Calculus Work Done – Variable Force SOLUTIONS

1) A 20-foot rope weighing 1 pound per foot hangs over the edge of a 50-foot tall building's roof. How much work is done lifting the rope to the roof?

$$W = \int_0^{20} 1(20 - x)\,dx = \left[20x - \frac{x^2}{2}\right]_0^{20} = (400 - 200) - (0 - 0) = 200 \text{ ft-lbs.}$$

A) A 3-pound bucket is attached to the end of the rope described above. How much work is done lifting the bucket to the roof?

$W = 3(20) = 60$ ft-lbs.

B) The bucket holds 50 pounds of sand, but leaks so that when the bucket is at the top of the building there is only 40 pounds of sand in it. How much work is done lifting the sand?

$$W = \int_0^{20} \left(50 - \frac{1}{2}x\right) dx = \left[50x - \frac{x^2}{4}\right]_0^{20} = (1000 - 100) - (0 - 0) = 900 \text{ ft-lbs.}$$

C) What is the total work done lifting the rope, bucket, and sand?

$W = 200 + 60 + 900 = 1160$ ft-lbs.

2) A bucket weighing 3 pounds holds 5 gallons of water, which weighs 8 pounds per gallon. It is to be raised from the ground to a platform 50 feet above the ground by a cable that weighs 2.5 pounds per foot. The bucket starts to leak as it is raised: the leak becomes increasingly worse as the bucket is raised, so that there is only 3 gallons in the bucket when it reaches the platform. The amount of water in the bucket can be modeled by the equation $y = 5e^{(\ln(0.6)/50)x}$, where x is the height above the ground while the bucket is being lifted.

A) How much work is done lifting the bucket from the ground to the platform?

$W = 3(50) = 150$ ft-lbs.

B) How much work is done lifting the cable from the ground to the platform?

$$W = \int_0^{50} 2.5(50 - x)\,dx = \left[125x - 1.25x^2\right]_0^{50} = (6250 - 3125) - (0 - 0) = 3125 \text{ ft-lbs.}$$

C) How much work is done lifting the water from the ground to the platform?

$$W = \int_0^{50} 8\left(5e^{(\ln 0.6/50)x}\right) dx = 40\int_0^{50} e^{(\ln 0.6/50)x}\,dx \qquad u = \frac{\ln 0.6}{50}x \qquad du = \frac{\ln 0.6}{50}dx$$

$$= 40\int_0^{\ln 0.6} e^u \left(\frac{50}{\ln 0.6}\right) du = \frac{2000}{\ln 0.6}\left[e^u\right]_0^{\ln 0.6} = \frac{2000}{\ln 0.6}\left[0.6 - (1)\right] \approx 1566.092 \text{ ft - lbs.}$$

D) What is the total work done lifting the water, bucket, and cable from the ground to the platform?