

Calculus Chapter 7 Review Solutions

$$1) \int_0^5 (t^2 - 0.2t^3) dt = \left[\frac{t^3}{3} - \frac{t^4}{20} \right]_0^5 = \left(\frac{125}{3} - \frac{625}{20} \right) - (0 - 0) = \frac{2500 - 1875}{60} = \frac{625}{60} = 10\frac{5}{12} \text{ ft}$$

$$2) \int_0^7 (4 + 0.001t^4) dt = \left[4t + \frac{t^5}{5000} \right]_0^7 = \left(28 + \frac{16807}{5000} \right) - (0 - 0) \approx 31.361 \text{ gal.}$$

$$3) \int_0^{100} (21 - e^{0.03x}) dx = \left[21x - \frac{e^{0.03x}}{0.03} \right]_0^{100} = \left(2100 - \frac{100}{3} e^3 \right) - \left(0 - \frac{100}{3} \right) = \frac{6400}{3} - \frac{100e^3}{3} \approx 1464 \text{ billboards}$$

$$4) \int_0^2 (11 - 4x) dx = \left[11x - 2x^2 \right]_0^2 = (22 - 8) - (0 - 0) = 14 \text{ g}$$

$$5) \int_0^{24} \left(600 - 300 \cos\left(\frac{\pi t}{12}\right) \right) dx = \left[600x - \frac{3600}{\pi} \sin\left(\frac{\pi t}{12}\right) \right]_0^{24} = (14400 - 0) - (0 - 0) = 14,400 \text{ kWh}$$

$$6) \int_1^2 \left(x - \frac{1}{x^2} \right) dx = \left[\frac{x^2}{2} + \frac{1}{x} \right]_1^2 = \left(2 + \frac{1}{2} \right) - \left(\frac{1}{2} - 1 \right) = 1$$

$$7) \quad x + 1 = 3 - x^2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0 \Rightarrow x = -2, 1$$

$$\int_{-2}^1 (3 - x^2 - (x + 1)) dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^1 = \left(2 - \frac{1}{2} - \frac{1}{3} \right) - \left(-4 - 2 + \frac{8}{3} \right) = \frac{9}{2}$$

$$8) \quad \sqrt{x} + \sqrt{y} = 1 \Rightarrow \sqrt{y} = 1 - \sqrt{x} \Rightarrow (\sqrt{y})^2 = (1 - \sqrt{x})^2 \Rightarrow y = 1 - 2\sqrt{x} + x$$

$$\int_0^1 (1 - 2\sqrt{x} + x) dx = \left[x - 2\left(\frac{x^{3/2}}{3/2}\right) + \frac{x^2}{2} \right]_0^1 = \left(1 - \frac{4}{3} + \frac{1}{2} \right) - (0 - 0 - 0) = \frac{1}{6}$$

$$9) \quad \int_0^3 2y^2 dy = \left[\frac{2y^3}{3} \right]_0^3 = (18 - 0) = 18$$

$$10) \quad y^2 - 4 = y + 16 \Rightarrow y^2 - y - 20 = 0 \Rightarrow (y - 5)(y + 4) = 0 \Rightarrow y = -4, 5$$

$$\frac{1}{4} \int_{-4}^5 (y + 16 - (y^2 - 4)) dy = \frac{1}{4} \left[20y + \frac{y^2}{2} - \frac{y^3}{3} \right]_{-4}^5 = \frac{1}{4} \left[\left(100 + \frac{25}{2} - \frac{125}{3} \right) - \left(-80 + 8 + \frac{64}{3} \right) \right] = 30.375$$

$$11) \quad \int_0^{\pi/4} (x - \sin x) dx = \left[\frac{x^2}{2} + \cos x \right]_0^{\pi/4} = \left(\frac{\pi^2}{32} + \frac{\sqrt{2}}{2} \right) - (0 + 1) \approx 0.0155$$

$$12) \quad \int_0^{\pi} (2 \sin x - \sin 2x) dx = \left[-2 \cos x + \frac{1}{2} \cos 2x \right]_0^{\pi} = \left(2 + \frac{1}{2} \right) - \left(-2 + \frac{1}{2} \right) = 4$$

13) Solved with a graphing calculator:

$$\cos x = 4 - x^2 \Rightarrow x \approx -2.128, 2.128$$

$$\int_{-2.128}^{2.128} (4 - x^2 - \cos x) dx \approx 8.902$$

14) Solved with a graphing calculator:

$$\sec^2 x = 3 - |x| \Rightarrow x \approx -0.826, 0.826$$

$$\int_{-0.826}^{0.826} (3 - |x| - \sec^2 x) dx \approx 2.104$$

$$15) \quad 1 + \cos x = 2 - \cos x \Rightarrow 2 \cos x = 1 \Rightarrow \cos x = \frac{1}{2} \Rightarrow y = -\frac{\pi}{3}, \frac{\pi}{3}$$

$$\int_{-\pi/3}^{\pi/3} (1 + \cos x - (2 - \cos x)) dx = [-x + 2 \sin x]_{-\pi/3}^{\pi/3} = \left(-\frac{\pi}{3} + 2 \left(\frac{\sqrt{3}}{2} \right) \right) - \left(-\left(-\frac{\pi}{3} \right) + 2 \left(-\frac{\sqrt{3}}{2} \right) \right) = 2\sqrt{3} - \frac{2\pi}{3}$$

$$16) \quad \int_{\pi/3}^{5\pi/3} (2 - \cos x - (1 + \cos x)) dx = [x - 2 \sin x]_{\pi/3}^{5\pi/3} = \left(\frac{5\pi}{3} - 2 \left(-\frac{\sqrt{3}}{2} \right) \right) - \left(\frac{\pi}{3} - 2 \left(\frac{\sqrt{3}}{2} \right) \right) = \frac{4\pi}{3} + 2\sqrt{3}$$

17) Solved with a graphing calculator:

$$x^3 - x = \frac{x}{x^2 + 1} \Rightarrow x \approx -1.189, 0, 1.189$$

$$\int_{-1.189}^0 \left(x^3 - x - \frac{x}{x^2 + 1} \right) dx + \int_0^{1.189} \left(\frac{x}{x^2 + 1} - (x^3 - x) \right) dx \approx 1.296$$

18) Solved with a graphing calculator:

$$3^{1-x^2} = \frac{x^2 - 3}{10} \Rightarrow x \approx -1.893, 1.893$$

$$\int_{-1.893}^{1.893} \left(3^{1-x^2} - \frac{x^2 - 3}{10} \right) dx \approx 5.731$$

$$19) \quad \int_{-\pi}^{\pi} (x \sin x - (-x \sin x)) dx = 2 \int_{-\pi}^{\pi} x \sin x dx \quad \begin{array}{ll} u = x & dv = \sin x dx \\ du = dx & v = -\cos x \end{array}$$

$$2 \int_{-\pi}^{\pi} x \sin x dx = 2 \left\{ [-x \cos x]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -\cos x dx \right\} = 2 \left\{ [-x \cos x]_{-\pi}^{\pi} + [\sin x]_{-\pi}^{\pi} \right\}$$

$$= 2 \left\{ (-\pi \cos \pi - (-(-\pi) \cos(-\pi))) + \sin \pi - \sin(-\pi) \right\} = 2 \{ \pi + \pi + 0 - 0 \} = 4\pi$$

$$20) \quad \int_{-1}^1 \pi(3x^4)^2 dx = \pi \int_{-1}^1 9x^8 dx = 9\pi \left[\frac{x^9}{9} \right]_{-1}^1 = 9\pi \left[\frac{1}{9} - \left(-\frac{1}{9} \right) \right] = 2\pi$$

$$21a) \quad \pi \int_0^4 \left((2\sqrt{x})^2 - x^2 \right) dx = \pi \int_0^4 (4x - x^2) dx = \pi \left[2x^2 - \frac{x^3}{3} \right]_0^4 = \pi \left[\left(32 - \frac{64}{3} \right) - (0 - 0) \right] = \frac{32}{3} \pi$$

$$21b) \quad \pi \int_0^4 \left(y^2 - \left(\frac{y^2}{4} \right)^2 \right) dy = \pi \int_0^4 \left(y^2 - \frac{y^4}{16} \right) dy = \pi \left[\frac{y^3}{3} - \frac{y^5}{80} \right]_0^4 = \pi \left[\left(\frac{64}{3} - \frac{1024}{80} \right) - (0 - 0) \right] = \frac{128}{15} \pi$$

$$21c) \quad \pi \int_0^4 \left(\left(4 - \frac{y^2}{4} \right)^2 - (4 - y)^2 \right) dy = \pi \int_0^4 \left(16 - 2y^2 + \frac{y^4}{16} - (16 - 8y + y^2) \right) dy = \pi \int_0^4 \left(8y - 3y^2 + \frac{y^4}{16} \right) dy$$

$$= \pi \left[4y^2 - y^3 + \frac{y^5}{80} \right]_0^4 = \pi \left[\left(64 - 64 + \frac{1024}{80} \right) - (0 - 0) \right] = \frac{64}{5} \pi$$

$$21d) \quad \pi \int_0^4 \left((4 - x)^2 - (4 - 2\sqrt{x})^2 \right) dx = \pi \int_0^4 \left(16 - 8x + x^2 - (16 - 16\sqrt{x} + 4x) \right) dx = \pi \int_0^4 (16\sqrt{x} - 12x + x^2) dx$$

$$= \pi \left[\frac{16x^{3/2}}{3/2} - 6x^2 + \frac{x^3}{3} \right]_0^4 = \pi \left[\left(\frac{256}{3} - 96 + \frac{64}{3} \right) - (0 - 0 + 0) \right] = \frac{32}{3} \pi$$

$$22a) \quad \pi \int_0^2 \left((\sqrt{2y})^2 \right) dy = \pi \int_0^2 (2y) dx = \pi [y^2]_0^2 = \pi [4 - 0] = 4\pi$$

$$22b) \quad \pi \int_0^k \left((\sqrt{2y})^2 \right) dy = \pi \int_0^k (2y) dx = \pi [y^2]_0^k = \pi [k^2 - 0] = k^2 \pi$$

$$22c) \quad V = k^2 \pi \quad \frac{dV}{dt} = 2k\pi \frac{dk}{dt} \quad 2 = 2(1)\pi \frac{dk}{dt} \Rightarrow \frac{dk}{dt} = \frac{1}{\pi} \text{ units per sec.}$$

$$23) \quad \frac{4x^2}{121} + \frac{y^2}{12} = 1 \Rightarrow 48x^2 + 121y^2 = 1452 \Rightarrow y = \sqrt{\frac{1452 - 48x^2}{121}}$$

$$V = \pi \int_{-11/2}^{11/2} \left(\sqrt{\frac{1452 - 48x^2}{121}} \right)^2 dx \approx 276 \text{ in}^3$$

$$24) \quad \int_0^\pi \pi (\sin x)^2 dx = \frac{\pi^2}{2}$$

$$25) \quad \pi \int_0^{\ln 2} \left((e^{x/2})^2 - 1^2 \right) dx = \pi \int_0^{\ln 3} (e^x - 1) dx = \pi [e^x - x]_0^{\ln 3} = \pi [(e^{\ln 3} - \ln 3) - (e^0 - 0)] = \pi (2 - \ln 3)$$

26)

$$\sqrt{4-x^2} = \sqrt{3} \Rightarrow x = 1$$

$$\begin{aligned} 2\pi \int_{-2}^{-1} (\sqrt{4-x^2})^2 dx + \pi (\sqrt{3})^2 (2) &= 2\pi \int_{-2}^{-1} (4-x^2) dx + 6\pi \\ &= 2\pi \left[4x - \frac{x^3}{3} \right]_{-2}^{-1} + 6\pi = 2\pi \left[\left(-4 - \left(-\frac{1}{3} \right) \right) - \left(-8 - \left(-\frac{8}{3} \right) \right) \right] + 6\pi + 2\pi \left[\frac{5}{3} \right] = \frac{28}{3}\pi \end{aligned}$$

27) Set up by hand, use GC for antiderivative.

$$9 - x^2 = 0 \Rightarrow \pm 3$$

$$\int_{-3}^3 \sqrt{1+(-2x)^2} dx = \int_{-3}^3 \sqrt{1+4x^2} dx \approx 19.494$$

28) Set up by hand, use GC for antiderivative.

$$x^3 - x = x - x^3 \Rightarrow 2x^3 - 2x = 0 \Rightarrow x = 0, \pm 1$$

$$\int_{-1}^1 \sqrt{1+(3x^2-1)^2} dx + \int_{-1}^1 \sqrt{1+(1-3x^2)^2} dx \approx 5.245$$

29) Set up by hand, use GC for antiderivative.

$$y' = 3x^2 - 6x \quad y' = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0, 2$$

$$y'' = 6x - 6 \quad y''(0) = -6 \Rightarrow \text{local max} \quad y''(2) = 6 \Rightarrow \text{local min}$$

$$\int_0^2 \sqrt{1+(3x^2-6x)^2} dx \approx 4.592 \quad \frac{4.592 \text{ units}}{2 \text{ units/sec}} = 2.296 \text{ sec.}$$

30a)

$$y = k \sin x \quad y' = k \cos x \quad \int_0^{2\pi} \sqrt{1+(k \cos x)^2} dx = \int_0^{2\pi} \sqrt{1+k^2 \cos^2 x} dx$$

$$y = \sin kx \quad y' = \frac{1}{k} \cos kx \quad \int_0^{2\pi} \sqrt{1+\left(\frac{1}{k} \cos kx\right)^2} dx = \int_0^{2\pi} \sqrt{1+\frac{\cos^2 kx}{k^2}}$$

#30a is not complete

31)

$$\begin{aligned} F' &= \sqrt{x^4-1} \quad \int_2^5 \sqrt{1+(\sqrt{x^4-1})^2} dx = \int_2^5 \sqrt{x^4} dx = \int_2^5 x^2 dx \\ &= \left[\frac{x^3}{3} \right]_2^5 = \frac{125}{3} - \frac{8}{3} = \frac{117}{3} = 39 \end{aligned}$$

32a)

$$W = \int_0^{40} (100) dx = [100x]_0^{40} = 4000 - 0 = 4000 \text{ J}$$

32b)

$$W = \int_0^{40} 0.8(40-x) dx = [32x - 0.4x^2]_0^{40} = (1280 - 0.4(1600)) - (0 - 0) = 640 \text{ J}$$

32c)

$$4000 \text{ J} + 640 \text{ J} = 4,640 \text{ J}$$

$$33) \quad W = \int_0^{4750} 8 \left(800 - \frac{400}{4750} x \right) dx = 22,800,000 \text{ ft} \cdot \text{lb}$$

$$34) \quad 80 = k(0.3) \Rightarrow k = \frac{800}{3} \quad W = \int_0^{0.3} \frac{800}{3} x dx = \left[\frac{400}{3} x^2 \right]_0^{0.3} = 12 - 0 = 12 \text{ J}$$

$$\int_{0.3}^{1.3} \frac{800}{3} x dx = \left[\frac{400}{3} x^2 \right]_{0.3}^{1.3} = 225 \frac{1}{3} - 12 = 213 \frac{1}{3} \text{ J}$$

$$36) \quad \int_{-8}^{-2} 0.04\pi \left(\sqrt{64 - y^2} \right)^2 (-y) dy \approx 113.097 \text{ in} \cdot \text{lb}$$

$$37) \quad \int_0^2 80(4y)(2 - y) dy \approx 426.667 \text{ ft} \cdot \text{lb}$$

38) Note: 57 lbs per cubic FOOT!

$$\text{Base:} \quad \frac{5.75}{12} \times \frac{3.5}{12} \times \frac{10}{12} \times 57 \approx 6.638 \text{ lb}$$

$$5.75'' \text{ wide side:} \quad \int_0^{5/6} 57 \left(\frac{5.75}{12} \right) \left(\frac{5}{6} - y \right) dy \approx 9.484 \text{ lb}$$

$$3.5'' \text{ wide side:} \quad \int_0^{5/6} 57 \left(\frac{3.5}{12} \right) \left(\frac{5}{6} - y \right) dy \approx 5.773 \text{ lb}$$

$$39) \quad x^{1/2} + y^{1/2} = \sqrt{6} \Rightarrow y = \left(\sqrt{6} - \sqrt{x} \right)^2 = 6 - 2\sqrt{6x} + x$$

$$\int_0^6 \left(6 - 2\sqrt{6x} + x \right)^2 dx = 14.4$$