

A diagram showing a hand holding a string attached to a small object, moving in a circular path indicated by a dashed green line.

# THE BIG IDEA

Centripetal force keeps an object in circular motion.

Which moves faster on a merry-go-round, a horse near the outside rail or one near the inside rail? While a hamster rotates its cage about an axis, does the hamster rotate or does it revolve about the same axis? We begin to answer these questions by discussing the difference between rotation and revolution.



## 10.1 Rotation and Revolution

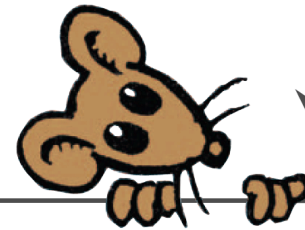


**Two types of circular motion are rotation and revolution.**

## 10.1 Rotation and Revolution

An **axis** is the straight line around which rotation takes place.

- When an object turns about an *internal* axis—that is, an axis located within the body of the object—the motion is called **rotation**, or *spin*.
- When an object turns about an *external* axis, the motion is called **revolution**.



You rotate about an internal axis when you spin. You revolve around an external axis when you circle about that axis.



## 10.1 Rotation and Revolution

The Ferris wheel turns about an axis.

The Ferris wheel rotates, while the riders revolve about its axis.



## 10.1 Rotation and Revolution

Earth undergoes both types of rotational motion.

- It revolves around the sun once every  $365 \frac{1}{4}$  days.
- It rotates around an axis passing through its geographical poles once every 24 hours.

## 10.1 Rotation and Revolution

**CONCEPT  
CHECK**

What are two types of circular motion?

## 10.2 Rotational Speed

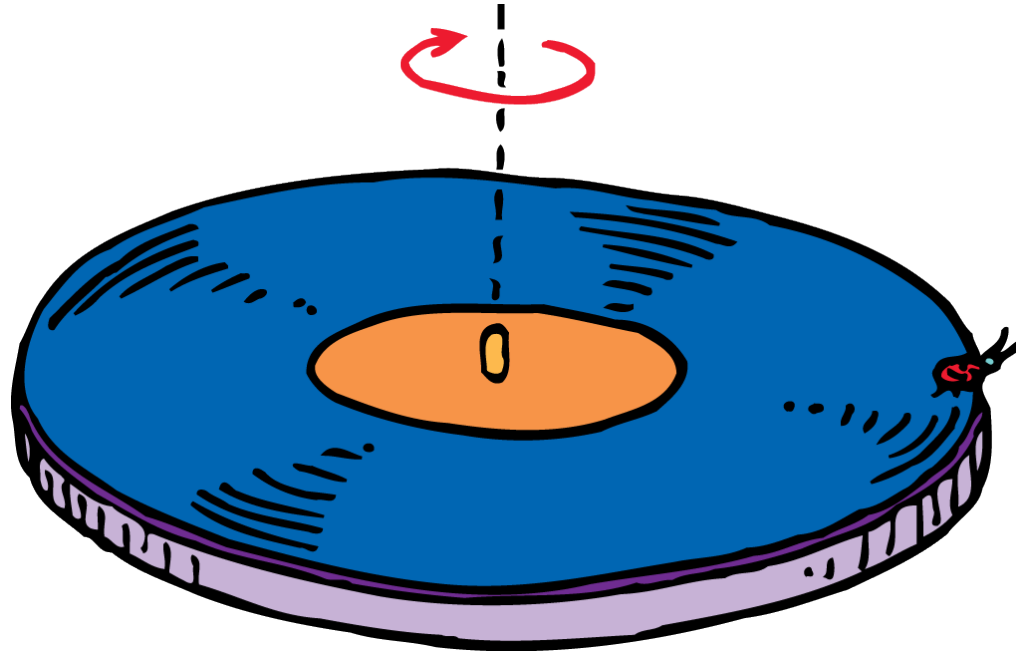


**Tangential speed depends on rotational speed and the distance from the axis of rotation.**



## 10.2 Rotational Speed

The turntable *rotates* around its axis while a ladybug sitting at its edge *revolves* around the same axis.



## 10.2 Rotational Speed

Which part of the turntable moves faster—the outer part where the ladybug sits or a part near the orange center?

It depends on whether you are talking about linear speed or rotational speed.

## 10.2 Rotational Speed

### Types of Speed

**Linear speed** is the distance traveled per unit of time.

- A point on the outer edge of the turntable travels a greater distance in one rotation than a point near the center.
- The linear speed is greater on the outer edge of a rotating object than it is closer to the axis.
- The speed of something moving along a circular path can be called **tangential speed** because the direction of motion is always tangent to the circle.

## 10.2 Rotational Speed

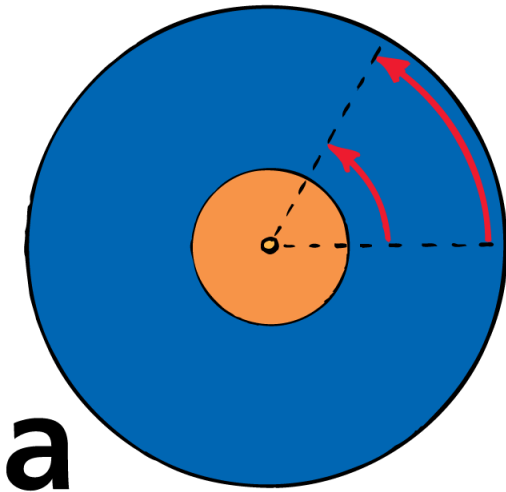
**Rotational speed** (sometimes called angular speed) is the number of rotations per unit of time.

- All parts of the rigid turntable rotate about the axis *in the same amount of time*.
- All parts have the same rate of rotation, or the same *number of rotations per unit of time*. It is common to express rotational speed in revolutions per minute (RPM).

## 10.2 Rotational Speed

All parts of the turntable rotate at the same rotational speed.

- a. A point farther away from the center travels a longer path in the same time and therefore has a greater tangential speed.

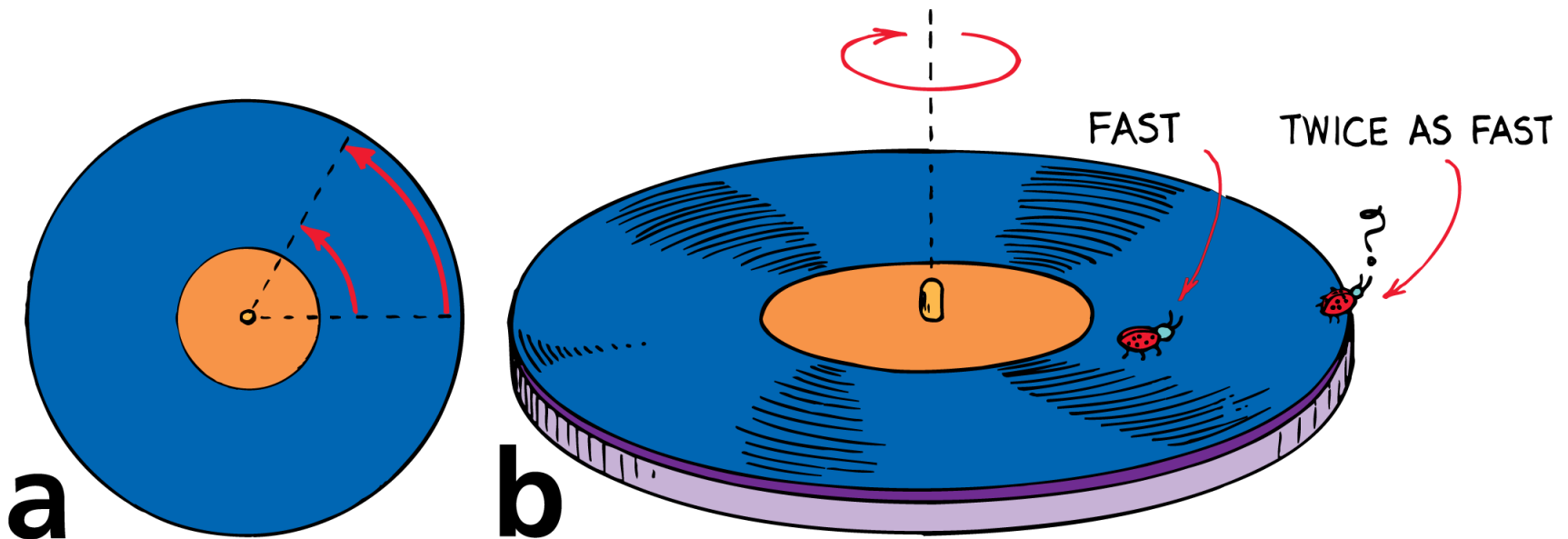


**a**

## 10.2 Rotational Speed

All parts of the turntable rotate at the same rotational speed.

- A point farther away from the center travels a longer path in the same time and therefore has a greater tangential speed.
- A ladybug sitting twice as far from the center moves twice as fast.





## 10.2 Rotational Speed

### Tangential and Rotational Speed

Tangential speed and rotational speed are related. Tangential speed is directly proportional to the rotational speed and the radial distance from the axis of rotation.

Tangential speed  $\sim$  radial distance  $\times$  rotational speed

We use the symbol  
 $\sim$  to mean directly  
proportional.



## 10.2 Rotational Speed

In symbol form,

$$v \sim r\omega$$

where  $v$  is tangential speed and  $\omega$  (pronounced oh MAY guh) is rotational speed.

You move faster if the rate of rotation increases (bigger  $\omega$ ).

- You also move faster if you are farther from the axis (bigger  $r$ ).

## 10.2 Rotational Speed

At the axis of the rotating platform, you have no tangential speed, but you do have rotational speed. You rotate in one place.

As you move away from the center, your tangential speed increases while your rotational speed stays the same.

Move out twice as far from the center, and you have twice the tangential speed.

## 10.2 Rotational Speed

### think!

At an amusement park, you and a friend sit on a large rotating disk. You sit at the edge and have a rotational speed of 4 RPM and a linear speed of 6 m/s. Your friend sits halfway to the center. What is her rotational speed? What is her linear speed?

## 10.2 Rotational Speed

### think!

At an amusement park, you and a friend sit on a large rotating disk. You sit at the edge and have a rotational speed of 4 RPM and a linear speed of 6 m/s. Your friend sits halfway to the center. What is her rotational speed? What is her linear speed?

### Answer:

Her rotational speed is also 4 RPM, and her linear speed is 3 m/s.

## 10.2 Rotational Speed

### Railroad Train Wheels

How do the wheels of a train stay on the tracks?

The train wheels stay on the tracks because their rims are slightly tapered.

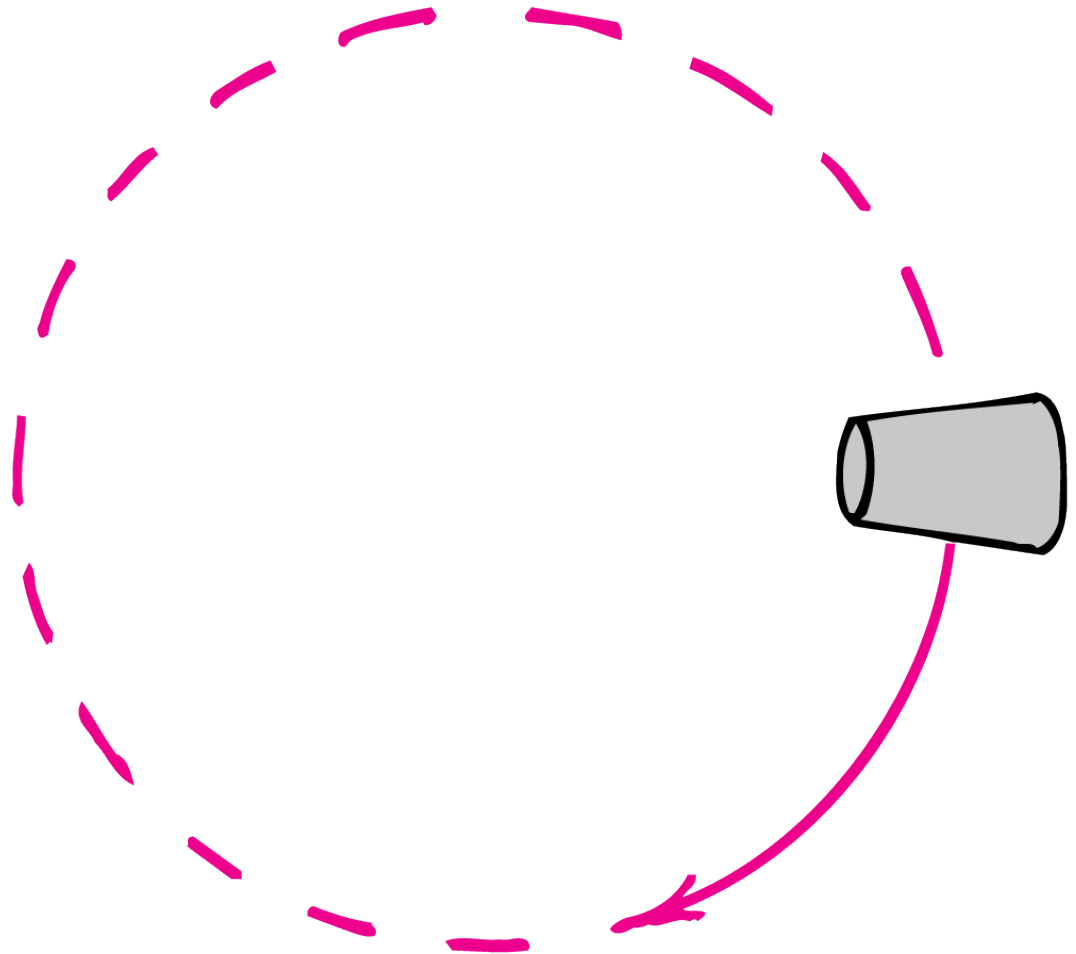


## 10.2 Rotational Speed

A curved path occurs when a tapered cup rolls. The wider part of the cup travels a greater distance per revolution.

## 10.2 Rotational Speed

A tapered cup rolls in a curve because the wide part of the cup rolls faster than the narrow part.



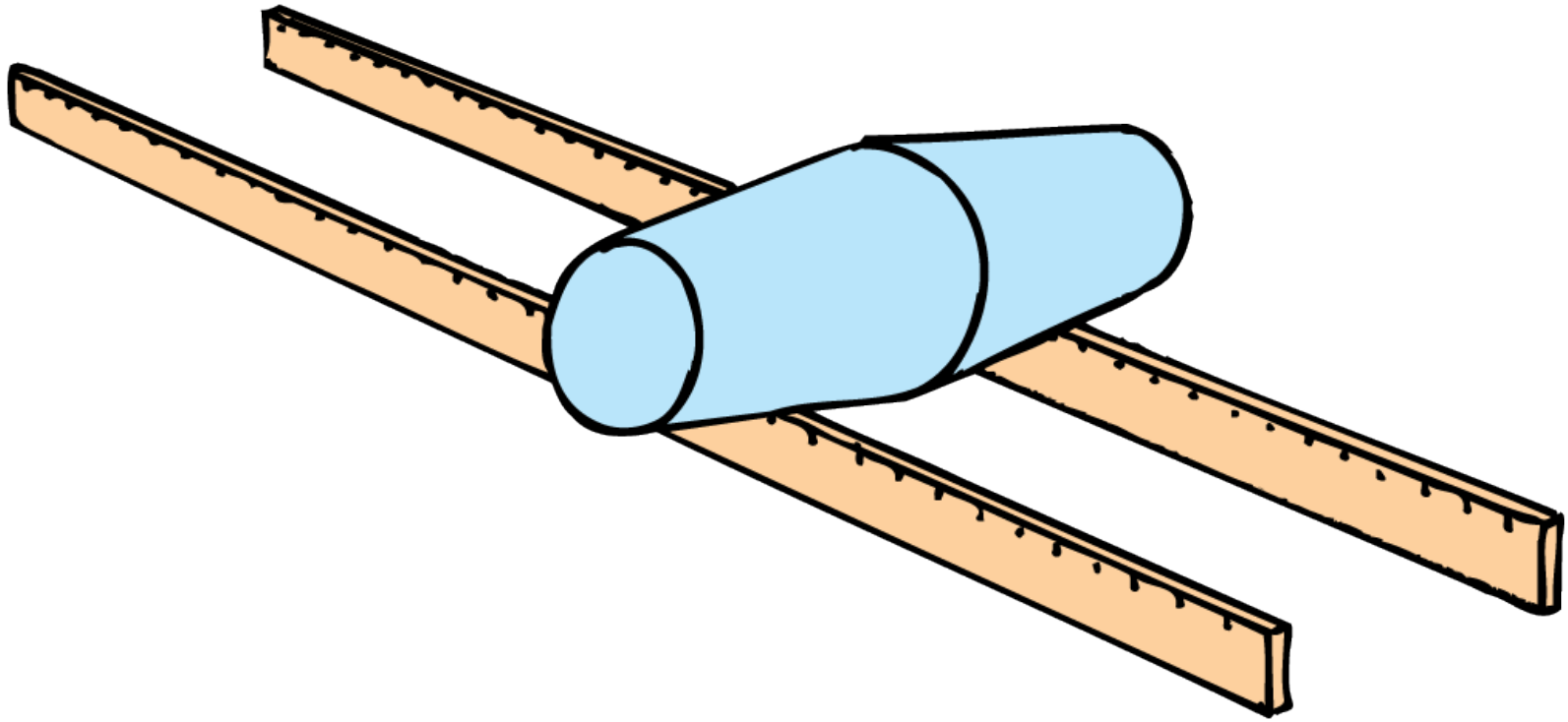
## 10.2 Rotational Speed

Fasten a pair of cups together at their wide ends and roll the pair along a pair of parallel tracks.

- The cups will remain on the track.
- They will center themselves whenever they roll off center.

## 10.2 Rotational Speed

A pair of cups fastened together will stay on the tracks as they roll.



## 10.2 Rotational Speed

When the pair rolls to the left of center, the wider part of the left cup rides on the left track while the narrow part of the right cup rides on the right track.

This steers the pair toward the center.

If it “overshoots” toward the right, the process repeats, this time toward the left, as the wheels tend to center themselves.

## 10.2 Rotational Speed

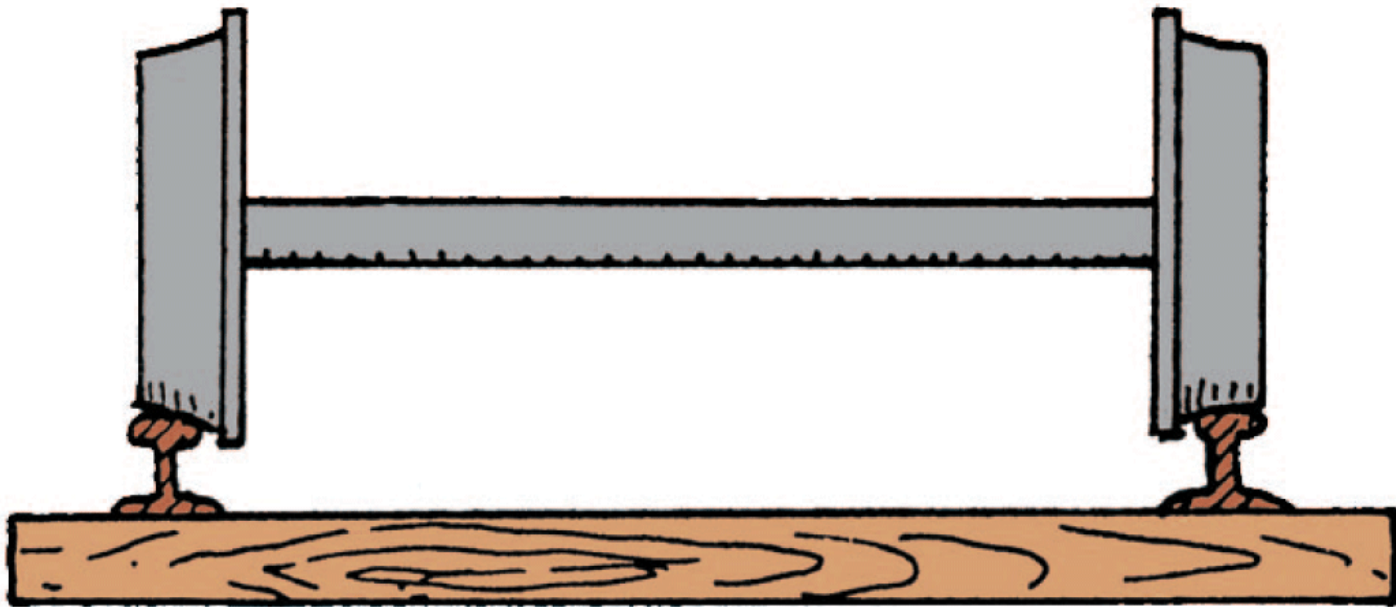
The wheels of railroad trains are similarly tapered. This tapered shape is essential on the curves of railroad tracks.

- On any curve, the distance along the outer part is longer than the distance along the inner part.
- When a vehicle follows a curve, its outer wheels travel faster than its inner wheels. This is not a problem because the wheels roll independent of each other.
- For a train, however, pairs of wheels are firmly connected like the pair of fastened cups, so they rotate together.



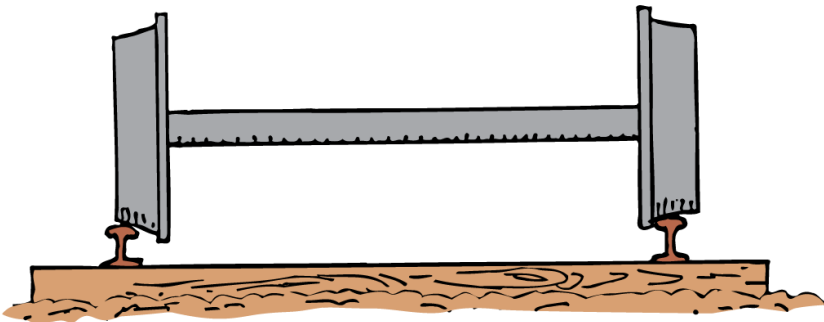
## 10.2 Rotational Speed

The tapered shape of railroad train wheels (shown exaggerated here) is essential on the curves of railroad tracks.



## 10.2 Rotational Speed

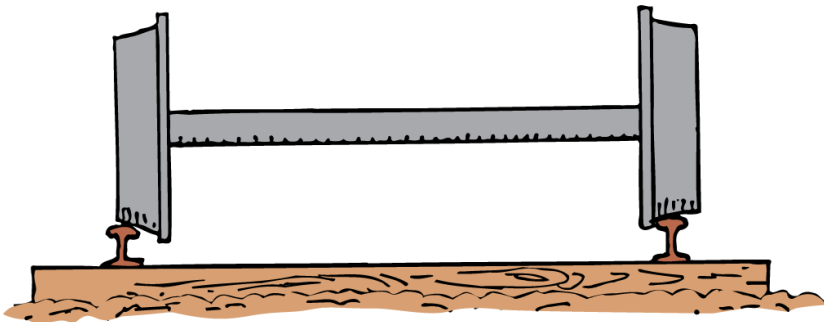
When a train rounds a curve, the wheels have different linear speeds for the same rotational speed.



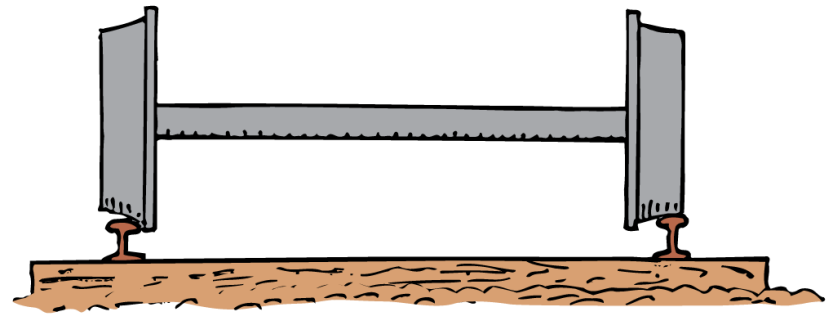
**a** Narrow part of left wheel goes slower, so wheels curve to left.

## 10.2 Rotational Speed

When a train rounds a curve, the wheels have different linear speeds for the same rotational speed.



**a** Narrow part of left wheel goes slower, so wheels curve to left.



**b** Wide part of left wheel goes faster, so wheels curve to right.

## 10.2 Rotational Speed

### think!

Train wheels ride on a pair of tracks. For straight-line motion, both tracks are the same length. But which track is longer for a curve, the one on the outside or the one on the inside of the curve?

## 10.2 Rotational Speed

### think!

Train wheels ride on a pair of tracks. For straight-line motion, both tracks are the same length. But which track is longer for a curve, the one on the outside or the one on the inside of the curve?

### *Answer:*

The outer track is longer—just as a circle with a greater radius has a greater circumference.

## 10.2 Rotational Speed

### CONCEPT CHECK

What is the relationship among tangential speed, rotational speed, and radial distance?



## 10.3 Centripetal Force



**The centripetal force on an object depends on the object's tangential speed, its mass, and the radius of its circular path.**

## 10.3 Centripetal Force

Velocity involves both speed and direction.

- When an object moves in a circle, even at constant speed, the object still undergoes acceleration because its direction is changing.
- This change in direction is due to a net force (otherwise the object would continue to go in a straight line).
- Any object moving in a circle undergoes an acceleration that is directed to the center of the circle—a *centripetal acceleration*.

## 10.3 Centripetal Force

*Centripetal* means “toward the center.”

The force directed toward a fixed center that causes an object to follow a circular path is called a **centripetal force**.

## 10.3 Centripetal Force

### Examples of Centripetal Forces

If you whirl a tin can on the end of a string, you must keep pulling on the string—exerting a centripetal force.

The string transmits the centripetal force, pulling the can from a straight-line path into a circular path.

## 10.3 Centripetal Force

The force exerted on a whirling can is toward the center. No outward force acts on the can.



## 10.3 Centripetal Force

Centripetal forces can be exerted in a variety of ways.

- The “string” that holds the moon on its almost circular path, for example, is gravity.
- Electrical forces provide the centripetal force acting between an orbiting electron and the atomic nucleus in an atom.
- Anything that moves in a circular path is acted on by a centripetal force.

## 10.3 Centripetal Force

Centripetal force is not a basic force of nature, but is the label given to any force that is directed toward a fixed center.

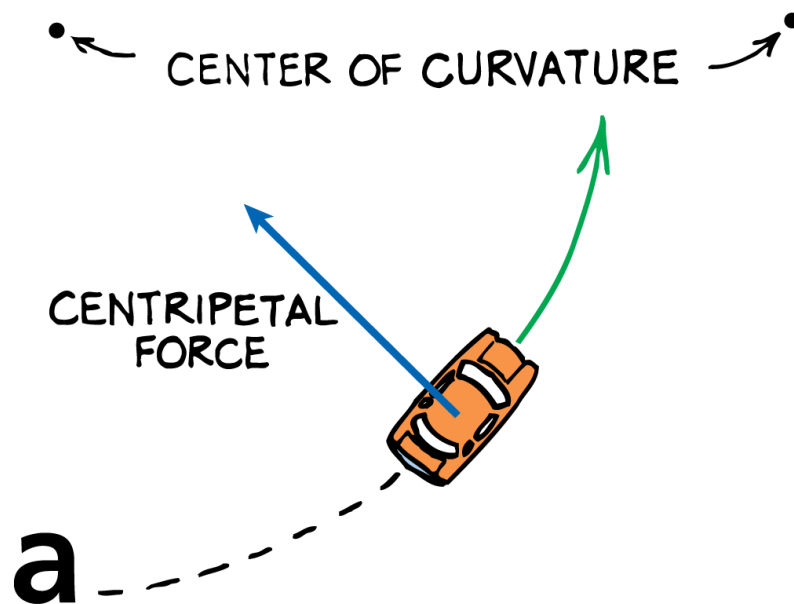
If the motion is circular and executed at constant speed, this force acts at right angles (tangent) to the path of the moving object.



## 10.3 Centripetal Force

Centripetal force holds a car in a curved path.

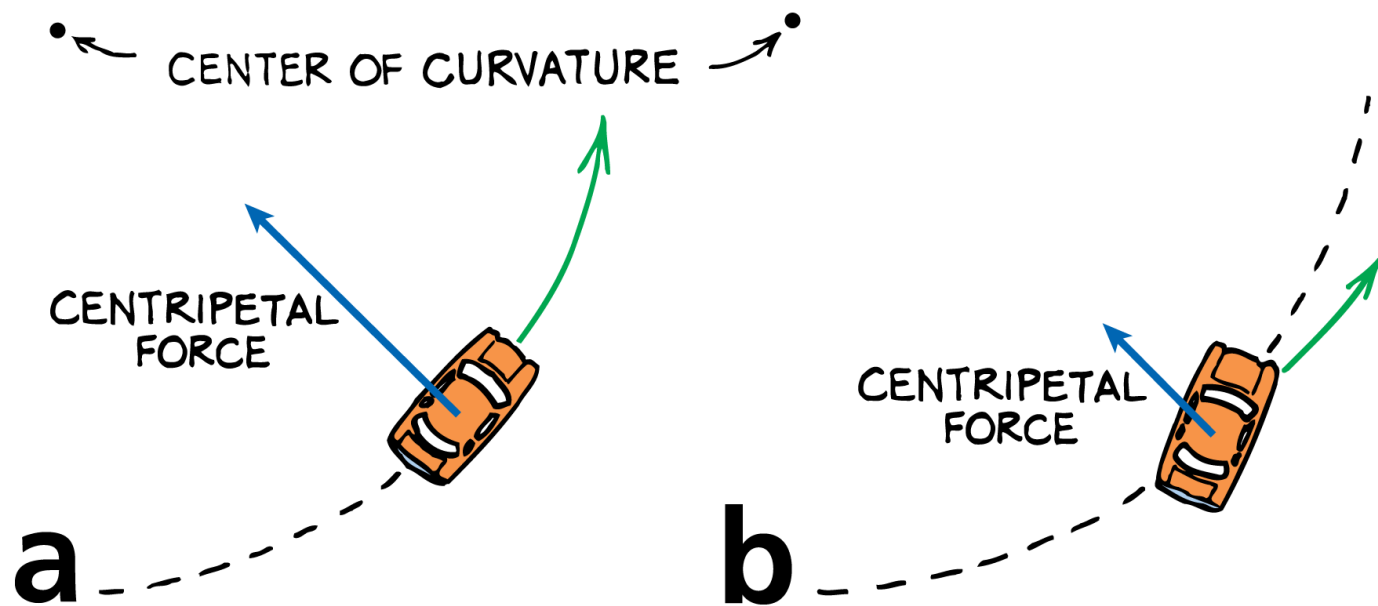
- a. For the car to go around a curve, there must be sufficient friction to provide the required centripetal force.



## 10.3 Centripetal Force

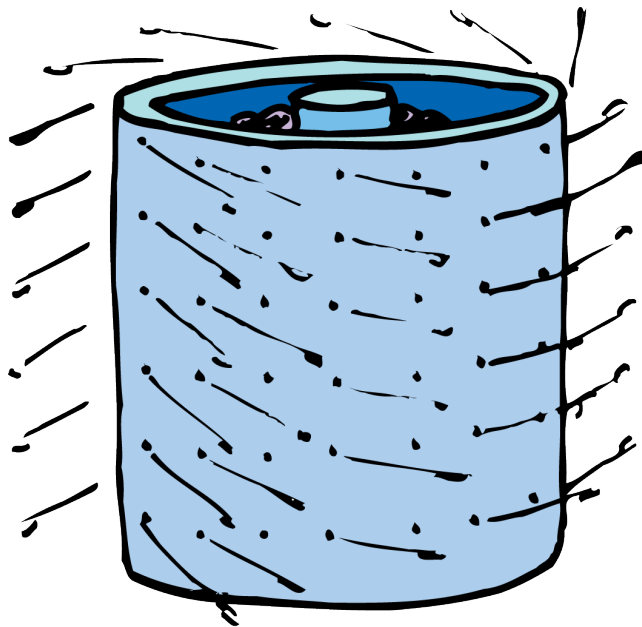
Centripetal force holds a car in a curved path.

- For the car to go around a curve, there must be sufficient friction to provide the required centripetal force.
- If the force of friction is not great enough, skidding occurs.



## 10.3 Centripetal Force

The clothes in a washing machine are forced into a circular path, but the water is not, and it flies off tangentially.



Mixtures are separated in a centrifuge according to their *densities*. That's how cream is separated from milk, and lighter plasma is separated from heavier blood corpuscles.



## 10.3 Centripetal Force

### Calculating Centripetal Forces

Greater speed and greater mass require greater centripetal force. Traveling in a circular path with a smaller radius of curvature requires a greater centripetal force.

$$\text{Centripetal force} = \frac{\text{mass} \times \text{speed}^2}{\text{radius of curvature}}$$

Centripetal force,  $F_c$ , is measured in newtons when  $m$  is expressed in kilograms,  $v$  in meters/second, and  $r$  in meters.

## 10.3 Centripetal Force

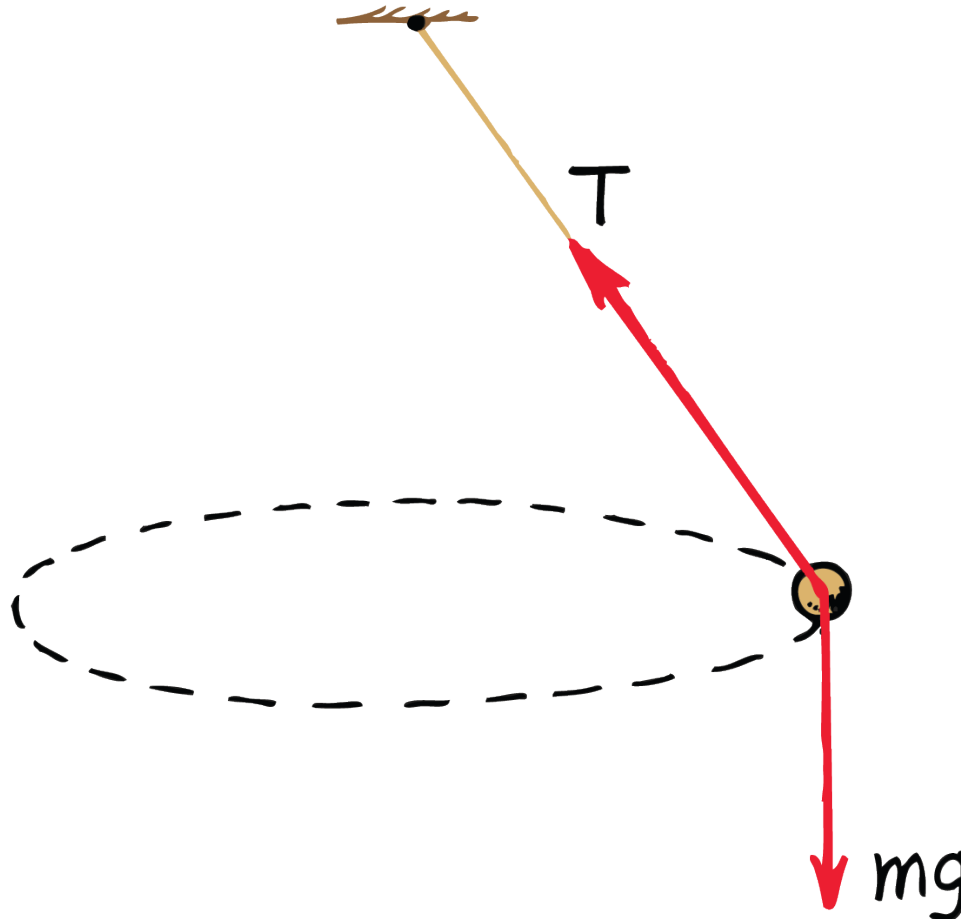
### Adding Force Vectors

A conical pendulum is a bob held in a circular path by a string attached above.

This arrangement is called a conical pendulum because the string sweeps out a cone.

## 10.3 Centripetal Force

The string of a conical pendulum sweeps out a cone.



## 10.3 Centripetal Force

Only two forces act on the bob:  $\mathbf{mg}$ , the force due to gravity, and  $\mathbf{T}$ , tension in the string.

- Both are vectors.

## 10.3 Centripetal Force

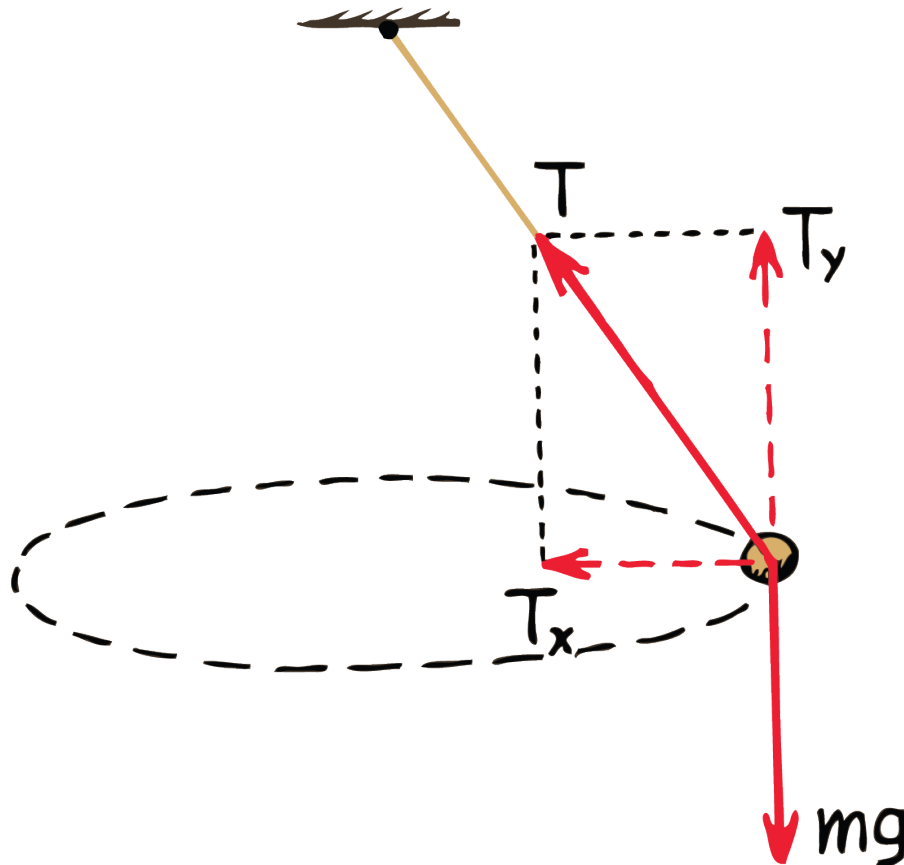
The vector  $\mathbf{T}$  can be resolved into two perpendicular components,  $\mathbf{T}_x$  (horizontal), and  $\mathbf{T}_y$  (vertical).

If vector  $\mathbf{T}$  were replaced with forces represented by these component vectors, the bob would behave just as it does when it is supported only by  $\mathbf{T}$ .



## 10.3 Centripetal Force

The vector  $\mathbf{T}$  can be resolved into a horizontal ( $\mathbf{T}_x$ ) component and a vertical ( $\mathbf{T}_y$ ) component.



## 10.3 Centripetal Force

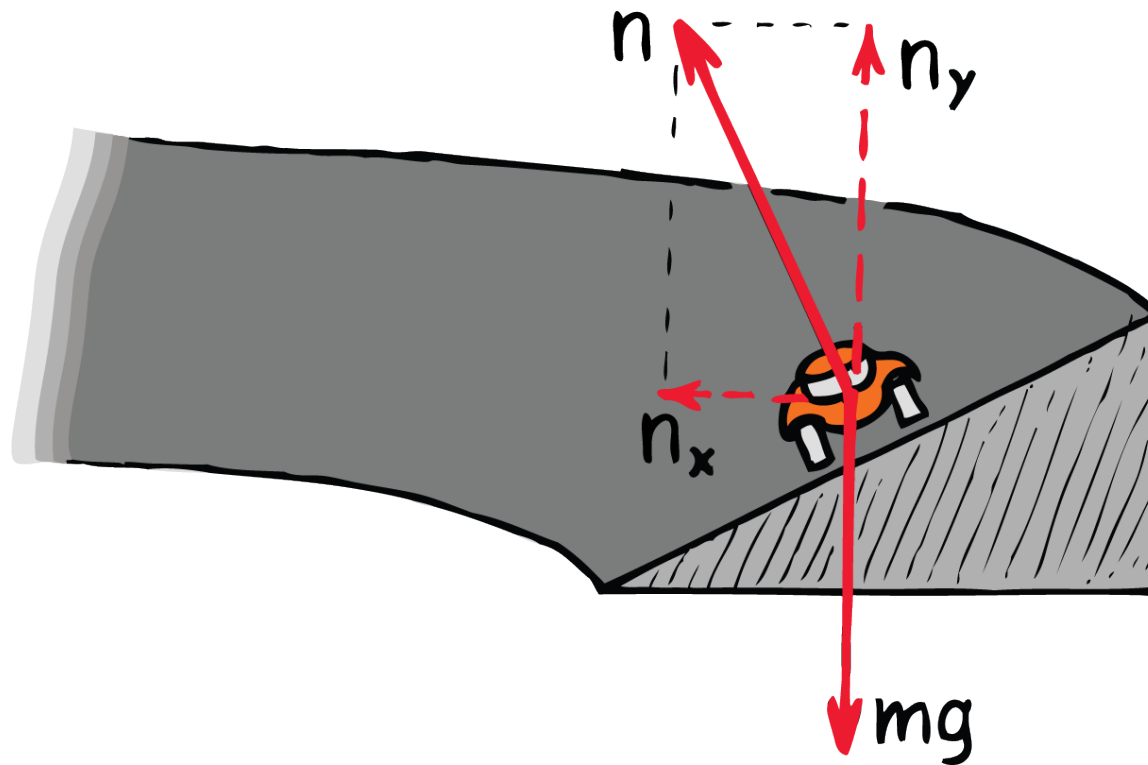
Since the bob doesn't accelerate vertically, the net force in the vertical direction is zero.

Therefore  $T_y$  must be equal and opposite to  $mg$ .

$T_x$  is the net force on the bob—the centripetal force. Its magnitude is  $mv^2/r$ , where  $r$  is the radius of the circular path.

## 10.3 Centripetal Force

Centripetal force keeps the vehicle in a circular path as it rounds a banked curve.



## 10.3 Centripetal Force

Suppose the speed of the vehicle is such that the vehicle has no tendency to slide down the curve or up the curve.

At that speed, friction plays no role in keeping the vehicle on the track.

Only two forces act on the vehicle, one  $\mathbf{mg}$ , and the other the normal force  $\mathbf{n}$  (the support force of the surface). Note that  $\mathbf{n}$  is resolved into  $\mathbf{n}_x$  and  $\mathbf{n}_y$  components.

## 10.3 Centripetal Force

Again,  $\mathbf{n}_y$  is equal and opposite to  $\mathbf{mg}$ , and  $\mathbf{n}_x$  is the centripetal force that keeps the vehicle in a circular path.

Whenever you want to identify the centripetal force that acts on a circularly moving object, it will be the net force that acts exactly along the radial direction—toward the center of the circular path.

## 10.3 Centripetal Force

**CONCEPT  
CHECK**

What factors affect the centripetal force acting on an object?

## 10.4 Centripetal and Centrifugal Forces



The “centrifugal-force effect” is attributed not to any real force but to inertia—the tendency of the moving body to follow a straight-line path.

## 10.4 Centripetal and Centrifugal Forces

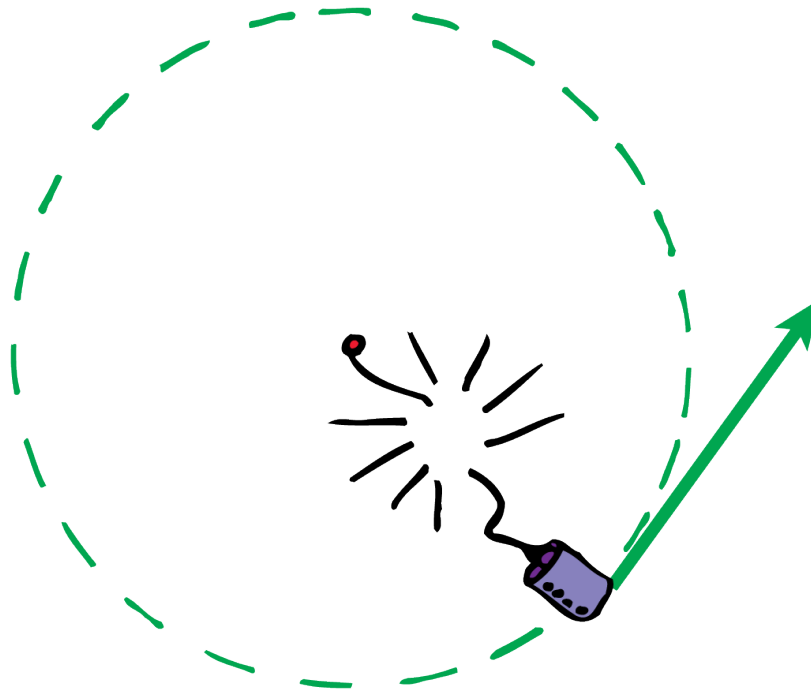
Sometimes an outward force is also attributed to circular motion.

This apparent outward force on a rotating or revolving body is called **centrifugal force**. *Centrifugal* means “center-fleeing,” or “away from the center.”



## 10.4 Centripetal and Centrifugal Forces

When the string breaks, the whirling can moves in a straight line, tangent to—not outward from the center of—its circular path.



## 10.4 Centripetal and Centrifugal Forces

In the case of the whirling can, it is a common misconception to state that a centrifugal force pulls outward on the can.

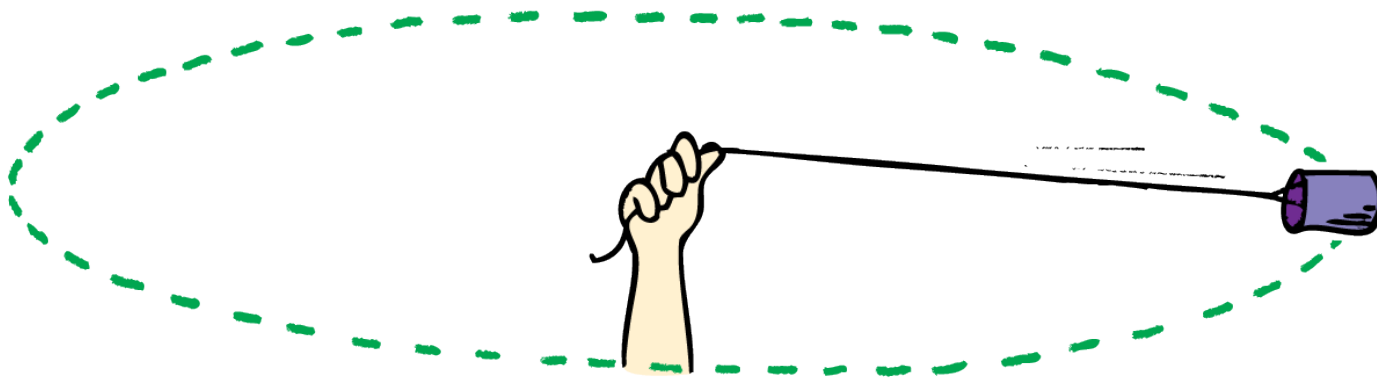
In fact, when the string breaks the can goes off in a tangential straight-line path because *no* force acts on it.

So when you swing a tin can in a circular path, there is *no* force pulling the can outward.

Only the force from the string acts on the can to pull the can inward. The outward force is *on the string*, not on the can.

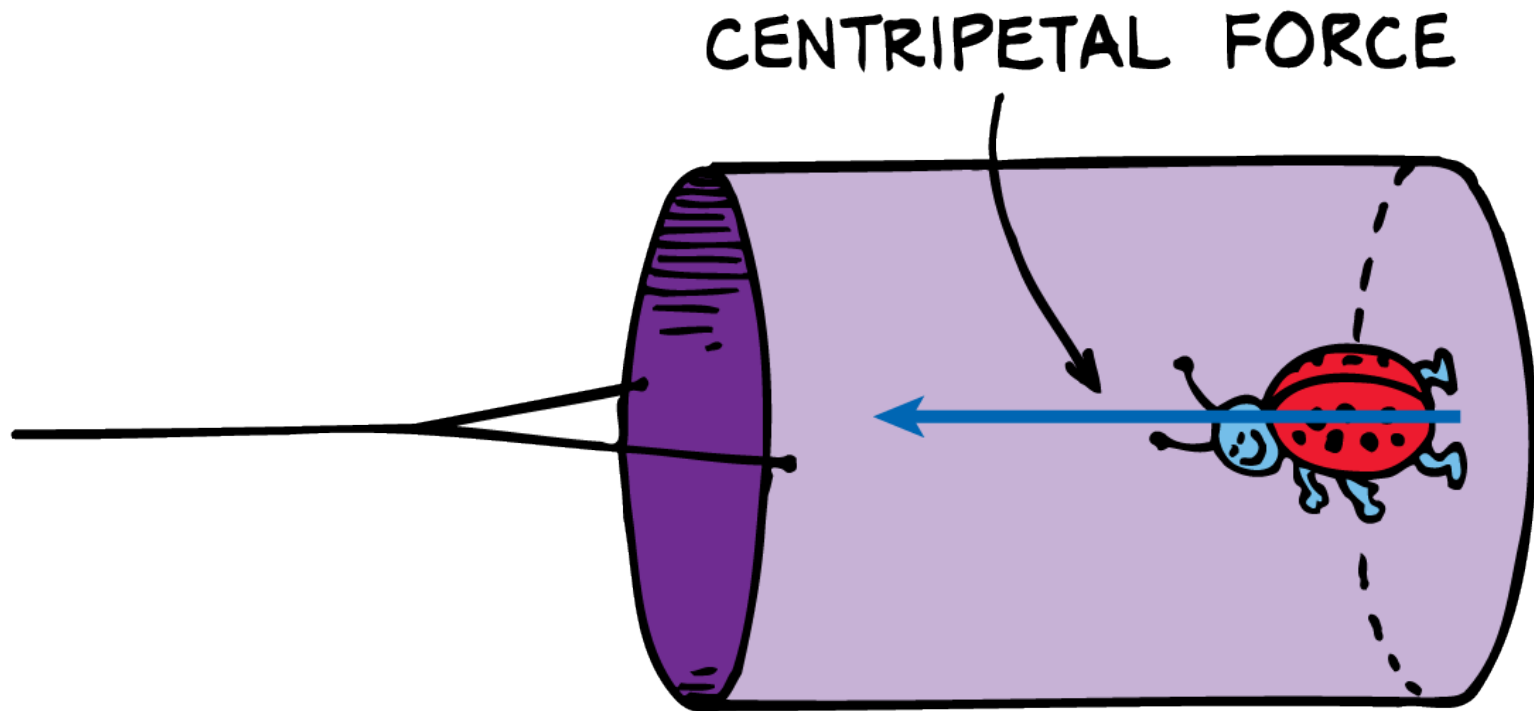
## 10.4 Centripetal and Centrifugal Forces

The only force that is exerted *on* the whirling can (neglecting gravity) is directed *toward* the center of circular motion. This is a *centripetal* force. No outward force acts on the can.



## 10.4 Centripetal and Centrifugal Forces

The can provides the centripetal force necessary to hold the ladybug in a circular path.



## 10.4 Centripetal and Centrifugal Forces

The can presses against the bug's feet and provides the centripetal force that holds it in a circular path.

The ladybug in turn presses against the floor of the can.

Neglecting gravity, the *only* force exerted *on the ladybug* is the force of the can on its feet.

From our outside stationary frame of reference, we see there is no centrifugal force exerted on the ladybug.

## 10.4 Centripetal and Centrifugal Forces

**CONCEPT  
CHECK**

What causes the “centrifugal-force effect”?

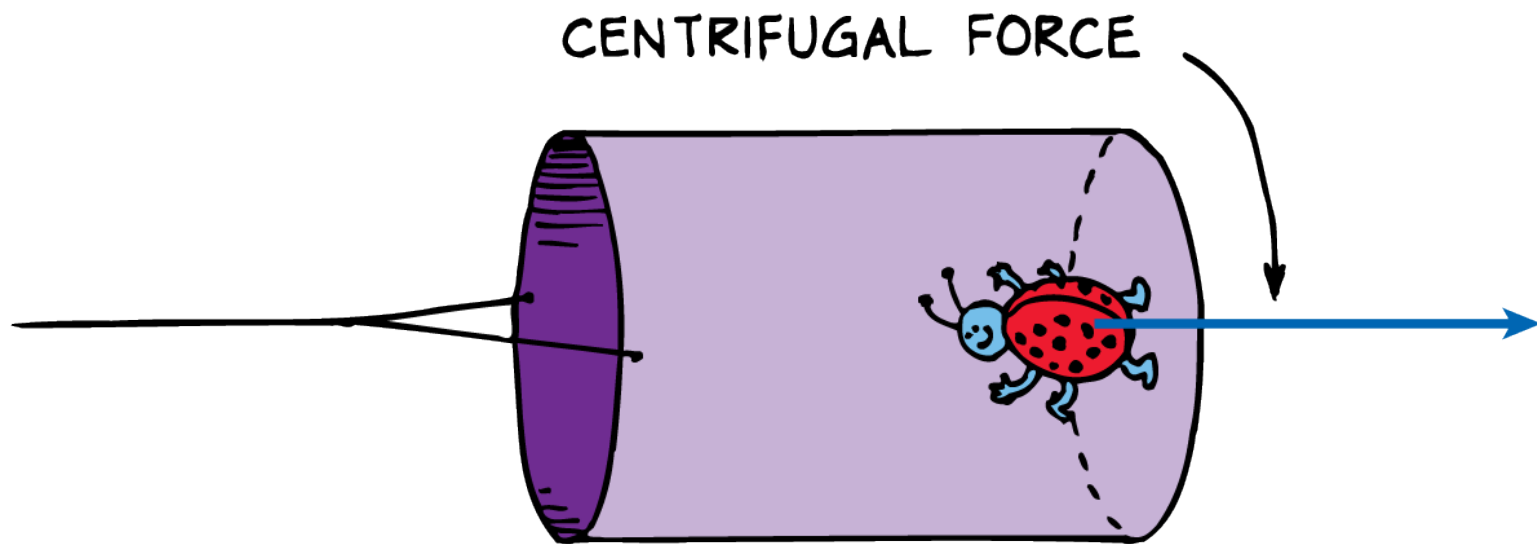
## 10.5 Centrifugal Force in a Rotating Reference Frame



**Centrifugal force is an effect of rotation. It is not part of an interaction and therefore it cannot be a true force.**

## 10.5 Centrifugal Force in a Rotating Reference Frame

From the reference frame of the ladybug inside the whirling can, the ladybug is being held to the bottom of the can by a force that is directed away from the center of circular motion.





## 10.5 Centrifugal Force in a Rotating Reference Frame

From a stationary frame of reference outside the whirling can, we see there is *no* centrifugal force acting on the ladybug inside the whirling can.

However, we do see centripetal force acting on the can, producing circular motion.

## 10.5 Centrifugal Force in a Rotating Reference Frame

Nature seen from the reference frame of the rotating system is different.

In the rotating frame of reference of the whirling can, both centripetal force (supplied by the can) *and* centrifugal force act *on the ladybug*.

## 10.5 Centrifugal Force in a Rotating Reference Frame

The centrifugal force appears as a force in its own right, as real as the pull of gravity.

However, there is a fundamental difference between the gravity-like centrifugal force and actual gravitational force.

Gravitational force is always an interaction between one mass and another. The gravity we feel is due to the interaction between our mass and the mass of Earth.

## 10.5 Centrifugal Force in a Rotating Reference Frame

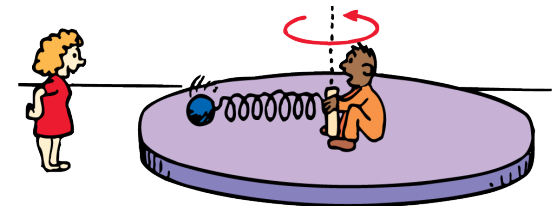
In a rotating reference frame the centrifugal force has no agent such as mass—there is no interaction counterpart.

For this reason, physicists refer to centrifugal force as a *fictitious force*, unlike gravitational, electromagnetic, and nuclear forces. Nevertheless, to observers who are in a rotating system, centrifugal force is very real. Just as gravity is ever present at Earth's surface, centrifugal force is ever present within a rotating system.

## 10.5 Centrifugal Force in a Rotating Reference Frame

### think!

A heavy iron ball is attached by a spring to a rotating platform, as shown in the sketch. Two observers, one in the rotating frame and one on the ground at rest, observe its motion. Which observer sees the ball being pulled outward, stretching the spring? Which observer sees the spring pulling the ball into circular motion?



## 10.5 Centrifugal Force in a Rotating Reference Frame

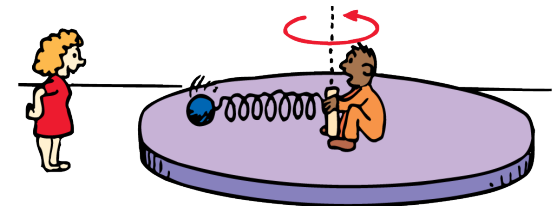
### think!

A heavy iron ball is attached by a spring to a rotating platform, as shown in the sketch. Two observers, one in the rotating frame and one on the ground at rest, observe its motion. Which observer sees the ball being pulled outward, stretching the spring? Which observer sees the spring pulling the ball into circular motion?

### Answer:

The observer in the reference frame of the rotating platform states that centrifugal force pulls radially outward on the ball, which stretches the spring. The observer in the rest frame states that centripetal force supplied by the stretched spring pulls the ball into circular motion.

*(Only the observer in the rest frame can identify an action-reaction pair of forces; where action is spring-on-ball, reaction is ball-on-spring. The rotating observer can't identify a reaction counterpart to the centrifugal force because there isn't any.)*



## 10.5 Centrifugal Force in a Rotating Reference Frame

**CONCEPT  
CHECK**

Why is centrifugal force not considered a true force?

## Assessment Questions

1. Whereas a rotation takes place about an axis that is internal, a revolution takes place about an axis that is
  - a. external.
  - b. at the center of gravity.
  - c. at the center of mass.
  - d. either internal or external.



## Assessment Questions

1. Whereas a rotation takes place about an axis that is internal, a revolution takes place about an axis that is
  - a. external.
  - b. at the center of gravity.
  - c. at the center of mass.
  - d. either internal or external.

Answer: A

## Assessment Questions

2. When you roll a tapered cup across a table, the path of the cup curves because the wider end rolls
- a. slower.
  - b. at the same speed as the narrow part.
  - c. faster.
  - d. in an unexplained way.

## Assessment Questions

2. When you roll a tapered cup across a table, the path of the cup curves because the wider end rolls
- a. slower.
  - b. at the same speed as the narrow part.
  - c. faster.
  - d. in an unexplained way.

Answer: C

## Assessment Questions

3. When you whirl a tin can in a horizontal circle overhead, the force that holds the can in the path acts
- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.

## Assessment Questions

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- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.

Answer: A

## Assessment Questions

4. When you whirl a tin can in a horizontal circle overhead, the force that the can exerts on the string acts
- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.

## Assessment Questions

4. When you whirl a tin can in a horizontal circle overhead, the force that the can exerts on the string acts
- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.

Answer: B

## Assessment Questions

5. A bug inside a can whirled in a circle feels a force of the can on its feet. This force acts
- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.



## Assessment Questions

5. A bug inside a can whirled in a circle feels a force of the can on its feet. This force acts
- a. in an inward direction.
  - b. in an outward direction.
  - c. in either an inward or outward direction.
  - d. parallel to the force of gravity.

Answer: A