IB Math SL

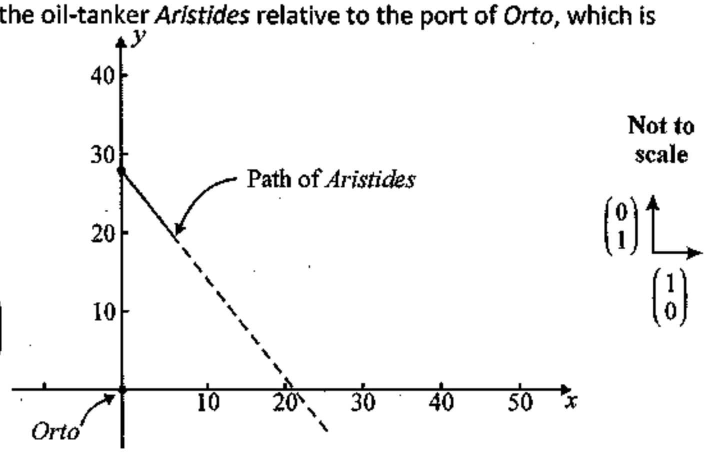
Ch. 11 – Vector Applications

Name Key

Hour____

1. In this question the vector $\begin{pmatrix}1\\0\end{pmatrix}$ km represents a displacement due east, and the vector $\begin{pmatrix}0\\1\end{pmatrix}$ km represents a displacement due north.

The diagram shows the path of the oil-tanker *Aristides* relative to the port of *Orto*, which is situated at the point (0, 0).



The position of the *Aristides* is given by the vector equation

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\28\end{pmatrix}+t\begin{pmatrix}6\\-8\end{pmatrix}$$

at a time t hours after 12:00.

(a) Find the position of the *Aristides* at 13:00.

$\begin{pmatrix}0\\28\end{pmatrix}+1\begin{pmatrix}6\\-8\end{pmatrix}=\boxed{\begin{pmatrix}6\\20\end{pmatrix}}$

(2)

(b) Find

(i) the velocity vector; $\boxed{\begin{pmatrix}6\\-8\end{pmatrix}}$

(ii) the speed of the *Aristides*. $\sqrt{6^2+(-8)^2}=\boxed{10\text{km/hr}}$

(4)

Another ship, the cargo-vessel *Boadicea*, is stationary, with position vector $\begin{pmatrix}18\\4\end{pmatrix}$ km.

(c) Show that the two ships will collide, and find the time of collision.

$\begin{pmatrix}18\\4\end{pmatrix}=\begin{pmatrix}0\\28\end{pmatrix}+t\begin{pmatrix}6\\-8\end{pmatrix}$

$18=6t$ $\quad$ $4=28-8t$ $\quad$ 15:00

$t=3$ $\quad$ $t=3$

Same place after 3 hrs.

(4)

2. In this question, distance is in kilometers, time is in hours. A balloon is moving at a constant height with a speed of 18 km h^{-1}, in the direction of the vector $\begin{pmatrix}3\\4\\0\end{pmatrix}$. At time $t = 0$, the balloon is at point B with coordinates (0, 0, 5).

(a) Show that the position vector $\boldsymbol{b}$ of the balloon at time t is given by

$$\boldsymbol{b} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}.$$

unit vector of $\begin{pmatrix}3\\4\\0\end{pmatrix} = \frac{1}{\sqrt{9+16}}\begin{pmatrix}3\\4\\0\end{pmatrix} = \frac{1}{5}\begin{pmatrix}3\\4\\0\end{pmatrix} = \begin{pmatrix}0.6\\0.8\\0\end{pmatrix}$

Magnitude needs to be 18: $18\begin{pmatrix}0.6\\0.8\\0\end{pmatrix} = \begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$

$$\boxed{\vec{b} = \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}}$$

(6)

At time $t = 0$, a helicopter goes to deliver a message to the balloon. The position vector $\boldsymbol{h}$ of the helicopter at time t is given by

$$\boldsymbol{h} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}49\\32\\0\end{pmatrix} + t\begin{pmatrix}-48\\-24\\6\end{pmatrix}$$

(b) (i) Write down the coordinates of the starting position of the helicopter. $\boxed{\begin{pmatrix}49\\32\\0\end{pmatrix}}$

(ii) Find the speed of the helicopter.

$\sqrt{(48)^2+(24)^2+(6)^2} = \boxed{54 \text{ km/hr}}$

(4)

(c) The helicopter reaches the balloon at point R.

(i) Find the time the helicopter takes to reach the balloon.

$\begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix} = \begin{pmatrix}49\\32\\0\end{pmatrix} + t\begin{pmatrix}-48\\-24\\6\end{pmatrix} \leftarrow$ $5 = 6t$

$t = 5/6$ hr

(ii) Find the coordinates of R.

$\begin{pmatrix}0\\0\\5\end{pmatrix} + \frac{5}{6}\begin{pmatrix}10.8\\14.4\\0\end{pmatrix} = \boxed{\begin{pmatrix}9\\12\\5\end{pmatrix}}$

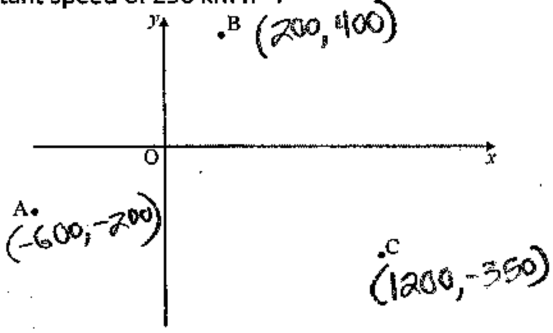
(5)
(Total 15 marks)

3. In this question the vector $\begin{pmatrix}1\\0\end{pmatrix}$ represents a displacement of 1 km east,

and the vector $\begin{pmatrix}0\\1\end{pmatrix}$ represents a displacement of **1** km north.

The diagram below shows the positions of towns A, B and C in relation to an airport O, which is at the point (0, 0). An aircraft flies over the three towns at a constant speed of 250 km h^{-1}.

Town A is 600 km west and 200 km south of the airport.
Town B is 200 km east and 400 km north of the airport.
Town C is 1200 km east and 350 km south of the airport.



(a) (i) Find $\overrightarrow{AB}$. $\begin{pmatrix}200\\400\end{pmatrix} - \begin{pmatrix}-600\\-200\end{pmatrix} = \boxed{\begin{pmatrix}800\\600\end{pmatrix}}$

(ii) Show that the vector of length one unit in the direction of $\overrightarrow{AB}$ is $\begin{pmatrix}0.8\\0.6\end{pmatrix}$.

$$\frac{1}{\sqrt{800^2+600^2}}\begin{pmatrix}800\\600\end{pmatrix} = \frac{1}{1000}\begin{pmatrix}800\\600\end{pmatrix} = \begin{pmatrix}0.8\\0.6\end{pmatrix}$$

(4)

An aircraft flies over town A at 12:00, heading towards town B at 250 km h^{-1}.

Let $\begin{pmatrix}p\\q\end{pmatrix}$ be the velocity vector of the aircraft. Let t be the number of hours in flight after 12:00.

The position of the aircraft can be given by the vector equation $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-600\\-200\end{pmatrix} + t\begin{pmatrix}p\\q\end{pmatrix}$.

(b) (i) Show that the velocity vector is $\begin{pmatrix}200\\150\end{pmatrix}$.

$$250\begin{pmatrix}0.8\\0.6\end{pmatrix} = \begin{pmatrix}200\\150\end{pmatrix}$$

(ii) Find the position of the aircraft at 13:00.

$$\begin{pmatrix}-600\\-200\end{pmatrix} + t\begin{pmatrix}200\\150\end{pmatrix} = \boxed{\begin{pmatrix}-400\\-50\end{pmatrix}}$$

(iii) At what time is the aircraft flying over town B?

$$\begin{pmatrix}200\\400\end{pmatrix} = \begin{pmatrix}-600\\-200\end{pmatrix} + t\begin{pmatrix}200\\150\end{pmatrix}$$

$$200 = -600 + 200t$$

$$t = 4, \boxed{16{:}00}$$

(6)

4. *In this question, distance is in metres.*

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(a) (i) Write down the coordinates of A. $(3,-4,0)$

(ii) Find the speed of the airplane in ms^{-1}. *[4 marks]*

$\sqrt{(-2)^2+3^2+1^2} = \sqrt{14} \approx \boxed{3.74 \text{ m/s}}$

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

$\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + 7\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -11 \\ 17 \\ 7 \end{pmatrix}$ $\boxed{(-11,17,7)}$

(ii) Find the distance the airplane has travelled during the seven seconds. *[5 marks]*

From $^{A}(3,-4,0)$ to $^{B}(-11,17,7)$

$\overrightarrow{AB} = \begin{pmatrix} -11 \\ 17 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -14 \\ 21 \\ 7 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{14^2+21^2+7^2} \approx \boxed{26.2 \text{ m}}$

(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}$.

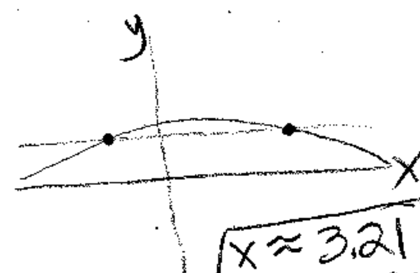
The angle between the flight paths of Airplane 1 and Airplane 2 is $40°$. Find the two values of a. Hint: Solve by graphing *[7 marks]*

$\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \, \& \, \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$

$\cos 40 = \dfrac{2+6+a}{\sqrt{4+9+1}\,\sqrt{1+4+a^2}}$

$\cos 40 = \dfrac{a+8}{\sqrt{14}\,\sqrt{5+a^2}}$

Solve by graphing



$\boxed{\begin{matrix} x \approx 3.21 \\ x \approx -0.990 \end{matrix}}$

5. NO CALCULATOR Problem

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane t seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

(a) Find the speed of Ryan's airplane. [3]

$$\left|\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}\right| = \sqrt{16+4+16} = \boxed{6 \text{ m/s}}$$

(b) Find the height of Ryan's airplane after two seconds. [2]

$$\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 10 \\ 8 \end{pmatrix} \leftarrow \text{Height} \qquad \boxed{8\text{m}}$$

The position of Jack's airplane s seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$.

(c) Show that the paths of the airplanes are perpendicular. [5]

$$\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix} = 0$$

$$-16-12+28=0$$

$$0=0$$

The two airplanes collide at the point $(-23, 20, 28)$.

(d) How long after Ryan's airplane takes off does Jack's airplane take off? [5]

$$\begin{pmatrix} -23 \\ 20 \\ 28 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} \qquad \begin{pmatrix} -23 \\ 20 \\ 28 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$$

$$4t=28 \qquad 7s=28$$

$$t=7\text{ sec} \qquad s=4\text{ sec}$$

3sec later