

Calculus Review for Chapter 6 Solutions

$$1) \int_0^{\pi/3} \sec^2 \theta \, d\theta = \left[\tan \theta \right]_0^{\pi/3} = \tan \frac{\pi}{3} - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$$

$$2) \int_1^2 \left(x + \frac{1}{x^2} \right) dx = \left[\frac{x^2}{2} - \frac{1}{x} \right]_1^2 = \left(2 - \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right) = 2$$

$$3) \int_0^1 \frac{36 dx}{(2x+1)^3} \quad u = 2x+1 \quad du = 2 dx$$

$$36 \left(\frac{1}{2} \right) \int_1^3 \frac{du}{u^3} = 18 \left[-\frac{1}{2u^2} \right]_1^3 = 18 \left[-\frac{1}{18} - \left(-\frac{1}{2} \right) \right] = -1 + 9 = 8$$

$$4) \int_{-1}^1 2x \sin(1-x^2) \, dx \quad u = 1-x^2 \quad du = -2x \, dx$$

$$-\int_0^0 \sin u \, du = 0$$

$$5) \int_0^{\pi/2} 5 \sin^{3/2} x \cos x \, dx \quad u = \sin x \quad du = \cos x \, dx$$

$$5 \int_0^1 u^{3/2} \, du = 5 \left[\frac{u^{5/2}}{5/2} \right]_0^1 = 5 \left(\frac{2}{5}(1) - \frac{2}{5}(0) \right) = 2$$

$$6) \int_{1/2}^4 \frac{x^2 + 3x}{x} \, dx = \int_{1/2}^4 (x + 3) \, dx = \left[\frac{x^2}{2} + 3x \right]_{1/2}^4 = (8 + 12) - \left(\frac{1}{8} + \frac{3}{2} \right) = 18 \frac{3}{8}$$

$$7) \int_0^{\pi/4} e^{\tan x} \sec^2 x \, dx \quad u = \tan x \quad du = \sec^2 x \, dx$$

$$\int_0^1 e^u \, du = \left[e^u \right]_0^1 = (e^1 - e^0) = e - 1$$

$$8) \int_1^e \frac{\sqrt{\ln r}}{r} \, dr \quad u = \ln r \quad du = \frac{1}{r} \, dr$$

$$\int_0^1 \sqrt{u} \, du = \left[\frac{u^{3/2}}{3/2} \right]_0^1 = \left(\frac{2}{3}(1) - \frac{2}{3}(0) \right) = \frac{2}{3}$$

$$9) \int_0^1 \frac{x}{x^2 + 5x + 6} \, dx \quad \frac{x}{x^2 + 5x + 6} = \frac{A}{x+3} + \frac{B}{x+2} \Rightarrow x = A(x+2) + B(x+3)$$

$$x = -3: \quad -3 = -A \Rightarrow A = 3 \quad x = -2: \quad -2 = B$$

$$\int_0^1 \frac{x}{x^2 + 5x + 6} \, dx = 3 \int_0^1 \frac{dx}{x+3} - 2 \int_0^1 \frac{dx}{x+2}$$

$$= 3 \left[\ln|x+3| \right]_0^1 - 2 \left[\ln|x+2| \right]_0^1 = 3(\ln 4 - \ln 3) - 2(\ln 3 - \ln 2) = 3 \ln 4 - 5 \ln 3 + 2 \ln 2 = \ln \frac{4^3 (2^2)}{3^5}$$

$$\begin{aligned}
 10) \quad & \int_1^2 \frac{2x+6}{x^2-3x} dx \quad \frac{2x+6}{x^2-3x} = \frac{A}{x} + \frac{B}{x-3} \Rightarrow 2x+6 = A(x-3) + Bx \\
 & x=3: 12=3B \Rightarrow B=4 \quad x=0: 6=-3A \Rightarrow A=-2 \\
 & \int_1^2 \frac{2x+6}{x^2-3x} dx = -2 \int_1^2 \frac{dx}{x} + 4 \int_1^2 \frac{dx}{x-3} dx \\
 & = -2 \left[\ln|x| \right]_1^2 + 4 \left[\ln|x-3| \right]_1^2 = -2(\ln 2 - \ln 1) + 4(\ln 1 - \ln 2) = -2 \ln 2 - 4 \ln 2 = \ln 2^{-6}
 \end{aligned}$$

$$\begin{aligned}
 11) \quad & \int \frac{\cos x}{2 - \sin x} dx \quad u = 2 - \sin x \quad du = -\cos x dx \\
 & - \int \frac{du}{u} = -\ln|u| + C = -\ln|2 - \sin x| + C
 \end{aligned}$$

$$\begin{aligned}
 12) \quad & \int \frac{dx}{\sqrt[3]{3x+4}} \quad u = 3x+4 \quad du = 3 dx \\
 & \frac{1}{3} \int u^{-1/3} du = \frac{1}{3} \left(\frac{u^{2/3}}{2/3} \right) + C = \frac{1}{2} (3x+4)^{2/3} + C
 \end{aligned}$$

$$\begin{aligned}
 13) \quad & \int \frac{t dt}{t^2+5} \quad u = t^2+5 \quad du = 2t dt \\
 & \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|t^2+5| + C
 \end{aligned}$$

$$\begin{aligned}
 14) \quad & \int \frac{1}{\theta^2} \sec \frac{1}{\theta} \tan \frac{1}{\theta} d\theta \quad u = \frac{1}{\theta} \quad du = -\frac{1}{\theta^2} d\theta \\
 & - \int \sec u \tan u du = -\sec u + C = -\sec \frac{1}{\theta} + C
 \end{aligned}$$

$$\begin{aligned}
 15) \quad & \int \frac{\tan(\ln y)}{y} dy \quad u = \ln y \quad du = \frac{1}{y} dy \\
 & \int \tan u du = \ln|\sec u| + C = \ln|\sec(\ln y)| + C
 \end{aligned}$$

$$\begin{aligned}
 16) \quad & \int e^x \sec(e^x) dx \quad u = e^x \quad du = e^x dx \\
 & \int \sec u du = \ln|\sec u + \tan u| + C = \ln|\sec(e^x) + \tan(e^x)| + C
 \end{aligned}$$

$$\begin{aligned}
 17) \quad & \int \frac{dx}{x \ln x} \quad u = \ln x \quad du = \frac{1}{x} dx \\
 & \int \frac{1}{u} du = \ln|u| + C = \ln|\ln x| + C
 \end{aligned}$$

$$18) \int \frac{dt}{t\sqrt{t}} = \int t^{-3/2} dt = \frac{t^{-1/2}}{-1/2} + C = -\frac{2}{\sqrt{t}} + C$$

$$19) \int x^3 \cos x \, dx$$

x^3	$\cos x$
$3x^2$	$\sin x$
$6x$	$-\cos x$
6	$-\sin x$
0	$\cos x$

$$\int x^3 \cos x \, dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + C$$

$$20) \int x^4 \ln x \, dx \quad \begin{array}{ll} u = \ln x & dv = x^4 \, dx \\ du = \frac{1}{x} \, dx & v = \frac{x^5}{5} \end{array}$$

$$\int x^4 \ln x \, dx = x^5 \ln x - \frac{1}{5} \int x^4 \, dx + C = x^5 \ln x - \frac{x^5}{25} + C$$

$$21) \int e^{3x} \sin x \, dx \quad \begin{array}{ll} u = e^{3x} & dv = \sin x \, dx \\ du = 3e^{3x} \, dx & v = -\cos x \end{array}$$

$$\int e^{3x} \sin x \, dx = -e^{3x} \cos x - (-3) \int e^{3x} \cos x \, dx + C \quad \begin{array}{ll} u = e^{3x} & dv = \cos x \, dx \\ du = 3e^{3x} \, dx & v = \sin x \end{array}$$

$$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3 \left(e^{3x} \sin x - 3 \int e^{3x} \sin x \, dx \right) + C$$

$$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3e^{3x} \sin x - 9 \int e^{3x} \sin x \, dx + C$$

$$10 \int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3e^{3x} \sin x + C$$

$$\int e^{3x} \sin x \, dx = \frac{e^{3x}}{10} (3 \sin x - \cos x) + C$$

$$22) \int x^2 e^{-3x} \, dx$$

x^2	e^{-3x}
$2x$	$-\frac{1}{3} e^{-3x}$
2	$\frac{1}{9} e^{-3x}$
0	$-\frac{1}{27} e^{-3x}$

$$\int x^2 e^{-3x} \, dx = -\frac{1}{3} x^2 e^{-3x} - \frac{2}{9} x e^{-3x} - \frac{2}{27} e^{-3x} + C$$

$$23) \quad \int \frac{25}{x^2 - 25} dx \quad \frac{25}{x^2 - 25} = \frac{A}{x+5} + \frac{B}{x-5} \Rightarrow 25 = A(x-5) + B(x+5)$$

$$x = 5 : 25 = 10B \Rightarrow B = \frac{5}{2} \quad x = -5 : 25 = -10A \Rightarrow A = -\frac{5}{2}$$

$$\int \frac{25}{x^2 - 25} dx = -\frac{5}{2} \int \frac{dx}{x+5} + \frac{5}{2} \int \frac{dx}{x-5} = -\frac{5}{2} \ln|x+5| + \frac{5}{2} \ln|x-5| + C$$

$$24) \quad \int \frac{5x+2}{2x^2+x-1} dx \quad \frac{5x+2}{2x^2+x-1} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 5x+2 = A(x+1) + B(2x-1)$$

$$x = -1 : -3 = -3B \Rightarrow B = 1 \quad x = \frac{1}{2} : \frac{9}{2} = \frac{3}{2}A \Rightarrow A = 3$$

$$\int \frac{5x+2}{2x^2+x-1} dx = 3 \int \frac{dx}{2x-1} + \int \frac{dx}{x+1} = 3\left(\frac{1}{2}\right) \ln|2x-1| + \ln|x+1| + C$$

$$25) \quad dy = \left(1 + x + \frac{x^2}{2}\right) dx \Rightarrow y = \int \left(1 + x + \frac{x^2}{2}\right) dx$$

$$y = x + \frac{x^2}{2} + \frac{x^3}{6} + C \quad 1 = 0 + 0 + 0 + C \Rightarrow C = 1$$

$$y = x + \frac{x^2}{2} + \frac{x^3}{6} + 1$$

$$26) \quad dy = \left(x + \frac{1}{x}\right)^2 dx \Rightarrow y = \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx$$

$$y = \frac{x^3}{3} + 2x - \frac{1}{x} + C \quad 1 = \frac{1}{3} + 2 - \frac{1}{1} + C \Rightarrow C = -\frac{1}{3}$$

$$y = \frac{x^3}{3} + 2x - \frac{1}{x} - \frac{1}{3}$$

$$27) \quad dy = \frac{1}{t+4} dt \Rightarrow y = \int \frac{1}{t+4} dt$$

$$y = \ln|t+4| + C \quad 2 = \ln 1 + C \Rightarrow C = 2$$

$$y = \ln|t+4| + 2$$

$$28) \quad dy = \csc 2\theta \cot 2\theta d\theta \Rightarrow y = \int \csc 2\theta \cot 2\theta d\theta$$

$$y = -\frac{1}{2} \csc 2\theta + C \quad 1 = -\frac{1}{2} \csc \frac{\pi}{2} + C \Rightarrow 1 = -\frac{1}{2}(1) + C \Rightarrow C = \frac{3}{2}$$

$$y = -\frac{1}{2} \csc 2\theta + \frac{3}{2}$$

$$\begin{aligned}
 31) \quad & \frac{dy}{y+2} = dx \Rightarrow \int \frac{dy}{y+2} = \int dx \\
 & \ln|y+2| = x + C \quad \ln 4 = 0 + C \Rightarrow C = \ln 4 \\
 & \ln|y+2| = x + \ln 4 \Rightarrow y+2 = e^{x+\ln 4} \Rightarrow y = 4e^x - 2
 \end{aligned}$$

$$\begin{aligned}
 32) \quad & \frac{dy}{y+1} = (2x+1)dx \Rightarrow \int \frac{dy}{y+1} = \int (2x+1)dx \\
 & \ln|y+1| = x^2 + x + C \quad \ln 2 = 1 - 1 + C \Rightarrow C = \ln 2 \\
 & \ln|y+1| = x^2 + x + \ln 2 \Rightarrow y+1 = e^{x^2+x+\ln 2} \Rightarrow y = 2e^{x^2+x} - 1
 \end{aligned}$$