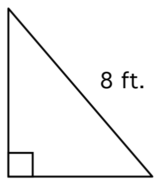
Calculus Related Rates Problems Worksheet

1) An 8-foot ladder is leaning against a wall. The top of the ladder is sliding down the wall at the rate of 2 feet per second. How fast is the bottom of the ladder moving along the ground at the point in time when the bottom of the ladder is 4 feet from the wall?

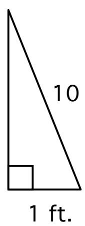


$$x\frac{dx}{dt}+y\frac{dy}{dt}=c\frac{dc}{dt}$$

$$4\frac{dx}{dt}+\left(4\sqrt{3}\right)\left(-2\right)=8\left(0\right)\Rightarrow\frac{dx}{dt}=2\sqrt{3}\text{ ft/sec}\approx 3.464\text{ ft/sec}$$

2) A 10-foot ladder is standing vertically against the side of a house. The base of the ladder is pulled away from the side of the house at the rate of 1 foot per second. How fast will the top of the ladder by falling down the side of the house 1 second after the base begins being pulled away from the house? After 3 seconds? After 8 seconds?

After 1 second:

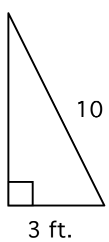


$$x\frac{dx}{dt}+y\frac{dy}{dt}=c\frac{dc}{dt}$$

$$1\left(1\right)+\left(\sqrt{99}\right)\frac{dy}{dt}=10\left(0\right)\Rightarrow\frac{dy}{dt}=\frac{-1}{\sqrt{99}}\text{ ft/sec}\approx -0.101\text{ ft/sec}$$

The ladder is sliding down at approximately 0.101 ft/sec

After 3 seconds:

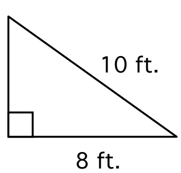


$$x\frac{dx}{dt}+y\frac{dy}{dt}=c\frac{dc}{dt}$$

$$3\left(1\right)+\left(\sqrt{91}\right)\frac{dy}{dt}=10\left(0\right)\Rightarrow\frac{dy}{dt}=\frac{-3}{\sqrt{91}}\text{ ft/sec}\approx -0.314\text{ ft/sec}$$

The ladder is sliding down at approximately 0.314 ft/sec

After 8 seconds:



$$x\frac{dx}{dt}+y\frac{dy}{dt}=c\frac{dc}{dt}$$

$$8\left(1\right)+\left(6\right)\frac{dy}{dt}=10\left(0\right)\Rightarrow\frac{dy}{dt}=\frac{-8}{6}\text{ ft/sec}\approx -1.333\text{ ft/sec}$$

The ladder is sliding down at approximately 1.333 ft/sec

3) Find the rate of change of the radius of a sphere at the point in time when the radius is 6 feet if the volume is increasing at the rate of 8π cubic feet per second.

$$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$8\pi = 4\pi(6)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{18}$$

The radius is changing at approximately 0.056 ft/sec, or 0.667 in/sec

4) The width of a rectangle is increasing at a rate of 2 cm/sec, while the length increases at 3 cm/sec. At what rate is the area increasing when $w = 4$ cm and $l = 5$ cm?

$$A = lw \qquad \frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt} \qquad \frac{dA}{dt} = 5(2) + 4(3) = 22$$

The area is increasing at a rate of 22 cm^2/sec

5) Two cars leave from the same position at the same time. One travels due North at 30 mph, while the other travels due East at 40 mph. At what rate is the distance between the cars changing after 1 hour?

$$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt} \qquad 40(40) + 30(30) = 50\frac{dc}{dt} \quad \frac{dc}{dt} = \frac{2500}{50} = 50$$

The distance between the cars is changing at a rate of 50 mph.

6) A spherical balloon is inflated at a rate of 100 cm^3/sec. How fast is the radius changing when the diameter of the balloon is 50 cm? (Volume of a sphere: $V = \frac{4}{3}\pi r^3$)

$$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

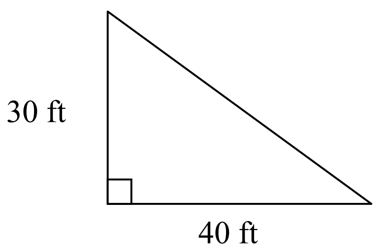
$$100 = 4\pi(25)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{100}{2500\pi} = \frac{1}{25\pi}$$

7) Two cars, one going due East at a rate of 90 km/hr and the other going due South at a rate of 60 km/hr, are traveling toward the intersection of the two roads. At what rate are the cars approaching each other at the instant when the first car is 0.2 km and the second car is 0.15 km from the intersection?

$$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt} \qquad -90(0.2) + (-60)(0.15) = 0.25\frac{dc}{dt} \quad \frac{dc}{dt} = \frac{-27}{.25} = -108$$

The distance is changing at a rate of 108 km/hr

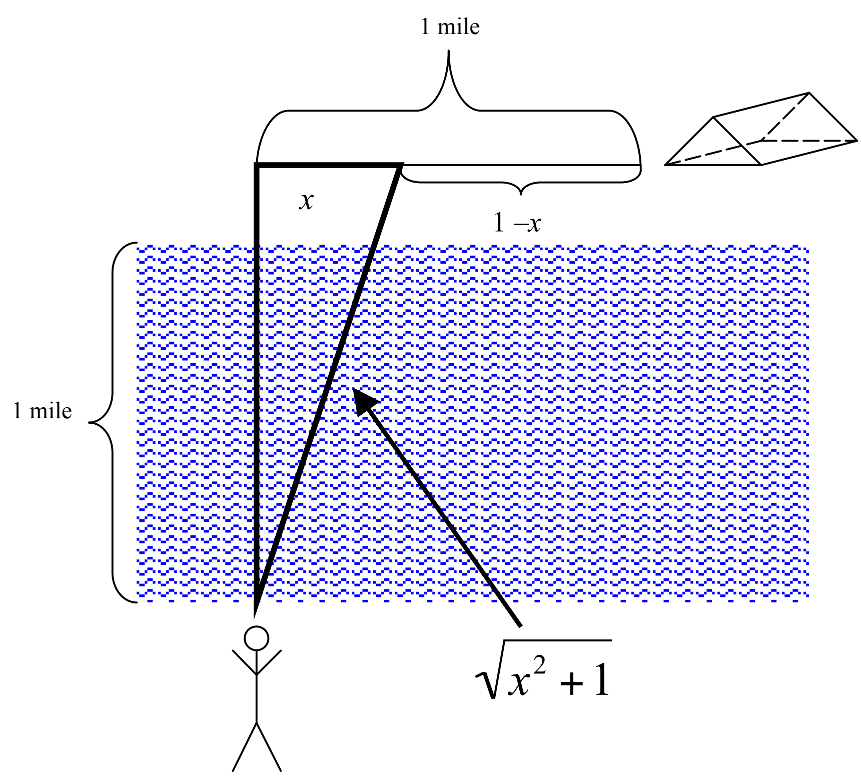
8) A large red balloon is rising at the rate of 20 ft/sec. The balloon is 10 ft above the ground at the point in time that the back end of a green car is directly below the bottom of the balloon. The car is traveling at 40 ft/sec. What is the rate of change of the distance between the bottom of the balloon and the point on the ground directly below the back of the car one second after the back of the car is directly below the balloon?



$$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt}$$

$$40(40) + (30)(20) = 50\frac{dc}{dt} \Rightarrow \frac{dc}{dt} = \frac{2200}{50} \text{ ft/sec} = 44 \text{ ft/sec}$$

Bonus (review of optimization): You are standing at the edge of a slow-moving river that is one mile wide and wish to return to your campground on the opposite side of the river. You can swim at 2 mph and walk at 3 mph. You must first swim across the river to any point on the opposite bank. From there you must walk to the campground, which is one mile from the point directly across the river from where you start your swim. What route will take the least amount of time?



$$T(x) = \frac{\sqrt{x^2+1}}{2} + \frac{1-x}{3}$$

$$T'(x) = \frac{x}{2\sqrt{x^2+1}} - \frac{1}{3} = \frac{3x - 2\sqrt{x^2+1}}{6\sqrt{x^2+1}}$$

$$\frac{3x - 2\sqrt{x^2+1}}{6\sqrt{x^2+1}} = 0 \Rightarrow 3x = 2\sqrt{x^2+1} \Rightarrow 9x^2 = 4(x^2+1)$$

$$9x^2 = 4x^2 + 4 \Rightarrow x^2 = \frac{4}{5} \Rightarrow x = \sqrt{\frac{4}{5}} \approx 0.894$$