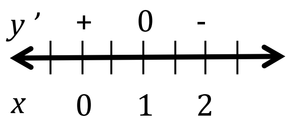
Calculus Worksheet *First Derivative Test* *Name*________________________________

Use analytic methods to find A) the local extrema, B) the intervals on which the function is increasing, and C) the intervals on which the function is decreasing.

1) $f(x) = -2x^2 + 4x + 3$

$f'(x) = -4x + 4$

$-4x + 4 = 0 \Rightarrow x = 1$



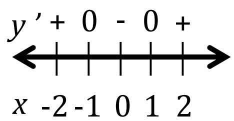
Increasing (-∞,1]
Decreasing [1,∞)
abs. max of 5 when x = 1

2) $f(x) = (x-1)^2(x+2)$

$f'(x) = (x+2)\left[2(x-1)(1)\right] + (x-1)^2(1)$

$= 3x^2 - 3$

$3x^2 - 3 = 0 \Rightarrow x = \pm 1$



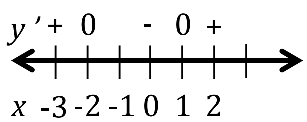
Increasing (-∞,-1] and [1,∞)
Decreasing [-1, 1]
rel. max of 4 when x = -1
rel. min of 0 when x = 1

3) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$6x^2 + 6x - 12 = 0 \Rightarrow 6(x-1)(x+2) = 0$

$\Rightarrow x = 1, -2$

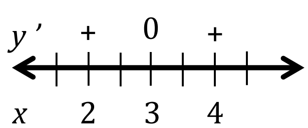


Increasing (-∞,-2] and [1,∞)
Decreasing [-2, 1]
rel. max of 20 when x = -2
rel. min of -7 when x = 1

4) $f(x) = (x-3)^3$

$f'(x) = 3(x-3)^2(1)$

$3(x-3)^2 = 0 \Rightarrow x = 3$

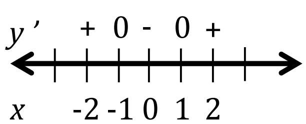


Increasing (-∞,3] and [3,∞)
Decreasing nowhere
No extremes

5) $f(x) = \dfrac{x^5 - 5x}{5}$

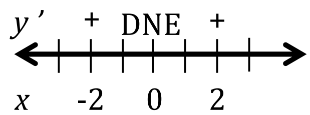
$f'(x) = x^4 - 1$

$x^4 - 1 = 0 \Rightarrow x = \pm 1$



Increasing (-∞, -1] and [1,∞)
Decreasing [-1, 1]
Rel max of 4/5 at x = -1
Rel min of -4/5 at x = 1

6) $f(x) = x^{1/3} + 1$

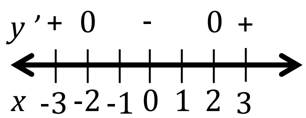


Increasing (-∞, 0] and [0,∞)
Decreasing nowhere
No extrema

$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

$f'(x)$ does not exist at $x = 0$

7) $f(x) = x^3 - 12x$

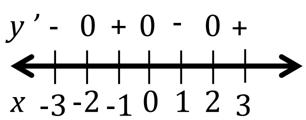


Increasing (-∞, -2] and [2,∞)
Decreasing [-2, 2]
Rel max of 16 at x = -2
Rel min of -16 at x = 2

$f'(x) = 3x^2 - 12$

$3x^2 - 12 = 0 \Rightarrow x = \pm 2$

8) $f(x) = \frac{1}{4}x^4 - 2x^2$



Increasing [-2, 0] and [2,∞)
Decreasing (-∞,- 2],[0,2]
Rel max of 0 at x = 0
Rel min of -4 at x = 2
Rel min of -4 at x = -2

$f'(x) = x^3 - 4x$

$x^3 - 4x = 0 \Rightarrow x = 0, \pm 2$

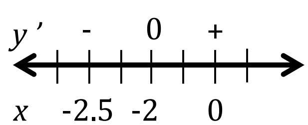
9) $f(x) = x\sqrt{x+3}$

Implied domain: $x \geq -3$

$f'(x) = x\left(\frac{1}{2}(x+3)^{-1/2}(1)\right) + 1\sqrt{x+3}$

$= \frac{x}{2\sqrt{x+3}} + \sqrt{x+3} = \frac{3x+6}{2\sqrt{x+3}}$

$3x + 6 = 0 \Rightarrow x = -2$



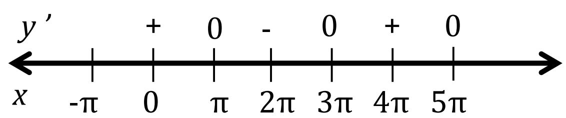
Increasing [-2,∞)
Decreasing [-3, -2]
Rel min of -2 at x = -2
This is also an absolute min

10)

$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$

$$\frac{1}{2}\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}\ldots$$

$$x = \pi, 3\pi, 5\pi$$

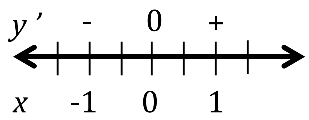


Increasing [0, π] and [3π, 4π]
Decreasing [π, 3π]
Rel max of 1 at x = π (also abs)
Rel min of -1 at x = 3π (also abs)

11) $f(x)=\sqrt{x^2+1}$

$$f'(x)=\frac{1}{2}\left(x^2+1\right)^{-1/2}\left(2x\right)=\frac{x}{\sqrt{x^2+1}}$$

$$\frac{x}{\sqrt{x^2+1}}=0 \Rightarrow x=0$$



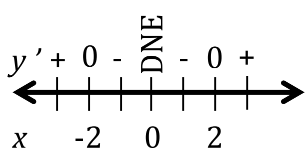
Increasing [0,∞)
Decreasing (-∞, 0]
Rel min of 1 at x = 0
(This is also an abs min)

12) $f(x)=x+\dfrac{4}{x}$

$$f'(x)=1-\frac{4}{x^2}$$

$$1-\frac{4}{x^2}=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2$$

$f'(x)$ does not exist at $x=0$



Increasing (-∞, -2], [2,∞)
Decreasing [-2, 0], [0, 2]
Rel max of -4 at x = -2
Rel min of 4 at x = 2