Calculus Mid-Term Review KEY

1) $\lim_{x\to 3}\dfrac{x^2-5}{x+3}=\dfrac{4}{6}=\dfrac{2}{3}$

2) $\lim_{x\to 3}\dfrac{x^2-9}{x-3}=6$

For #3 – 5, use the following piecewise-defined function:

$$f(x)=\begin{cases} x^2-2, & x<1 \\ -1, & x=1 \\ x-2, & x>1 \end{cases}$$

3) $\lim_{x\to 1^-} f(x)=-1$

4) $\lim_{x\to 1^+} f(x)=-1$

5) $\lim_{x\to 1} f(x)=-1$

6) Find $\dfrac{dy}{dx}$ if $y=x^2+6x-\dfrac{1}{x^3}$

$$\frac{dy}{dx}=2x+6+\frac{3}{x^4}$$

7) Find $\dfrac{dy}{dx}$ if $y=x^2\cos x$

$$\frac{dy}{dx}=2x\cos x-x^2\sin x$$

8) Find $\dfrac{dy}{dx}$ if $y=\dfrac{x^2+1}{3x-5}$

$$\frac{dy}{dx}=\frac{(3x-5)(2x)-(x^2+1)3}{(3x-5)^2}=\frac{3x^2-10x-3}{(3x-5)^2}$$

9) Find $\dfrac{d^2y}{dx^2}$ if $y=x^3+2x^2-8x+21$

$$\frac{dy}{dx}=3x^2+4x-8$$

$$\frac{d^2y}{dx^2}=6x+4$$

10) Find $\dfrac{d^2y}{dx^2}$ if $y=\tan x$

$$\frac{dy}{dx}=\sec^2 x$$

$$\frac{d^2y}{dx^2}=2\sec x(\sec x\tan x)=2\sec^2 x\tan x$$

11) Find an equation for the line **tangent** to $y=x^3+x^2-3x+1$ at $x=1$

$$\frac{dy}{dx}=3x^2+2x-3 \quad (1,0) \quad y=2(x-1)$$

12) Find an equation for the line **normal** to $y=x^3+x^2-3x+1$ at $x=1$

$$\frac{dy}{dx}=3x^2+2x-3 \quad (1,0) \quad y=-\frac{1}{2}(x-1)$$

13) Find $\dfrac{dy}{dx}$ if $y=\cos^3 x$

$$\frac{dy}{dx}=-3\cos^2 x\sin x$$

14) Find $\dfrac{dy}{dx}$ if $y=\ln(\tan x)$

$$\frac{dy}{dx}=\frac{\sec^2 x}{\tan x}=\frac{1}{\cos x\sin x}$$

15) Find $\dfrac{dy}{dx}$ if $y=e^{\sec x}$

$$\frac{dy}{dx}=e^{\sec x}\sec x\tan x$$

16) Find $\dfrac{dy}{dx}$ if $x^2+y^2=2y$

$$2x+2y\frac{dy}{dx}=2\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{2x}{2-2y}=\frac{x}{1-y}$$

17) Find $\dfrac{dy}{dx}$ if $x=\tan t$ and $y=\cos t$

$$\frac{dy}{dx}=\frac{-\sin t}{\sec^2 t}=-\sin t\cos^2 t$$

For #18 – 21, use the following function: $f(x) = -\frac{1}{3}x^3 - 2x^2 + 5x$

$f'(x) = -x^2 - 4x + 5 = -1(x+5)(x-1)$

$f'(x) = 0 \Rightarrow x = -5, x = 1$

18) Find the local maximum value.

y = 8/3 when x = 1

19) Find the local minimum value.

y = -200/3 when x = -5

20) Find the intervals where the function is increasing.

[-5, 1]

21) Find the absolute maximum value.

None

22) If $f''(x) = (x+3)(x-2)$, where is the function concave down?

(-3, 2)

23) A 17-foot ladder is sliding down a wall at 2.5 inches per second. How fast is the base of the ladder moving away from the wall when the base is 8 feet from the wall?

$$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt}$$

$$8\frac{dx}{dt} + 15(-2.5) = 17(0)$$

$$\frac{dx}{dt} = \frac{37.5}{8} = 4.725 \text{ in/sec}$$