

$$7) \lim_{n \rightarrow \infty} \frac{3n^2 - 1}{2n^2 + 1} = \lim_{n \rightarrow \infty} \frac{3 - \frac{1}{n^2}}{2 + \frac{1}{n^2}} = \frac{3}{2}, \text{ so the sequence converges.}$$

$$8) \lim_{n \rightarrow \infty} (-1)^n \frac{3n - 1}{n + 2} = \lim_{n \rightarrow \infty} (-1)^n \frac{3 - \frac{1}{n}}{1 + \frac{2}{n}} = \pm 3, \text{ so the sequence diverges.}$$

$$9) \lim_{t \rightarrow 0} \frac{t - \ln(1 + 2t)}{t^2} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{t \rightarrow 0} \frac{1 - \frac{2}{1 + 2t}}{2t} = \infty, \text{ so the limit does not exist.}$$

$$10) \lim_{t \rightarrow 0} \frac{\tan 3t}{\tan 5t} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{t \rightarrow 0} \frac{3 \sec^2(3t)}{5 \sec^2(5t)} = \frac{3}{5}$$

$$11) \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 0} \frac{x \cos x + \sin x}{\sin x} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 0} \frac{-x \sin x + \cos x + \cos x}{\cos x} = 2$$

$$12) \lim_{x \rightarrow 1} x^{1/(1-x)} \quad 1^\infty \quad y = x^{1/(1-x)} \quad \ln y = \frac{1}{1-x} \ln x \quad \lim_{x \rightarrow 1} \frac{\ln x}{1-x} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 1} \frac{\frac{1}{x}}{-1} = -1$$

$$\lim_{x \rightarrow 1} x^{1/(1-x)} = e^{-1}$$

$$13) \lim_{x \rightarrow \infty} x^{1/x} \quad \infty^0 \quad y = x^{1/x} \quad \ln y = \frac{1}{x} \ln x \quad \lim_{x \rightarrow \infty} \frac{\ln x}{x} \quad \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

$$\lim_{x \rightarrow \infty} x^{1/x} = e^0 = 1$$

$$14) \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x \quad 1^\infty \quad y = \left(1 + \frac{3}{x}\right)^x \quad \ln y = x \ln \left(1 + \frac{3}{x}\right) \quad \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{3}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{3}{x}} \left(-\frac{3}{x^2}\right)}{-\frac{1}{x^2}} = 3$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x = e^3$$

$$15) \lim_{r \rightarrow \infty} \frac{\cos r}{\ln r} = 0$$

$$16) \lim_{\theta \rightarrow \pi/2} \left(\theta - \frac{\pi}{2} \right) \sec \theta \quad (\text{RW}) \lim_{\theta \rightarrow \pi/2} \frac{\theta - \frac{\pi}{2}}{\cos \theta} = \frac{0}{0} \quad (\text{L'H}) \lim_{\theta \rightarrow \pi/2} \frac{1}{-\sin \theta} = -1$$

$$17) \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{1}{\ln x} \right) \quad (\text{RW}) \lim_{x \rightarrow 1} \frac{\ln x - x + 1}{(x-1) \ln x} = \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} = (\text{RW}) \lim_{x \rightarrow 1} \frac{1-x}{x-1+x \ln x}$$

$$\frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 1} \frac{-1}{1+1+\ln x} = -\frac{1}{2}$$

$$18) \lim_{x \rightarrow 0^+} \left(1 + \frac{1}{x} \right)^x = \infty^0 \quad y = \left(1 + \frac{1}{x} \right)^x \quad \ln y = x \left(\ln \left(1 + \frac{1}{x} \right) \right) \quad \lim_{x \rightarrow 0^+} x \left(\ln \left(1 + \frac{1}{x} \right) \right) = 0 \cdot \infty$$

$$(\text{RW}) \lim_{x \rightarrow 0^+} \frac{\ln \left(1 + \frac{1}{x} \right)}{\frac{1}{x}} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow 0^+} \frac{\frac{1}{1 + \frac{1}{x}} \left(-\frac{1}{x^2} \right)}{\left(-\frac{1}{x^2} \right)} = (\text{RW}) \lim_{x \rightarrow 0^+} \frac{1}{1 + \frac{1}{x}} = 0$$

$$\lim_{x \rightarrow 0^+} y = e^0 = 1$$

$$19) \lim_{\theta \rightarrow 0^+} (\tan \theta)^\theta = 0^0 \quad y = (\tan \theta)^\theta \quad \ln y = \theta \ln(\tan \theta) \quad \lim_{\theta \rightarrow 0^+} \theta \ln(\tan \theta) = 0 \cdot \infty$$

$$(\text{RW}) \lim_{\theta \rightarrow 0^+} \frac{\ln(\tan \theta)}{\frac{1}{\theta}} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{\theta \rightarrow 0^+} \frac{\frac{1}{\tan \theta} (\sec^2 \theta)}{\left(-\frac{1}{\theta^2} \right)} = (\text{RW}) \lim_{\theta \rightarrow 0^+} \frac{-\theta^2}{\sin \theta \cos \theta} = \frac{0}{0}$$

$$(\text{L'H}) \lim_{\theta \rightarrow 0^+} \frac{-2\theta}{\sin \theta (-\sin \theta) + \cos \theta \cos \theta} = 0 \quad \lim_{\theta \rightarrow 0^+} y = e^0 = 1$$

$$20) \lim_{\theta \rightarrow \infty} \theta^2 \sin \left(\frac{1}{\theta} \right) = \infty \cdot 0 \quad (\text{RW}) \lim_{\theta \rightarrow \infty} \frac{\sin \left(\frac{1}{\theta} \right)}{\frac{1}{\theta^2}} = \frac{0}{0} \quad (\text{L'H}) \lim_{\theta \rightarrow \infty} \frac{\left(\frac{-1}{\theta^2} \right) \left(\cos \left(\frac{1}{\theta} \right) \right)}{\left(-\frac{2}{\theta^3} \right)} = (\text{RW}) \lim_{\theta \rightarrow \infty} \left(\frac{\theta^2}{2} \cos \left(\frac{1}{\theta} \right) \right) = \infty$$

$$21) \lim_{x \rightarrow \infty} \frac{x^3 - 3x^2 + 1}{2x^2 + x - 3} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{3x^2 - 6x}{4x + 1} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{6x - 6}{4} = \infty$$

$$22) \lim_{x \rightarrow \infty} \frac{3x^2 - x + 1}{x^4 - x^3 + 2} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{6x + 1}{4x^3 - 3x^2} = \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{6}{12x^2 - 6x} = 0$$

$$23) \lim_{x \rightarrow \infty} \frac{x}{5x} = \frac{1}{5}. \text{ As this is a nonzero constant, they grow at the same rate.}$$

$$24) \lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3 x} = (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{x \ln 2}}{\frac{1}{x \ln 3}} = \frac{\ln 3}{\ln 2}. \text{ As this is a nonzero constant, they grow at the same rate.}$$

$$25) \lim_{x \rightarrow \infty} \frac{x}{x + \frac{1}{x}} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{x}{\frac{x^2 + 1}{x}} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{x^2}{x^2 + 1} = (\text{L'H}) \lim_{x \rightarrow \infty} \frac{2x}{2x} = 1$$

As this is a nonzero constant, they grow at the same rate.

$$26) \lim_{x \rightarrow \infty} \frac{x}{xe^{-x}} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{xe^x}{100x} = (\text{L'H}) \lim_{x \rightarrow \infty} \frac{xe^x + e^x}{100} = \infty$$

$\frac{x}{100}$ grows faster than xe^{-x}

$$27) \lim_{x \rightarrow \infty} \frac{x}{\tan^{-1} x} = \infty$$

x grows faster than $\tan^{-1} x$

$$28) \lim_{x \rightarrow \infty} \frac{\csc^{-1} x}{\frac{1}{x}} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{\sin^{-1}\left(\frac{1}{x}\right)}{\frac{1}{x}} = (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{1 - \frac{1}{x^2}}} \left(-\frac{1}{x^2}\right)}{-\frac{1}{x^2}} = 1$$

$\csc^{-1} x$ grows at the same rate as $\frac{1}{x}$

$$29) \lim_{x \rightarrow \infty} \frac{x^{\ln x}}{x^{\log_2 x}} = \frac{\infty}{\infty} \quad (\text{RW}) \lim_{x \rightarrow \infty} \frac{x^{\ln x}}{\frac{\ln x}{x^{\ln 2}}} = (\text{RW}) \lim_{x \rightarrow \infty} \left(\frac{x}{x^{\ln 2}}\right)^{\ln x} = 0$$

$x^{\ln x}$ grows slower than $x^{\log_2 x}$

$$30) \lim_{x \rightarrow \infty} \frac{3^{-x}}{2^{-x}} = \frac{0}{0} \quad (\text{RW}) \lim_{x \rightarrow \infty} \frac{2^x}{3^x} = (\text{RW}) \lim_{x \rightarrow \infty} \left(\frac{2}{3}\right)^x = 0$$

3^{-x} grows slower than 2^{-x}

$$31) \quad \lim_{x \rightarrow \infty} \frac{\ln 2x}{\ln x^2} \quad \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{2}{2x}}{\frac{2}{x^2}} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{2x^2}{4x^2} = \frac{1}{2}$$

$\ln 2x$ grows at the same rate as $\ln x^2$

$$32) \quad \lim_{x \rightarrow \infty} \frac{10x^3 + 2x}{e^x} \quad \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{30x^2 + 2}{e^x} \quad \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{60x}{e^x} \quad \frac{\infty}{\infty} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{60}{e^x} = 0$$

$10x^3 + 2x$ grows slower than e^x

$$33) \quad \lim_{x \rightarrow \infty} \frac{\tan^{-1} \frac{1}{x}}{\frac{1}{x}} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \left(\frac{1}{x}\right)^2} \left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{1}{1 + \left(\frac{1}{x}\right)^2} = 1$$

$\tan^{-1} \frac{1}{x}$ grows at the same rate as $\frac{1}{x}$

$$34) \quad \lim_{x \rightarrow \infty} \frac{\sin^{-1} \frac{1}{x}}{\frac{1}{x^2}} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{1 - \left(\frac{1}{x^2}\right)}} \left(-\frac{1}{x^2}\right)}{\left(-\frac{2}{x^3}\right)} = (\text{RW}) \lim_{x \rightarrow \infty} \frac{x}{2\sqrt{1 - \left(\frac{1}{x^2}\right)}} = \infty$$

$\sin^{-1} \frac{1}{x}$ grows faster than $\frac{1}{x^2}$

$$35a) \quad \lim_{x \rightarrow 0} \frac{2^{\sin x} - 1}{e^x - 1} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 0} \frac{2^{\sin x} (\ln 2) (\cos x)}{e^x} = \ln 2$$

$$35b) \quad f(x) = \begin{cases} \frac{2^{\sin x} - 1}{e^x - 1}, & x \neq 0 \\ \ln 2 & x = 0 \end{cases}$$

$$36a) \quad \lim_{x \rightarrow 0^+} x \ln x \quad (\text{RW}) \lim_{x \rightarrow 0^+} \frac{\ln x}{\frac{1}{x}} \quad \frac{0}{0} \quad (\text{L'H}) \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\left(-\frac{1}{x^2}\right)} = (\text{RW}) \lim_{x \rightarrow 0^+} (-x) = 0$$

$$36b) \quad f(x) = \begin{cases} x \ln x, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$37) \quad \int_1^{\infty} \frac{dx}{x^{3/2}} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^{3/2}} = \lim_{b \rightarrow \infty} \left[-\frac{2}{\sqrt{x}} \right]_1^b = \lim_{b \rightarrow \infty} \left[-\frac{2}{\sqrt{b}} - (-2) \right] = 2$$

$$38) \quad \int_1^{\infty} \frac{dx}{x^2 + 7x + 12} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^2 + 7x + 12} \quad \frac{1}{x^2 + 7x + 12} = \frac{A}{x+3} + \frac{B}{x+4} = \frac{1}{x+3} - \frac{1}{x+4}$$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x^2 + 7x + 12} = \lim_{b \rightarrow \infty} \int_1^b \left(\frac{1}{x+3} - \frac{1}{x+4} \right) dx = \lim_{b \rightarrow \infty} \left[\ln|x+3| - \ln|x+4| \right]_1^b = \lim_{b \rightarrow \infty} \left[\ln \left| \frac{x+3}{x+4} \right| \right]_1^b$$

$$= \lim_{b \rightarrow \infty} \left[\ln \left| \frac{b+3}{b+4} \right| - \ln \left(\frac{4}{5} \right) \right] = (\text{RW}) \lim_{b \rightarrow \infty} \left[\ln \left| \frac{1 + \frac{3}{b}}{1 + \frac{4}{b}} \right| - \ln \left(\frac{4}{5} \right) \right] = -\ln \left(\frac{4}{5} \right)$$

$$39) \quad \int_{-\infty}^{-1} \frac{3dx}{3x - x^2} = \lim_{b \rightarrow -\infty} \int_b^{-1} \frac{3dx}{3x - x^2} \quad \frac{3}{3x - x^2} = \frac{A}{x} + \frac{B}{3-x} = \frac{1}{x} + \frac{1}{3-x}$$

$$\lim_{b \rightarrow -\infty} \int_b^{-1} \frac{3dx}{3x - x^2} = \lim_{b \rightarrow -\infty} \int_b^{-1} \left(\frac{1}{x} + \frac{1}{3-x} \right) dx = \lim_{b \rightarrow -\infty} \left[\ln|x| - \ln|3-x| \right]_b^{-1} = \lim_{b \rightarrow -\infty} \left[\ln \left| \frac{x}{3-x} \right| \right]_b^{-1}$$

$$= \lim_{b \rightarrow -\infty} \left[\ln \left(\frac{1}{4} \right) - \ln \left| \frac{-b}{3+b} \right| \right] = (\text{RW}) \lim_{b \rightarrow -\infty} \left[\ln \left(\frac{1}{4} \right) - \ln \left| \frac{-1}{\frac{3}{-b} + 1} \right| \right] = \ln \left(\frac{1}{4} \right)$$

$$40) \quad \int_0^3 \frac{dx}{\sqrt{9-x^2}} = \lim_{b \rightarrow 3} \int_0^b \frac{dx}{\sqrt{9-x^2}} = (\text{RW}) \lim_{b \rightarrow 3} \int_0^b \frac{dx}{\sqrt{9 \left(1 - \frac{x^2}{9} \right)}} = (\text{RW}) \lim_{b \rightarrow 3} \int_0^b \frac{dx}{3 \sqrt{1 - \left(\frac{x}{3} \right)^2}}$$

$$= \lim_{b \rightarrow 3} \left[\sin^{-1} \frac{x}{3} \right]_0^b = \lim_{b \rightarrow 3} \left(\sin^{-1} \left(\frac{b}{3} \right) - \sin^{-1} 0 \right) = \frac{\pi}{2}$$

$$41) \quad \int_0^1 \ln(x) dx = \lim_{b \rightarrow 0^+} \int_b^1 \ln(x) dx \quad \begin{array}{ll} u = \ln x & dv = dx \\ du = \frac{1}{x} dx & v = x \end{array} \quad \int \ln(x) dx = x \ln x - x + C$$

$$\lim_{b \rightarrow 0^+} \int_b^1 \ln(x) dx = \lim_{b \rightarrow 0^+} \left[x \ln x - x \right]_b^1 = \lim_{b \rightarrow 0^+} \left[0 - 1 - (b \ln b - b) \right]_0^b$$

$$= -1 - \lim_{b \rightarrow 0^+} \left(\frac{\ln b}{\frac{1}{b}} \right) = -1 - (\text{L'H}) \lim_{b \rightarrow 0^+} \left(\frac{\frac{1}{b}}{-\frac{1}{b^2}} \right) = -1$$

$$42) \quad \int_{-1}^1 \frac{dy}{y^{2/3}} = \lim_{a \rightarrow 0} \int_{-1}^a \frac{dy}{y^{2/3}} + \lim_{b \rightarrow 0} \int_b^1 \frac{dy}{y^{2/3}} = \lim_{a \rightarrow 0} \left[3y^{1/3} \right]_{-1}^a + \lim_{b \rightarrow 0} \left[3y^{1/3} \right]_b^1$$

$$= \lim_{a \rightarrow 0} \left[3a^{1/3} - (-3) \right] + \lim_{b \rightarrow 0} \left[3 - 3b^{1/3} \right] = 3 + 3 = 6$$

$$\begin{aligned}
43) \quad \int_{-2}^0 \frac{d\theta}{(\theta+1)^{3/5}} &= \lim_{a \rightarrow -1} \int_{-2}^a \frac{d\theta}{(\theta+1)^{3/5}} + \lim_{b \rightarrow -1} \int_b^0 \frac{d\theta}{(\theta+1)^{3/5}} = \lim_{a \rightarrow -1} \left[\frac{5}{2} (\theta+1)^{2/5} \right]_{-2}^a + \lim_{b \rightarrow -1} \left[\frac{5}{2} (\theta+1)^{2/5} \right]_b^0 \\
&= \lim_{a \rightarrow -1} \left[\frac{5}{2} (a+1)^{2/5} - \frac{5}{2} \right] + \lim_{b \rightarrow -1} \left[\frac{5}{2} - \frac{5}{2} (b+1)^{2/5} \right] = -\frac{5}{2} + \frac{5}{2} = 0
\end{aligned}$$

$$\begin{aligned}
44) \quad \int_3^\infty \frac{2dx}{x^2-2x} &= \lim_{b \rightarrow \infty} \int_3^b \frac{2dx}{x^2-2x} \quad \frac{2}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2} = -\frac{1}{x} + \frac{1}{x-2} \\
\lim_{b \rightarrow \infty} \int_3^b \frac{2dx}{x^2-2x} &= \lim_{b \rightarrow \infty} \int_3^b \left(-\frac{1}{x} + \frac{1}{x-2} \right) dx = \lim_{b \rightarrow \infty} \left[-\ln|x| + \ln|x-2| \right]_3^b = \lim_{b \rightarrow \infty} \left[\ln \left| \frac{x-2}{x} \right| \right]_3^b \\
&= \lim_{b \rightarrow \infty} \left[\ln \left| \frac{b-2}{b} \right| - \ln \left(\frac{1}{3} \right) \right] = (\text{RW}) \lim_{b \rightarrow \infty} \left[\ln \left| \frac{1 - \frac{2}{b}}{1} \right| - \ln \frac{1}{3} \right] = 0 - \ln \left(\frac{1}{3} \right) = -\ln \frac{1}{3} = \ln 3
\end{aligned}$$

$$\begin{aligned}
45) \quad \int_0^\infty x^2 e^{-x} dx &= \lim_{a \rightarrow \infty} \int_0^a x^2 e^{-x} dx \quad \int x^2 e^{-x} dx = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c \\
&= \lim_{a \rightarrow \infty} \left[-x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \right]_0^a = \lim_{a \rightarrow \infty} \left[-a^2 e^{-a} - 2a e^{-a} - 2e^{-a} - (0 - 0 - 2) \right] = 2
\end{aligned}$$

$$\begin{aligned}
46) \quad \int_{-\infty}^0 x e^{3x} dx &= \lim_{b \rightarrow -\infty} \int_b^0 x e^{3x} dx \quad \begin{array}{l} u = x \\ du = dx \end{array} \quad \begin{array}{l} dv = e^{3x} dx \\ v = \frac{1}{3} e^{3x} \end{array} \quad \int x e^{3x} dx = \frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C \\
\lim_{b \rightarrow -\infty} \int_b^0 x e^{3x} dx &= \lim_{b \rightarrow -\infty} \left[\frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} \right]_b^0 = \lim_{b \rightarrow -\infty} \left[0 - \frac{1}{9} - \left(\frac{1}{3} b e^{3b} - \frac{1}{9} e^{3b} \right) \right] = -\frac{1}{9}
\end{aligned}$$

$$\begin{aligned}
47) \quad \int_{-\infty}^\infty \frac{dx}{e^x + e^{-x}} &= \lim_{b \rightarrow -\infty} \int_b^0 \frac{dx}{e^x + e^{-x}} + \lim_{a \rightarrow \infty} \int_0^a \frac{dx}{e^x + e^{-x}} \quad \int \frac{dx}{e^x + e^{-x}} = (\text{RW}) \int \frac{dx}{\frac{e^{2x} + 1}{e^x}} = (\text{RW}) \int \frac{e^x dx}{e^{2x} + 1} \\
&= \lim_{b \rightarrow -\infty} \int_b^0 \frac{e^x dx}{e^{2x} + 1} + \lim_{a \rightarrow \infty} \int_0^a \frac{e^x dx}{e^{2x} + 1} \quad \begin{array}{l} u = e^x \\ du = e^x dx \end{array} \\
&= \lim_{b \rightarrow -\infty} \int_{e^b}^1 \frac{du}{u^2 + 1} + \lim_{a \rightarrow \infty} \int_1^{e^a} \frac{du}{u^2 + 1} = \lim_{b \rightarrow -\infty} \left[\tan^{-1} u \right]_{e^b}^1 + \lim_{a \rightarrow \infty} \left[\tan^{-1} u \right]_1^{e^a} \\
&= \lim_{b \rightarrow -\infty} \left[\tan^{-1} 1 - \tan^{-1} e^b \right] + \lim_{a \rightarrow \infty} \left[\tan^{-1} e^a - \tan^{-1} 1 \right] = \left(\frac{\pi}{4} - \left(-\frac{\pi}{2} \right) \right) + \left(\frac{\pi}{2} - \frac{\pi}{4} \right) = \frac{\pi}{2}
\end{aligned}$$

$$\begin{aligned}
48) \quad \int_{-\infty}^\infty \frac{4dx}{x^2 + 16} &= (\text{RW}) \int_{-\infty}^\infty \frac{4dx}{\left(\frac{x}{4} \right)^2 + 1} \quad \begin{array}{l} u = \frac{x}{4} \\ du = \frac{1}{4} dx \end{array} \\
&= \lim_{b \rightarrow -\infty} \int_{b/4}^0 \frac{du}{u^2 + 1} + \lim_{a \rightarrow \infty} \int_0^{a/4} \frac{du}{u^2 + 1} = \lim_{b \rightarrow -\infty} \left[\tan^{-1} u \right]_{b/4}^0 + \lim_{a \rightarrow \infty} \left[\tan^{-1} u \right]_0^{a/4} \\
&= \lim_{b \rightarrow -\infty} \left[0 - \tan^{-1} \frac{b}{4} \right] + \lim_{a \rightarrow \infty} \left[\tan^{-1} \frac{a}{4} - 0 \right] = -\left(\left(-\frac{\pi}{2} \right) \right) + \left(\frac{\pi}{2} \right) = \pi
\end{aligned}$$

