Calculus   Lesson 8.4 Improper Integrals SOLUTIONS

Calculate the values of the integrals (if they converge).
Problems from Calculus, 2nd Edition by Deborah Hughes-Hallett

1) $y = \int_1^\infty e^{-2x}dx = \lim_{b\to\infty}\int_1^b e^{-2x}dx = \lim_{b\to\infty}\left[-\frac{1}{2}e^{-2x}\right]_1^b = \lim_{b\to\infty}\left[-\frac{1}{2}e^{-2b}+\frac{1}{2}e^{-2}\right] = \frac{1}{2e^2}$

2) $y = \int_1^\infty \frac{x}{4+x^2}dx = \lim_{b\to\infty}\left[\int_1^b \frac{x}{4+x^2}dx\right] = \lim_{b\to\infty}\left[\frac{1}{2}\ln\left(4+x^2\right)\right]_1^b = \lim_{b\to\infty}\left[\frac{1}{2}\ln\left(4+b^2\right)-\frac{1}{2}\ln 5\right] = \infty$

3) $y = \int_{-\infty}^0 \frac{e^x}{1+e^x}dx = \lim_{a\to-\infty}\left[\int_a^0 \frac{e^x}{1+e^x}dx\right] = \lim_{a\to-\infty}\left[\ln\left(1+e^x\right)\right]_a^0 = \lim_{a\to-\infty}\left[\ln 2-\ln(1+e^a)\right] = \ln 2$

4) $y = \int_{\pi/4}^{\pi/2} \frac{\sin x}{\sqrt{\cos x}}dx = \lim_{b\to\pi/2}\int_{\pi/4}^b \frac{\sin x}{\sqrt{\cos x}}dx = \lim_{b\to\pi/2}\left[-2\sqrt{\cos x}\right]_{\pi/4}^b = \lim_{b\to\pi/2}\left[-2\sqrt{\cos b}+2\sqrt{\cos\frac{\pi}{4}}\right]_{\pi/4}^b = 2\sqrt{\frac{\sqrt{2}}{2}}$

5) $y = \int_{-1}^1 \frac{1}{x}dx = \lim_{a\to0}\int_{-1}^a \frac{dx}{x} + \lim_{b\to0}\int_b^1 \frac{dx}{x} = \lim_{a\to0}\left[\ln|x|\right]_{-1}^a + \lim_{b\to0}\left[\ln|x|\right]_b^1 = \lim_{a\to0}\left[\ln|a|-0\right]_{-1}^a + \lim_{b\to0}\left[0-\ln|b|\right]_b^1$

6) $y = \int_3^\infty \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\int_3^b \frac{dx}{x(\ln x)^2} = \lim_{b\to\infty}\left[\frac{-1}{\ln x}\right]_3^b = \lim_{b\to\infty}\left[\frac{-1}{\ln b}-\frac{-1}{\ln 3}\right] = \frac{1}{\ln 3}$