Calculus Disk/Washer Method Expanded Solutions

Use the disk or washer method to find the volume of the solid generated when the region bounded by the following is revolved about the x-axis.

1) $y = 2x,\ y = 0,\ x = 3$

$$V = \pi \int_0^3 (2x)^2\, dx = \pi \left[\frac{4x^3}{3} \right]_0^3 = \pi[36 - 0] = 36\pi$$

2) $y = 2 - 2x,\ y = 0,\ x = 0$

$$V = \pi \int_0^1 (2-2x)^2\, dx = \pi \left[4x - 4x^2 + \frac{4x^3}{3} \right]_0^1 = \pi \left[4 - 4 + \frac{4}{3} - (0 - 0\_0) \right] = \frac{4}{3}\pi$$

3) $y = e^{-x},\ y = 0,\ x = 0,\ y = \ln 4$

$$V = \pi \int_0^{\ln 4} \left(e^{-x}\right)^2 dx = \pi \left[-\frac{1}{2} e^{-2x} \right]_0^{\ln 4} = \pi \left[-\frac{1}{2} e^{-2\ln 4} - \left(-\frac{1}{2} e^{0} \right) \right] = \pi \left[-\frac{1}{2} \left(\frac{1}{e^{\ln 16}} \right) + \frac{1}{2} \right] = \pi \left[-\frac{1}{2} \left(\frac{1}{16} \right) + \frac{1}{2} \right] = \frac{15}{32}\pi$$

4) $y = \sqrt{25 - x^2},\ y = 0 \quad \sqrt{25 - x^2} = 0 \Rightarrow 25 - x^2 = 0 \Rightarrow x = \pm 5$

$$V = \pi \int_{-5}^{5} \left(\sqrt{25 - x^2} \right)^2 dx = \pi \left[25x - \frac{x^3}{3} \right]_{-5}^{5} = \pi \left[125 - \frac{125}{3} - \left(-125 + \frac{125}{3} \right) \right] = \frac{500}{3}\pi$$

5) $y = \sec x,\ y = 0,\ x = 0,\ x = \dfrac{\pi}{4}$

$$V = \pi \int_0^{\pi/4} (\sec x)^2\, dx = \pi [\tan x]_0^{\pi/4} = \pi \left[\tan \frac{\pi}{4} - \tan 0 \right] = \pi[1 - 0] = \pi$$

6) $y = x,\ y = 2\sqrt{x} \quad x = 2\sqrt{x} \Rightarrow x^2 = 4x \Rightarrow x^2 - 4x = 0 \Rightarrow x = 0, 4$

$$V = \pi \int_0^4 \left[\left(2\sqrt{x}\right)^2 - x^2 \right] dx = \pi \left[2x^2 - \frac{x^3}{3} \right]_0^4 = \pi \left[2(16) - \frac{64}{3} - (0 - 0) \right] = \frac{32}{3}\pi$$

7) $y = x,\ y = \sqrt[4]{x} \quad x = \sqrt[4]{x} \Rightarrow x^4 = x \Rightarrow x^4 - x = 0 \Rightarrow x = 0, 1$

$$V = \pi \int_0^1 \left[\left(\sqrt[4]{x}\right)^2 - x^2 \right] dx = \pi \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right]_0^1 = \pi \left[\frac{2}{3} - \frac{1}{3} - (0 - 0) \right] = \frac{\pi}{3}$$

8) $y = e^{x/2},\ y = e^{-x/2},\ x = \ln 2,\ x = \ln 3$

$$V = \pi \int_{\ln 2}^{\ln 3} \left[\left(e^{x/2}\right)^2 - \left(e^{-x/2}\right)^2 \right] dx = \pi \left[e^x - \left(-e^{-x}\right) \right]_{\ln 2}^{\ln 3} = \pi \left[e^{\ln 3} + e^{-\ln 3} - \left(e^{\ln 2} + e^{-\ln 2} \right) \right] = \pi \left[3 + \frac{1}{3} - \left(2 + \frac{1}{2} \right) \right] = \frac{5\pi}{6}$$

9) $y = x,\ y = x + 2,\ x = 0,\ x = 4$

$$V = \pi \int_0^4 \left[(x+2)^2 - x^2 \right] dx = \pi \int_0^4 \left[x^2 + 4x + 4 - x^2 \right] dx = \pi \left[2x^2 + 4x \right]_0^4 = \pi[32 + 16 - (0 + 0)] = 48\pi$$

10) $y=\sqrt{\sin x},\ y=1,\ x=0 \quad \sqrt{\sin x}=1 \Rightarrow \sin x=1 \Rightarrow x=\dfrac{\pi}{2}$

$$V=\pi\int_0^{\pi/2}\left[(1)^2-\left(\sqrt{\sin x}\right)^2\right]dx=\pi\left[x+\cos x\right]_0^{\pi/2}=\pi\left[\frac{\pi}{2}+\cos\frac{\pi}{2}-(0+\cos 0)\right]=\frac{\pi^2}{2}-\pi$$

Use the disk or washer method to find the volume of the solid generated when the region bounded by the following is revolved about the *y*-axis.

11) $y=x,\ y=2x,\ y=6 \quad x=2x \Rightarrow x=0$

$$V=\pi\int_0^6\left[y^2-\left(\frac{y}{2}\right)^2\right]dy=\pi\left[\left(\frac{3}{4}\right)\frac{y^3}{3}\right]_0^6=\pi\left[54-0\right]=54\pi$$

12) $y=x^3,\ y=0,\ x=2$

$$V=\pi\int_0^8\left[(2)^2-\left(\sqrt[3]{y}\right)^2\right]dy=\pi\left[4y-\frac{y^{5/3}}{5/3}\right]_0^8=\pi\left[32-\frac{3}{5}(32)-(0-0)\right]=\frac{64}{5}\pi$$

13) $x=\sqrt{4-y^2},\ x=0 \quad \sqrt{4-y^2}=0 \Rightarrow 4-y^2=0 \Rightarrow y=\pm 2$

$$V=\pi\int_{-2}^2\left[\left(\sqrt{4-y^2}\right)^2\right]dy=\pi\left[4y-\frac{y^3}{3}\right]_{-2}^2=\pi\left[8-\frac{8}{3}-\left(-8+\frac{8}{3}\right)\right]=\frac{32}{3}\pi$$