Calculus Volume of a Solid of Revolution – Shells Method

Use the method of cylindrical shells to determine the volume of the solid of revolution.

1) The area between $y = x^2 - 5x + 4$ and the x-axis, rotated about the y-axis.

$$x^2 - 5x + 4 = 0 \Rightarrow x = 1, 4$$

$$2\pi \int_1^4 x\left(-x^2 + 5x - 4\right)dx = 2\pi\left[-\frac{x^4}{4} + \frac{5x^3}{3} - 2x^2\right]_1^4 = \frac{45\pi}{2}$$

2) The area between $y = x^2 - 7x + 10$ and the x-axis, rotated about the y-axis.

$$x^2 - 7x + 10 = 0 \Rightarrow x = 2, 5$$

$$2\pi \int_2^5 x\left(-x^2 + 7x - 10\right)dx = 2\pi\left[-\frac{x^4}{4} + \frac{7x^3}{3} - 5x^2\right]_2^5 = \frac{63\pi}{2}$$

3) The area between $y = x^2 + 8x + 12$ and the x-axis, rotated about the y-axis.

$$x^2 + 8x + 12 = 0 \Rightarrow x = -2, -6$$

$$2\pi \int_{-2}^{-6} x\left(-x^2 - 8x - 12\right)dx = 2\pi\left[-\frac{x^4}{4} - \frac{8x^3}{3} - 6x^2\right]_{-2}^{-6} = \frac{256\pi}{3}$$

4) The area between $x = y^2 - 10y + 16$ and the y-axis, rotated about the x-axis.

$$y^2 - 10y + 16 = 0 \Rightarrow x = 2, 8$$

$$2\pi \int_2^8 y\left(-y^2 + 10y - 16\right)dy = 2\pi\left[-\frac{y^4}{4} + \frac{10y^3}{3} - 8y^2\right]_2^8 = 360\pi$$

5) The area between $x = y^2 + 6y + 5$ and the y-axis, rotated about the x-axis.

$$y^2 + 6y + 5 = 0 \Rightarrow x = -1, -5$$

$$2\pi \int_{-1}^{-5} y\left(-y^2 - 6y - 5\right)dy = 2\pi\left[-\frac{y^4}{4} - 2y^3 - \frac{5y^2}{2}\right]_{-1}^{-5} = 64\pi$$

6) The region between $y = x^2$, $y = 1/x$, and $y = 4$ rotated about the x-axis.

$$x^2 = \frac{1}{x} \Rightarrow x^3 = 1 \Rightarrow x = 1 \quad x = 1 \Rightarrow y = 1$$

$$2\pi \int_1^4 y\left(\sqrt{y} - \frac{1}{y}\right)dy = 2\pi \int_1^4 \left(y^{3/2} - 1\right)dy = 2\pi\left[\frac{2}{5}y^{5/2} - y\right]_1^4 = \frac{94\pi}{5}$$