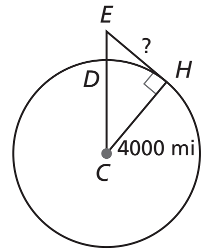
ARE YOU READY? PAGE 743

1. C
2. E
3. B
4. A
5. total # of students = 192 + 208 + 216 + 184 = 800
$\left(\frac{192}{800}\right) \cdot 100\% = 24\%$
6. $\frac{216}{800} \cdot 100\% = 27\%$
7. $\frac{208 + 216}{800} \cdot 100\% = 53\%$
8. 11%(400,000) = 44,000
9. 27%(400,000) = 108,000
10. 19% + 13% = 32%
11. 32%(400,000) = 128,000
12. $11y - 8 = 8y + 1$
$3y - 8 = 1$
$3y = 9$
$y = 3$
13. $12x + 32 = 10 + x$
$11x + 32 = 10$
$11x = -22$
$x = -1$
14. $z + 30 = 10z - 15$
$30 = 9z - 15$
$45 = 9z$
$z = 5$
15. $4y + 18 = 10y + 15$
$18 = 6y + 15$
$3 = 6y$
$y = \frac{1}{2}$
16. $-2x - 16 = x + 6$
$-16 = 3x + 6$
$-22 = 3x$
$x = -\frac{22}{3}$
17. $-2x - 11 = -3x - 1$
$x - 11 = -1$
$x = 10$
18. $17 = x^2 - 32$
$49 = x^2$
$x = \pm 7$
19. $2 + y^2 = 18$
$y^2 = 16$
$y = \pm 4$
20. $4x^2 + 12 = 7x^2$
$12 = 3x^2$
$4 = x^2$
$x = \pm 2$
21. $188 - 6x^2 = 38$
$-6x^2 = -150$
$x^2 = 25$
$x = \pm 5$

11-1 LINES THAT INTERSECT CIRCLES, PAGES 746–754

CHECK IT OUT! PAGES 747–750

1. chords: $\overline{QR}$, $\overline{ST}$; tangent: $\overleftrightarrow{UV}$; radii: $\overline{PQ}$, $\overline{PS}$, $\overline{PT}$; secant: $\overleftrightarrow{ST}$; diameter: $\overline{ST}$
2. radius of circle C: 3 − 2 = 1
radius of circle D: 5 − 2 = 3
point of tangency: (2, −1)
equation of tangent line: $y = -1$
3. **1 Understand the Problem**
The answer will be the length of an imaginary segment from the summit of Mt. Kilimanjaro to the Earth's horizon.
2 Make a Plan
Let C be the center of the Earth, E be the summit of Mt. Kilimanjaro, and H be a point on the horizon. Find the length of $\overline{EH}$, which is tangent to circle C at H. By Thm. 11-1-1, $\overline{EH} \perp \overline{CH}$. So $\triangle CHE$ is a right $\triangle$.



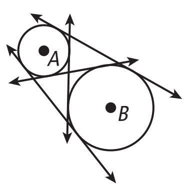
3 Solve
$ED = 19{,}340$ ft
$= \frac{19{,}340}{5280} \approx 3.66$ mi
$EC = CD + ED$
$\approx 4000 + 3.66 = 4003.66$ mi
$EC^2 \approx EH^2 + CH^2$
$4003.66^2 \approx EH^2 + 4000^2$
$29{,}293.40 \approx EH^2$
$171 \text{ mi} \approx EH$
4 Look Back
The problem asks for the distance to the nearest mile. Check that the answer is reasonable by using the Pythagorean Thm. Is $171^2 + 4000^2 \approx 4004^2$? Yes, 16,029,241 ≈ 16,032,016.

4a. By Thm. 11-1-3,
$RS = RT$
$\frac{x}{4} = x - 6.3$
$x = 4x - 25.2$
$-3x = -25.2$
$x = 8.4$
$RS = \frac{(8.4)}{2} = 2.1$

b. By Thm.11-1-3,
$RS = RT$
$n + 3 = 2n - 1$
$3 = n - 1$
$4 = n$
$RS = (4) + 3 = 7$

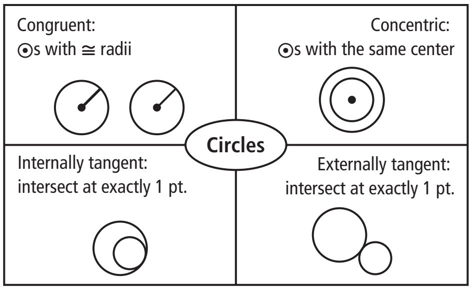
THINK AND DISCUSS, PAGE 750

1. 4 lines



2. No; if line is tangent to the circle with the larger radius, it will not intersect the circle with the smaller radius. If the line is tangent tothe circle with the smaller radius, it will intersect the circle with the larger radius at 2 points.
3. No; a circle consists only of those points which are a given distance from the center.
4. By Thm. 11-1-1, m∠PQR = 90°. So by Triangle Sum Theorem m∠PRQ = 180 − (90 + 59) = 31°.

5.



EXERCISES, PAGES 751–754

GUIDED PRACTICE, PAGE 751

1. secant **2.** concentric

3. congruent

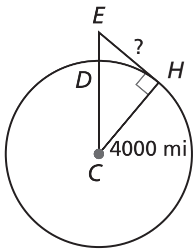
4. chord: $\overline{EF}$; tangent: m; radii: $\overline{DE}$, $\overline{DF}$; secant: ℓ; diameter: $\overline{EF}$

5. chord: $\overline{QS}$; tangent: $\overleftrightarrow{ST}$; radii: $\overline{PQ}$, $\overline{PR}$, $\overline{PS}$; secant: $\overleftrightarrow{QS}$; diameter: $\overline{QS}$

6. radius of circle A: $4 - 1 = 3$
radius of circle B: $4 - 2 = 2$
point of tangency: $(-1, 4)$
equation of tangent line: $y = 4$

7. radius of circle R: $4 - 2 = 2$
radius of circle S: $4 - 2 = 2$
point of tangency: $(1, 2)$
equation of tangent line: $x = 1$

8. **1 Understand the Problem**
The answer will be the length of an imaginary segment from the ISS to the Earth's horizon.
2 Make a Plan
Let C be the center of the Earth, let E be ISS, and let H be the point on the horizon. Find the length of $\overline{EH}$, which is tangent to circle C at H. By Thm. 11-1-1, $\overline{EH} \perp \overline{CH}$. So CHE is a right △.



3 Solve
$EC = CD + ED$
$\approx 4000 + 240 = 4240$ mi
$EC^2 \approx EH^2 + CH^2$
$4240^2 \approx EH^2 + 4000^2$
$1{,}977{,}60 \approx EH^2$
$1406 \text{ mi} \approx EH$
4 Look Back
The problem asks for the distance to the nearest mile. Check that answer is reasonable by using the Pythagorean Thm. Is $1406^2 + 4000^2 \approx 4240^2$? Yes, $17{,}976{,}836 \approx 17{,}977{,}600$.

9. By Thm. 11-1-3,
$JK = JL$
$4x - 1 = 2x + 9$
$2x - 1 = 9$
$2x = 10$
$x = 5$
$JK = 4(5) - 1 = 19$

10. By Thm. 11-1-3,
$ST = SU$
$y - 4 = \frac{3}{4}y$
$4y - 16 = 3y$
$y - 16 = 0$
$y = 16$
$ST = (16) - 4 = 12$

PRACTICE AND PROBLEM SOLVING, PAGES 752–754

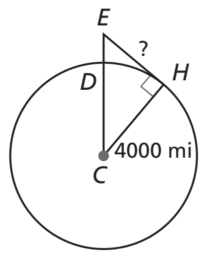
11. chords: $\overline{RS}$, $\overline{VW}$; tangent: ℓ; radii: $\overline{PV}$, $\overline{PW}$; secant: $\overleftrightarrow{VW}$; diameter: $\overline{VW}$

12. chords: $\overline{AC}$, $\overline{DE}$; tangent: $\overleftrightarrow{CF}$; radii: $\overline{BA}$, $\overline{BC}$; secant: $\overleftrightarrow{DE}$; diameter: $\overline{AC}$

13. radius of circle C: $2 - 0 = 2$
radius of circle D: $4 - 0 = 4$
point of tangency: $(-4, 0)$
equation of tangent line: $x = -4$

14. radius of circle M: $3 - 2 = 1$
radius of circle N: $5 - 2 = 3$
point of tangency: $(2, 1)$
equation of tangent line: $y = 1$

15. **1 Understand the Problem**
The answer will be the length of an imaginary segment from the summit of Olympus Mons to Mars' horizon.
2 Make a Plan
Let C be the center of Mars, let E be summit of Olympus Mons, and let H be a point on the horizon. Find the length of $\overline{EH}$, which is tangent to circle C at H. By Thm. 11-1-1, $\overline{EH} \perp \overline{CH}$. So triangle CHE is a right triangle.



3 Solve
$EC = CD + ED$
$\approx 3397 + 25 = 3422$ km
$EC^2 \approx EH^2 + CH^2$
$3422^2 \approx EH^2 + 3397^2$
$170{,}475 \approx EH^2$
$413 \text{ km} \approx EH$
4 Look Back
The problem asks for the distance to the nearest km. Check that the answer is reasonable by using the Pythagorean Thm. Is $413^2 + 3397^2 \approx 3422^2$? Yes, $11{,}710{,}178 \approx 11{,}710{,}084$.

16. By Thm. 11-1-3,
$AB = AC$
$2x^2 = 8x$
Since $x \neq 0$,
$2x = 8$
$x = 4$
$AB = 2(4)^2 = 32$

17. By Thm. 11-1-3,
$RS = RT$
$y = \frac{y^2}{7}$
$7y = y^2$
Since $y \neq 0$,
$7 = y$
$RT = \frac{(7)^2}{7} = 7$

18. S (true if circles are identical)

19. N **20.** N

21. A

22. S (if chord passes through center)

23. $\overline{AC}$ **24.** $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, $\overline{PD}$

25. $\overline{AC}$

26. By Thm. 11-1-1, Thm 11-1-3, the definition of a circle, and SAS, $\triangle PQR \cong \triangle PQS$; so $\overrightarrow{PQ}$ bisects $\angle RPS$. Therefore, $m\angle QPR = \frac{1}{2}(42) = 21°$.
By Thm 11-1-1, $\triangle PQR$ is a right $\triangle$. By the Sum,
$m\angle PQR + m\angle PRQ + m\angle QPR = 180$
$m\angle PQR + 90 + 21 = 180$
$m\angle PQR = 180 - (90 + 21) = 69°$
$m\angle PQS = 2m\angle PQR = 2(69) = 138°$

27. By Thm 11-1-1, $m\angle R = m\angle S = 90°$.
By Quad. Sum Thm.,
$m\angle P + m\angle Q + m\angle R + m\angle S = 360$
$x + 3x + 90 + 90 = 360$
$4x + 180 = 360$
$4x = 180$
$x = 45$
$m\angle P = 45°$

28a. The perpendicular segment from a point to a line is the shortest segment from the point to the line.

b. *B* **c.** radius

d. line $\ell \perp \overline{AB}$

29. Let *E* be any point on the line *m* other than *D*. It is given that line $m \perp \overline{CD}$. So $\triangle CDE$ is a right $\triangle$ with hypotenuse $\overline{CE}$. Therefore, $CE > CD$. Since $\overline{CD}$ is a radius, *E* must lie in exterior of circle *C*. Thus *D* is only a point on the line *m* that is also on circle *C*. So the line *m* is tangent to circle *C*.

30. Since 2 points determine a line, draw auxiliary segments $\overline{PA}$, $\overline{PB}$, and $\overline{PC}$. Since $\overline{AB}$ and $\overline{AC}$ are tangents to circle *P*, $\overline{AB} \perp \overline{PB}$ and $\overline{AC} \perp \overline{PC}$. So $\triangle ABP$ and $\triangle ACP$ are right ⦣. $\overline{PB} \cong \overline{PC}$ since they are both radii of circle *P*, and $\overline{PA} \cong \overline{PA}$ by Reflex. Prop. of $\cong$. Therefore, $\triangle ABP \cong \triangle ACP$ by HL $\cong$ and $\overline{AB} \cong \overline{AC}$ by CPCTC.

31.
$QR = QS = 5$
$QT^2 = QR^2 + RT^2$
$(ST + 5)^2 = 5^2 + 12^2$
$ST + 5 = 13$
$ST = 8$

32.
$AB = AD$
$23 = x$
$AC = AE$
$23 + x - 5 = x + DE$
$23 + 23 - 5 = 23 + DE$
$41 = 23 + DE$
$DE = 18$

33. $JK = JL$ and $JL = JM$, so, $JK = JM$
$JK = JM$
$6y - 2 = 30 - 2y$
$8y = 32$
$y = 4$
$JL = JM = 30 - 2(4) = 22$

34. Point of tangency must be $(x, 2)$, where
$x - 2 = \pm 3$
$x = 5$ or -1.
Possible points of tangency are (5, 2) and (−1, 2).

35a. *BCDE* is a rectangle; by Thm. 11-1-1, $\angle BCD$ and $\angle EDC$ are right ⦣. It is given that $\angle DEB$ is a right $\angle$. $\angle CBE$ must also be a right by Quad. Sum Thm. Thus, *BCDE* has 4 right ⦣ and is a rectangle.

b. $BE = CE = 17$ in.
$AE = AD - DE = AD - BC = 5 - 3 = 2$ in.

c. $AB^2 = AE^2 + BE^2$
$= 2^2 + 17^2$
$AB^2 = 293$
$AB = \sqrt{293} \approx 17.1$ in.

36. Not possible; if it were possible, $\triangle XBC$ would contain 2 right ⦣. which contradicts $\triangle$ Sum Thm.

37. By Thm 11-1-1, $\angle R$ and $\angle S$ are right ⦣. By Quad. Sum Thm.,
$\angle P + \angle Q + \angle R + \angle S = 360$
$\angle P + \angle Q + 90 + 90 = 360$
$\angle P + \angle Q = 180$
By definition, $\angle P$ and $\angle Q$ are supplementary angles.

TEST PREP, PAGE 754

38. C
$AD^2 = AB^2 + BD^2$
$= 10^2 + 3^2 = 109$
$AD = \sqrt{109} \approx 10.4$ cm

39. G
$-2 - (-4) = 2$. So, $(3, -4)$ lies on circle *P*;
$y = -4$ meets circle *P* only at $(3, -4)$. So it is tangent to circle *P*.

40. B
$$\frac{\pi(5)^2}{\pi(6)^2} = \frac{25\pi}{36\pi} = \frac{25}{36}$$

CHALLENGE AND EXTEND, PAGE 754

41. Since 2 points determine a line, draw auxiliary segments $\overline{GJ}$ and $\overline{GK}$. It is given that $\overline{GH} \perp \overline{JK}$, so, $\angle GHJ$ and $\angle GHK$ are right ⦣. Therefore, $\triangle GHJ$ and $\triangle GHK$ are right ⦣. $\overline{GH} \cong \overline{GH}$ by Reflex. Prop. of $\cong$, and $\overline{GJ} \cong \overline{GK}$ because they are radii of circle *G*. Thus $\triangle GHJ \cong \triangle GHK$ by HL, and $\overline{JH} \cong \overline{KH}$ by CPCTC.

42. By Thm. 11-1-1, $\angle C$ and $\angle D$ are right ⦣. So *BCDE* is a rectangle, $CE = DB = 2$, and $BE = DC = 12$. Therefore, $\triangle ABE$ is a right with leg lengths $5 - 2 = 3$ and 12. So
$$AB = \sqrt{AE^2 + BE^2} = \sqrt{3^2 + 12^2} = \sqrt{153} = 3\sqrt{17}$$

43. Draw a segment from *X* to the center *C* of the wheel. $\angle XYC$ is a right angle and $m\angle YXC = \frac{1}{2}(70) = 35°$. So
$$\tan 35° = \frac{13}{XY}$$
$$XY = \frac{13}{\tan 35°} \approx 18.6 \text{ in.}$$

SPIRAL REVIEW, PAGE 754

44. $14 + 6.25h > 12.5 + 6.5h$
$1.5 > 0.25h$
$6 > h$
Since h is positive, $0 < h < 6$.

45. $P = \frac{LM + PR}{LR} = \frac{10 + (16 + 4)}{10 + 6 + 4 + 16 + 4} = \frac{30}{40} = \frac{3}{4}$

46. $P = \frac{LP}{LR} = \frac{10 + 6 + 4}{40} = \frac{20}{40} = \frac{1}{2}$

47. $P = \frac{MN + PR}{LR} = \frac{6 + (16 + 4)}{40} = \frac{26}{40} = \frac{13}{20}$

48. $P = \frac{QR}{LR} = \frac{4}{40} = \frac{1}{10}$

CONNECTING GEOMETRY TO DATA ANALYSIS: CIRCLE GRAPHS, PAGE 755

TRY THIS, PAGE 755

1. **Step 1** Add all the amounts.
$18 + 10 + 8 = 36$
Step 2 Write each part as a fraction of the whole.
novels: $\frac{18}{36}$ reference: $\frac{10}{36}$ textbooks: $\frac{8}{36}$
Step 3 Multiply each fraction by 360° to calculate central ∠ measure.
novels: $\frac{18}{36}(360) = 180°$ reference: $\frac{10}{36}(360) = 100°$
textbooks: $\frac{8}{36}(360) = 80°$
Step 4 Match a circle graph to the data.
The data match graph D.

2. **Step 1** Add all the amounts.
$450 + 120 + 900 + 330 = 1800$
Step 2 Write each part as a fraction of the whole.
travel: $\frac{450}{1800}$ meals: $\frac{120}{1800}$ lodging: $\frac{900}{1800}$
other: $\frac{330}{1800}$
Step 3 Multiply each fraction by 360° to calculate central ∠ measure.
travel: $\frac{450}{1800}(360) = 90°$ meals: $\frac{120}{1800}(360) = 24°$
lodging: $\frac{900}{1800}(360) = 180°$ other: $\frac{330}{1800}(360) = 66°$
Step 4 Match a circle graph to the data.
The data match graph C.

3. **Step 1** Add all the amounts.
$190 + 375 + 120 + 50 = 735$
Step 2 Write each part as a fraction of the whole.
food: $\frac{190}{735}$ health: $\frac{375}{735}$ training: $\frac{120}{735}$ other: $\frac{50}{735}$
Step 3 Multiply each fraction by 360° to calculate central ∠ measure.
food: $\frac{190}{735}(360) \approx 93°$ health: $\frac{375}{735}(360) \approx 184°$
training: $\frac{120}{735}(360) \approx 59°$ other: $\frac{50}{735}(360) \approx 24°$
Step 4 Match a circle graph to the data.
The data match graph B.

11-2 ARCS AND CHORDS, PAGES 756–763

CHECK IT OUT! PAGES 756–759

1a. $m\angle FMC = (0.03 + 0.09 + 0.10 + 0.11)360° = 108°$

b. $m\angle \widehat{AHB} = (1 - 0.25)360° = 270°$

c. $m\angle EMD = (0.10)360° = 36°$

2a. $m\angle JPK = 25°$ (Vert. ∡ Thm.)
$m\widehat{JK} = 25°$
$m\angle KPL + m\angle LPM + m\angle MPN = 180°$
$m\angle KPL + 40° + 25° = 180°$
$m\angle KPL = 115°$
$m\widehat{KL} = 115°$
$m\widehat{JKL} = m\widehat{JK} + m\widehat{KL}$
$= 25° + 115° = 140°$

b. $m\widehat{LK} = m\widehat{KL} = 115°$
$m\angle KPN = 180°$
$m\widehat{KJN} = 180°$
$m\widehat{LJN} = m\widehat{LK} + m\widehat{KJN}$
$= 180° + 115°$

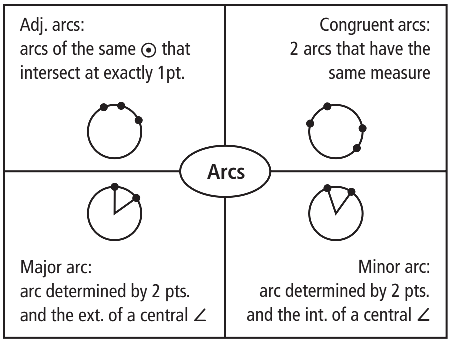
3a. $m\angle RPT = m\angle SPT$
$RT = ST$
$6x = 20 - 4x$
$10x = 20$
$x = 2$
$RT = 6(2) = 12$

b. $m\angle CAD = m\angle EBF$ (11-2-2(3))
$mCD = mEF$
$25y = 30y - 20$
$20 = 5y$
$y = 4$
$mCD = m\angle CAD = 25(4) = 100°$

4. **Step 1** Draw radius $\overline{PQ}$.
$PQ = 10 + 10 = 20$
Step 2 Use Pythagorean and 11-2-3.
$PT^2 + QT^2 = PQ^2$
$10^2 + QT^2 = 20^2$
$QT^2 = 300$
$QT = \sqrt{300} = 10\sqrt{3}$
Step 3 Find QR.
$QR = 2\left(10\sqrt{3}\right) = 20\sqrt{3} \approx 34.6$

THINK AND DISCUSS, PAGE 759

1. The arc measures between 90° and 180°.
2. if arcs are on 2 different circles with different radii
3.



EXERCISES, PAGES 760–763

GUIDED PRACTICE, PAGE 760

1. semicircle
2. Vertex is the center of the circle.
3. major arc
4. minor arc
5. $m\angle PAQ = 0.45(360) = 162°$
6. $m\angle VAU = 0.07(360) = 25.2°$
7. $m\angle SAQ = (0.06 + 0.11)360 = 61.2°$
8. $m\overset{\frown}{UT} = m\angle UAT = 0.1(360) = 36°$
9. $m\overset{\frown}{RQ} = m\angle RAQ = 0.11(360) = 39.6°$
10. $m\overset{\frown}{UPT} = (1 - 0.1)360 = 324°$
11. $m\overset{\frown}{DE} = m\angle DAE = 90°$
    $m\overset{\frown}{EF} = m\angle EAF = m\angle BAC = 90 - 51 = 39°$
    $m\overset{\frown}{DF} = m\overset{\frown}{DE} + m\overset{\frown}{EF} = 90 + 39 = 129°$
12. $m\overset{\frown}{DEB} = m\angle DAE + m\angle EAB = 90 + 180 = 270°$
13. $m\angle HGJ + m\angle JGL = m\angle HGL$
    $72 + m\angle JGL = 180$
    $m\angle JGL = 108°$
    $m\overset{\frown}{JL} = 108°$
14. $m\overset{\frown}{HLK} = m\angle HGL + m\angle LGK = 180 + 30 = 210°$
15. $QR = RS$ (Thm. 11-2-2(1))
    $8y - 8 = 6y$
    $2y = 8$
    $y = 4$
    $QR = 8(4) - 8 = 24$
16. $m\angle CAD = m\angle EBF$ (Thm. 11-2-2(3))
    $45 - 6x = -9x$
    $3x = -45$
    $x = -15$
    $m\angle EBF = -9(-15) = 135°$
17. **Step 1** Draw radius $\overline{PR}$.
    $PR = 5 + 8 = 13$
    Step 2 Use the Pythagorean Thm. and Thm. 11-2-3.
    Let the intersection of $\overline{PQ}$ and $\overline{RS}$ be T.
    $PT^2 + RT^2 = PR^2$
    $5^2 + RT^2 = 13^2$
    $RT = 12$
    Step 3 Find RS.
    $RS = 2(12) = 24$
18. **Step 1** Draw radius $\overline{CE}$.
    $CE = 50 + 20 = 70$
    Step 2 Use the Pythagorean Thm. and Thm. 11-2-3.
    Let the intersection of $\overline{CD}$ and $\overline{EF}$ be G.
    $CG^2 + EG^2 = CE^2$
    $50^2 + EG^2 = 70^2$
    $RG = \sqrt{2400} = 20\sqrt{6}$
    Step 3 Find EF.
    $EF = 2\left(20\sqrt{6}\right) = 40\sqrt{6} \approx 98.0$

PRACTICE AND PROBLEM SOLVING, PAGES 761–762

19. $m\angle ADB = \frac{35}{35 + 39 + 29}(360) = \frac{35}{103}(360) \approx 122.3°$
20. $m\angle ADC = \frac{29}{103}(360) = 101.4°$
21. $m\overset{\frown}{AB} = m\angle ADB \approx 122.3°$
22. $m\overset{\frown}{BC} = m\angle BDC = \frac{39}{103}(360) \approx 136.3°$
23. $m\overset{\frown}{ACB} = 360 - m\angle ADB \approx 360 - 122.3 = 237.7°$
24. $m\overset{\frown}{CAB} = 360 - m\angle BDC \approx 360 - 136.3 = 223.7°$
25. $m\overset{\frown}{MP} = m\angle MJP$
    $= m\angle MJQ - m\angle PJQ$
    $= 180 - 28 = 152°$
26. $mQNL = mQNM + mML$
    $= m\angle QJM + m\angle MJL$
    $= 180 + 28 = 208°$
27. $m\overset{\frown}{WT} = m\overset{\frown}{WS} + m\overset{\frown}{ST}$
    $= m\angle WXS + m\angle SXT$
    $= 55 + 100 = 155°$
28. $m\overset{\frown}{WTV} = m\overset{\frown}{WS} + m\overset{\frown}{STV}$
    $= m\angle WXS + m\angle SXV$
    $= 55 + 180 = 235°$
29. $m\angle CAD = m\angle EBF$ (Thm. 11-2-2(3))
    $10x - 63 = 7x$
    $3x = 63$
    $x = 21$
    $m\angle CAD = 10(21) - 63 = 147°$
30. $m\overset{\frown}{JK} = m\overset{\frown}{LM}$ (Thm. 11-2-2(2))
    $4y + y = y + 68$
    $y = 17$
    $m\overset{\frown}{JK} = 4(17) + 17 = 85°$

31. $AC = AB = 2.4 + 1.7 = 4.1$
Let $\overline{AB}$ and $\overline{CD}$ meet at E.
$AE^2 + CE^2 = AC^2$
$2.4^2 + CE^2 = 4.1^2$
$CE = \sqrt{11.05}$
$CD = 2\sqrt{11.05} \approx 6.6$

32. $PR = PQ = 2(3) = 6$
Let $\overline{PQ}$ and $\overline{RS}$ meet at T.
$PT^2 + RT^2 = PR^2$
$3^2 + RT^2 = 6^2$
$RT = \sqrt{27} = 3\sqrt{3}$
$RS = 2(3\sqrt{3}) = 6\sqrt{3} \approx 10.4$

33. F; the ∠ measures between 0° and 180°. So it could be right or obtuse.

34. F; Endpts. of a diameter determine 2 ≅ arcs measuring exactly 180°.

35. T (Thm. 11-2-4)

36. Check students' graphs.

37. Let $m\angle AEB = 3x$, $m\angle BEC = 4x$, and $m\angle CED = 5x$.
$m\angle AEB + m\angle BEC + m\angle CED = 180$
$3x + 4x + 5x = 180$
$12x = 128$
$x = 15$
$m\angle AEB = 3(15) = 45°$ $m\angle BEC = 4(15) = 60°$
$m\angle CED = 5(15) = 75°$

38. $m\widehat{JL} + m\widehat{JK} + m\widehat{KL} = 360$
$7x - 18 + 4x - 2 + 6x + 6 = 360$
$17x = 374$
$x = 22$
$m\widehat{JL} = 7(22) - 18 = 136°$

39. $m\angle QPR = 180$
$10x = 180$
$x = 18$
$m\angle SPT = 6(18) = 108°$

40.

Statements	Reasons
1. $\overline{BC} \cong \overline{DE}$	1. Given.
2. $\overline{AB} \cong \overline{AD}$ and $\overline{AC} \cong \overline{AE}$	2. All radii of a circle are ≅.
3. $\triangle BAC \cong \triangle DAE$	3. SSS
4. $\angle BAC \cong \angle DAE$	4. CPCTC
5. $m\angle BAC = m\angle DAE$	5.
6. $m\widehat{BC} = m\widehat{DE}$	6. Definition of arc measures
7. $\widehat{BC} \cong \widehat{DE}$	7. Definition of arcs

41.

Statements	Reasons
1. $\widehat{BC} \cong \widehat{DE}$	1. Given
2. $m\widehat{BC} = m\widehat{DE}$	2. Definition of ≅ arcs
3. $m\angle BAC = m\angle DAE$	3. Definition of arc measures
4. $\angle BAC \cong \angle DAE$	4.

42.

Statements	Reasons
1. $\overline{CD} \perp \overline{EF}$	1. Given
2. Draw radii $\overline{CE}$ and $\overline{CF}$.	2. 2 points determine a line.
3. $\overline{CE} \cong \overline{CF}$	3. All radii of a circle are congruent.
4. $\overline{CM} \cong \overline{CM}$	4. Reflex. Prop. of ≅
5. $\angle CMF$ and $\angle CME$ are rt. ∠s.	5. Def. of ⊥
6. $\triangle CMF$ and $\triangle CME$ are rt. △s.	6. Def. of a rt. △
7. $\triangle CMF \cong \triangle CME$	7. HL Steps 3, 4
8. $\overline{FM} \cong \overline{EM}$	8. CPCTC
9. $\overline{CD}$ bisects $\overline{EF}$	9. Def. of a bisector
10. $\angle FCD \cong \angle ECD$	10. CPCTC
11. $m\angle FCD = m\angle ECD$	11. Def. of ≅ ∠s
12. $m\widehat{FD} = m\widehat{ED}$	12. Def. of arc measures
13. $\widehat{FD} \cong \widehat{ED}$	13. Def. of arcs
14. $\overline{CD}$ bisects $\widehat{EF}$.	14. Def. of a bisector

43.

Statements	Reasons
1. $\overline{JK}$ is the ⊥ bis. of $\overline{GH}$	1. Given
2. A is equidistant from G and H.	2. Def. of the center of circle
3. A lies on the ⊥ bis. of $\overline{GH}$.	3. Perpendicular Bisector Theorem
4. $\overline{JK}$ is a diameter of circle A.	4. Def. of diam.

44. The circle is divided into eight ≅ sectors, each with central ∠ measure 45°. So possible measures of the central congruent are multiples of 45° between 0(45) = 0° and 8(45) = 360°. So there are three different sizes of angles: 135°, 90°, and 45°.

45. Solution A is incorrect because it assumes that ∠BGC is a right ∠.

46. To make a circle graph, draw a circle and then draw central ∠s that measure 0.4(360) = 144°, 0.35(360) = 126°, 0.15(360) = 54°, and 0.1(360) = 36°.

47a. $AC = \frac{1}{2}(27) = 13.5$ in.
$AD = AB - DB = 13.5 - 7 = 6.5$ in.

b. $CD^2 + AD^2 = AC^2$
$CD^2 + 6.5^2 = 13.5^2$
$CD = \sqrt{140} = 2\sqrt{35} \approx 11.8$ in.

c. By Theorem 11-2-3, $\overline{AB}$ bisects $\overline{CE}$. So $CE = 2CD = 4\sqrt{35} \approx 23.7$ in.

TEST PREP, PAGE 763

48. D
$m\widehat{WT} = 90 + 18 = 108°$ $m\widehat{VR} = 180 - 41 = 139°$
$m\widehat{UW} = 90°$ $m\widehat{TV} = 180 - 18 = 162°$

49. F
$CE = \frac{1}{2}(10) = 5$
$AE^2 + 5^2 = 6^2$
$AE = \sqrt{11} \approx 3.3$

50. 90

$\overline{AP}$ is a horizontal radius, and $\overline{BP}$ is a vertical radius. So $m\widehat{AB} = m\angle APB = 90°$.

CHALLENGE AND EXTEND, PAGE 763

51. $AD = AB = 4 + 2 = 6$

$\cos BAD = \frac{4}{6} = \frac{2}{3}$

$m\widehat{BD} = m\angle BAD = \cos^{-1}\left(\frac{2}{3}\right) \approx 48.2°$

52. 2 points determine 2 distinct arcs.
3 points determine 6 arcs.
4 points determine 12 arcs.
5 points determine 20 arcs.
⋮
n points determine $n(n - 1)$ arcs.

53a. $\pi \rightarrow 180°$. So $\frac{\pi}{2} \rightarrow 90°$, $\frac{\pi}{3} \rightarrow 60°$, and $\frac{\pi}{4} \rightarrow 45°$

b. $135° \rightarrow \frac{135}{180}(\pi) = \frac{3\pi}{4}$

$270° \rightarrow \frac{270}{180}(\pi) = \frac{3\pi}{2}$

SPIRAL REVIEW, PAGE 763

54. $(3x)^3\left(2y^2\right)\left(3^{-2}y^2\right)$

$\left(27x^3\right)\left(2y^2\right)\left(\frac{1}{9}y^2\right)$

$6x^3y^4$

55. $a^4b^3(-2a)^{-4}$

$a^4b^3\left(\frac{1}{16}a^{-4}\right)$

$\frac{1}{16}b^3$

56. $\left(-2r^3s^2\right)\left(3ts^2\right)^2$

$-2t^3s^2\left(9t^2s^4\right)$

$-18t^5s^6$

57. $3 = 1 + 2$
$7 = 3 + 4$
$13 = 7 + 6$
$21 = 13 + 8$
$21 + 10 =$ **31**

58. *C*, *E*, *G*, *I*, *K*, ***M***

59. $6 = 1 + 5$
$15 = 6 + 9$
$15 + 13 =$ **28**

60. $\angle NPQ$ and $\angle NMQ$ are right ∡ (Thm. 11-1-2). So $\angle NMQ = 90°$.

61. $PQ = MQ$ (Thm. 11-1-3)
$2x = 4x - 9$
$9 = 2x$
$x = 4.5$
$MQ = 4(4.5) - 9 = 9$

CONSTRUCTION, PAGE 763

1. O is on the ⊥ bisector of $\overline{PQ}$. So by Conv. of ⊥ Bisector, $OP = OQ$. Similarly, O is on ⊥ bisector of $\overline{QR}$. So $OQ = OR$. Thus, $\overline{OP}$, $\overline{OQ}$, and $\overline{OR}$ are radii of circle O, and circle O contains Q and R.

11-3 SECTOR AREA AND ARC LENGTH, PAGES 764–769

CHECK IT OUT! PAGES 764–766

1a. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(1)^2\left(\frac{90}{360}\right) = \frac{1}{4}\pi \text{ m}^2 \approx 0.79 \text{ m}^2$

b. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(16)^2\left(\frac{36}{360}\right) = 25.6\pi \text{ in.}^2 \approx 80.42 \text{ in.}^2$

2. $A = \pi r^2\left(\frac{m}{360}\right)$

$= \pi(360)^2\left(\frac{180}{360}\right)$

$\approx 203{,}575 \text{ ft}^2$

3. Step 1 Find the area of sector *RST*.

$A = \pi r^2\left(\frac{m}{360}\right) = \pi(4)^2\left(\frac{90}{360}\right) = 4\pi \text{ m}^2$

Step 2 Find the area of △*RST*.

$A = \frac{1}{2}bh = \frac{1}{2}(4)(4) = 8 \text{ m}^2$

Step 3

area of segment = area of sector *RST* − area of △*RST*

$= 4\pi - 8 \approx 4.57 \text{ m}^2$

4a. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(6)\left(\frac{40}{360}\right) = \frac{4}{3}\pi \text{ m} \approx 4.19 \text{ m}$

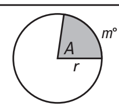
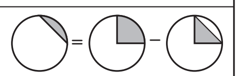
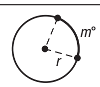
b. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(4)\left(\frac{135}{360}\right) = 3\pi \text{ cm} \approx 9.42 \text{ cm}$

THINK AND DISCUSS, PAGE 766

1. An arc measure is measured in degrees. An arc length is measured in linear units.

2. the radius and central ∠ of the sector

3.

	Formula	Diagram
Area of a Sector	$A = \pi r^2\left(\frac{m°}{360°}\right)$	
Area of a Segment	Area of seg. = area of sector − area of △	
Arc Length	$L = 2\pi r\left(\frac{m°}{360°}\right)$	

EXERCISES, PAGES 767–769

GUIDED PRACTICE, PAGE 767

1. segment

2. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(6)^2\left(\frac{90}{360}\right) = 9\pi \text{ m}^2 \approx 28.27 \text{ m}^2$

3. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(8)^2\left(\frac{135}{360}\right) = 24\pi \text{ cm}^2 \approx 75.40 \text{ cm}^2$

4. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(2)^2\left(\frac{20}{360}\right) = \frac{2}{9}\pi \text{ ft}^2 \approx 0.70 \text{ ft}^2$

5. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(3)^2\left(\frac{150}{360}\right) = \frac{15}{4}\pi \text{ mi}^2 \approx 12 \text{ mi}^2$

6. **Step 1** Find area of sector ABC.
$A = \pi r^2\left(\frac{m}{360}\right) = \pi(3)^2\left(\frac{90}{360}\right) = \frac{9}{4}\pi \text{ in.}^2$
Step 2 Find area of $\triangle ABC$.
$A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = \frac{9}{2} \text{ in.}^2$
Step 3
area of segment = area of sector ABC − area of $\triangle ABC$
$= \frac{9}{4}\pi - \frac{9}{2} \approx 2.57 \text{ in.}^2$

7. **Step 1** Find the area of sector DEF.
$A = \pi r^2\left(\frac{m}{360}\right) = \pi(20)^2\left(\frac{60}{360}\right) = \frac{200}{3}\pi \text{ m}^2$
Step 2 Find the area of $\triangle DEF$.
$A = \frac{1}{2}bh = \frac{1}{2}(20)\left(10\sqrt{3}\right) = 100\sqrt{3} \text{ m}^2$
Step 3
area of segment = area of sector DEF − area of $\triangle DEF$
$= \frac{200}{3}\pi - 100\sqrt{3} \approx 36.23 \text{ m}^2$

8. **Step 1** Find the area of sector ABC.
$A = \pi r^2\left(\frac{m}{360}\right) = \pi(6)^2\left(\frac{45}{360}\right) = \frac{9}{2}\pi \text{ cm}^2$
Step 2 Find the area of $\triangle ABC$.
$A = \frac{1}{2}bh = \frac{1}{2}(6)\left(3\sqrt{2}\right) = 9\sqrt{2} \text{ cm}^2$
Step 3
area of segment = area of sector ABC − area of $\triangle ABC$
$= \frac{9}{2}\pi - 9\sqrt{2} \approx 1.41 \text{ cm}^2$

9. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(16)\left(\frac{45}{360}\right) = 4\pi \text{ ft} \approx 12.57 \text{ ft}$

10. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(9)\left(\frac{120}{360}\right) = 6\pi \text{ m} \approx 18.85 \text{ m}$

11. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(6)\left(\frac{20}{360}\right) = \frac{2}{3}\pi \text{ in.} \approx 2.09 \text{ in.}$

PRACTICE AND PROBLEM SOLVING, PAGES 767–769

12. $A = \pi(20)^2\left(\frac{150}{360}\right) = \frac{500}{3}\pi \text{ m}^2 \approx 523.60 \text{ m}^2$

13. $A = \pi(9)^2\left(\frac{100}{360}\right) = \frac{45}{2}\pi \text{ in}^2 \approx 70.69 \text{ in.}^2$

14. $A = \pi(2)^2\left(\frac{47}{360}\right) = \frac{47}{90}\pi \text{ ft}^2 \approx 1.64 \text{ ft}^2$

15. $A = \pi(20)^2\left(\frac{180}{360}\right) = 200\pi \approx 628 \text{ in.}^2$

16. $\triangle ABC$ is a 45°-45°-90° triangle. So central angle measures 90°.
area of segment = area of sector ABC − area of $\triangle ABC$
$= \pi(10)^2\left(\frac{90}{360}\right) - \frac{1}{2}(10)(10)$
$= 25\pi - 50 \approx 28.54 \text{ m}^2$

17. $m\angle KLM = m\widehat{KM} = 120°$
area of segment = area of sector KLM − area of $\triangle KLM$
$= \pi(5)^2\left(\frac{120}{360}\right) - \frac{1}{2}\left(\frac{5}{2}\right)\left(5\sqrt{3}\right)$
$= \frac{25}{3}\pi - \frac{25}{4}\sqrt{3} \approx 15.35 \text{ in.}^2$

18. area of segment = area of sector RST − area of $\triangle RST$
$= \pi(1)^2\left(\frac{60}{360}\right) - \frac{1}{2}(1)\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{1}{6}\pi - \frac{1}{4}\sqrt{3} \approx 0.09 \text{ in.}^2$

19. $L = 2\pi(5)\left(\frac{50}{360}\right) = \frac{25}{18}\pi \text{ mm} \approx 4.36 \text{ mm}$

20. $L = 2\pi(1.5)\left(\frac{160}{360}\right) = \frac{4}{3}\pi \text{ m} \approx 4.19 \text{ m}$

21. $L = 2\pi(2)\left(\frac{9}{360}\right) = \frac{1}{10}\pi \text{ ft} \approx 0.31 \text{ ft}$

22. $P = 2\pi(3)\left(\frac{180}{360}\right) + 3\left(2\pi(1)\left(\frac{180}{360}\right)\right) = 6\pi \approx 18.8 \text{ in.}$

23. never

24. sometimes (if radii of arcs are equal)

25. always

26.
$A = \pi r^2\left(\frac{m}{360}\right)$
$9\pi = \pi r^2\left(\frac{90}{360}\right)$
$36 = r^2$
$r = 6$

27.
$L = 2\pi r\left(\frac{m}{360}\right)$
$8\pi = 2\pi r\left(\frac{120}{360}\right)$
$24\pi = 2\pi r$
$r = 12$

28a. $L \approx 2\left(\frac{22}{7}\right)(7)\left(\frac{90}{360}\right) = 11 \text{ in.}$

b. $L = 2\pi(7)\left(\frac{90}{360}\right) = \frac{7}{2}\pi \approx 10.99557429 \text{ in.}$

c. overestimate, since $L < 10.996 < 11$

29a. $L = 2\pi(2.5)\left(\frac{90}{360}\right) = \frac{5}{4}\pi \approx 3.9 \text{ ft}$

b. $4.5 = 2\pi(2.5)\left(\frac{m}{360}\right)$
$\frac{9}{10\pi} = \frac{m}{360}$
$m = \frac{324}{\pi} \approx 103°$

30. The area of sector BAC is $\frac{45}{360} = \frac{1}{8}$ area of circle A.
So if area of circle A is 24 in^2, the area of the sector will automatically be $\frac{1}{8}(24) = 3 \text{ in.}^2$
So we need only to solve $\pi r^2 = 24$ for r.
$\pi r^2 = 24$
$r^2 = \frac{24}{\pi}$
$r = \sqrt{\frac{24}{\pi}} \approx 2.76 \text{ in.}$

31. If the length of the arc is L and its degree measure is m, then
$L = 2\pi r\left(\frac{m}{360}\right)$
$360L = 2\pi rm$
$r = \frac{360L}{2\pi m} = \frac{180L}{\pi m}.$

TEST PREP, PAGE 769

32. B
$A = \pi(8)^2\left(\frac{90}{360}\right) = 16\pi$

33. G
$A = 2\pi(8)\left(\frac{90}{360}\right) = 4\pi$

34. 43.98
$A = \pi(12)^2\left(\frac{35}{360}\right) \approx 43.98$

CHALLENGE AND EXTEND, PAGE 769

35. $A = \pi(5)^2\left(\frac{40}{360}\right) - \pi(2)^2\left(\frac{40}{360}\right) = \frac{25}{9}\pi - \frac{4}{9}\pi = \frac{7}{3}\pi$

36a. $V = Bh = \left(\pi(4)^2\left(\frac{30}{360}\right)\right)(3) = 4\pi \approx 12.6 \text{ in.}^3$

b. $B = 2\left(\pi(4)^2\left(\frac{30}{360}\right)\right) = \frac{8}{3}\pi$

$L = 2(3)(4) + 2\left(\pi(4)\left(\frac{30}{360}\right)\right)(3) = 24 + \pi$

$A = \frac{8}{3}\pi + 24 + 2\pi \approx 38.7 \text{ in.}^2$

37a. $A(\odot) = \pi(2)^2 = 4\pi$

$A(\text{red}) = 4\pi(1)^2\left(\frac{45}{360}\right) = \frac{1}{2}\pi$

$P(\text{red}) = \frac{\frac{1}{2}\pi}{4\pi} = \frac{1}{8}$

b. $A(\text{blue}) = 4\left(\pi(2)^2\left(\frac{45}{360}\right) - \pi(1)^2\left(\frac{45}{360}\right)\right) = \frac{3}{2}\pi$

$P(\text{blue}) = \frac{\frac{3}{2}\pi}{4\pi} = \frac{3}{8}$

c. $P(\text{red or blue}) = \frac{\frac{1}{2}\pi + \frac{3}{2}\pi}{4\pi} = \frac{1}{2}$

SPIRAL REVIEW, PAGE 769

38. $8x - 2y = 6$
$2y = 8x - 6$
$y = 4x - 3$
The line is ∥.

39. slope $= \frac{2 - 0}{1\frac{1}{2} - \frac{1}{2}} = 2$
The line is neither ∥ nor ⊥.

40. $y = mx + 1$
$0 = m(4) + 1$
$m = -\frac{1}{4}$
The line is ⊥.

41. $V = \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

42. $S = 4\pi = 4\pi r^2$
$r^2 = 1$
$r = 1$ cm
$C = 2\pi(1) = 2\pi$ cm

43. $m\angle KLJ = m\angle KLH - (m\angle GLJ - m\angle GLH)$
$10x - 28 = 180 - (90 - (2x + 2))$
$10x - 28 = 90 + 2x + 2$
$8x = 120$
$x = 15$
$m\angle KLJ = 10(15) - 28 = 122°$

44. $m\widehat{KJ} = m\angle KLJ = 122°$

45. $m\widehat{JFH} = m\widehat{JF} + m\widehat{FG} + m\widehat{GH}$
$= m\angle JLF + m\angle FLG + m\angle GLH$
$= 180 + 90 + 2(15) + 2 = 302°$

11A MULTI-STEP TEST PREP, PAGE 770

1. Let ℓ represent the length of the stand. Radius $r = 14$ in. Form a right △ with leg lengths $14 - 10 = 4$ in. and $\frac{1}{2}\ell$ in., and hypotenuse length 14 in.

$4^2 + \left(\frac{1}{2}\ell\right)^2 = 14^2$
$16 + \frac{1}{4}\ell^2 = 196$
$\frac{1}{4}\ell^2 = 180$
$\ell^2 = 720$
$\ell = \sqrt{720} = 12\sqrt{5} \approx 26.8$ in.

2. Let E be on $\overline{AD}$ with $\overline{BE}$ $\overline{CD}$ and $BE = CD$. △ABE is a right △. So
$AE^2 + BE^2 = AB^2$
$(4 - 2)^2 + CD^2 = 15^2$
$CD^2 = 221$
$CD = \sqrt{221} \approx 14.9$ in.

3. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(4)\left(\frac{60}{360}\right) = \frac{4}{3}\pi \approx 4.2$ in.

4. $L = 2\pi r\left(\frac{m}{360}\right)$
$\frac{4}{3}\pi = 2\pi(2)\left(\frac{m}{360}\right)$
$\frac{1}{3} = \frac{m}{360}$
$m = \frac{360}{3} = 120°$

11A READY TO GO ON? PAGE 771

1. chord: $\overline{PR}$; tangent: m; radii: $\overline{QP}$, $\overline{QR}$, $\overline{QS}$; secant: $\overleftrightarrow{PR}$; diameter: $\overline{PR}$

2. chords: $\overline{BD}$, $\overline{BE}$; tangent: $\overleftrightarrow{BC}$; radii: $\overline{AB}$, $\overline{AE}$; secant: $\overleftrightarrow{BD}$; diameter: $\overline{BE}$

3. Let x be the distance to the horizon. The building forms a right △ with leg lengths x mi and 4000 mi, and hypotenuse length $\left(4000 + \frac{732}{5280}\right)$ mi.

$x^2 + (4000)^2 = \left(4000 + \frac{732}{5280}\right)^2$
$x^2 \approx 1109.11$
$x \approx 33$ mi

4. $m\widehat{BC} + m\widehat{CD} = m\widehat{BD}$
$m\widehat{BD} + m\angle CAD = m\angle BAD$
$m\widehat{BD} + 49 = 90$
$m\widehat{BD} = 41°$

5. $m\widehat{BED} = m\widehat{BFE} + m\widehat{ED}$
$= m\angle BAE + m\angle EAD$
$= 180 + 90 = 270°$

6. $m\widehat{SR} + m\widehat{RQ} = m\widehat{SQ}$
$m\widehat{SR} + m\angle RPQ = m\angle SPQ$
$m\widehat{SR} + 71 = 180$
$m\widehat{SR} = 109°$

7. $m\widehat{SQU} + m\widehat{UT} + m\widehat{TS} = 360$
$m\widehat{SQU} + m\angle UPT + m\angle TPS = 360$
$m\widehat{SQU} + 40 + 71 = 360$
$m\widehat{SQU} = 249°$

8. Let $JK = 2x$; then
$x^2 + 4^2 = (4 + 3)^2$
$x^2 = 33$
$x = \sqrt{33}$
$JK = 2\sqrt{33} \approx 11.5$

9. Let $XY = 2x$; then
$x^2 + 4^2 = 8^2$
$x^2 = 48$
$x = 4\sqrt{3}$
$XY = 8\sqrt{3} \approx 13.9$

10. $A = \pi(22)^2\left(\frac{80}{360}\right) = \frac{968}{9}\pi \approx 338 \text{ cm}^2$

11. $\widehat{AB} = 2\pi(4)\left(\frac{150}{360}\right) = \frac{10}{3}\pi \text{ ft} \approx 10.47 \text{ ft}$

12. $\widehat{EF} = 2\pi(2.4)\left(\frac{75}{360}\right) = \pi \text{ cm} \approx 3.14 \text{ cm}$

13. arc length $= 2\pi(5)\left(\frac{44}{360}\right) = \frac{11}{9}\pi \text{ in.} \approx 3.84 \text{ in.}$

11. arc length $= 2\pi(46)\left(\frac{180}{360}\right) = 46\pi \text{ m} \approx 144.51 \text{ m}$

11-4 INSCRIBED ANGLES, PAGES 772–779

CHECK IT OUT! PAGES 773–775

1a. $m\angle ABC = \frac{1}{2}m\widehat{ADC}$
$135 = \frac{1}{2}m\widehat{ADC}$
$m\widehat{ADC} = 270°$

b. $m\angle DAE = \frac{1}{2}mDE$
$= \frac{1}{2}(72) = 36°$

2. $m\angle ABD = \frac{1}{2}m\widehat{AD} = \frac{1}{2}(86) = 43°$
$m\angle BAC = \frac{1}{2}m\widehat{BC}$
$60 = \frac{1}{2}m\widehat{BC}$
$m\widehat{BC} = 120°$

3a. $\angle ABC$ is a right angle
$m\angle ABC = 90$
$8z - 6 = 90$
$8z = 96$
$z = 12$

b. $m\angle EDF = m\angle EGF$
$2x + 3 = 75 - 2x$
$4x = 72$
$x = 18$
$m\angle EDF = 2(18) + 3$
$= 39°$

4. Step 1 Find the value of x.
$m\angle K + m\angle M = 180$
$33 + 6x + 4x - 13 = 180$
$10x = 160$
$x = 16$

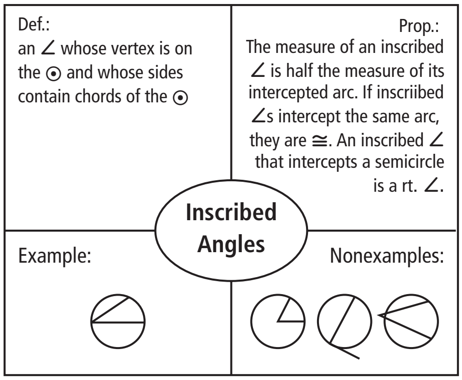
Step 2 Find the measure of each ∠.
$m\angle K = 33 + 6(16) = 129°$
$m\angle L = \frac{9(16)}{2} = 72°$
$m\angle M = 4(16) - 13 = 51°$
$m\angle J + m\angle L = 180$
$m\angle J + 72 = 180$
$m\angle J = 108°$

THINK AND DISCUSS, PAGE 775

1. No; a quadrilateral can be inscribed in a circle if and only if its opposite ∠s are supplementary.

2. An arc that is $\frac{1}{4}$ of a circle measures 90°. If the arc measures 90°, then the measure of the inscribed ∠ is $\frac{1}{2}(90) = 45°$.

3.



EXERCISES, PAGES 776–779

GUIDED PRACTICE, PAGE 776

1. inscribed

2. $m\angle DEF = \frac{1}{2}m\widehat{DF}$
$= \frac{1}{2}(78) = 39°$

3. $m\angle EFG = \frac{1}{2}m\widehat{EG}$
$29 = \frac{1}{2}m\widehat{EG}$
$m\widehat{EG} = 58°$

4. $m\angle JNL = \frac{1}{2}m\widehat{JKL}$
$102 = \frac{1}{2}m\widehat{JKL}$
$m\widehat{JKL} = 204°$

5. $m\angle LKM = \frac{1}{2}m\widehat{LM}$
$= \frac{1}{2}(52) = 26°$

6. $m\angle QTR + m\angle Q + m\angle R = 180$
$m\angle QTR + \frac{1}{2}m\widehat{RS} + m\angle S = 180$
$m\angle QTR + \frac{1}{2}(90) + 25 = 180$
$m\angle QTR + 70 = 180$
$m\angle QTR = 110°$

7. $\angle DEF$ is a right ∠
$\frac{4x}{5} = 90$
$4x = 450$
$x = 112.5$

8. $\angle FHG$ is a right ∠. So $\triangle FGH$ is a 45°-45°-90° triangle.
$m\angle GFH = 45$
$3y + 6 = 45$
$3y = 39$
$y = 13$

9. $m\angle XYZ = m\angle XWZ$
$7y - 3 = 4 + 6y$
$y = 7$
$m\angle XYZ = 7(7) - 3 = 46°$

10. Step 1 Find the value of x.

$$\begin{aligned} m\angle P + m\angle R &= 180 \\ 5x + 20 + 7x - 8 &= 180 \\ 12x &= 168 \\ x &= 14 \end{aligned}$$

Step 2 Find the measures.

$m\angle P = 5(14) + 20 = 90°$
$m\angle Q = 10(14) = 140°$
$m\angle R = 7(14) - 8 = 90°$

$$\begin{aligned} m\angle S + m\angle Q &= 180 \\ m\angle S + 140 &= 180 \\ m\angle S &= 40° \end{aligned}$$

11. Step 1 Find the value of z.

$$\begin{aligned} m\angle A + m\angle C &= 180 \\ 4z - 10 + 10 + 5z &= 180 \\ 9z &= 180 \\ z &= 20 \end{aligned}$$

Step 2 Find the measures.

$m\angle A = 4(20) - 10 = 70°$
$m\angle B = 6(20) - 5 = 115°$
$m\angle C = 10 + 5(20) = 110°$

$$\begin{aligned} m\angle B + m\angle D &= 180 \\ 115 + m\angle D &= 180 \\ m\angle D &= 65° \end{aligned}$$

PRACTICE AND PROBLEM SOLVING, PAGES 776–778

12. $m\angle MNL = \frac{1}{2}m\overset{\frown}{ML}$
$43 = \frac{1}{2}m\overset{\frown}{ML}$
$m\overset{\frown}{ML} = 86°$

13. $m\angle KMN = \frac{1}{2}m\overset{\frown}{KN}$
$= \frac{1}{2}(95)$
$= 47.5°$

14. $m\angle EJH = \frac{1}{2}m\overset{\frown}{EGH}$
$139 = \frac{1}{2}m\overset{\frown}{EGH}$
$m\overset{\frown}{EGH} = 278°$

15. $m\angle GFH = \frac{1}{2}m\overset{\frown}{GH}$
$= \frac{1}{2}(95.2)$
$= 47.6°$

16. $m\angle ADC = m\angle ADB + m\angle BDC$
$= \frac{1}{2}mAB + m\angle BEC$
$= \frac{1}{2}(44) + 40 = 62°$

17. $m\angle SRT = 90$
$3y^2 - 18 = 90$
$3y^2 = 108$
$y^2 = 36$
$y = \pm 6$

18. $\triangle JKL$ is a 30°-60°-90°
$6z - 4 = 60$
$6z = 64$
$z = \frac{64}{6} = 10\frac{2}{3}$

19. $m\angle ADB = m\angle ACB$
$2x^2 = 10x$
$2x = 10 \quad (x \neq 0)$
$x = 5$
$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$
$2(5)^2 = \frac{1}{2}m\overset{\frown}{AB}$
$50 = \frac{1}{2}m\overset{\frown}{AB}$
$m\overset{\frown}{AB} = 2(50) = 100°$

20. $\angle MPN$ and $\angle MNP$ are complementary.

$$\begin{aligned} m\angle MPN + m\angle MNP &= 90 \\ \frac{11x}{3} + 3x - 10 &= 90 \\ 20x - 30 &= 270 \\ 20x &= 300 \\ x &= 15 \end{aligned}$$

$m\angle MPN = \frac{11(15)}{3} = 55°$

21. Step 1 Find the value of x.

$$\begin{aligned} m\angle B + m\angle D &= 180 \\ \frac{x}{2} + \frac{x}{4} + 30 &= 180 \\ \frac{3x}{4} &= 150 \\ 3x &= 600 \\ x &= 200 \end{aligned}$$

Step 2 Find the measures.

$m\angle B = \frac{200}{2} = 100°$
$m\angle D = \frac{200}{4} + 30 = 80°$
$m\angle E = 200 - 59 = 141°$

$$\begin{aligned} m\angle C + m\angle E &= 180 \\ m\angle C + 141 &= 180 \\ m\angle C &= 39° \end{aligned}$$

22. Step 1 Find the value of x.

$$\begin{aligned} m\angle U + m\angle W &= 180 \\ 14 + 4x + 6x - 14 &= 180 \\ 10x &= 180 \\ x &= 18 \end{aligned}$$

Step 2 Find the value of y.

$$\begin{aligned} m\angle T + m\angle V &= 180 \\ 12y - 5 + 15y - 4 &= 180 \\ 27y &= 189 \\ y &= 7 \end{aligned}$$

Step 3 Find the measures.

$m\angle T = 12(7) - 5 = 79°$
$m\angle U = 14 + 4(18) = 86°$
$m\angle V = 15(7) - 4 = 101°$
$m\angle W = 6(18) - 14 = 94°$

23. always

24. never

25. sometimes (if opposite ∠s are supplementary)

26. $m\angle ABC = \frac{1}{2}m\overset{\frown}{AC} = \frac{1}{2}m\angle ADC = \frac{1}{2}(112) = 56°$

27. Let S be any point on the major arc from P to R.

$$\begin{aligned} m\angle PQR + m\angle PSR &= 180 \\ m\angle PQR + \tfrac{1}{2}m\overset{\frown}{PQR} &= 180 \\ m\angle PQR + \tfrac{1}{2}(130) &= 180 \\ m\angle PQR &= 180 - 65 = 115° \end{aligned}$$

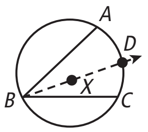
28. By the definition of an arc measure, $m\overset{\frown}{JK} = m\angle JHK$. Also, the measure of an ∠ inscribed in a circle is half the measure of the intercepted arc. So $m\angle JLK = \frac{1}{2}m\overset{\frown}{JK}$. Multiplying both sides of equation by 2 gives $2m\angle JLK = m\overset{\frown}{JK}$. By substitution, $m\angle JHK = 2m\angle JLK$.

29a. $m\angle BAC = \frac{1}{2}m\overset{\frown}{BC} = \frac{1}{2}\left(\frac{1}{6}(360)\right) = 30°$

b. $m\angle CDE = \frac{1}{2}mCAE = \frac{1}{2}\left(\frac{4}{6}(360)\right) = 120°$

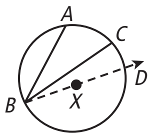
c. $\angle FBC$ is inscribed in a semicircle. So it must be a right ∠; therefore, $\triangle FBC$ is a right ∠. (Also, $m\angle CFD =$ 30°. So, $\triangle FBC$ is a 30°-60°-90° △.)

30.



Since any 2 points determine a line, draw $\overrightarrow{BX}$.
Let D be a point where $\overrightarrow{BX}$ intercepts $\widehat{AC}$. By Case 1 of the Inscribed ∠ Thm., $m\angle ABD = \frac{1}{2}m\widehat{AD}$ and $m\angle DBC = \frac{1}{2}m\widehat{DC}$. By Add., Distrib., and Trans. Props. of =, $m\angle ABD + m\angle DBC = \frac{1}{2}m\widehat{AD} + \frac{1}{2}m\widehat{DC}$ $= \frac{1}{2}(m\widehat{AD} + m\widehat{DC})$. Thus, by ∠ Add. Post. and Arc Add. Post., $m\angle ABC = \frac{1}{2}mAC$.

31.



Since any 2 points determine a line, draw $\overrightarrow{BX}$.
Let D be pt. where $\overrightarrow{BX}$ intercepts $\widehat{ACB}$. By Case 1 of Inscribed ∠ Thm., $m\angle ABD = \frac{1}{2}m\widehat{AD}$ and $m\angle CBD$ $= \frac{1}{2}m\widehat{CD}$. By Subtr., Distrib., and Trans. Props. of =, $m\angle ABD - m\angle CBD = \frac{1}{2}m\widehat{AD} - \frac{1}{2}m\widehat{CD}$ $= \frac{1}{2}(m\widehat{AD} - m\widehat{CD})$. Thus by ∠ Add. Post. and Arc Add. Post., $m\angle ABC = \frac{1}{2}mAC$.

32. By the Inscribed ∠ Thm., $m\angle ACD = \frac{1}{2}m\widehat{AB}$ and $m\angle ADB = \frac{1}{2}m\widehat{AB}$. By Substitution, $m\angle ACD = m\angle ADB$, and therefore, $\angle ACD \cong \angle ADB$.

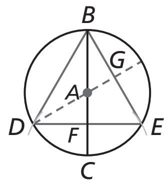
33. $m\angle J = \frac{1}{2}m\widehat{KLM} = \frac{1}{2}(216) = 108°$
$m\angle J + m\angle L = 180$
$108 + m\angle L = 180$
$m\angle L = 72°$
$m\angle M = \frac{1}{2}m\widehat{JKL} = \frac{1}{2}(198) = 99°$
$m\angle K + m\angle M = 180$
$m\angle K + 99 = 180$
$m\angle K = 81°$

34. $\overline{PR}$ is a diag. of $PQRS$. $\angle Q$ is an inscribed right angle. So its intercepted arc is a semicircle. Thus, $\overline{PR}$ is a diameter of the circle.

35a. $AB^2 + AC^2 = BC^2$, so by Conv. of Pythag. Thm., $\triangle ABC$ is a right △ with right $\angle A$. Since $\angle A$ is an inscribed right ∠, it intercepts a semicircle. This means that $\overline{BC}$ is a diameter.

b. $m\angle ABC = \sin^{-1} - \left(\frac{14}{18}\right) \approx 51.1°$.
Since $m\angle ABC = \frac{1}{2}m\widehat{AC}$, $m\widehat{AC} = 102°$.

36.



Draw a diagram through D and A. Label the intersection of $\overline{BC}$ and $\overline{DE}$ as F and the intersection of $\overrightarrow{DA}$ and $\overline{BE}$ as G. Since $\overline{BC}$ is a diameter of the circle, it is a bisector of chord $\overline{DE}$. Thus, $\overline{DF} \cong \overline{EF}$, and $\angle BFD$ and $\angle BFE$ are ≅ right ∠s. $\overline{BF} \cong \overline{BF}$ by Reflex. Prop. of ≅. Thus, $\triangle BFD \cong \triangle BFE$ by SAS. $\overline{BD} \cong \overline{BE}$ by CPCTC. By Trans. Prop. of ≅, $\overline{BE} \cong \overline{ED}$. Thus, by definition, $\triangle DBE$ is equilateral.

37. Agree; the opposite ∠s of a quadrilateral are congruent. So the ∠ opposite the 30° ∠ also measures 30°. Since this pair of the opposite ∠s are not supplementary, the quadrilateral cannot be inscribed in a circle.

38. Check students' constructions.

TEST PREP, PAGE 778

39. D
$m\angle BAC + m\angle ABC + m\angle ACB = 180$
$m\angle BAC + \frac{1}{2}mAC + m\angle ACB = 180$
$m\angle BAC + \frac{1}{2}(76) + 61 = 180$
$m\angle BAC + 38 + 61 = 180$
$m\angle BAC = 81°$

40. H
$\frac{1}{2}m\widehat{XY} = m\angle XCY = \frac{1}{2}m\angle XCZ$
$m\widehat{XY} = m\angle XCZ = 60°$

41. C
Let $m\angle A = 4x$ and $m\angle C = 5x$.
$m\angle A + m\angle C = 180$
$4x + 5x = 180$
$9x = 180$
$x = 20$
$m\angle A = 4(20) = 80°$

42. F
$m\angle STR = 180 - (m\angle TRS + m\angle TSR)$
$= 180 - \left(\frac{1}{2}m\widehat{PS} + \frac{1}{2}m\widehat{QR}\right)$
$= 180 - (42 + 56) = 82°$
$m\angle QPR = \frac{1}{2}m\widehat{QR} = 56°$
$m\angle QPR = \frac{1}{2}m\widehat{QR} = 56°$
$m\angle PQS = \frac{1}{2}m\widehat{PS} = 42°$

CHALLENGE AND EXTEND, PAGE 779

43. If an ∠ is inscribed in a semicircle, the measure of the intercepted arc is 180°. The measure of ∠ is $\frac{1}{2}(180) = 90°$. So the angle is a right ∠. Conversely, if an inscribed angle is a right angle, then it measures 90° and its intercepted arc measures $2(90) = 180°$. An arc that measures 180° is a semicircle.

44. Suppose the quadrilateral *ABCD* is inscribed in a circle. Then $m\angle A = \frac{1}{2}m\widehat{BCD}$ and $m\angle C = \frac{1}{2}m\widehat{DAB}$. By Add., Distrib., and Trans. Prop. of =, $m\angle A + m\angle C = \frac{1}{2}m\widehat{BCD} + \frac{1}{2}m\widehat{DAB} = \frac{1}{2}(m\widehat{BCD} + m\widehat{DAB})$. $m\widehat{BCD} + m\widehat{DAB} = 360°$. So by subst., $m\angle A + m\angle C = \frac{1}{2}(360) = 180°$. Thus, $\angle A$ and $\angle C$ are supplementary. A similar proof shows that $\angle B$ and $\angle D$ are supplementary.

45. $\overline{RQ}$ is a diameter. So $\angle P$ is a right $\angle$.
$m\widehat{PQ} = 2m\angle R = 2\tan^{-1}\left(\frac{7}{3}\right) \approx 134°$

46. Draw $\overline{AC}$ and $\overline{DE}$.
$m\frac{1}{2}m\overline{CE} = 19°$
$m\angle ACD = \frac{1}{2}m\widehat{AD} = 36°$
$m\angle ABC + 19 + 36 = 180$
$m\angle ABC = 125°$
$m\angle ABD + 125 = 180$
$m\angle ABD = 55°$

47. Check students' constructions.

SPIRAL REVIEW, PAGE 779

48. $\begin{cases} a + b + c = 12 \quad (1) \\ 30a + 22.5b + 15c = 255 \quad (2) \\ 30a = 15c \quad (3) \end{cases}$

Substitute (3) in (2):
$15c + 22.5b + 15c = 255$
$3b + 4c = 34 \quad (4)$
Substitute $\frac{1}{15}(3)$ in 2(1):
$c + 2b + 2c = 24$
$2b + 3c = 24 \quad (5)$
$3(5) - 2(4)$:
$6b + 9c - 6b - 8c = 72 - 68$
$c = 4$
Substitute in (3):
$30a = 15(4) = 60$
$a = 2$
Substitute in (5):
$2b + 3(4) = 24$
$2b = 12$
$b = 6$

49. $m = \dfrac{\frac{1}{2} - (-6)}{8 - 4\frac{1}{2}} = \dfrac{6\frac{1}{2}}{3\frac{1}{2}} = \dfrac{13}{7}$

50. $m = \dfrac{-2 - (-8)}{0 - (-9)} = \dfrac{6}{9} = \dfrac{2}{3}$

51. $m = \dfrac{6 - (-14)}{11 - 3} = \dfrac{20}{8} = \dfrac{5}{2}$

52. By Vert. ∡ Thm., $\angle RWV \cong \angle SWT$. So by Thm. 11-2-2 (1),
$RV = ST$
$2z + 15 = 9z + 1$
$14 = 7z$
$z = 2$
$\widehat{RV} \cong \widehat{ST}$ and $\widehat{RS} \cong \widehat{VT}$. So by substitution,
$2m\widehat{ST} + 2m\widehat{VT} = 360$
$m\widehat{ST} + m\widehat{VT} = 180$
$m\widehat{ST} + 31(2) + 2 = 180$
$m\widehat{ST} + 64 = 180$
$m\widehat{ST} = 116°$

53. Let $BD = 2x$; then
$1.5^2 + x^2 = (1 + 1.5)^2$
$2.25 + x^2 = 6.25$
$x^2 = 4$
$x = 2.$
$\triangle ABD$ has base 2(2) = 4 m and height 1.5 m.
So $A = \frac{1}{2}bh = \frac{1}{2}(4)(1.5) = 3\text{ m}^2$.

CONSTRUCTION, PAGE 779

1. Yes; $\overline{CR}$ is a radius of circle *C*. If a line is tangent to a circle, then it is ⊥ to the radius at the point of tangency.

11-5 TECHNOLOGY LAB: EXPLORE ANGLE RELATIONSHIPS IN CIRCLES, PAGES 780–781

TRY THIS, PAGE 781

1. The relationship is the same. Both types of ∡ have a measure equal to half the measure of the arc.
2. A tangent line must be ⊥ to the radius at the point of tangency. If construction is done without this ⊥ relationship, then tangent line created is not guaranteed to intersect circle at only one point. If a line is ⊥ to a radius of a circle at a point on the circle, then line is tangent to circle.
3. The relationship remains the same. The measure of ∠ is half difference of intercepted arcs.
4. An ∠ whose vertex is *on* the circle (inscribed ∡ and ∡ created by a tangent and a secant intersecting at the point of tangency) will have a measure equal to half its intercepted arc.
   An ∠ whose vertex is *inside* the circle (∡ created by intersecting secants or chords) will have a measure equal to half sum of its intercepted arcs.
   An ∠ whose vertex is *outside* the circle (∡ created by secants and/or tangents that intersect outside circle) will have a measure equal to half its intercepted arc.
5. No; it is a means to discover relationships and make conjectures.

11-5 ANGLE RELATIONSHIPS IN CIRCLES, PAGES 782–789

CHECK IT OUT! PAGES 782–785

1a. $m\angle STU = \frac{1}{2}m\widehat{ST}$
$= \frac{1}{2}(166) = 83°$

b. $m\angle QSR = \frac{1}{2}m\widehat{SR}$
$71 = \frac{1}{2}m\widehat{SR}$
$m\widehat{SR} = 142°$

2a. $m\angle ABD = \frac{1}{2}(m\widehat{AD} + m\widehat{CE})$
$= \frac{1}{2}(65 + 37)$
$= \frac{1}{2}(102) = 51°$

b. $m\angle RNP = \frac{1}{2}(m\widehat{MQ} + m\widehat{RP})$
$= \frac{1}{2}(91 + 225)$
$= \frac{1}{2}(316) = 158°$
$m\angle RNM + m\angle RNP = 180$
$m\angle RNM + 158 = 180$
$m\angle RNM = 22°$

3. $m\angle L = \frac{1}{2}(m\widehat{JN} - m\widehat{KN})$
$25 = \frac{1}{2}(83 - x)$
$50 = 83 - x$
$x = 33$

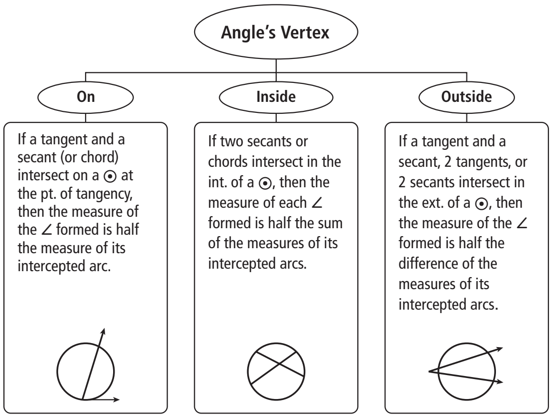
4. $m\angle ACB = \frac{1}{2}(m\widehat{AEB} - m\widehat{AB})$
$= \frac{1}{2}(225 - 135) = 45°$

5. Step 1 Find $m\widehat{PR}$.
$m\angle Q = \frac{1}{2}(m\widehat{MS} - mPR)$
$26 = \frac{1}{2}(80 - m\widehat{PR})$
$52 = 80 - m\widehat{PR}$
$m\widehat{PR} = 28°$
Step 2 Find $m\widehat{LP}$.
$m\widehat{LP} + m\widehat{PR} = mLR$
$m\widehat{LP} + 28 = 100$
$m\widehat{LP} = 72°$

THINK AND DISCUSS, PAGE 786

1. For both chords and secants that intersect in the interior of a circle, the measure of ∠ formed is half the sum of the measures of their intercepted arcs.

2.



EXERCISES, PAGES 786–789

GUIDED PRACTICE, PAGES 786–787

1. $m\angle DAB = \frac{1}{2}m\widehat{AB}$
$= \frac{1}{2}(140) = 70°$

2. $27 = \frac{1}{2}m\widehat{AC}$
$m\widehat{AC} = 54°$

3. $61 = \frac{1}{2}m\widehat{PN}$
$m\widehat{PN} = 122°$

4. $m\angle MNP = \frac{1}{2}(238)$
$= 119°$

5. $m\angle STU = \frac{1}{2}(m\widehat{SU} + m\widehat{VW})$
$= \frac{1}{2}(104 + 30)$
$= \frac{1}{2}(134) = 67°$

6. $m\angle HFG = \frac{1}{2}(m\widehat{EJ} + m\widehat{GH})$
$= \frac{1}{2}(59 + 23)$
$= \frac{1}{2}(82) = 41°$

7. $m\angle NPL = \frac{1}{2}(m\widehat{KM} + m\widehat{NL})$
$= \frac{1}{2}(61 + 111)$
$= \frac{1}{2}(172) = 86°$
$m\angle NPK + m\angle NPL = 180$
$m\angle NPK + 86 = 180$
$m\angle NPK = 94°$

8. $x = \frac{1}{2}(161 - 67) = \frac{1}{2}(94) = 47$

9. $x = \frac{1}{2}(238 - 122) = \frac{1}{2}(116) = 58$

10. $27 = \frac{1}{2}(x - 40)$
$54 = x - 40$
$x = 94°$

11. $m\angle S = \frac{1}{2}(m\widehat{ACB} - m\widehat{AB})$
$38 = \frac{1}{2}((360 - x) - x)$
$76 = 360 - 2x$
$2x = 360 - 76 = 284$
$x = 142°$

12. $m\angle E = \frac{1}{2}(m\widehat{BF} - m\widehat{DF})$
$50 = \frac{1}{2}(150 - mDF)$
$100 = 150 - m\widehat{DF}$
$m\widehat{DF} = 50°$

13. $m\widehat{BC} + m\widehat{CD} + m\widehat{DF} + m\widehat{BF} = 360$
$64 + m\widehat{CD} + 50 + 150 = 360$
$m\widehat{CD} + 264 = 360$
$m\widehat{CD} = 96°$

14. $m\angle NPQ = \frac{1}{2}(m\widehat{JK} + m\widehat{PN})$
$79 = \frac{1}{2}(48 + m\widehat{PN})$
$158 = 48 + m\widehat{PN}$
$m\widehat{PN} = 110°$

15. $m\widehat{KN} + m\widehat{PN} + m\widehat{JP} + m\widehat{JL} = 360$
$m\widehat{KN} + 110 + 86 + 48 = 360$
$m\widehat{KN} + 244 = 360$
$m\widehat{KN} = 116°$

PRACTICE AND PROBLEM SOLVING, PAGES 787–788

16. $m\angle BCD = \frac{1}{2}m\overset{\frown}{BC} = \frac{1}{2}(112) = 56°$

17. $m\angle ABC = \frac{1}{2}(360 - 112) = \frac{1}{2}(248) = 124°$

18. $m\angle XZW = \frac{1}{2}m\overset{\frown}{XZ} = \frac{1}{2}(180) = 90°$

19. $m\overset{\frown}{XZV} = m\overset{\frown}{XZ} + m\overset{\frown}{ZV}$
$= 180 + 2m\angle VXZ$
$= 180 + 2(40) = 260°$

20. $m\angle QPR = \frac{1}{2}(m\overset{\frown}{QR} + m\overset{\frown}{ST}) = \frac{1}{2}(31 + 98) = 64.5°$

21. $m\angle ABC = 180 - m\angle ABD$
$= 180 - \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{CE})$
$= 180 - \frac{1}{2}(100 + 45) = 107.5°$

22. $m\angle MKJ = 180 - m\angle MKL$
$= 180 - \frac{1}{2}(m\overset{\frown}{JN} + m\overset{\frown}{ML})$
$= 180 - \frac{1}{2}(38.5 + 51.5) = 135°$

23. $x = \frac{1}{2}(170 - (360 - (170 + 135)))$
$= \frac{1}{2}(170 - 55) = 57.5°$

24. $x = \frac{1}{2}(220 - 140) = 40°$

25. $x = \frac{1}{2}((180 - (20 + 104)) - 20)$
$= \frac{1}{2}(56 - 20) = 18°$

26. $m\angle ABV = \frac{1}{2}(180 - m\overset{\frown}{AB}) = \frac{1}{2}(180 - 48) = 66°$

27. $m\angle DJH = \frac{1}{2}(m\overset{\frown}{DH} + m\overset{\frown}{EG})$
$180 - 89 = \frac{1}{2}(137 + m\overset{\frown}{EG})$
$91 = \frac{1}{2}(137 + m\overset{\frown}{EG})$
$182 = 137 + m\overset{\frown}{EG}$
$m\overset{\frown}{EG} = 45°$

28. $m\angle DJE = \frac{1}{2}(m\overset{\frown}{DE} + m\overset{\frown}{GH})$
$89 = \frac{1}{2}(m\overset{\frown}{DE} + 61)$
$178 = m\overset{\frown}{DE} + 61$
$m\overset{\frown}{DE} = 117°$

29. **Step 1** Find $m\angle P$.
$m\angle P = \frac{1}{2}m\overset{\frown}{LR} = \frac{1}{2}(170) = 85°$
Step 2 Find $m\angle QPR$.
$m\angle QPR + 50 + 85 = 180$
$m\angle QPR = 45°$
Step 3 Find $m\overset{\frown}{PR}$.
$m\angle QPR = \frac{1}{2}m\overset{\frown}{PR}$
$45 = \frac{1}{2}m\overset{\frown}{PR}$
$m\overset{\frown}{PR} = 90°$

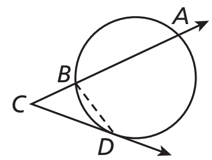
30. $50 = \frac{1}{2}(m\overset{\frown}{PS} - m\overset{\frown}{ML})$
$= \frac{1}{2}(m\overset{\frown}{PM} - m\overset{\frown}{ML})$
$= \frac{1}{2}m\overset{\frown}{LP}$
$m\overset{\frown}{LP} = 100°$

31. $m\angle ABC = \frac{1}{2}m\overset{\frown}{AB}$
$x = \frac{1}{2}m\overset{\frown}{AB}$
$m\overset{\frown}{AB} = 2x°$

32. $m\angle ABD + m\angle ABC = 180$
$m\angle ABD + x = 180$
$m\angle ABD = (180 - x)°$

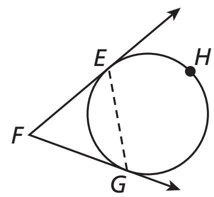
33. $m\overset{\frown}{AEB} + m\overset{\frown}{AB} = 360$
$m\overset{\frown}{AEB} + 2x = 360$
$m\overset{\frown}{AEB} = (360 - 2x)°$

34.



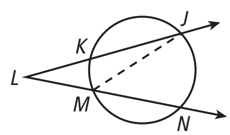
Since 2 points determine a line, draw $\overline{BD}$. By Ext. ∠ Thm., $m\angle ABD = m\angle C + m\angle BDC$. So $m\angle C = m\angle ABD - m\angle BDC$. $m\angle ABD = \frac{1}{2}m\overset{\frown}{AD}$ by Inscribed ∠ Thm., and $m\angle BDC = \frac{1}{2}m\overset{\frown}{BD}$ by Thm. 11-5-1. By substitution, $m\angle C = \frac{1}{2}m\overset{\frown}{AD} - \frac{1}{2}m\overset{\frown}{BD}$. Thus, by Distrib. Prop. of =, $m\angle C = \frac{1}{2}(m\overset{\frown}{AD} - m\overset{\frown}{BD})$.

35.



Since 2 points determine a line, draw $\overline{EG}$. By Ext. ∠ Thm., $m\angle DEG = m\angle F + m\angle EGF$. So $m\angle F = m\angle DEG - m\angle EGF$. $m\angle DEG = \frac{1}{2}m\overset{\frown}{EHG}$ and $m\angle EGF = \frac{1}{2}m\overset{\frown}{EG}$ by Theorem 11-5-1. By substitution, $m\angle F = \frac{1}{2}m\overset{\frown}{EHG} - \frac{1}{2}m\overset{\frown}{EG}$. Thus, by Distrib. Prop. of =, $m\angle F = \frac{1}{2}(m\overset{\frown}{EHF} - m\overset{\frown}{EG})$.

36.



Since 2 points determine a line, draw $\overline{JM}$. By Ext. ∠ Thm., $m\angle JMN = m\angle L + m\angle KJM$. So $m\angle L = m\angle JMN - m\angle KJM$. $m\angle JMN = \frac{1}{2}m\overset{\frown}{JN}$ and $m\angle KJM = \frac{1}{2}m\overset{\frown}{KM}$ by Inscribed ∠ Thm. By substitution, $m\angle F = \frac{1}{2}m\overset{\frown}{JN} - \frac{1}{2}m\overset{\frown}{KM}$. Thus, by Distrib. Prop. of =, $m\angle F = \frac{1}{2}(m\overset{\frown}{JN} - m\overset{\frown}{KM})$.

37. $m\angle 1 > m\angle 2$ because $m\angle 1 = \frac{1}{2}(m\overset{\frown}{AB} + m\overset{\frown}{CD})$ and $m\angle 2 = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$. Since $m\overset{\frown}{CD} > 0$, the expression for $m\angle 1$ is greater.

38. When a tangent and secant intersect on the circle, the measure of ∠ formed is half the measure of the intercepted arc.

When 2 secants intersect inside the circle, the measure of each ∠ formed is half the sum of the measures of the intercepted arcs, or $\frac{1}{2}(90° + 90°)$.

When 2 secants intersect inside the circle, the measure of each ∠ formed is half the difference of the measures of the intercepted arcs, or $\frac{1}{2}(270° - 90°)$.

39. $m\widehat{AC} + m\widehat{AD} + m\widehat{CD} = 360$
$2x - 10 + x + 160 = 360$
$3x = 210$
$x = 70$
$m\angle B = \frac{1}{2}(m\widehat{AC} - m\widehat{AD}) = \frac{1}{2}(2(70) - 10 - 70) = 30°$
$m\angle C = \frac{1}{2}m\widehat{AD} = \frac{1}{2}(70) = 35°$
$m\angle A = 180 - (m\angle B + m\angle C)$
$= 180 - (30 + 35) = 115°$

40. $m\angle B = \frac{1}{2}(m\widehat{AD} - m\widehat{DE})$
$4x = \frac{1}{2}(18x - 15 - (8x + 1))$
$8x = 10x - 16$
$16 = 2x$
$x = 8$
$m\angle B = 4(8) = 32°$
$m\widehat{AD} = 18(8) - 15 = 129°$
$m\angle C = \frac{1}{2}(m\widehat{AED} - m\widehat{AD}) = \frac{1}{2}(231 - 129) = 51°$
$m\angle A = 180 - (m\angle B - m\angle C)$
$= 180 - (32 + 51) = 97°$

41a. $m\angle BHC = m\widehat{BC} = \frac{1}{6}(360) = 60°$

b. $m\angle EGD = \frac{1}{2}(m\widehat{DE} - m\widehat{DAE})$
$= \frac{1}{2}(60 + 3(60)) = 120°$

c. $m\angle CED = m\angle EDF = \frac{1}{2}(60) = 30°$ and $m\angle EGD > 90°$, so $\triangle EGD$ is an obtuse isosceles.

TEST PREP, PAGE 789

42. C
$m\angle DCE = \frac{1}{2}(m\widehat{AF} + m\widehat{DE})$
$= \frac{1}{2}(58 + 100) = 79°$

43. J

44. 56°
$m\angle JLK = \frac{1}{2}(m\widehat{MN} - m\widehat{JK})$
$45 = \frac{1}{2}(146 - m\widehat{JK})$
$90 = 146 - m\widehat{JK}$
$m\widehat{JK} = 56°$

CHALLENGE AND EXTEND, PAGE 789

45. Case 1: Assume $\overline{AB}$ is a diameter of the circle. Then $m\widehat{AB} = 180°$ and $\angle ABC$ is a right ∠. Thus, $m\angle ABC = \frac{1}{2}m\widehat{AB}$.
Case 2: Assume $\overline{AB}$ is not a diameter of the circle. Let X be the center of the circle and draw radii $\overline{XA}$ and $\overline{XB}$. $\overline{XA} \cong \overline{XB}$. So $\triangle AXB$ is isosceles. Thus, $\angle XAB \cong \angle XBA$, and $2m\angle XBA + m\angle AXB = 180$. This means that $m\angle XBA = 90 - \frac{1}{2}m\angle AXB$. By Thm. 11-1-1, $\angle XBC$ is a right ∠. So $m\angle XBA + m\angle ABC = 90$ or $m\angle ABC = 90 - m\angle XBA$. By substituting, $m\angle ABC = 90 - (90 - \frac{1}{2}m\angle AXB)$ $= \frac{1}{2}m\angle AXB$. $m\angle AXB = m\widehat{AB}$ because $\angle AXB$ is a central ∠. Thus, $m\angle ABC = \frac{1}{2}m\widehat{AB}$.

46. Since $m\widehat{WY} = 90°$, $m\angle YXW = 90°$ because it is a central ∠. By Thm. 11-1-1, $\angle XYZ$ and $\angle XWZ$ are right ⦣. The sum of measures of the ⦣ of a quadrilateral is 360°. So $m\angle WZY = 90°$. Thus, all 4 ⦣ of $WXYZ$ are right ⦣. So $WXYZ$ is a rectangle. $XY \cong XW$ because they are radii. By Thm. 6-5-3, $WXYZ$ is a rhombus. Since $WXYZ$ is a rectangle and a rhombus, it must also be a square by Theorem 6-5-6.

47. $m\angle V = \frac{1}{2}(x - 21) = \frac{1}{2}(124 - 50)$
$x - 21 = 124 - 50 = 74$
$x = 74 + 21 = 95°$

48. Step 1 Find $m\angle CED$.
$m\angle DCE + m\angle ECJ = 180$
$m\angle DCE + 135 = 180$
$m\angle DCE = 45°$
$m\angle CDE = m\angle FDG = 82°$
$m\angle CED + m\angle DCE + m\angle CDE = 180$
$m\angle CED + 45 + 82 = 180$
$m\angle CED = 53°$
Step 2 Find $m\widehat{GH}$.
$m\angle CED = \frac{1}{2}(m\widehat{GH} + mKL)$
$53 = \frac{1}{2}(m\widehat{GH} + 27)$
$106 = m\widehat{GH} + 27$
$m\widehat{GH} = 79°$

SPIRAL REVIEW, PAGE 789

49. $g(7) = 2(7)^2 - 15(7) - 1 = 98 - 105 - 1 = -8$; yes

50. $f(7) = 29 - 3(7) = 29 - 21 = 8$; no

51. $-\frac{7}{8}(7) = -\frac{49}{8}$; no

52. $V = \frac{1}{3}Bh$
$= \frac{1}{3}\left(\frac{1}{2}aP\right)h$
$= \frac{1}{3}\left(\frac{1}{2}\left(2\sqrt{3}\right)(24)\right)(7)$
$= 56\sqrt{3} \approx 97.0 \text{ m}^3$

53. $L = \pi r\ell$
$60\pi = \pi(6)\ell$
$\ell = 10$ cm
$r^2 + h^2 = \ell^2$
$6^2 + h^2 = 10^2$
$h = 8$ cm
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(6)^2(8) = 96\pi$ cm^3
≈ 310.6 cm^3

54. $S = \frac{1}{2}P\ell + s^2$
$1200 = \frac{1}{2}(96)\ell + 576$
$624 = 48\ell$
$\ell = 13$ in.
$\left(\frac{1}{2}s\right)^2 + h^2 = \ell^2$
$12^2 + h^2 = 13^2$
$h = 5$ in.
$V = \frac{1}{3}Bh = \frac{1}{3}(576)(5) = 960$ in.^{3}

55. $m\angle BCA = \frac{1}{2}m\widehat{BA} = \frac{1}{2}(74) = 37°$

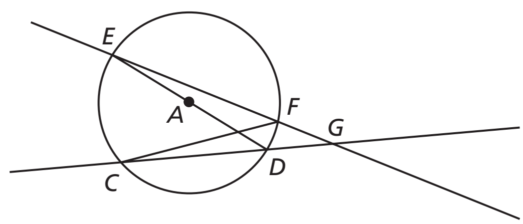
56. $m\angle DCB = \frac{1}{2}m\widehat{DB} = m\angle DAB = 67°$
$m\angle BDC = 90°$ ($\overline{BC}$ is a diam.)
$m\angle DBC = 180 - (90 + 67) = 23°$

57. $m\angle ADC = \frac{1}{2}m\widehat{AC} = \frac{1}{2}(180 - 74) = 53°$

11-6 TECHNOLOGY LAB: EXPLORE SEGMENT RELATIONSHIPS IN CIRCLES, PAGES 790–791

ACTIVITY 1, TRY THIS, PAGE 790

1.



2. $\angle CGF$ and $\angle EGD$; $\angle FCG$ and $\angle DEG$; $\triangle CFG$ and $\triangle EDG$ are ~ ⚠ by AA ~ Post.

3. $\frac{GC}{GF} = \frac{GE}{GD}$; $GC \cdot GD = GE \cdot GF$

ACTIVITY 2, TRY THIS, PAGE 791

4. The products of the segment lengths for a tangent and a secant are similar to the products of the segment lengths for 2 secants because for a tangent there is only 1 segment. Thus, the "whole segment" multiplied by the "external segment" becomes the square of the tangent seg.

5. 2 tangent segments from same external point will have = lengths, so the circle segments are ≅.

6. In diagram, the circle A is given with tangents $\overline{DC}$ and $\overline{DE}$ from D. Since 2 points determine a line, draw radii $\overline{AC}$, $\overline{AE}$, and $\overline{AD}$. $\overline{AC} \cong \overline{AE}$ because all radii are ≅. $\overline{AD} \cong \overline{AD}$ by Reflex. Prop. of ≅. $\angle ACD$ and $\angle AED$ are right ⚠ because they are each formed by a radius and a tangent intersecting at point of tangency. Thus, $\triangle ACD$ and $\triangle AED$ are right ⚠. $\triangle ACD \cong \triangle AED$ by HL. Therefore, $\overline{DC} \cong \overline{DE}$ by CPCTC.

ACTIVITY 3, TRY THIS, PAGE 791

7. $\angle DGE$ and $\angle FGC$; $\angle GDE$ and $\angle GFC$; $\angle GED$ and $\angle GCF$; $\triangle DGE$ and $\triangle FGC$ are ~ ⚠ by AA ~.

8. When 2 secants or 2 chords of a circle intersect, 4 segments will be formed, each with the point of intersection as 1 endpoint. The product of segment lengths on 1 secant/chord will equal the product of segment lengths on the other secant/chord.

If a secant and a tangent of a circle intersect on an exterior point, 3 segments will be formed, each with exterior point as an endpoint. The product of the secant segment lengths will equal square of tangent segment length.

11-6 SEGMENT RELATIONSHIPS IN CIRCLES, PAGES 792–798

CHECK IT OUT! PAGES 792–794

1. $AE \cdot EB = CE \cdot ED$
$6(5) = x(8)$
$30 = 8x$
$x = 3.75$
$AB = 6 + 5 = 11$
$CD = (3.75) + 8 = 11.75$

2. original disk: $PR = 11\frac{1}{3}$ in.
New disk:
$AQ \cdot QB = PQ \cdot QR$
$6(6) = 3(QR)$
$36 = 3QR$
$QR = 12$ in.
$PR = 12 + 3 = 15$ in.
change in $PR = 15 - 11\frac{1}{3} = 3\frac{2}{3}$ in.

3. $GH \cdot GJ = GK \cdot GL$
$13(13 + z) = 9(9 + 30)$
$169 + 13z = 81 + 270$
$13z = 182$
$z = 14$
$GJ = 13 + (14) = 27$
$GL = 9 + 30 = 39$

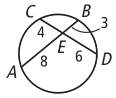
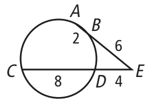
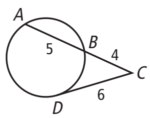
4. $DE \cdot DF = DG^2$
$7(7 + y) = 10^2$
$49 + 7y = 100$
$7y = 51$
$y = 7\frac{2}{7}$

THINK AND DISCUSS, PAGE 795

1. Yes; in this case, chords intersect at center of the circle. So segments of the chords are all radii, and theorem simplifies to $r^2 = r^2$.

2. 2

3.

	Theorem	Diagram	Example
Chord-Chord	If 2 chords intersect in the int. of a ⊙, then the products of the lengths of the segs. of the chords are =.		$3 \cdot 8 = 4 \cdot 6$
Secant-Secant	If 2 secants intersect in the ext. of a ⊙, then the product of the lengths of 1 secant seg. and its external seg. = the product of the lengths of the other secant seg. and its external seg.		$8 \cdot 6 = 12 \cdot 4$
Secant-Tangent	If a secant and a tangent intersect in the ext. of a ⊙, then the product of the lengths of the secant seg. and its ext. seg. = the length of the tangent seg. squared.		$9 \cdot 4 = 6^2$

EXERCISES, PAGES 795–798

GUIDED PRACTICE, PAGES 795–796

1. tangent segment

2. $HK \cdot KJ = LK \cdot KM$
$8(3) = 4(y)$
$24 = 4y$
$y = 6$
$LM = 4 + (6) = 10$
$HJ = 8 + 3 = 11$

3. $AE \cdot EB = CE \cdot ED$
$x(4) = 6(6)$
$4x = 36$
$x = 9$
$AB = (9) + 4 = 13$
$CD = 6 + 6 = 12$

4. $PS \cdot SQ = RS \cdot ST$
$z(10) = 6(8)$
$10z = 48$
$z = 4.8$
$PQ = 10 + (4.8) = 14.8$
$RT = 6 + 8 = 14$

5. Let the diameter be d ft.
$GF \cdot FH = EF \cdot (d - EF)$
$25(25) = 20(d - 20)$
$625 = 20d - 400$
$225 = 20d$
$d = 51\frac{1}{4}$ ft

6. $AC \cdot BC = EC \cdot DC$
$(x + 7.2)7.2 = (9 + 7.2)7.2$
$x + 7.2 = 9 + 7.2$
$x = 9$
$AC = (9) + 7.2 = 16.2$
$EC = 9 + 7.2 = 16.2$

7. $PQ \cdot PR = PS \cdot PT$
$5(5 + y) = 6(6 + 7)$
$25 + 5y = 78$
$5y = 53$
$y = 10.6$
$PR = 5 + (10.6) = 15.6$
$PT = 6 + 7 = 13$

8. $GH \cdot GJ = GK \cdot GL$
$10(10 + 10.7) = 11.5(11.5 + x)$
$207 = 132.25 + 11.5x$
$74.75 = 11.5x$
$x = 6.5$
$GJ = 10 + 10.7 = 20.7$
$GL = 11.5 + (6.5) = 18$

9. $AB \cdot AC = AD^2$
$2(2 + 6) = x^2$
$16 = x^2$
$x = 4$ (since $x > 0$)

10. $MP \cdot MQ = MN^2$
$3(3 + y) = 4^2$
$9 + 3y = 16$
$3y = 7$
$y = 2\frac{1}{3}$

11. $RT \cdot RU = RS^2$
$3(3 + 8) = x^2$
$33 = x^2$
$x = \sqrt{33}$ (since $x > 0$)

PRACTICE AND PROBLEM SOLVING, PAGES 796–797

12. $DH \cdot HE = FH \cdot HG$
$3(4) = 2(y)$
$12 = 2y$
$y = 6$
$DE = 3 + 4 = 7$
$FG = 2 + (6) = 8$

13. $JK \cdot KL = MN \cdot KN$
$10(x) = 6(7)$
$10x = 42$
$x = 4.2$
$JL = 10 + (4.2) = 14.2$
$MN = 6 + 7 = 13$

14. $UY \cdot YV = WY \cdot YZ$
$8(5) = x(11)$
$40 = 11x$
$x = 3\frac{7}{11}$
$UV = 8 + 5 = 13$
$WZ = 3\frac{7}{11} + 11 = 14\frac{7}{11}$

15. $590(590) = 225.4(d - 225.4)$
$348{,}100 = 225.4d - 50{,}805.16$
$398{,}905.16 = 225.4d$
$d \approx 1770$ ft

16. $AB \cdot AC = AD \cdot AE$
$5(5 + x) = 6(6 + 6)$
$25 + 5x = 72$
$5x = 47$
$x = 9.4$
$AC = 5 + (9.4) = 14.4$
$AE = 6 + 6 = 12$

17. $HL \cdot JL = NL \cdot ML$
$(y + 10)10 = (18 + 9)9$
$10y + 100 = 243$
$10y = 143$
$y = 14.3$
$HL = (14.3) + 10 = 24.3$
$NL = 18 + 9 = 27$

18. $PQ \cdot PR = PS \cdot PT$
$3(3 + 6) = 2(2 + x)$
$27 = 4 + 2x$
$23 = 2x$
$x = 11.5$
$PR = 3 + 6 = 9$
$PT = 2 + (11.5) = 13.5$

19. $UW \cdot UX = UV^2$
$6(6 + 8) = z^2$
$84 = z^2$
$z = \sqrt{84} = 2\sqrt{21}$ (since $z > 0$)

20. $BC \cdot BD = BA^2$
$2(2 + x) = 5^2$
$4 + 2x = 25$
$2x = 21$
$x = 10.5$

21. $GE \cdot GF = GH^2$
$8(8 + 12) = y^2$
$160 = y^2$
$y = 4\sqrt{10}$ (since $y > 0$)

22a. $RM \cdot MS = PM \cdot MQ$
$10(MS) = 12(12) = 144$
$MS = 14.4$ cm

b. $RS = RM + MS = 10 + 14.4 = 24.4$ cm

23a. $PM \cdot MQ = RM \cdot MS$
$PM^2 = 4(13 - 4) = 36$
$PM = 6$ in.

b. $PQ = 2PM = 2(6) = 12$ in.

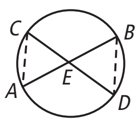
24. Step 1 Find x.
$AB \cdot AC = AD \cdot AE$
$5(5 + 5.4) = 4(4 + x)$
$52 = 16 + 4x$
$36 = 4x$
$x = 9$
Step 2 Find y.
$AB \cdot AC = AF^2$
$52 = y^2$
$y = \sqrt{52} = 2\sqrt{13}$

25. Step 1 Find x.
$NQ \cdot QM = PQ \cdot QR$
$4(x) = 3.2(10) = 32$
$x = 8$
Step 2 Find y.
$KM \cdot KN = KL^2$
$6(6 + (8) + 4) = y^2$
$108 = y^2$
$y = \sqrt{108} = 6\sqrt{3}$

26. $SP^2 = SE \cdot (SE + d)$
$= 6000(14{,}000) = 84{,}000{,}000$
$SP = \sqrt{84{,}000{,}000} = 2000\sqrt{21} \approx 9165$ mi

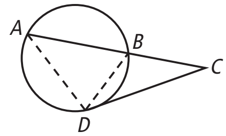
27. Solution B is incorrect. The first step should be $AC \cdot BC = DC^2$, not $AB \cdot BC = DC^2$.

28.



Since 2 points determine a line, draw $\overline{AC}$ and $\overline{BD}$. $\angle ACD \cong \angle DBA$ because they both intercept $\widehat{AD}$. $\angle CEA \cong \angle BED$ by Vert. ∡ Thm. Therefore, $\triangle ECA \sim \triangle EBD$ by AA $\sim$. Corresponding sides are proportional. So $\frac{AE}{ED} = \frac{CE}{EB}$. By Cross Products Property, $AE \cdot EB = CE \cdot ED$.

29.



Since 2 points determine a line, draw $\overline{AD}$ and $\overline{BD}$. $m\angle CAD = \frac{1}{2}m\widehat{BD}$ by Inscribed ∠ Thm. $m\angle BDC = \frac{1}{2}m\widehat{BD}$ by Thm. 11-5-1. Thus, $\angle CAD \cong \angle BDC$. Also, $\angle C \cong \angle C$ by Reflex. Prop. of $\cong$. Therefore, $\triangle CAD \sim \triangle CDB$ by AA $\sim$. Corresponding sides are proportional. So $\frac{AC}{DC} = \frac{DC}{BC}$. By Cross Products Property, $AC \cdot BC = DC^2$.

30. Yes; $PR \cdot PQ = PT \cdot PS$, and it is given that $PQ = PS$. So $PR = PT$. Subtracting the $\cong$ segments from each of these shows that $\overline{QR} \cong \overline{ST}$.

31. Method 1: By Secant-Tangent Product Theorem, $BC^2 = 12(4) = 48$, and so $BC = \sqrt{48} = 4\sqrt{3}$.
Method 2: By Thm. 11-1-1, ∠ABC is a right ∠. By Pythagorean Thm., $BC^2 + 4^2 = 8^2$. Thus $BC^2 = 64 - 16 = 48$, and $BC = 4\sqrt{3}$.

32a. $AE \cdot EC = BE \cdot DE$
$5.2(4) = 3(DE)$
$20.8 = 3DE$
$DE \approx 6.9$ cm

b. diameter $= BD = BE + DE \approx 3 + 6.9 \approx 9.9$ cm

c. $OE = OB - BE \approx \frac{1}{2}(9.933) - 3 \approx 1.97$ cm

TEST PREP, PAGE 798

33. B
$PQ^2 = PR \cdot PS = 6(6 + 8) = 84$
$PQ = \sqrt{84} \approx 9.2$

34. F
$PT \cdot PU = PQ^2$
$7(7 + UT) = 84$
$7 + UT = 12$
$UT = 5$

35. $CE = ED = 6$, and by Chord-Chord Product Thm., $3(EF) = 6(6) = 36$. So $EF = 12$, $FB = 12 + 3 = 15$, and radius $AB = \frac{1}{2}(15) = 7.5$.

CHALLENGE AND EXTEND, PAGE 798

36a. Step 1 Find the length y of the chord formed by $\overline{KM}$.
$KL^2 = 144 = 8(8 + y)$
$18 = 8 + y$
$y = 10$
Step 2 Find the value of x.
$6(6 + x) = 8(8 + (10))$
$36 + 6x = 144$
$6x = 108$
$x = 18$

b. $KL^2 + LM^2 \stackrel{?}{=} KM^2$
$12^2 + (6 + 18)^2 \stackrel{?}{=} (8 + 10 + 8)^2$
$720 > 676$
$\triangle KLM$ is acute.

37. Let R be center of the circle; then $\triangle PQR$ is a right $\angle$. So

$$PR^2 = PQ^2 + QR^2$$
$$= 6^2 + 4^2 = 52$$
$$PR = \sqrt{52} = 2\sqrt{13} \text{ in.}$$

Let S be the intersection of $\overline{PR}$ with circle R, so that $SR = 4$ in.; then the distance from P to the circle $= PS = 2\sqrt{13} - 4 \approx 3.2$ in.

38. By Chord-Chord Product Thm.,

$$(c + a)(c - a) = b \cdot b$$
$$c^2 - a^2 = b^2$$
$$c^2 = a^2 + b^2$$

39.

$$GJ \cdot HJ = FJ^2$$
$$(y + 6)y = 10^2 = 100$$
$$y^2 + 6y - 100 = 0$$
$$y = \frac{-6 \pm \sqrt{6^2 - 4(1)(-100)}}{2}$$
$$= \frac{-6 + \sqrt{436}}{2} \quad \text{(since } y > 0\text{)}$$
$$= -3 + \sqrt{109} \approx 7.44$$

SPIRAL REVIEW, PAGE 798

40.

$$P = \frac{\text{\# favorable outcomes}}{\text{\# trials}}$$
$$0.035 = \frac{14}{n}$$
$$n = \frac{14}{0.035} = 400$$

41. $P = \frac{36}{50} = 0.72 = 72\%$

42. $\overrightarrow{BA}$ and $\overrightarrow{CD}$ **43.** $\overrightarrow{CD}$ **44.** $\overline{BC}$

45. $A = \pi(12)^2\left(\frac{55}{360}\right) = 22\pi \text{ ft}^2 \approx 69.12 \text{ ft}^2$

46. $L = 2\pi(12)\left(\frac{55}{360}\right) = 3\frac{2}{3}\pi \text{ ft} \approx 11.52 \text{ ft}$

47. The area of sector YZX is $40\pi - 22\pi = 18\pi \text{ ft}^2$

$$A = \pi r^2\left(\frac{m}{360}\right)$$
$$18\pi = \pi(12)^2\left(\frac{m}{360}\right)$$
$$\frac{m}{360} = \frac{18}{144} = \frac{1}{8}$$
$$m = \frac{1}{8}(360) = 45°$$

11-7 CIRCLES IN THE COORDINATE PLANE, PAGES 799–805

CHECK IT OUT! PAGES 799–801

1a.

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 0)^2 + (y - (-3))^2 = 8^2$$
$$x^2 + (y + 3)^2 = 64$$

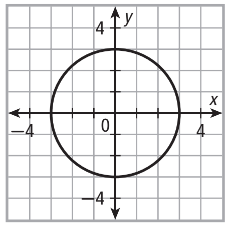
b. $r = \sqrt{(2 - 2)^2 + (3 - (-1))^2} = \sqrt{16} = 4$

$$(x - 2)^2 + (y - (-1))^2 = 4^2$$
$$(x - 2)^2 + (y + 1)^2 = 16$$

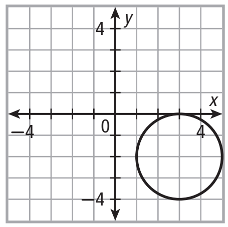
2a. Step 1 Make a table of values.
Since the radius is $\sqrt{9} = 3$, use ± 3 and values in between for x-values.

x	–3	–2	–1	0	–1	–2	–3
y	0	±2.2	±2.8	±3	±2.8	±2.2	0

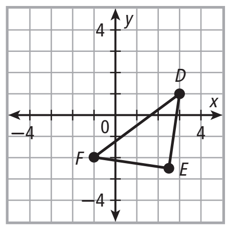
Step 2 Plot points and connect them to form a circle.



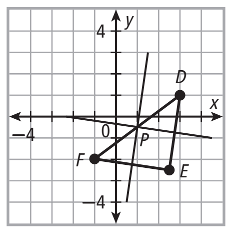
b. The equation of given circle can be written as $(x - 3)^2 + (y - (-2))^2 = 2^2$
So $h = 3$, $k = -2$, and $r = 2$. The center is (3, –2) and the radius is 2. Plot point (3, –2). Then graph a circle having this center and radius 2.



3. Step 1 Plot 3 given points.
Step 2 Connect D, E, and F to form a $\triangle$.



Step 3 Find a point that is equidistant from 3 points by constructing $\perp$ bisectors of 2 sides of $\triangle DEF$.

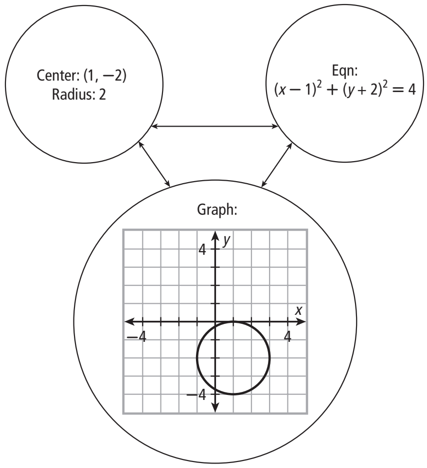


The $\perp$ bisectors of sides of $\triangle DEF$ intersect at a point that is equidistant from D, E, and F.

The intersection of the $\perp$ bisectors is $P(2, -1)$. P is the center of circle passing through D, E, and F.

THINK AND DISCUSS, PAGE 801

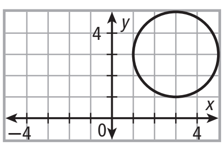
1. $x^2 + y^2 = r^2$
2. First find the center by finding the midpoint of the diameter. By the Midpoint Formula, the center of the circle is $(-1, 4)$. The radius is half the length of the diameter. So $r = 2$. The equation is $(x + 1)^2 + (y - 4)^2 = 4$.
3. No; a radius represents length, and length cannot be negative.
4.



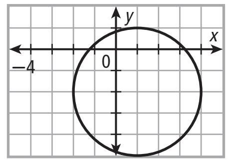
EXERCISES, PAGES 802–805

GUIDED PRACTICE, PAGE 802

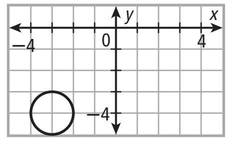
1. $(x - h)^2 + (y - k)^2 = r^2$
   $(x - 3)^2 + (y - (-5))^2 = 12^2$
   $(x - 3)^2 + (y + 5)^2 = 144$
2. $(x - h)^2 + (y - k)^2 = r^2$
   $(x - (-4))^2 + (y - 0)^2 = 7^2$
   $(x + 4)^2 + y^2 = 49$
3. $r = \sqrt{(2 - 4)^2 + (0 - 0)^2} = \sqrt{4} = 2$
   $(x - 2)^2 + (y - 0)^2 = 2^2$
   $(x - 2)^2 + y^2 = 4$
4. $r = \sqrt{(2 - (-1))^2 + (-2 - 2)^2} = \sqrt{25} = 5$
   $(x - (-1))^2 + (y - 2)^2 = 5^2$
   $(x + 1)^2 + (y - 2)^2 = 25$
5. The equation of given circle can be written as $(x - 3)^2 + (y - 3)^2 = 2^2$
   So $h = 3$, $k = 3$, and $r = 2$. The center is $(3, 3)$ and radius is 2. Plot point $(3, 3)$. Then graph a circle having this center and radius 2.



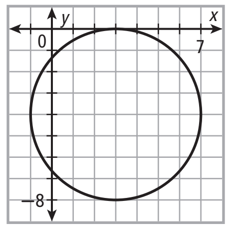
6. The equation of given circle can be written as $(x - 1)^2 + (y - (-2))^2 = 3^2$
   So $h = 1$, $k = -2$, and $r = 3$. The center is $(1, -2)$ and radius is 3. Plot point $(1, -2)$. Then graph a circle having this center and radius 3.



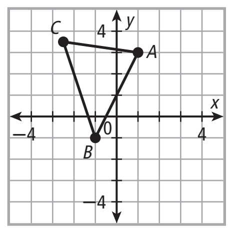
7. The equation of given circle can be written as $(x - (-3))^2 + (y - (-4))^2 = 1^2$
   So $h = -3$, $k = -4$, and $r = 1$. The center is $(-3, -4)$ and radius is 1. Plot point $(-3, -4)$. Then graph a circle having this center and radius 1.



8. The equation of given circle can be written as $(x - 3)^2 + (y - (-4))^2 = 2^2$
   So $h = 3$, $k = -4$, and $r = 2$. The center is $(3, -4)$ and radius is 4. Plot point $(3, -4)$. Then graph a circle having this center and radius 4.



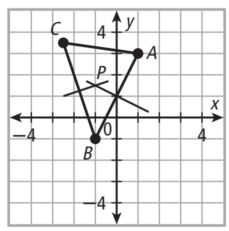
9a. **Step 1** Plot the 3 given points.
Step 2 Connect A, B, and C to form a $\triangle$.



Step 3 Find a point that is equidistant from 3 points by constructing $\perp$ bisectors of 2 sides of $\triangle ABC$.

The $\perp$ bisectors of sides of $\triangle ABC$ intersect at a point that is equidistant from A, B, and C.

The intersection of $\perp$ bisectors is $P(-2, 3)$. P is center of circle passing through A, B, and C.



b. There are approximately 10 units across the circle. So the diameter is approximately 10 ft.

PRACTICE AND PROBLEM SOLVING, PAGES 802–804

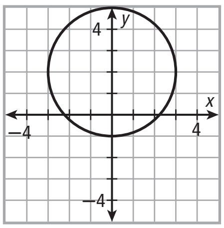
10. $(x-h)^2+(y-k)^2=r^2$
$(x-(-12))^2+(y-(-10))^2=8^2$
$(x+12)^2+(y+10)^2=64$

11. $(x-h)^2+(y-k)^2=r^2$
$(x-1.5)^2+(y-(-2.5))^2=\left(\sqrt{3}\right)^2$
$(x-1.5)^2+(y+2.5)^2=3$

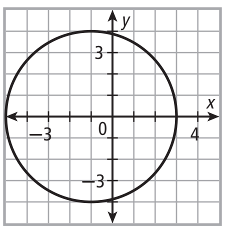
12. $r=\sqrt{(2-1)^2+(2-1)^2}=\sqrt{2}$
$(x-1)^2+(y-1)^2=\left(\sqrt{2}\right)^2$
$(x-1)^2+(y-1)^2=2$

13. $r=\sqrt{(-5-1)^2+(1-(-2))^2}=\sqrt{45}=3\sqrt{5}$
$(x-1)^2+(y-(-2))^2=\left(3\sqrt{5}\right)^2$
$(x-1)^2+(y+2)^2=45$

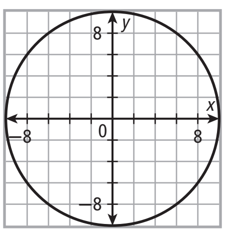
14. The equation of given circle can be written as $(x-0)^2+(y-2)^2=3^2$.
So $h=0$, $k=2$, and $r=3$. The center is (0, 2) and radius is 3. Plot point (0, 2). Then graph a circle having this center and radius 3.



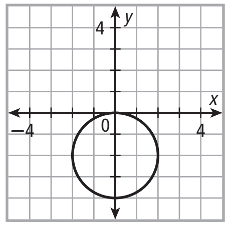
15. The equation of given circle can be written as $(x-(-1))^2+(y-0)^2=4^2$.
So $h=-1$, $k=0$, and $r=4$. The center is (−1, 0) and radius is 4. Plot point (−1, 0). Then graph a circle having this center and radius 4.



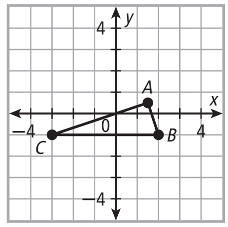
16. The equation of given circle can be written as $(x-0)^2+(y-0)^2=10^2$.
So $h=0$, $k=0$, and $r=10$. The center is (0, 0) and radius is 10. Then graph a circle having origin as center and radius 10.



17. The equation of given circle can be written as $(x-0)^2+(y-(-2))^2=2^2$.
So $h=0$, $k=-2$, and $r=2$. The center is (0, −2) and radius is 2. Plot point (0, −2). Then graph a circle having this center and radius 2.



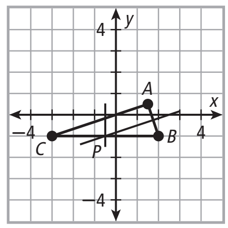
18a. Step 1 Plot the 3 given points.
Step 2 Connect *A*, *B*, and *C* to form a △.



Step 3 Find a point that is equidistant from the 3 points by constructing ⊥ bisectors of the 2 sides of △*ABC*.

The ⊥ bisectors of sides of △*ABC* intersect at a point that is equidistant from *A*, *B*, and *C*.

The intersection of ⊥ bisectors is *P*(−1, −2). *P* is center of the circle passing through *A*, *B*, and *C*.



b. There are approximately 10 units across the circle. So the diameter is approximately 10 ft.

19. The circle has the center (1, −2) and radius 2.
$(x-1)^2+(y-(-2))^2=2^2$
$(x-1)^2+(y+2)^2=4$

20. The circle has the center (−1, 1) and radius 4.
$(x-(-1))^2+(y-1)^2=4^2$
$(x+1)^2+(y-1)^2=16$

21a. $r=\sqrt{24^2+32^2}=40$
$d=2(40)=80$ units or 80 ft

b. $x^2+y^2=40^2$
$x^2+y^2=1600$

22. F; $r=\sqrt{7}$

23. T; $(-1-2)^2+(-3+3)^2=9$

24. F; center is (6, −4), in fourth quadrant.

25. T; $\left(0, 4\pm\sqrt{3}\right)$ lie on *y*-axis and ⊙.

26. F; the equation is $x^2+y^2=3^2=9$.

27a. Possible answer: 28 units^2

b. $r = 3$, so $A = \pi(3)^2 = 9\pi \approx 28.3$ units^2

c. Check students' work.

28. slope of radius from center (4, −6) to pt. (1, −10) is
$m = \frac{-10-(-6)}{1-4} = \frac{-4}{-3} = \frac{4}{3}$
tangent has slope $-\frac{3}{4}$, and eqn.
$y - (-10) = -\frac{3}{4}(x-1)$ or $y + 10 = -\frac{3}{4}(x-1)$

29a. $r = 3$
$E(-3, 5 - 2(3)) = E(-3, -1)$
$G(0 - 2(3), 2) = G(-6, 2)$

b. $d = 2(3) = 6$

c. center is $(0 - 3, 2) = (-3, 2)$
$(x - (-3))^2 + (y - 2)^2 = 3^2$
$(x + 3)^2 + (y - 2)^2 = 9$

30. $(x - 2)^2 + (x + 3)^2 = 81$
$(x - 2)^2 + (x - (-3))^2 = 9^2$
center (2, −3), radius 9

31. $x^2 + (y + 15)^2 = 25$
$(x - 0)^2 + (y - (-15))^2 = 5^2$
center (0, −15), radius 5

32. $(x + 1)^2 + y^2 = 7$
$(x - (-1))^2 + (y - 0)^2 = (\sqrt{7})^2$
center (−1, 0), radius $\sqrt{7}$

33. $r = 3$; $A = \pi(3)^2 = 9\pi$; $C = 2\pi(3) = 6\pi$

34. $r = \sqrt{7}$; $A = \pi(\sqrt{7})^2 = 9\pi$; $C = 2\pi(\sqrt{7}) = 2\sqrt{7}\pi$

35. $r = \sqrt{(2 - (-1))^2 + (-1 - 3)^2} = 5$
$A = \pi(5)^2 = 25\pi$; $C = 2\pi(5) = 10\pi$

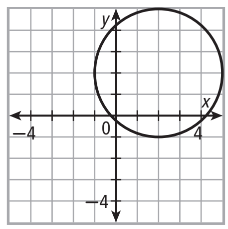
36. Graph is a single point, (0, 0).

37. The epicenter (x, y) is a solution of the equations of 3 circles.
Let 1 unit represent 100 mi.
seismograph A:
$(x + 2)^2 + (y - 2)^2 = 3^2$
$x^2 + 4x + 4 + y^2 - 4y + 4 = 9$
$x^2 + 4x + y^2 - 4y = 1$ (1)
seismograph B:
$(x - 4)^2 + (y + 1)^2 = 6^2$
$x^2 - 8x + 16 + y^2 + 2y + 1 = 36$
$x^2 - 8x + y^2 + 2y = 19$ (2)
seismograph C:
$(x - 1)^2 + (y + 5)^2 = 5^2$
$x^2 - 2x + 1 + y^2 + 10y + 25 = 25$
$x^2 - 2x + y^2 + 10y = -1$ (3)
(1) − (2):
$12x - 6y = -18$
$2x + 3 = y$ (4)
(1) − (3):
$6x - 14y = 2$
$3x = 7y + 1$ (5)
(4) in (5):
$3x = 7(2x + 3) + 1$
$3x = 14x + 22$
$-11x = 22$
$x = -2$
$y = 2(-2) + 3 = -1$
The location of epicenter is (−200, −100).

38. The circle has a radius of 5. So 5 is tangent to x-axis if the center has y-coordinate $k = \pm 5$.

39. $d = \sqrt{(5 - (-3))^2 + (-2 - (-2))^2} = 8$; $r = 4$
center $= \left(\frac{-3+5}{2}, \frac{-2+(-2)}{2}\right) = (1, -2)$
equation is $(x - 1)^2 + (y + 2)^2 = 16$

40.



The locus is a circle with a radius 3 centered at (2, 2).

41. The point does not lie on circle P because it is not a solution to the equation $(x - 2)^2 + (y - 1)^2 = 9$. Since $(3 - 2)^2 + ((-1) - 1)^2 < 9$, the point lies in the interior of circle P.

TEST PREP, PAGE 804

42. C
(−2, 0) lies on the circle.

43. H
$(x - (-3))^2 + (y - 5)^2 = (1 - (-3))^2 + (5 - 5)^2$
$(x + 3)^2 + (y - 5)^2 = 16$

44. A
distances from statues to fountain:
$\sqrt{(4 - (-1))^2 + (-2 - (-2))^2} = 5$
$\sqrt{(-1 - (-1))^2 + (3 - (-2))^2} = 5$
$\sqrt{(-5 - (-1))^2 + (-5 - (-2))^2} = 5$

CHALLENGE AND EXTEND, PAGE 805

45a. $r = \sqrt{(1-2)^2 + (-2-(-4))^2 + (-5-3)^2} = \sqrt{69}$

$(x-2)^2 + (y-(-4))^2 + (z-3)^2 = \left(\sqrt{69}\right)^2$

$(x-2)^2 + (y+4)^2 + (z-3)^2 = 69$

b. 15; if 2 segments are tangent to a circle or sphere from same exterior point, then segments are ≅.

46. $x + y = 5$

$y = 5 - x$

Substitute in equation of a circle:

$x^2 + (5-x)^2 = 25$

$x^2 + 25 - 10x + x^2 = 25$

$2x^2 - 10x = 0$

$2x(x-5) = 0$

$x = 0$ or 5

The point of intersection are $(0, 5-(0)) = (0, 5)$ and $(5, 5-(5)) = (5, 0)$.

47. Given the line is ⊥ to a line through (3, 4) with slope −0.5. For point of tangency,

$y = 2x + 3$ (1)

and

$y - 4 = -0.5(x-3)$

$2(y-4) = 3 - x$

$2y - 8 = 3 - x$

$x = 11 - 2y$ (2)

(2) in (1):

$y = 2(11 - 2y) + 3$

$y = 25 - 4y$

$5y = 25$

$y = 5$

$x = 11 - 2(5) = 1$

Point of tangency is (1, 5). So $r^2 = (1-3)^2 + (5-4)^2 = 5$.

The equation of the circle is

$(x-3)^2 + (y-4)^2 = 5$

SPIRAL REVIEW, PAGE 805

48. $\dfrac{2x^2 - 2(4x^2+1)}{2}$

$x^2 - (4x^2 + 1)$

$x^2 - 4x^2 - 1$

$-3x^2 - 1$

49. $\dfrac{18a + 4(9a+3)}{6}$

$3a + \frac{2}{3}(9a + 3)$

$3a + 6a + 2$

$9a + 2$

50. $3(x + 3y) - 4(3x + 2y) - (x - 2y)$

$3x + 9y - 12x - 8y - x + 2y$

$-10x + 3y$

51. By the Isosc. △ Thm.,

$m\angle D = m\angle F$

$7x + 4 = 60$

$7x = 56$

$x = 8$

52. $DE = EF$

$2y + 10 = 4y - 1$

$11 = 2y$

$y = 5.5$

53. $180 - 142 = 38 = \frac{1}{2}(m\widehat{LK} + m\widehat{JQ})$

$m\widehat{LKQ} = m\widehat{LK} + m\widehat{KJ} + m\widehat{JQ}$

$= 2(38) + 88 = 164°$

$m\widehat{LNQ} = 360 - 164 = 196°$

54. $m\angle NMP = \frac{1}{2}(m\widehat{LKQ} - m\widehat{NP})$

$= \frac{1}{2}(164 - 50) = 57°$

11B MULTI-STEP TEST PREP, PAGE 806

1. $m\angle AGB = \frac{1}{2}m\widehat{AB} = \frac{1}{2}\left(\frac{1}{12}(360)\right) = 15°$

2. $m\angle KAE = 90°$; the angle is inscribed in a semicircle. So it is a right ∠.

3. $m\angle KMJ = \frac{1}{2}(m\widehat{KJ} + m\widehat{BE}) = \frac{1}{2}(30 + 90) = 60°$

4. $ME = 22 - 4.8$

$ME \approx 17.2$

$KM \cdot ME = JM \cdot MB$

$4.8 \cdot 17.2 = 6.4x$

$82.56 = 6.4x$

$12.9 \approx x$

$MB \approx 12.9$ ft

5. $(x-20)^2 + (y-14)^2 = 11^2 = 121$

6. $L(20, 14+11) = L(20, 25)$

$C(20+11, 14) = C(31, 14)$

$F(20, 14-11) = F(20, 3)$

$I(20-11, 14) = I(9, 14)$

11B READY TO GO ON? PAGE 807

1. $m\angle BAC = \frac{1}{2}m\widehat{BC} = \frac{1}{2}(102) = 51°$

2. $m\widehat{CD} = 2m\angle CAD = 2(38) = 76°$

3. $\angle FGH$ is inscribed in a semicircle. So $m\angle FGH = 90°$.

4. $m\widehat{JGF} = 310°$

5. $m\angle RST = \frac{1}{2}mST = \frac{1}{2}(266) = 133°$

6. $m\angle AEC = \frac{1}{2}(m\widehat{AC} + m\widehat{BD}) = \frac{1}{2}(130 + 22) = 76°$

7. $m\angle MPN = \frac{1}{2}((360 - 102) - 102) = 78°$

8. $AE \cdot EB = CE \cdot ED$

$2(6) = x(3)$

$12 = 3x$

$x = 4$

$AB = 2 + 6 = 8$

$CD = (4) + 3 = 7$

9. $FH \cdot GH = KH \cdot JH$

$(y+3)3 = (3+4)4$

$3y + 9 = 28$

$3y = 19$

$y = 6\frac{1}{3}$

$FH = \left(6\frac{1}{3}\right) + 3 = 9\frac{1}{3}$

$KH = 3 + 4 = 7$

10. $RU \cdot (d - UR) = SU \cdot UT$

$3.9(d - 3.9) = 6.1(6.1)$

$3.9d - 15.21 = 37.21$

$3.9d = 52.42$

$d \approx 13.44$ m

11. $(x-(-2))^2 + (y-(-3))^2 = 3^2$

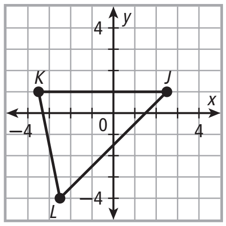
$(x+2)^2 + (y+3)^2 = 9$

12. $r = \sqrt{(1-4)^2 + (1-5)^2} = 5$

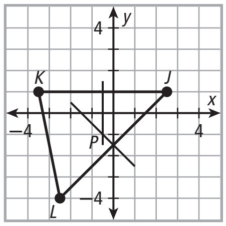
$(x-4)^2 + (y-5)^2 = 5^2$

$(x-4)^2 + (y-5)^2 = 25$

13. Step 1 Plot the 3 given points.
Step 2 Connect J, K, and L to form a $\triangle$.



Step 3 Find a point that is equidistant from the 3 points by constructing $\perp$ bisectors of 2 sides of $\triangle JKL$.



The $\perp$ bisectors of the sides of $\triangle JKL$ intersect at a point that is equidistant from A, B, and C.

The intersection of the $\perp$ bisectors is $P(-1, -2)$. P is the center of the circle passing through J, K, and L.

STUDY GUIDE: REVIEW, PAGES 810–813

VOCABULARY, PAGE 810

1. segment of a circle
2. central angle
3. major arc
4. concentric circles

LESSON 11-1, PAGE 810

5. chords: $\overline{QS}$, $\overline{UV}$; tangent: ℓ; radii: $\overline{PQ}$, $\overline{PS}$; secant: $\overleftrightarrow{UV}$; diameter: $\overline{QS}$

6. chords: $\overline{KH}$, $\overline{MN}$; tangent: $\overleftrightarrow{KL}$; radii: $\overline{JH}$, $\overline{JK}$, $\overline{JM}$, $\overline{JN}$; secant: $\overleftrightarrow{MN}$; diameters: $\overline{KH}$, $\overline{MN}$

7. $AB = BC$
$9x - 2 = 7x + 4$
$2x = 6$
$x = 3$
$AB = 9(3) - 2 = 25$

8. $EF = EG$
$5y + 32 = 8 - y$
$6y = -24$
$y = -4$
$EG = 8 - (-4) = 12$

9. $JK = JL$
$8m - 5 = 2m + 4$
$6m = 9$
$m = 1.5$
$JK = 8(1.5) - 5 = 7$

10. $WX = WY$
$0.8x + 1.2 = 2.4x$
$1.2 = 1.6x$
$x = 0.75$
$WY = 2.4(0.75) = 1.8$

LESSON 11-2, PAGE 811

11. $m\widehat{KM} = m\angle KGL + m\angle LGM$
$= m\angle KGL + m\angle HGJ$
$= 30 + 51 = 81°$

12. $m\widehat{HMK} = m\angle HGL + m\angle LGK$
$= 180 + 30 = 210°$

13. $m\widehat{JK} = m\angle JGK$
$= m\angle HGL - m\angle HGJ - m\angle KGL$
$= 180 - 51 - 30 = 99°$

14. $m\widehat{MJK} = 360 - m\widehat{KM}$
$= 360 - 81 = 279°$

15. Let $ST = 2x$.
$x(x) = 4(7 + 11)$
$x^2 = 72$
$x = \sqrt{72} = 6\sqrt{2}$
$ST = 2\left(6\sqrt{2}\right) \approx 17.0$

16. Let $CD = 2x$.
$x^2 = 2.5(2.5 + 5)$
$x(x) = 18.75$
$x = \sqrt{18.75} = 2.5\sqrt{3}$
$CD = 2\left(2.5\sqrt{3}\right) \approx 8.7$

LESSON 11-3, PAGE 811

17. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(12)^2\left(\frac{30}{360}\right) = 12\pi \text{ in.}^2 \approx 37.70 \text{ in.}^2$

18. $A = \pi r^2\left(\frac{m}{360}\right) = \pi(1)^2\left(\frac{90}{360}\right) = \frac{1}{4}\pi \text{ m}^2 = 0.79 \text{ m}^2$

19. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(18)\left(\frac{160}{360}\right) = 16\pi \text{ cm} \approx 50.27 \text{ cm}$

20. $L = 2\pi r\left(\frac{m}{360}\right) = 2\pi(2)\left(\frac{270}{360}\right) = 3\pi \text{ ft} \approx 9.42 \text{ ft}$

LESSON 11-4, PAGE 812

21. $m\widehat{JL} = 2m\angle JNL = 2(82) = 164°$

22. $m\angle MKL = \frac{1}{2}m\widehat{ML} = \frac{1}{2}(64) = 32°$

23. $\angle B$ is inscribed in a semicircle is and therefore, a right $\angle$.
$3x + 12 = 90$
$3x = 78$
$x = 26$

24. $m\angle RSP = \frac{1}{2}m\widehat{RP} = m\angle RQP$
$3y + 3 = 5y - 21$
$24 = 2y$
$y = 12$
$m\angle RSP = 3(12) + 3 = 39°$

LESSON 11-5, PAGE 812

25. $m\widehat{MR} = 2m\angle PMR = 2(41) = 82°$

26. $m\angle QMR = \frac{1}{2}m\widehat{QR} = \frac{1}{2}(360 - 120 - 82) = 79°$

27. $m\angle GKH = \frac{1}{2}(m\widehat{FJ} + m\widehat{GH}) = \frac{1}{2}(41 + 93) = 67°$

28. $m\angle BXC = \frac{1}{2}(m\widehat{AD} + m\widehat{BC})$
$= \frac{1}{2}\left(\frac{2}{16}(360) + \frac{6}{16}(360)\right) = 90°$

LESSON 11-6, PAGE 813

29. $BA \cdot AC = DA \cdot AE$
$3(y) = 7(5)$
$3y = 35$
$y = 11\frac{2}{3}$
$BC = 3 + \left(11\frac{2}{3}\right) = 14\frac{2}{3}$
$DE = 7 + 5 = 12$

30. $QP \cdot PR = SP \cdot PT$
$z(10) = 15(8)$
$10z = 120$
$z = 12$
$QR = (12) + 10 = 22$
$ST = 15 + 8 = 23$

31. $GJ \cdot HJ = LJ \cdot KJ$
$(4 + 6)6 = (x + 5)5$
$60 = 5x + 25$
$35 = 5x$
$x = 7$
$GJ = 4 + 6 = 10$
$LJ = (7) + 5 = 12$

32. $AB \cdot AC = AD \cdot AE$
$4(4 + y) = 5(5 + 5)$
$16 + 4y = 50$
$4y = 34$
$y = 8\frac{1}{2}$
$AC = 4 + \left(8\frac{1}{2}\right) = 12\frac{1}{2}$
$AE = 5 + 5 = 10$

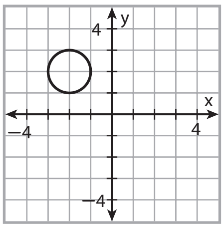
LESSON 11-7, PAGE 813

33. $(x - (-4))^2 + (y - (-3))^2 = 3^2$
$(x + 4)^2 + (y + 3)^2 = 9$

34. $r = \sqrt{(-2 - (-2))^2 + (-2 - 0)^2} = 2$
$(x - (-2))^2 + (y - 0)^2 = 2^2$
$(x + 2)^2 + y^2 = 4$

35. $(x - 1)^2 + (y - (-1))^2 = 4^2$
$(x - 1)^2 + (y + 1)^2 = 16$

36. $(x + 2)^2 + (y - 2)^2 = 1$
$(x - (-2))^2 + (y - 2)^2 = 1^2$
Graph a circle with center $(-2, 2)$ and radius 1.



CHAPTER TEST, PAGE 814

1. chord: $\overline{EC}$; tangent: $\overleftrightarrow{AB}$; radii: $\overline{DE}$, $\overline{DC}$; secant: $\overleftrightarrow{EC}$; diameter: $\overline{EC}$

2. Let x be distance to the horizon.
$(4000)^2 + x^2 = (4006.25)^2$
$x^2 = 50{,}039.0625$
$x \approx 224$ mi

3. $m\overset{\frown}{JK} = m\angle JPK$
$= m\angle JPL - m\angle KPL$
$= m\angle MPN - m\angle KPL$
$= 84 - 65 = 19°$

4. Let $UV = 2x$.
$x(x) = 6(9 + 15)$
$x^2 = 144$
$x = 12$ (since $x > 0$)
$UV = 2(12) = 24$

5. $A = \pi(8)^2\left(\frac{135}{360}\right) = 24\pi \text{ cm}^2 \approx 75.40 \text{ cm}^2$

6. $A = 2\pi(8)\left(\frac{135}{360}\right) = 6\pi \text{ cm} \approx 18.85 \text{ cm}$

7. $m\overset{\frown}{SR} = 2m\angle SPR = 2(47) = 94°$

8. $m\angle PTQ = \frac{1}{2}(m\overset{\frown}{PQ} + m\overset{\frown}{SR}) = \frac{1}{2}(58 + 94) = 76°$
$m\angle QTR = 180 - m\angle PTQ = 180 - 76 = 104°$

9. $m\angle ABC = 180 - \frac{1}{2}m\overset{\frown}{AB} = 180 - \frac{1}{2}(128) = 116°$

10. $m\angle MKL = \frac{1}{2}(m\overset{\frown}{JN} + m\overset{\frown}{ML}) = \frac{1}{2}(118 + 58) = 88°$
$m\angle MKL = 180 - m\angle MKL = 180 - 88 = 92°$

11. $m\angle CSD = \frac{1}{2}(m\overset{\frown}{CD} - m\overset{\frown}{AB})$
$42 = \frac{1}{2}(124 - m\overset{\frown}{AB})$
$84 = 124 - m\overset{\frown}{AB}$
$m\overset{\frown}{AB} = 124 - 84 = 40°$

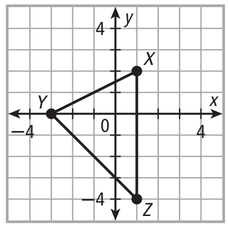
12. $z(2) = 6(4)$
$2z = 24$
$z = 12$
$EF = (12) + 2 = 14$
$GH = 4 + 6 = 10$

13. $6(6 + x) = 8(8 + 4)$
$36 + 6x = 96$
$6x = 60$
$x = 10$
$PR = 8 + 4 = 12$
$PT = 6 + (10) = 16$

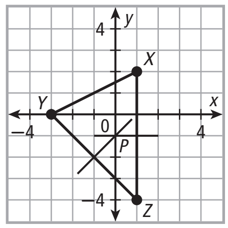
14. $4(4) = 2(d - 2)$
$16 = 2d - 4$
$20 = 2d$
$d = 10$ in.

15. $r = \sqrt{(-2 - 1)^2 + (4 - (-2))^2} = \sqrt{45} = 3\sqrt{5}$
$(x - 1)^2 + (y - (-2))^2 = (3\sqrt{5})^2$
$(x - 1)^2 + (y + 2)^2 = 45$

16. **Step 1** Plot the 3 given points.
Step 2 Connect X, Y, and Z to form a △.



Step 3 Find a point that is equidistant from 3 points by constructing ⊥ bisectors of the 2 sides of △XYZ.
The ⊥ bisectors of sides of △XYZ intersect at a point that is equidistant from X, Y, and Z.
The intersection of the ⊥ bisectors is $P(0, -2)$. P is center of the circle passing through X, Y, and Z.



COLLEGE ENTRANCE EXAM PRACTICE, PAGE 815

1. E
Draw $\overline{AD}$. $m\angle ADC$ is inscribed in a semicircle. So is a right ∠.
$m\overset{\frown}{AB} = 2m\angle ADB$
$= 2(90 - m\angle BDC)$
$= 2(90 - 30) = 120°$

2. A
Possible coordinates of the center are $(1 \pm 2, 3 \pm 2)$. $(-1, 1)$ is a possible center. The circle with this center and radius 2 is
$(x - (-1))^2 + (y - 1)^2 = 2^2$
$(x + 1)^2 + (y - 1)^2 = 4.$

3. E

$PK \cdot KM = LK \cdot KN$

$3(PM - 3) = 6(10 - 6)$

$3PM - 9 = 24$

$3PM = 33$

$PM = 11$

4. C

$L = 2\pi r\left(\frac{m\widehat{AC}}{360}\right)$

$= 2\pi(6)\left(\frac{2m\angle ABC}{360}\right)$

$= 12\pi\left(\frac{50}{360}\right) = \frac{5}{3}\pi$

5. B

Draw $\triangle$ with a base along the shaded area and the apex at the center of the circle. $\triangle$ is a 45°-45°-90° $\triangle$ with $b = 9\sqrt{2}$ and $h = 4.5\sqrt{2}$.

$A = \pi r^2\left(\frac{90}{360}\right) - \frac{1}{2}bh$

$= \pi(9)^2\left(\frac{1}{4}\right) - \frac{1}{2}\left(9\sqrt{2}\right)\left(4.5\sqrt{2}\right)$

$= \frac{81}{4}\pi - \frac{81}{2} \approx 23.12$

CHAPTERS 1–11, PAGES 818–819

1. C

$L = Ph + \frac{1}{2}P\ell$

$328 = 40(4) + \frac{1}{2}(40)\ell$

$328 = 160 + 20\ell$

$168 = 20\ell$

$\ell = 8.4$ ft

2. G

$A = (12 - 2)(3 - 0) - \frac{1}{2}(12 - 6)(3 - 0)$

$= 10(3) - \frac{1}{2}(6)(3)$

$= 30 - 9 = 21$ units^2

3. B

$m\angle EFD = m\widehat{ED} = 45°$

$m\angle FED = 180 - (45 + 90) = 45°$

Since F is the center, $\triangle EFC$ and $\triangle CFA$ are also 45°-45°-90° $\triangle$s. $\overline{FB} \perp \overline{BC}$. So $m\widehat{BC} = m\angle BFC = 45°$.

4. G

$L = 2\pi r\left(\frac{m\widehat{ED}}{360}\right)$

$6\pi = 2\pi r\left(\frac{45}{360}\right) = \frac{1}{4}\pi r$

$r = 24$ cm

$A = \pi(24)^2\left(\frac{45}{360}\right) = 72\pi$ cm^2

5. C

F is not on the circle. So $\overline{AF}$ is not a chord.

6. J

$KL = 18$ and $KJ = 24 \rightarrow \tan J = \frac{18}{24} = \frac{3}{4}$;

$KL^2 + KJ^2 \stackrel{?}{=} JL^2$

$18^2 + 24^2 \stackrel{?}{=} 30^2$

$324 + 576 = 900$

7. B

$m\angle P = \frac{1}{2}(m\widehat{RS} - m\widehat{QU})$

$= m\angle T - \frac{1}{2}m\widehat{QU}$

$29 = 50 - \frac{1}{2}m\widehat{QU}$

$\frac{1}{2}m\widehat{QU} = 21$

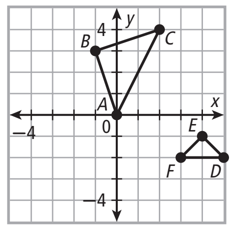
$m\widehat{QU} = 42°$

8. F

$PS \cdot PU = PR \cdot PQ$

$PS = \frac{PR \cdot PQ}{PU}$

9. C



From the diagram, $y = -3$.

10. G

$d = \sqrt{(3 - (-1))^2 + (-5 - 1)^2} = \sqrt{52} = 2\sqrt{13}$

$r = \sqrt{13}$

center $= \left(\frac{-1 + 3}{2}, \frac{1 + (-5)}{2}\right) = (1, -2)$

$(x - 1)^2 + (y - (-2))^2 = \left(\sqrt{13}\right)^2$

$(x - 1)^2 + (y + 2)^2 = 13$

11. D

The diagonals of a kite intersect at right $\angle$s. So the shortest segment to Q is from the intersection T; $\overline{TQ}$ is shortest segment.

12. H

$P = \frac{1}{2}(3b) = 3\left(\frac{1}{2}b\right)$

$A = \frac{1}{2}\left(\frac{1}{2}b\right)\left(\frac{1}{4}b\sqrt{3}\right) = \frac{1}{16}b^2\sqrt{3} = \frac{1}{4}\left(\frac{1}{2}b\left(\frac{1}{2}b\sqrt{3}\right)\right)$

The area is reduced by $\frac{1}{4}$.

13. C

Let the leg length be x m.

$A = \frac{1}{2}(x)(x)$

$36 = \frac{1}{2}x^2$

$72 = x^2$

$x = 6\sqrt{2}$

hypotenuse $= \left(6\sqrt{2}\right)\sqrt{2} = 12$ m

14. Let the side lengths be $4x$, $5x$, and $8x$ cm.

$P = 4x + 5x + 8x$

$38.25 = 17x$

$x = 2.25$

$4x = 4(2.25) = 9$ cm

15. 8

$x^2 = 4(16) = 64$

$x = 8$

16. If $\triangle HGJ \cong \triangle LMK$, $\overline{HG} \cong \overline{LM}$ and $\overline{HJ} \cong \overline{LK}$.

$\overline{HG} \cong \overline{LM}$
$HG = LM$
$4x + 5 = 13$
$4x = 8$
$x = 2$
Check: $HJ = 5(2) - 1 = 9 = KL$

17. $s = 2$, so $a = \sqrt{3}$ and $P = 6(2) = 12$
$A = \frac{1}{2}aP = \frac{1}{2}(\sqrt{3})(12) = 6\sqrt{3} \approx 10.4$

18. 15.71 mm
$L = \frac{1}{2}(2\pi(5)) = 5\pi \approx 15.71$ mm

19. 452.39 cm^2
$V = \frac{4}{3}\pi r^3$
$288\pi = \frac{4}{3}\pi r^3$
$216 = r^3$
$r = 6$ cm
$S = 4\pi(6)^2 = 144\pi \approx 452.39\ cm^2$

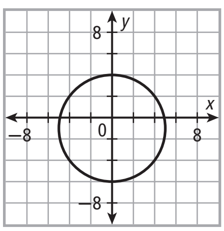
20. 3
$x = 6\cos 60° = 3$

21. Possible answer:
$180 - 7x = \frac{1}{2}(4x + 10 + 6x + 14)$
$180 - 7x = \frac{1}{2}(10x + 24)$
$180 - 7x = 5x + 12$
$-12x = -168$
$x = 14$

22a. 1st unit:
$V = 10(5)(9) = 450\ ft^3$
$\text{cost}/ft^3 = \frac{85}{450} \approx \$0.19$
2nd unit:
$V = 11(4)(8) = 352\ ft^3$
$\text{cost}/ft^3 = \frac{70}{352} \approx \$0.20$
The first unit has the lower price per ft^3.

b. $V \cdot \$0.25/ft^3 = \$100$
$V = 400$
Possible dimensions: 10 ft by 5 ft by 8 ft

23a. $x^2 + (y + 1)^2 = 25$
$(x - 0)^2 + (y - (-1))^2 = 5^2$
Plot the circle with center (0, −1) and radius 5.



b. slope of radius to (3, 3) $= \frac{3 - (-1)}{3 - 0} = \frac{4}{3}$
slope of tangent $= -\frac{3}{4}$
equation of tangent:
$y - 3 = -\frac{3}{4}(x - 3)$
$y - 3 = -\frac{3}{4}x + 2\frac{1}{4}$
$y = -\frac{3}{4}x + 5\frac{1}{4}$

24. The measure of the intercepted arc must be $> 0°$ and $< 180°$.

25a.

Statements	Reasons
1. $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle BCD \cong \angle ABD$	2. Alt. Int. ∠s Thm.
3. $m\angle BCD = m\angle ABD$	3. Def. of ≅ ∠s
4. $m\angle ACD = \frac{1}{2}m\widehat{AD}$, $m\angle BDC = \frac{1}{2}m\widehat{BC}$	4. Inscribed ∠ Thm.
5. $\frac{1}{2}m\widehat{BC} = \frac{1}{2}m\widehat{AD}$	5. Subst. Prop.
6. $m\widehat{BC} = m\widehat{AD}$	6. Mult. Prop. of =

b. Possible answer: Since $\overline{AB} \parallel \overline{CD}$, $m\widehat{BC} = m\widehat{AD}$ from part **a**. Also, by Inscribed ∠ Thm., $m\angle ACD = \frac{1}{2}m\widehat{AD}$ and $m\angle BDC = \frac{1}{2}m\widehat{BC}$. Then use Arc Add. Post. to get $m\angle ADC = \frac{1}{2}(m\widehat{AB} + m\widehat{BC})$ and $m\angle BCD = \frac{1}{2}(m\widehat{AB} + m\widehat{AD})$. Subst. $m\widehat{BC}$ for $m\widehat{AD}$, which gives $m\angle BCD = \frac{1}{2}(m\widehat{AB} + m\widehat{BC}) = m\angle ADC$. Since $ABCD$ has 1 pair of base ∠s, $ABCD$ is isosceles.

c. Possible answer: Since $\overline{AB} \parallel \overline{CD}$ and $ABCD$ is not a trapezoid, $\overline{AD} \parallel \overline{BC}$. So $ABCD$ must be a quadrilateral. Therefore, $\angle ADC \cong \angle ABC$. So by def. of ≅,
$m\angle ADC = m\angle ABC$. Also, since $ABCD$ can be inscribed in a circle, $\angle ADC$ and $\angle ABC$ are supplementary. So $m\angle ADC + m\angle ABC = 180°$.
Subst. and solve for $m\angle ABC$:
$m\angle ABC + m\angle ABC = 180$
$2m\angle ABC = 180$
$m\angle ABC = 90°$
Therefore, $\angle ABC$ is a right ∠. Since $ABCD$ is a quadrilateral, it follows that every ∠ is a rt. ∠, so $ABCD$ is a rectangle Therefore, if $ABCD$ is not a trapezoid, it must be a rectangle.