Calculus Worksheet on 4.1 – 4.3 Name ___________________________

Determine the absolute extrema of the function and the x-value in the closed interval where it occurs.

1) $f(x)=2(3-x),\ [-1,2]$ $f'(x)=-2$, so the graph is decreasing the entire interval.
Absolute maximum is (-1, 8); absolute minimum is (2, 2)

2) $f(x)=x^2+2x-4,\ [-1,1]$ $f'(x)=2x+2 \quad 2x+2=0 \Rightarrow x=-1$
Absolute maximum is (1, -1); absolute minimum is (-1, -5)

3) $f(x)=3x^{2/3}-2x,\ [-1,1]$ $f'(x)=2x^{-1/3}-2=\dfrac{2}{\sqrt[3]{x}}-2 \quad \dfrac{2}{\sqrt[3]{x}}-2=0 \Rightarrow \sqrt[3]{x}=1 \Rightarrow x=1$
Absolute maximum is (-1, 5); absolute minimum is (1, 1)

For the following: A) state whether or not the function satisfies the hypotheses of the MVT on the given interval, and B) if it does, find each value of c in the interval (a,b) that satisfies the equation.

4) $f(x)=x^2,\ [-2,1]$ A) Yes, it does B) $f'(x)=2x \quad 2c=\dfrac{1-4}{1-(-2)}=-1 \Rightarrow c=-\dfrac{1}{2}$

5) $f(x)=x(x^2-x-2),\ [-1,1]$ A) Yes, it does B) $f'(x)=3x^2-2x-2$

$3c^2-2c-2=\dfrac{-2-0}{1-(-1)}=-1 \Rightarrow 3c^2-2c-1=0 \Rightarrow c=1,-\dfrac{1}{3}$

6) $f(x)=x^{2/3},\ [0,1]$ A) Yes, it does B) $f'(x)=\dfrac{2}{3}x^{-1/3}=\dfrac{2}{3\sqrt[3]{x}}$

$\dfrac{2}{3\sqrt[3]{c}}=\dfrac{1-0}{1-0}=1 \Rightarrow \sqrt[3]{c}=\dfrac{2}{3} \Rightarrow c=\dfrac{8}{27}$

7) $f(x)=\dfrac{x+1}{x},\ \left[\dfrac{1}{2},2\right]$ A) Yes, it does B) $f'(x)=-\dfrac{1}{x^2}$

$-\dfrac{1}{c^2}=\dfrac{\frac{3}{2}-3}{2-\frac{1}{2}}=-1 \Rightarrow c^2=1 \Rightarrow c=\pm 1$, but on our interval $c=1$

8) $f(x)=\sqrt{x-2},\ [-1,1]$ A) No, it does not. The function is not defined on [-1,1]

9) $f(x)=x^3,\ [0,1]$ A) Yes, it does B) $f'(x)=3x^2$

$3c^2=\dfrac{1-0}{1-0}=1 \Rightarrow c^2=\dfrac{1}{3} \Rightarrow c=\pm\sqrt{\dfrac{1}{3}}$, but on our interval $c=\sqrt{\dfrac{1}{3}}$

Use analytic methods to find A) the local extrema, B) the intervals on which the function is increasing, and C) the intervals on which the function is decreasing.

10) $f(x) = -2x^2 + 4x + 3$ $\quad f'(x) = -4x + 4 \quad -4x + 4 = 0 \Rightarrow x = 1$

Increases on (-∞,1); decreases on (1,∞); local (and absolute) max at (1,5)

11) $f(x) = (x-1)^2(x+2)$ $\quad f'(x) = 2(x-1)(x+2) + (x-1)^2 = 3x^2 - 3 \quad 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

Increases on (-∞,-1) and (1,∞); decreases on (-1,1);
local max at (-1,4); local min at (1,0)

12) $f(x) = 2x^3 + 3x^2 - 12x$ $\quad f'(x) = 6x^2 + 6x - 12 \quad 6x^2 + 6x - 12 = 0 \Rightarrow x = -2, 1$

Increases on (-∞,-2) and (1,∞); decreases on (-2,1);
local max at (-2,20); local min at (1,-7)

13) $f(x) = (x-3)^3$ $\quad f'(x) = 3(x-3)^2 \quad 3(x-3)^2 = 0 \Rightarrow x = 3$

Increases on (-∞,∞), never decreases;
no local extrema

14) $f(x) = \dfrac{x^5 - 5x}{5}$ $\quad f'(x) = x^4 - 1 \quad x^4 - 1 = 0 \Rightarrow x = \pm 1$

Increases on (-∞,-1) and (1,∞); decreases on (-1,1);
local max at (-1,4/5); local min at (1,-4/5)

15) $f(x) = x^{1/3} + 1$ $\quad f'(x) = \frac{1}{3}x^{-2/3} \quad \frac{1}{3}x^{-2/3} = 0$ is impossible

Increases on (-∞,∞); never decreases;
no local or absolute extrema

For the following, find all relative extrema. Use the Second Derivative Test where applicable.

16) $f(x) = 6x - x^2$ $\quad f'(x) = 6 - 2x \quad f''(x) = -2 \quad 6 - 2x = 0 \Rightarrow x = 3$

local max at (3,9)

17) $f(x) = x^2 + 3x - 8$ $\quad f'(x) = 2x + 3 \quad f''(x) = 2 \quad 2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

local min at $\left(-\frac{3}{2}, -\frac{41}{4}\right)$

18) $f(x) = -(x-5)^2$ $\quad f'(x) = -2(x-5) \quad f''(x) = -2 \quad -2(x-5) = 0 \Rightarrow x = 5$

local max at $(5,0)$

19) $f(x) = x^{2/3} - 3$ $\quad f'(x) = \frac{2}{3}x^{-1/3} \quad f''(x) = -\frac{2}{9}x^{-4/3} \quad \frac{2}{3}x^{-1/3} = 0 \Rightarrow x = 0$

no local extrema

20) $f(x)=\sqrt{x^2+1}$

$f'(x)=\frac{1}{2}\left(x^2+1\right)^{-1/2}\left(2x\right)=\frac{x}{\sqrt{x^2+1}}$ $\quad \frac{x}{\sqrt{x^2+1}}=0 \Rightarrow x=0$

local min at (0,1)

21) $f(x)=x+\frac{4}{x}$

$f'(x)=1-\frac{4}{x^2}=\frac{x^2-4}{x^2}$ $\quad f''(x)=\frac{2x^3-2x\left(x^2-4\right)}{x^4}=\frac{8}{x^3}$ $\quad \frac{x^2-4}{x^2}=0 \Rightarrow x=\pm 2$

local max at (-2,-4); local min at (2,4)

For the following, find all relative extrema, points of inflection, and intervals of concavity. Then use a graphing calculator to graph the function and confirm you answers.

22) $f(x)=x^3-12x$

$f'(x)=3x^2-12$ $\quad f''(x)=6x$ $\quad 3x^2-12=0 \Rightarrow x=\pm 2$

local max at (-2,16); local min at (2,-16); pt of inflection at (0,0);
concave up on (-∞,0); concave down on (0,∞)

23) $f(x)=\frac{1}{4}x^4-2x^2$

$f'(x)=x^3-4x$ $\quad f''(x)=3x^2-4$ $\quad x^3-4x=0 \Rightarrow x=0,\pm 2$

local max at (0,0); local min at (-2,-4) and (2,-4);

pt of inflection at $x=\pm\sqrt{\frac{4}{3}}=\pm\frac{2\sqrt{3}}{3}$.concave up on $\left(-\infty,-\sqrt{\frac{4}{3}}\right)$ and $\left(\sqrt{\frac{4}{3}},\infty\right)$;

concave down on $\left(-\sqrt{\frac{4}{3}},\sqrt{\frac{4}{3}}\right)$

24) $f(x)=x\sqrt{x+3}$

$f'(x)=x\left(\frac{1}{2}\left(x+3\right)^{-1/2}\right)+\sqrt{x+3}=\frac{3x+6}{2\sqrt{x+3}}$

$f''(x)=\frac{3\left(2\sqrt{x+3}\right)-\left(3x+6\right)\left(x+3\right)^{-1/2}}{4\left(x+3\right)}=\frac{3x+12}{4\left(x+3\right)^{3/2}}$

$f'(x)=0 \Rightarrow 3x+6=0 \Rightarrow x=-2$

local min at (-2,-2); pt of inflection at (0,0);
concave up on (-∞,0); no point of inflection. Concave up for entire graph.

25) $f(x)=\sin\frac{x}{2}$ $\quad [0,4\pi]$

$f'(x)=\frac{1}{2}\cos\frac{x}{2}$ $\quad f''(x)=-\frac{1}{4}\sin\frac{x}{2}$ $\quad \frac{1}{2}\cos\frac{x}{2}=0 \Rightarrow x=\pi,3\pi$

local max at (π,1); local min at (3π,-1); pt of inflection at (2π,0);
concave up on (2π,4π); concave down on (0,2π)

26) $f(x)=\sec\left(x-\frac{\pi}{2}\right)$ $\quad [0,4\pi]$

$f'(x)=\sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)$

$f''(x)=\sec\left(x-\frac{\pi}{2}\right)\left[\sec^2\left(x-\frac{\pi}{2}\right)\right]+\sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)\left[\tan\left(x-\frac{\pi}{2}\right)\right]$

$\sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)=0 \Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$

local max at (-2,-4); local min at (2,4)

27) $f(x) = 2\sin x + \sin 2x\ [0, 2\pi]\ f'(x) = 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2} \quad f''(x) = \frac{2x^3 - 2x(x^2-4)}{x^4} = \frac{8}{x^3} \quad \frac{x^2-4}{x^2} = 0 \Rightarrow x = \pm 2$

local max at (-2,-4); local min at (2,4)