Calculus p. 481ff Solutions

1a) n^2 1b) $(n+1)^2$ 1c) 3

2a) $\left(\frac{1}{3}\right)^n$ 2b) $(-1)^{n+1}\left(\frac{1}{n}\right)$ or $\frac{(-1)^{n-1}}{n}$ 2c) 5×10^{-n} or $\frac{5}{10^n}$

#3-6: $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1} = 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$

3) $\sum_{n=1}^{\infty}-\left(\frac{1}{2}\right)^{n-1} = -1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}+\ldots$ different

4) $\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^{n} = 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ same

5) $\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^{n} = 1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ same

6) $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^{n-1}} = -1+\frac{1}{2}-\frac{1}{4}+\frac{1}{8}+\ldots$ different

7)

$$1+1.1+1.11+1.111+\ldots = \sum_{n=0}^{\infty}1+\left(\frac{1}{10}\right)^n$$

$$\lim_{n\to\infty}\left(1+\left(\frac{1}{10}\right)^n\right) = \lim_{n\to\infty}\left(\frac{10^n+1}{10^n}\right) = 1$$

Since the limit as n approaches ∞ is not 0, the series diverges.

8)

$$(2-1)+(1-1)+(1-1)+(1-1)+\ldots = 1$$

$$2-(1-1)-(1-1)-(1-1)-\ldots = 2$$

A series cannot have two different sums, so the series diverges.

9)

$$S_1 = \frac{1}{2}$$
$$S_2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$
$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$$
$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$$
$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} = \frac{31}{32}$$

The series converges to 1.

10)

$$S_1 = 3$$
$$S_2 = 3 + 0.5 = 3.5$$
$$S_3 = 3 + 0.5 + 0.55 = 3.55$$
$$S_4 = 3 + 0.5 + 0.55 + 0.555 = 3.555$$
$$S_5 = 3 + 0.5 + 0.55 + 0.555 + 0.5555 = 3.5555$$

The series converges to $3.\overline{5}$.

11) Converges, as this is geometric with $r < 1$ $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$

The sum is $\dfrac{1}{1 - \frac{2}{3}} = 3$.

12)

$$(1-2) + (3-4) + (5-6) + (7-8) + \ldots = -1(\infty) = -\infty$$
$$1 + (-2+3) + (-4+5) + (-6+7) + \ldots = 1 + (1)(\infty) = \infty$$

A series cannot have two different sums, so the series diverges.

13) Converges, as this is geometric with $r < 1$ $\left(r = \frac{2}{3}, a = \frac{5}{4}\right)$

The sum is $\dfrac{\frac{5}{4}}{1 - \frac{2}{3}} = \dfrac{15}{4}$.

14) Diverges, as this is geometric with $r > 1$ $\quad r = \frac{5}{4}, a = \frac{2}{3}$.

15) $\lim_{n\to\infty}\left[\cos(n\pi)\right] \neq 0$, so the series diverges.

16) Converges, as this is geometric with $r < 1$ $\quad \left(r = -\frac{1}{10}, a = 3\right)$: $\quad \sum_{n=0}^{\infty} 3(-0.1)^n$

The sum is $\dfrac{3}{1-\left(-\frac{1}{10}\right)} = \frac{30}{11}$.

17) Looking at the first several partial sums, you should see that this can be rewritten as a Geometric Series:

$$S_0 = 1$$

$$S_1 = 1 - \frac{\sqrt{2}}{2}$$

$$S_2 = 1 - \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2$$

$$S_3 = 1 - \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^3$$

$$S_n = 1 - \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^3 + \ldots + \left(-\frac{\sqrt{2}}{2}\right)^n \quad \sum_{n=0}^{\infty}\left(-\frac{\sqrt{2}}{2}\right)^n$$

Converges, as this is geometric with $|r| < 1$ $\quad \left(r = -\frac{\sqrt{2}}{2}, a = 1\right)$

The sum is $\dfrac{1}{1-\left(-\frac{\sqrt{2}}{2}\right)} = \dfrac{1}{\frac{2+\sqrt{2}}{2}} = \dfrac{2}{2+\sqrt{2}}\left(\dfrac{2-\sqrt{2}}{2-\sqrt{2}}\right) = 2 - \sqrt{2}$.

18) $\lim_{n\to\infty}\left(\frac{n}{n+1}\right) = 1$, so this series diverges.

19) Converges, as this is geometric with $r < 1$ $\quad \left(r = \frac{e}{\pi}, a = \frac{e}{\pi}\right)$

The sum is $\dfrac{\frac{e}{\pi}}{1-\frac{e}{\pi}} = \dfrac{\frac{e}{\pi}}{\frac{\pi - e}{\pi}} = \dfrac{e}{\pi - e}$.

20) Rewriting: $\sum_{n=0}^{\infty}\frac{5^n}{6^{n+1}}=\sum_{n=0}^{\infty}\frac{5^n}{6^n(6)}=\sum_{n=0}^{\infty}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^n$

Converges, as this is geometric with $r<1$ $\left(r=\frac{5}{6},a=\frac{1}{6}\right)$

The sum is $\frac{\frac{1}{6}}{1-\frac{5}{6}}=\frac{\frac{1}{6}}{\frac{1}{6}}=1.$

21) Converges when $|r|<1$ $(r=2x,a=1)$

The interval of convergence is $-1<2x<1\Rightarrow -\frac{1}{2}<x<\frac{1}{2}.$

The function is $y=\frac{1}{1-2x},-\frac{1}{2}<x<\frac{1}{2}$

22) Converges when $|r|<1$ $(r=-1(x+1),a=1)$

The interval of convergence is $-1<-x-1<1\Rightarrow -2<x<0.$

The function is $y=\frac{1}{1-(-x-1)}=\frac{1}{2+x}.-2<x<0$

23) Converges when $|r|<1$ $\left(r=\left(\frac{-x+3}{2}\right),a=1\right)$

The interval of convergence is $-1<\frac{-x+3}{2}<1\Rightarrow -2<-x+3<2\Rightarrow -5<-x<-1\Rightarrow 1<x<5.$

The function is $y=\frac{1}{1-\left(\frac{-x+3}{2}\right)}=\frac{1}{\frac{x-1}{2}}=\frac{2}{x-1},1<x<5$

24) Converges when $|r|<1$ $\left(r=\left(\frac{x-1}{2}\right),a=3\right)$

The interval of convergence is $-1<\frac{x-1}{2}<1\Rightarrow -2<x-1<2\Rightarrow -1<x<3.$

The function is $y=3\left(\frac{1}{1-\left(\frac{x-1}{2}\right)}\right)=3\left(\frac{1}{\frac{3-x}{2}}\right)=\frac{6}{3-x},-1<x<3$

25) Converges when $|r|<1 \quad (r=\sin x, a=1)$

The interval of convergence is $-1<\sin x<1 \Rightarrow x \neq \frac{\pi}{2}+n\pi$, n is an odd integer..

The function is $y=\frac{1}{1-\sin x}, x \neq \left(\frac{\pi}{2}+n\pi\right)$, n is an odd integer.

26) Converges when $|r|<1 \quad (r=\tan x, a=1)$

The interval of convergence is $-1<\tan x<1 \Rightarrow x \neq \frac{\pi}{4}+\frac{\pi}{2}n$, n is an integer. $\left(\frac{\pi}{4}+\frac{\pi}{2}n=\frac{\pi+2n\pi}{4}\right)$.

The function is $y=\frac{1}{1-\tan x}, x \neq \left(\frac{\pi+2n\pi}{4}\right)$, n is an integer.

27) The function is $y=\frac{1}{1-2x}, -\frac{1}{2}<x<\frac{1}{2}$

$$y'=\frac{-1}{(1-2x)^2}(-2)=\frac{2}{(1-2x)^2}, -\frac{1}{2}<x<\frac{1}{2}$$

28) The function is $y=\frac{1}{2+x}, -2<x<0$

$$y'=\frac{-1}{(2+x)^2}, -2<x<0$$

29) The function is $y=\frac{2}{x-1}, 1<x<5$

$$y'=\frac{-2}{(x-1)^2}, 1<x<5$$

30) The function is $y=\frac{6}{3-x}, \quad -1<x<3$

$$y'=\frac{-6}{(3-x)^2}(-1)=\frac{6}{(3-x)^2}, -1<x<3$$

31) The function is $y=\frac{1}{1-2x}, -\frac{1}{2}<x<\frac{1}{2}$

$$\frac{dy}{dx}=\frac{1}{1-2x} \Rightarrow y=\int \frac{dx}{1-2x} \quad \begin{matrix} u=1-2x \\ du=-2dx \end{matrix} \quad -\frac{1}{2}\int \frac{du}{u} \Rightarrow y=-\frac{1}{2}\ln|1-2x|, -\frac{1}{2}<x<\frac{1}{2}$$

32) The function is $y = \frac{1}{2+x}, -2 < x < 0$

$$\frac{dy}{dx} = \frac{1}{2+x} \Rightarrow y = \int \frac{dx}{2+x} \quad \begin{array}{c} u = 2+x \\ du = dx \end{array} \quad \int \frac{du}{u} \Rightarrow y = \ln|2+x|, -2 < x < 0$$

33) The function is $y = \frac{2}{x-1}, 1 < x < 5$

$$\frac{dy}{dx} = \frac{2}{x-1} \Rightarrow y = \int \frac{2dx}{x-1} \quad \begin{array}{c} u = x-1 \\ du = dx \end{array} \quad 2\int \frac{du}{u} \Rightarrow y = 2\ln|x-1|, 1 < x < 5$$ BOOK IS WRONG!

34) The function is $y = \frac{6}{3-x}, \quad -1 < x < 3$

$$\frac{dy}{dx} = \frac{6}{3-x} \Rightarrow y = \int \frac{6dx}{3-x} \quad \begin{array}{c} u = 3-x \\ du = -dx \end{array} \quad -6\int \frac{du}{u} \Rightarrow y = -6\ln|3-x|, -1 < x < 3$$

35a) $S_1 = 2, S_2 = 6, S_3 = 12, S_4 = 20$. The difference between partial sums is growing by $2n$.

35b) $S_1 = 1, S_2 = 0, S_3 = 1, S_4 = 0$. The partial sums will alternate between 1 and 0.

35c) $S_1 = -2, S_2 = 2, S_3 = -4, S_4 = 4, S_5 = -6, S_6 = 6$. The partial sums will alternate and continue to increase in absolute value.

36) $\lim_{x\to\infty} \frac{e^{n\pi}}{\pi^{ne}} = \lim_{x\to\infty} \left(\frac{e^{\pi}}{\pi^{e}}\right)^n = \infty$, since $\frac{e^{\pi}}{\pi^{e}} > 1$, therefore the series diverges.

37) $\frac{1}{1-x} = 20 \Rightarrow 1 = 20 - 20x \Rightarrow x = \frac{19}{20}$

38) You really think I'm going to answer this for you? :->

39a) $\frac{2}{1-r} = 5 \Rightarrow 2 = 5 - 5r \Rightarrow r = \frac{3}{5} \qquad \sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1}$

39b) $\frac{13/2}{1-r} = 5 \Rightarrow \frac{13}{2} = 5 - 5r \Rightarrow r = -\frac{3}{10} \qquad \sum_{n=1}^{\infty} \frac{13}{2}\left(-\frac{3}{10}\right)^{n-1}$

40) $0.\overline{21} = \frac{21}{99} = \frac{7}{33} \qquad \sum_{n=1}^{\infty} 21(.01)^n \qquad S = \frac{.21}{1-.01} = \frac{7}{33}$

41) $0.\overline{234} = \frac{234}{999} = \frac{26}{111} \qquad \sum_{n=1}^{\infty} 234(.001)^n \qquad S = \frac{.234}{1-.001} = \frac{26}{111}$

42) $\frac{7}{9}$ 43) $\frac{d}{9}$ 44) $\frac{6}{90}=\frac{1}{15}$ 45) $\frac{1413}{999}=\frac{157}{111}$

46)
$$100000n = 124123.\overline{123}$$
$$100n = 124.\overline{123}$$
$$99900n = 123999 \Rightarrow n = \frac{123999}{99900} = \frac{41333}{33300}$$

47)
$$1000000n = 3142857.\overline{142857}$$
$$n = 3.\overline{142857}$$
$$999999n = 3142854 \Rightarrow n = \frac{3142854}{999999} = \frac{349206}{111111} = \frac{116402}{37037} = \frac{3146}{1001} = \frac{22}{7}$$

48) $4+2\sum_{n=1}^{\infty}4(0.6)^n = 4+2\left(\frac{2.4}{1-0.6}\right) = 4+\frac{4.8}{0.4} = 16m$

49) $t=\sqrt{\frac{16}{4.9}} \approx 1.429s$

50) $\sum_{n=0}^{\infty}4\left(\frac{1}{2}\right)^n = \frac{4}{1-.5} = 8$