Calculus page 500 ff solutions

1) $f(x)=e^{-2x}$ $\quad f'(x)=-2e^{-2x}$ $\quad f''(x)=4e^{-2x}$

$f'''(x)=-8e^{-2x}$ $\quad f^{(4)}(x)=16e^{-2x}$

$$P_4(x)=1-2x+\frac{4x^2}{2!}-\frac{8x^3}{3!}+\frac{16x^4}{4!}=1-2x+2x^2-\frac{4}{3}x^3+\frac{2}{3}x^4$$

$$P_4(0.2)=1-2(0.2)+2(0.2)^2-\frac{4}{3}(0.2)^3+\frac{2}{3}(0.2)^4=1-0.4+0.08-\frac{0.032}{3}+\frac{0.0032}{3}\approx 0.6704$$

2)

$f(x)=\cos\left(\frac{\pi x}{2}\right)$ $\quad f'(x)=-\frac{\pi}{2}\sin\left(\frac{\pi x}{2}\right)$ $\quad f''(x)=-\frac{\pi^2}{4}\cos\left(\frac{\pi x}{2}\right)$

$f'''(x)=\frac{\pi^3}{8}\sin\left(\frac{\pi x}{2}\right)$ $\quad f^{(4)}(x)=\frac{\pi^4}{16}\cos\left(\frac{\pi x}{2}\right)$

$$P_4(x)=1-0x-\frac{\frac{\pi^2}{4}x^2}{2!}+0x^3+\frac{\frac{\pi^4}{16}x^4}{4!}=1-\frac{\pi^2}{8}x^2+\frac{\pi^4}{384}x^4$$

$$P_4(0.2)=1-\frac{\pi^2}{8}(0.2)^2+\frac{\pi^4}{384}(0.2)^4\approx 0.9511$$

At this point I realized I was doing it “the hard way”….and started using the Maclauren series chart.

3) $\sin x=x-\frac{x^3}{3!}$ $\quad 5\sin(-x)=5(-x)-\frac{5(-x)^3}{3!}=-5x+\frac{5}{6}x^3$

$$f(0.2)=-5(0.2)+\frac{5}{6}(0.2)^3\approx -0.9933$$

4) $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}$ $\quad \ln(1+x^2)=x^2-\frac{x^4}{2}+\frac{x^6}{3}$

$$f(0.2)=(0.2)^2-\frac{(0.2)^4}{2}+\frac{(0.2)^6}{3}\approx 0.0392$$

5)

$\frac{1}{1-x}=1+x+x^2+x^3$ $\quad (1-x)^{-2}=\frac{1}{1-2x+x^2}=\frac{1}{1-(2x-x^2)}$

$\frac{1}{1-(2x-x^2)}=1+(2x-x^2)+(2x-x^2)^2+(2x-x^2)^3+(2x-x^2)^4$ YOIKS - try old school

$f(x)=(1-x)^{-2}$ $\quad f'(x)=2(1-x)^{-3}$ $\quad f''(x)=6(1-x)^{-4}$

$f'''(x)=24(1-x)^{-5}$ $\quad f^{(4)}(x)=120(1-x)^{-6}$

$$P_4(x)=1+2x+\frac{6x^2}{2!}+\frac{24x^3}{3!}+\frac{120x^4}{4!}=1+2x+3x^2+4x^3+5x^4$$

$$f(0.2)=1+2(0.2)+3(0.2)^2+4(0.2)^3+5(0.2)^4\approx 1.561$$

6) $$\frac{x^5}{5!}-\frac{x^7}{7!}+\frac{x^9}{9!}-\cdots+(-1)^n\frac{x^{2n+5}}{(2n+5)!}+\cdots$$

7) $$x+x^2+\frac{x^3}{2!}+\cdots+\frac{x^{n+1}}{n!}+\cdots$$

8) $$\frac{1}{2}+\frac{1}{2}\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\frac{(2x)^6}{6!}+\cdots+(-1)^n\frac{(2x)^{2n}}{(2n)!}+\cdots\right)$$

$$=1-x^2+\frac{x^4}{3}-\frac{2x^6}{45}+\cdots+(-1)^n\frac{2^{2n-1}x^{2n}}{(2n)!}$$

9) $$\sin^2 x=\frac{1-\cos 2x}{2}$$

$$\frac{1}{2}-\frac{1}{2}\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\frac{(2x)^6}{6!}+\cdots+(-1)^n\frac{(2x)^{2n}}{(2n)!}+\cdots\right)$$

$$=x^2-\frac{x^4}{3}+\frac{2x^6}{45}+\cdots+(-1)^n\frac{2^{2n+1}x^{2n+2}}{(2n+2)!}$$

10) $$\frac{x^2}{1-2x}=x^2\left(\frac{1}{1-2x}\right)$$

$$x^2\left(1+2x+(2x)^2+\cdots+(2x)^n+\cdots\right)$$

$$=x^2+2x^3+4x^4+\cdots+2^nx^{n+2}+\cdots$$

15) $$xe^x=x+x^2+\frac{x^3}{2!}+\frac{x^4}{3!}+\ldots+\frac{x^{n+1}}{n!}+\ldots=\sum_{n=0}^{\infty}\frac{x^{n+1}}{n!}$$

$$f(x)=xe^x \qquad f'(x)=xe^x+e^x \qquad f''(x)=xe^x+e^x+e^x=xe^x+2e^x$$

$$f'''(x)=xe^x+e^x+2e^x=xe^x+3e^x \qquad f^{(4)}(x)=xe^x+4e^x$$

$$f^{(n+1)}=xe^x+(n+1)e^x$$

$$R_n(x)=\frac{ce^c+(n+1)e^c}{(n+1)!}\left(x^{n+1}\right)$$

$$\lim_{n\to\infty}\frac{ce^c+(n+1)e^c}{(n+1)!}\left(x^{n+1}\right)=\lim_{n\to\infty}\left(\frac{ce^c}{(n+1)!}+\frac{(n+1)e^c}{(n+1)!}\right)\left(x^{n+1}\right)=0$$

16) $\sin x - x + \frac{x^3}{3!} = \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots + (-1)^n \frac{x^{2n+5}}{(2n+5)!} + \cdots$

$$f'(x) = \cos x - 1 + \frac{3x^2}{3!} \qquad f''(x) = -\sin x + x$$

$$f'''(x) = -\cos x + 1 \qquad f^{(4)}(x) = \sin x \qquad f^{(5)}(x) = \cos x$$

$$f^{(n+1)} = (-1)$$

$$R_n(x) = \frac{ce^c + (n+1)e^c}{(n+1)!}\left(x^{n+1}\right)$$

$$\lim_{n\to\infty} \frac{ce^c + (n+1)e^c}{(n+1)!}\left(x^{n+1}\right) = \lim_{n\to\infty}\left(\frac{ce^c}{(n+1)!} + \frac{(n+1)e^c}{(n+1)!}\right)\left(x^{n+1}\right) = 0$$

STILL no flippin' clue on 15 and 16!

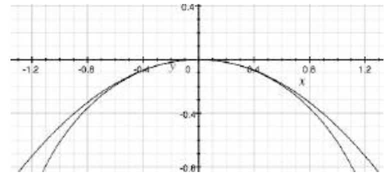
27)

$$f(x) = \ln(\cos x) \qquad f'(x) = \frac{1}{\cos x}\left(-\sin x\right) = -\tan x$$

$$f''(x) = -\sec^2 x$$

$$P_1(x) = \ln(\cos 0) + \left(-\tan 0\right)x = 0$$

$$P_2(x) = 0 + \frac{\left(-\sec^2 0\right)}{2!}x^2 = -\frac{x^2}{2}$$



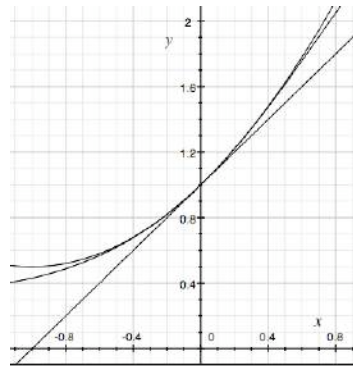
28)

$$f(x) = e^{\sin x} \qquad f'(x) = e^{\sin x}\left(\cos x\right)$$

$$f''(x) = e^{\sin x}\left(-\sin x\right) + e^{\sin x}\left(\cos^2 x\right)$$

$$P_1(x) = e^{\sin 0} + e^{\sin 0}\left(\cos 0\right)x = 1 + x$$

$$P_2(x) = 1 + x + \frac{\left(e^{\sin 0}\left(-\sin 0\right) + e^{\sin 0}\left(\cos^2 0\right)\right)}{2!}x^2 = 1 + x + \frac{x^2}{2}$$



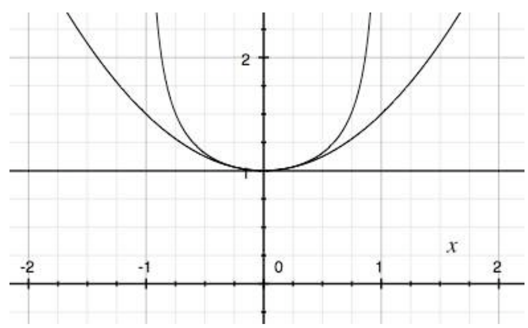
29)

$$f(x) = \left(1 - x^2\right)^{-1/2} \qquad f'(x) = -\frac{1}{2}\left(1 - x^2\right)^{-3/2}\left(-2x\right) = x\left(1 - x^2\right)^{-3/2}$$

$$f''(x) = x\left(-\frac{3}{2}\left(1 - x^2\right)^{-5/2}\right)\left(-2x\right) + \left(1 - x^2\right)^{-3/2}$$

$$P_1(x) = 1 + 0x = 1$$

$$P_2(x) = 1 + \frac{1}{2!}x^2 = 1 + \frac{x^2}{2}$$



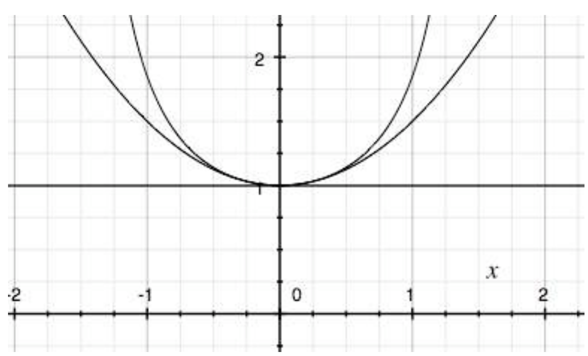
30)

$$f(x) = \sec x \qquad f'(x) = \sec x \tan x$$

$$f''(x) = \sec x\left(\sec^2 x\right) + \sec x \tan x\left(\tan x\right)$$

$$P_1(x) = 1 + 0x = 1$$

$$P_2(x) = 1 + \frac{1}{2!}x^2 = 1 + \frac{x^2}{2}$$

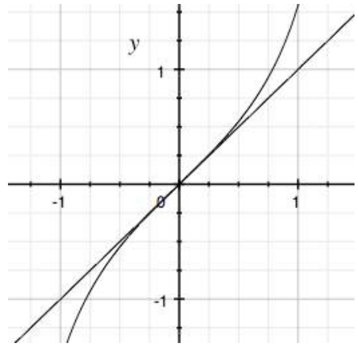


31) $f(x)=\tan x \qquad f'(x)=\sec^2 x$

$f''(x)=2\sec x(\sec x\tan x)$

$P_1(x)=0+x=x$

$P_2(x)=x+\frac{0}{2!}x^2=x$



32) $f(x)=(1+x)^k \qquad f'(x)=k(1+x)^{k-1}$

$f''(x)=k(k-1)(1+x)^{k-2} \qquad f'''(x)=k(k-1)(k-2)(1+x)^{k-3}$

$P_2(x)=1+kx+\frac{k(k-1)}{2}x^2 \qquad R_2(x)=\frac{k(k-1)(k-2)}{6}x^3$

When $k=3$, $|R_2(x)|<0.01 \Rightarrow x^3<0.01 \Rightarrow x<0.215$

33) $R_3(x)=\frac{x^4}{24}$

$|R_3(0.1)|=e^{0.1}\left(\frac{(0.1)^4}{24}\right)=4.605\times10^{-6}$

34) $P_3(x)=1+x+x^2+x^3$

$R_3(x)=x^4$

$|R_3(0.1)|=\left(\frac{1}{0.9}\right)(0.1^4)\approx1.111\times10^{-4}$

35a) No

35b) $\frac{dy}{dx}=1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+\ldots+(-1)^n\frac{x^{2n}}{n!}$

$y=2+x-\frac{x^3}{3}+\frac{x^5}{5\cdot2!}-\frac{x^7}{7\cdot3!}+\ldots+(-1)^{n+1}\frac{x^{2n+1}}{(2n+1)n!}$

35c) All real values of x.

36a)

$$\ln(1+(-x))=-x-\frac{(-x)^2}{2}+\frac{(-x)^3}{3}-\ldots+(-1)^{n-1}\frac{(-x)^n}{n}+\ldots$$

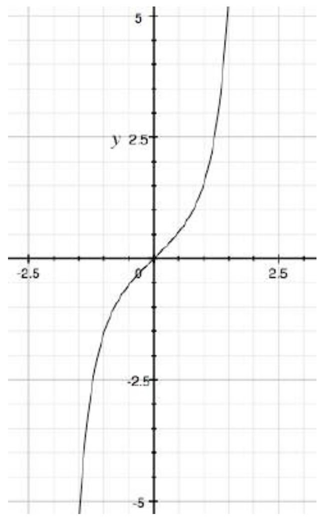
$$=-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots-\frac{x^n}{n}-\ldots$$

36b) $\ln\frac{1+x}{1-x} = \ln(1+x) - \ln(1-x) =$

$$\left[x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{(-x)^n}{n} + \ldots\right] - \left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^n}{n} - \ldots\right]$$

$$= 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \ldots + \frac{2x^{2n-1}}{2n-1} + ..$$

37a) The graph appears to be that of tan x.



37b) The graph appears to be that of sec x.