Calculus page 416 Solutions

1) $L = \int_{-1}^{2} \sqrt{1 + (2x)^2}\, dx \approx 6.126$

2) $L = \int_{-\pi/3}^{0} \sqrt{1 + (\sec^2 x)^2}\, dx \approx 2.057$

3) $L = \int_{0}^{\pi} \sqrt{1 + (\cos y)^2}\, dy \approx 3.820$

4) $L = \int_{-1/2}^{1/2} \sqrt{1 + \left(\frac{-y}{\sqrt{1 - y^2}}\right)^2}\, dy \approx 1.047$

5)
$$y^2 + 2y = 2x + 1 \Rightarrow x = \frac{1}{2}\left(y^2 + 2y - 1\right)$$
$$L = \int_{-1}^{3} \sqrt{1 + (y+1)^2}\, dy \approx 9.294$$

6)
$$\frac{dy}{dx} = \cos x - \left(-x \sin x + \cos x\right) = x \sin x$$
$$L = \int_{0}^{\pi} \sqrt{1 + (x \sin x)^2}\, dx \approx 4.698$$

7)
$$\frac{dy}{dx} = \tan x$$
$$L = \int_{0}^{\pi/6} \sqrt{1 + (\tan x)^2}\, dx \approx 0.549$$

8)
$$\frac{dx}{dy} = \sqrt{\sec^2 y - 1}$$
$$L = \int_{-\pi/3}^{\pi/4} \sqrt{1 + \left(\sqrt{\sec^2 y - 1}\right)^2}\, dy \approx 2.198$$

9) $L = \int_{-\pi/3}^{\pi/3} \sqrt{1 + (\sec x \tan x)^2}\, dx \approx 3.139$

10) $L = \int_{-3}^{3} \sqrt{1 + \left(\frac{e^x - e^{-x}}{2}\right)^2}\, dx \approx 20.036$

11) $$\frac{dy}{dx}=\frac{3}{2}\left(\frac{1}{3}\right)\left(x^2+2\right)^{1/2}\left(2x\right)=x\sqrt{x^2+2}$$

$$L=\int_0^3\sqrt{1+\left(x\sqrt{x^2+2}\right)^2}\,dx=\int_0^3\sqrt{1+x^2\left(x^2+2\right)}dx=\int_0^3\sqrt{x^4+2x^2+1}\,dx=\int_0^3\left(x^2+1\right)dx$$

$$=\left[\frac{x^3}{3}+x\right]_0^3=\left(9+3\right)-\left(0+0\right)=12$$

12) $$\frac{dy}{dx}=\frac{3}{2}\sqrt{x}$$

$$L=\int_0^4\sqrt{1+\left(\frac{3}{2}\sqrt{x}\right)^2}\,dx=\int_0^4\sqrt{1+\frac{9x}{4}}dx=\int_0^4\sqrt{\frac{4+9x}{4}}dx$$

$$=\int_0^4\frac{1}{2}\sqrt{4+9x}\,dx\qquad \begin{matrix}u=4+9x\\ du=9dx\end{matrix}\qquad \frac{1}{18}\int_4^{40}\sqrt{u}\,du$$

$$=\frac{1}{18}\left[\frac{2}{3}u^{3/2}\right]_4^{40}=\frac{1}{18}\left[\frac{2}{3}\left(40^{3/2}\right)-\frac{2}{3}\left(8\right)\right]$$

13) $$\frac{dx}{dy}=y^2-\frac{1}{4y^2}=\frac{4y^4-1}{4y^2}$$

$$L=\int_1^3\sqrt{1+\left(\frac{4y^4-1}{4y^2}\right)^2}\,dy=\int_1^3\sqrt{1+\left(\frac{16y^8-8y^4+1}{16y^4}\right)}dy=\int_1^3\sqrt{\frac{16y^4+16y^8-8y^4+1}{16y^4}}\,dy$$

$$=\int_1^3\frac{\sqrt{16y^8+8y^4+1}}{4y^2}dy=\int_1^3\frac{\sqrt{\left(4y^4+1\right)^2}}{4y^2}dy=\int_1^3\frac{4y^4+1}{4y^2}dy=\int_1^3\left(y^2+\frac{1}{4y^2}\right)dy$$

$$=\left[\frac{y^3}{3}-\frac{1}{4y}\right]_1^3=\left[\left(9-\frac{1}{12}\right)-\left(\frac{1}{3}-\frac{1}{4}\right)\right]=\frac{53}{6}$$

14) $$\frac{dx}{dy}=y^3-\frac{1}{4y^3}=\frac{4y^6-1}{4y^3}$$

$$L=\int_1^2\sqrt{1+\left(\frac{4y^6-1}{4y^3}\right)^2}\,dy=\int_1^2\sqrt{1+\left(\frac{16y^{12}-8y^6+1}{16y^6}\right)}dy=\int_1^2\sqrt{\frac{16y^6+16y^{12}-8y^6+1}{16y^6}}\,dy$$

$$=\int_1^2\frac{\sqrt{16y^{12}+8y^6+1}}{4y^3}dy=\int_1^2\frac{\sqrt{\left(4y^6+1\right)^2}}{4y^3}dy=\int_1^2\frac{4y^6+1}{4y^3}dy=\int_1^2\left(y^3+\frac{1}{4y^3}\right)dy$$

$$=\left[\frac{y^4}{4}-\frac{1}{8y^2}\right]_1^2=\left[\left(4-\frac{1}{32}\right)-\left(\frac{1}{4}-\frac{1}{8}\right)\right]=\frac{123}{32}$$

15) $\frac{dx}{dy}=\frac{y^2}{2}-\frac{1}{2y^2}=\frac{y^4-1}{2y^2}$

$$L=\int_1^2\sqrt{1+\left(\frac{y^4-1}{2y^2}\right)^2}\,dy=\int_1^2\sqrt{1+\left(\frac{y^8-2y^4+1}{4y^4}\right)}\,dy=\int_1^2\sqrt{\frac{4y^4+y^8-2y^4+1}{4y^4}}\,dy$$

$$=\int_1^2\frac{\sqrt{y^8+2y^4+1}}{2y^2}\,dy=\int_1^2\frac{\sqrt{\left(y^4+1\right)^2}}{2y^2}\,dy=\int_1^2\frac{y^4+1}{2y^2}\,dy=\int_1^2\left(\frac{y^2}{2}+\frac{1}{2y^2}\right)dy$$

$$=\left[\frac{y^3}{6}-\frac{1}{2y}\right]_1^2=\left[\left(\frac{4}{3}-\frac{1}{4}\right)-\left(\frac{1}{6}-\frac{1}{2}\right)\right]=\frac{17}{12}$$

16) $\frac{dy}{dx}=x^2+2x+1-\frac{4}{\left(4x+4\right)^2}=\left(x+1\right)^2-\frac{1}{4\left(x+1\right)^2}=\frac{4\left(x+1\right)^4-1}{4\left(x+1\right)^2}$

$$L=\int_0^2\sqrt{1+\left(\frac{4\left(x+1\right)^4-1}{4\left(x+1\right)^2}\right)^2}\,dx=\int_0^2\sqrt{1+\left(\frac{16\left(x+1\right)^8-8\left(x+1\right)^4+1}{16\left(x+1\right)^4}\right)}\,dx$$

$$=\int_0^2\sqrt{\frac{16\left(x+1\right)^4+16\left(x+1\right)^8-8\left(x+1\right)^4+1}{16\left(x+1\right)^4}}\,dx$$

$$=\int_0^2\frac{\sqrt{16\left(x+1\right)^8+8\left(x+1\right)^4+1}}{4\left(x+1\right)^2}\,dy=\int_0^2\frac{\sqrt{\left(4\left(x+1\right)^4+1\right)^2}}{4\left(x+1\right)^2}\,dy$$

$$=\int_0^2\frac{4\left(x+1\right)^4+1}{4\left(x+1\right)^2}\,dy=\int_0^2\left[\left(x+1\right)^2+\frac{1}{4\left(x+1\right)^2}\right]dx$$

$$=\left[\frac{\left(x+1\right)^3}{3}-\frac{1}{4\left(x+1\right)}\right]_0^2=\left[\left(9-\frac{1}{12}\right)-\left(\frac{1}{3}-\frac{1}{4}\right)\right]=\frac{53}{6}$$

17) $\frac{dx}{dy}=\sqrt{\sec^4 y-1}$

$$L=\int_{-\pi/4}^{\pi/4}\sqrt{1+\left(\sqrt{\sec^4 y-1}\right)^2}\,dy=\int_{-\pi/4}^{\pi/4}\sqrt{1+\sec^4 y-1}\,dy=\int_{-\pi/4}^{\pi/4}\sqrt{\sec^4 y}\,dy$$

$$=\int_{-\pi/4}^{\pi/4}\sec^2 y\,dy=\left[\tan y\right]_{-\pi/4}^{\pi/4}=\left[1-\left(-1\right)\right]=2$$

18) $\dfrac{dy}{dx} = \sqrt{3x^4 - 1}$

$$L = \int_{-2}^{-1} \sqrt{1 + \left(\sqrt{3x^4 - 1}\right)^2}\, dx = \int_{-2}^{-1} \sqrt{1 + 3x^4 - 1}\, dx = \int_{-2}^{-1} \sqrt{3x^4}\, dx$$

$$= \int_{-2}^{-1} x^2 \sqrt{3}\, dx = \left[\frac{x^3\sqrt{3}}{3}\right]_{-2}^{-1} = \left[\frac{-\sqrt{3}}{3} - \left(\frac{-8\sqrt{3}}{3}\right)\right] = \frac{7\sqrt{3}}{3}$$