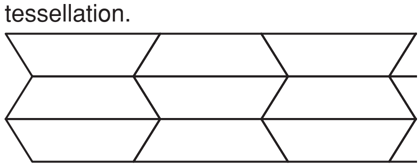


8. **Step 1** Rotate the trapezoid 180° about the midpoint of one side.

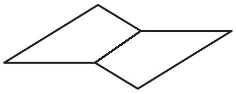
Step 2 Translate the resulting pair of trapezoids to make a row of trapezoids.



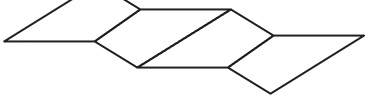
Step 3 Translate the row of trapezoids to make a tessellation.



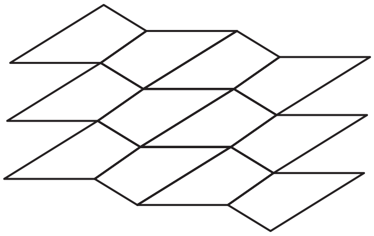
9. Step 1 Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



Step 3 Translate the row of quadrilaterals to make a tessellation.



10. Only the regular hexagons are used. The tessellation is regular.

11. Irregular quadrilaterals (rhombuses) are used. The tessellation is neither regular nor semiregular.

12. Two squares and three equilateral ⚠s meet at each vertex. The tessellation is semiregular.

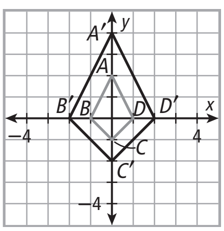
13. No; each internal ∠ of a regular octagon measures 135°, which is not a divisor of 360°.

14. Yes; the figures are similar and the image is not turned or flipped.

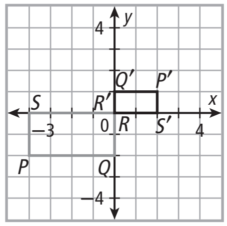
15. No; the figures are not similar.

16. Yes; the figures are similar and the image is not turned or flipped.

17. The dilation sends (x, y) to $(2x, 2y)$.
$A(0, 2) \rightarrow A'(2(0), 2(2)) = A'(0, 4)$
$B(-1, 0) \rightarrow B'(2(-1), 2(0)) = B'(-2, 0)$
$C(0, -1) \rightarrow C'(2(0), 2(-1)) = C'(0, -2)$
$D(1, 0) \rightarrow D'(2(1), 2(0)) = D'(2, 0)$



18. The dilation sends (x, y) to $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.
$P(-4, -2) \rightarrow P'\left(-\frac{1}{2}(-4), -\frac{1}{2}(-2)\right) = P'(2, 1)$
$Q(0, -2) \rightarrow Q'\left(-\frac{1}{2}(0), -\frac{1}{2}(-2)\right) = Q'(0, 1)$
$R(0, 0) \rightarrow R'\left(-\frac{1}{2}(0), -\frac{1}{2}(0)\right) = R'(0, 0)$
$S(-4, 0) \rightarrow S'\left(-\frac{1}{2}(-4), -\frac{1}{2}(0)\right) = S'(2, 0)$



STUDY GUIDE: REVIEW, PAGES 884–887

VOCABULARY, PAGE 884

1. regular tessellation
2. frieze pattern
3. isometry
4. composition of transformations

LESSON 12-1, PAGE 884

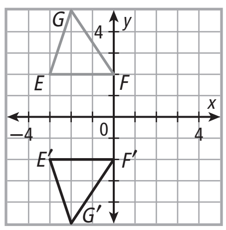
5. Yes; the image appears to be flipped.

6. No; the image appears to be shifted as well as flipped.

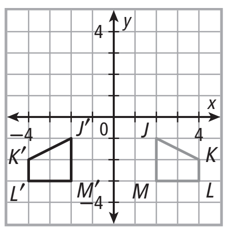
7. No; the figure appears to be turned.

8. Yes; the image appears to be flipped.

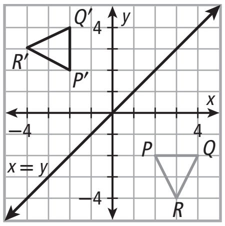
9. The image of (x, y) is $(x, -y)$.
$E(-3, 2) \rightarrow E'(-3, -2)$
$F(0, 2) \rightarrow F'(0, -2)$
$G(-2, 5) \rightarrow G'(-2, -5)$



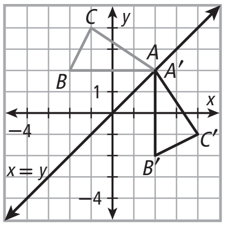
10. The image of (x, y) is $(-x, y)$.
$J(2, -1) \rightarrow J'(-2, -1)$
$K(4, -2) \rightarrow K'(-4, -2)$
$L(4, -3) \rightarrow L'(-4, -3)$
$M(2, -3) \rightarrow M'(-2, -3)$



11. The image of (x, y) is (y, x).
$P(2, -2) \rightarrow P'(-2, 2)$
$Q(4, -2) \rightarrow Q'(-2, 4)$
$R(3, -4) \rightarrow R'(-4, 3)$



12. The image of (x, y) is (y, x).
$A(2, 2) \rightarrow A'(2, 2)$
$B(-2, 2) \rightarrow B'(2, -2)$
$C(-1, 4) \rightarrow C'(4, -1)$



LESSON 12-2, PAGE 885

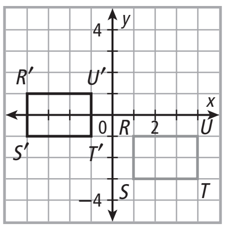
13. No; the image is smaller than the preimage.

14. Yes; the image appears to be ≅ to the preimage.

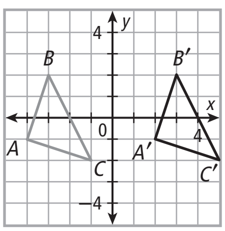
15. No; the image is flipped.

16. No; the image is smaller than the preimage.

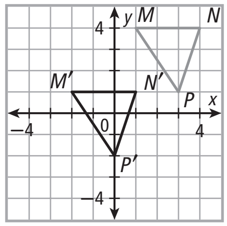
17. $R(1, -1) \rightarrow R'(1 - 5, -1 + 2) = R'(-4, 1)$
$S(1, -3) \rightarrow S'(1 - 5, -3 + 2) = S'(-4, -1)$
$T(4, -3) \rightarrow T'(4 - 5, -3 + 2) = T'(-1, -1)$
$U(4, -1) \rightarrow U'(4 - 5, -1 + 2) = U'(-1, 1)$



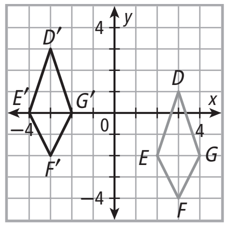
18. $A(-4, -1) \rightarrow A'(-4 + 6, -1) = A'(2, 1)$
$B(-3, 2) \rightarrow B'(-3 + 6, 2) = B'(3, 2)$
$C(-1, -2) \rightarrow C'(-1 + 6, -2) = C'(5, -2)$



19. $M(1, 4) \rightarrow M'(1 - 3, 4 - 3) = M'(-2, 1)$
$N(4, 4) \rightarrow N'(4 - 3, 4 - 3) = N'(1, 1)$
$P(3, 1) \rightarrow P'(3 - 3, 1 - 3) = P'(0, -2)$



20. $D(3, 1) \rightarrow D'(3 - 6, 1 + 2) = D'(-3, 3)$
$E(2, -2) \rightarrow E'(2 - 6, -2 + 2) = E'(-4, 0)$
$F(3, -4) \rightarrow F'(3 - 6, -4 + 2) = F'(-3, -2)$
$G(4, -2) \rightarrow G'(4 - 6, -2 + 2) = G'(-2, 0)$



LESSON 12-3, PAGE 885

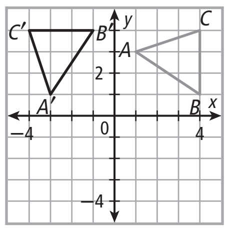
21. Yes; the image appears to be turned.

22. Yes; the image appears to be turned.

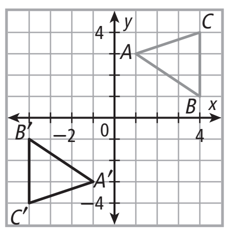
23. No; the image is smaller than the preimage.

24. No; the image appears to be flipped.

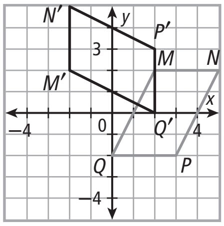
25. $(x, y) \rightarrow (-y, x)$
$A(1, 3) \rightarrow A'(-3, 1)$
$B(4, 1) \rightarrow B'(-1, 4)$
$C(4, 4) \rightarrow C'(-4, 4)$



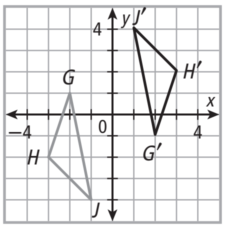
26. $(x, y) \rightarrow (-x, -y)$
$A(1, 3) \rightarrow A'(-1, -3)$
$B(4, 1) \rightarrow B'(-4, -1)$
$C(4, 4) \rightarrow C'(-4, -4)$



27. $(x, y) \rightarrow (-y, x)$
$M(2, 2) \rightarrow M'(-2, 2)$
$N(5, 2) \rightarrow N'(-2, 5)$
$P(3, -2) \rightarrow P'(2, 3)$
$Q(0, -2) \rightarrow Q'(2, 0)$

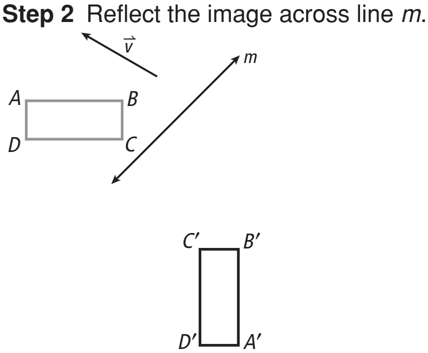


28. $(x, y) \rightarrow (-x, -y)$
$G(-2, 1) \rightarrow G'(2, -1)$
$H(-3, -2) \rightarrow H'(3, 2)$
$J(-1, -4) \rightarrow J'(1, 4)$

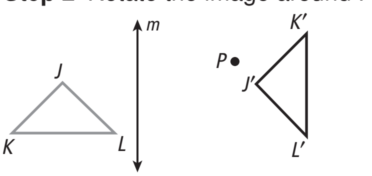


LESSON 12-4, PAGE 886

29. **Step 1** Translate *ABCD* along $\vec{v}$.
Step 2 Reflect the image across line *m*.

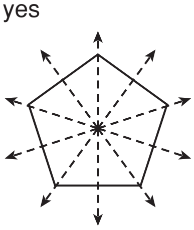


30. **Step 1** Reflect △*JKL* across line *m*.
Step 2 Rotate the image around *P*.

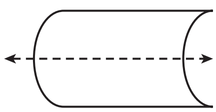


LESSON 12-5, PAGE 886

31. yes



32. yes



33. yes; 120°; order: 3

34. no

35. yes; 120°; order: 3

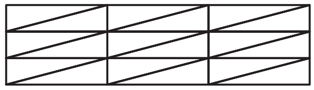
36. yes; 180°; order: 2

LESSON 12-6, PAGE 887

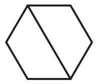
37. **Step 1** Rotate △ 180° about the midpoint of one side.

Step 2 Translate the resulting pair of △s to make a row of △s.

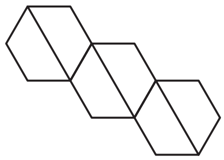
Step 3 Translate the row of △s to make a tessellation.



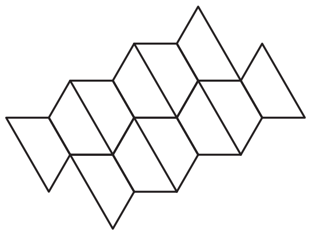
38. **Step 1** Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.

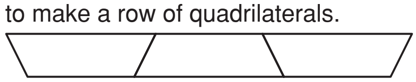


Step 3 Translate the row of quadrilaterals to make a tessellation.

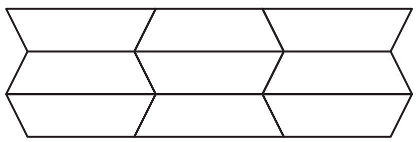


39. **Step 1** Rotate the quadrilateral 180° about the midpoint of one side.

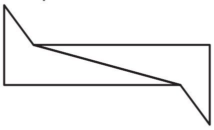
Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



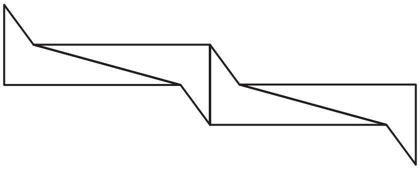
Step 3 Translate the row of quadrilaterals to make a tessellation.



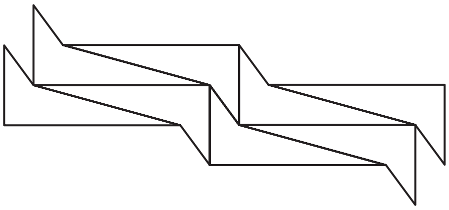
40. **Step 1** Rotate the quadrilateral 180° about the midpoint of one side.



Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



Step 3 Translate the row of quadrilaterals to make a tessellation.



41. Irregular pentagons are used. The tessellation is neither regular nor semiregular.

42. Two regular hexagons and two equilateral △ meet as each vertex. The tessellation is semiregular.

LESSON 12-7, PAGE 887

43. Yes; the figures appear to be similar, and the image is not flipped or turned.

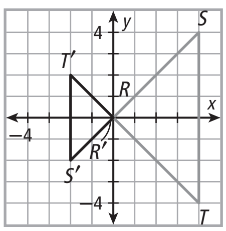
44. Yes; the figures appear to be similar, and the image is not flipped or turned.

45. The dilation sends (x, y) to $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.

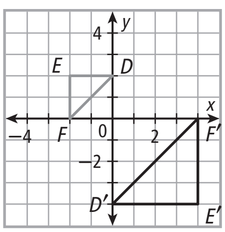
$R(0, 0) \rightarrow R'\left(-\frac{1}{2}(0), -\frac{1}{2}(0)\right) = R'(0, 0)$

$S(4, 4) \rightarrow S'\left(-\frac{1}{2}(4), -\frac{1}{2}(4)\right) = S'(-2, -2)$

$T(4, -4) \rightarrow T'\left(-\frac{1}{2}(4), -\frac{1}{2}(-4)\right) = T'(-2, 2)$



46. The dilation sends (x, y) to $(-2x, -2y)$.
$D(0, 2) \rightarrow D'(-2(0), -2(2)) = D'(0, -4)$
$E(-2, 2) \rightarrow E'(-2(-2), -2(2)) = E'(4, -4)$
$F(-2, 0) \rightarrow F'(-2(-2), -2(0)) = F'(4, 0)$



CHAPTER TEST, PAGE 888

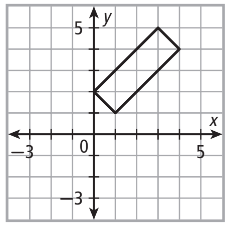
1. No; the image appears to be shifted, not flipped.

2. Yes; the figures are ≅ and the image is not flipped or turned.

3. No; the image is smaller than the preimage.

4. Yes; the figures are similar and the image is not flipped or turned.

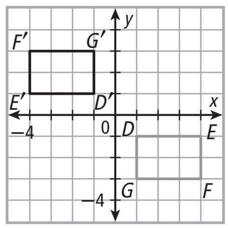
5. $(1, 3) \rightarrow (1 - 1, 3 - 1) = (0, 2)$
$(2, 2) \rightarrow (2 - 1, 2 - 1) = (1, 1)$
$(5, 5) \rightarrow (5 - 1, 5 - 1) = (4, 4)$
$(4, 6) \rightarrow (4 - 1, 6 - 1) = (3, 5)$



6. Yes; the figures appear ≅ and the image appears to be turned.

7. Yes; the figures appear ≅ and the image appears to be turned.

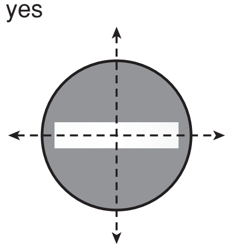
8. $(x, y) \rightarrow (-x, -y)$
$D(1, -1) \rightarrow D'(-1, 1)$
$E(4, -1) \rightarrow E'(-4, 1)$
$F(4, -3) \rightarrow F'(-4, 3)$
$G(1, -3) \rightarrow G'(-1, 3)$



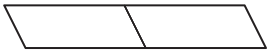
9. Rectangle *ABCD* with vertices *A*(3, −1), *B*(3, −2), *C*(1, −2), and *D*(1, −1) is reflected across the *y–axis*, and then its image is reflected across the *x–axis*. Describe a single transformation that moves the rectangle from its starting position to its final position.

10. yes

11. yes; 180°; order: 2

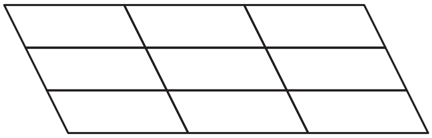


12. Step 1 Rotate the quadrilateral 180° about the midpoint of one side.

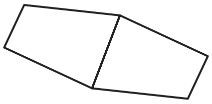


Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.

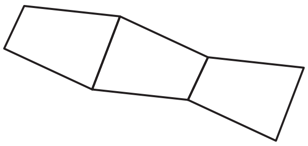
Step 3 Translate the row of quadrilaterals to make a tessellation.



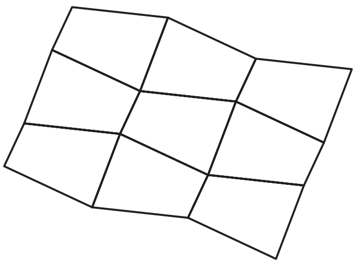
13. Step 1 Rotate the quadrilateral 180° about the midpoint of one side.



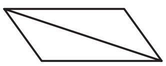
Step 2 Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.



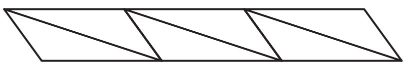
Step 3 Translate the row of quadrilaterals to make a tessellation.



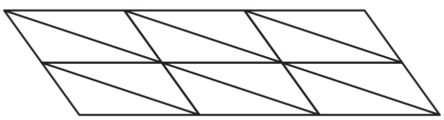
14. Step 1 Rotate the △ 180° about the midpoint of one side.



Step 2 Translate the resulting pair of ⚠ to make a row of ⚠.



Step 3 Translate the row of ⚠ to make a tessellation.



15. One square and two regular octagons meet at each vertex. The tessellation is semiregular.

16. Yes; the figures are similar, and the image is not turned or flipped.

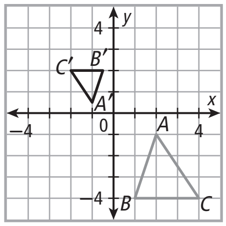
17. No; the figures are not similar.

18. The dilation sends (x, y) to $\left(-\frac{1}{2}x, -\frac{1}{2}y\right)$.

$A(2, -1) \rightarrow A'\left(-\frac{1}{2}(2), -\frac{1}{2}(-1)\right) = A'(-1, 0.5)$

$B(1, -4) \rightarrow B'\left(-\frac{1}{2}(1), -\frac{1}{2}(-4)\right) = S'(-0.5, 2)$

$C(4, -4) \rightarrow C'\left(-\frac{1}{2}(4), -\frac{1}{2}(-4)\right) = C'(-2, 2)$



COLLEGE ENTRANCE EXAM PRACTICE, PAGE 889

1. A
$f(-x) = (-x)^4 - 2 = x^4 - 2 = f(x)$

2. G
$(1, -3) \rightarrow (-1, -7) = (1 - 2, -3 - 4)$
$(-4, 5) \rightarrow (-4 - 2, 5 - 4) = (-6, 1)$

3. D
$(x, y) \rightarrow (x, -y)$
$(-2, -5) \rightarrow (-2, 5)$

4. H
$A(1, 4) \rightarrow (-1, 4) \rightarrow (-1, 4 - 6) = C(-1, -2)$
$B(4, 2) \rightarrow (-4, 2) \rightarrow (-4, 2 - 6) = D(-4, -4)$

5. C