Calculus Chapter 7 Review Problems

1) Find the area between $y = 2(x-4)^2 + 3$ and $y = x - 2$ on [-2,4]

$$A = \int_{-2}^{4} \left(\left(2x^2 - 16x + 35\right) - \left(x - 2\right) \right) dx = \left[\frac{2x^3}{3} - \frac{17x^2}{2} + 37x \right]_{-2}^{4} = 168$$

2) Find the area between $y = 2x - 1$ and $y = -\frac{3}{2}x^2 + \frac{7}{2}x + 8$

$$2x - 1 = -\frac{3}{2}x^2 + \frac{7}{2}x + 8 \Rightarrow x = -2, 3$$

$$A = \int_{-2}^{3} \left(\left(-\frac{3}{2}x^2 + \frac{7}{2}x + 8 \right) - \left(2x - 1\right) \right) dx = \left[-\frac{x^3}{2} + \frac{3x^2}{4} + 9x \right]_{-2}^{3} = \frac{125}{4}$$

3) Find the area between $y = 2x - 1$ and $y = 3 - 2x$ on [-4,2]

$$A = \int_{-4}^{2} \left(\left(3 - 2x\right) - \left(2x - 1\right) \right) dx = \left[4x - 2x^2 \right]_{-4}^{2} = 48$$

4) Find the area between $x = y^2 + 3$ and $y = 2x - 2$ from y = -4 to y = 3

$$A = \int_{-4}^{3} \left(\left(y^2 + 3\right) - \left(\frac{y+2}{2} \right) \right) dy = \left[\frac{y^3}{3} - \frac{y^2}{4} + 2y \right]_{-4}^{3} = \left(9 - \frac{9}{4} + 6\right) - \left(-\frac{64}{3} - 4 - 8 \right) = \frac{553}{12}$$

5) Find the area between $x = -2(y+2)^2 + 1$ and $y = x$

$$-2(y+2)^2 + 1 = y \Rightarrow y = -\frac{7}{2}, -1$$

$$A = \int_{-7/2}^{-1} \left(-2(y+2)^2 + 1 - y \right) dy = \left[-\frac{2y^3}{3} - \frac{9y^2}{2} - 7y \right]_{-7/2}^{-1} = \frac{125}{24}$$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

6) $y = \frac{1}{\sqrt{x+1}}, y = 0, x = 0, x = 3$

$$V = \pi \int_0^3 \left(\frac{1}{\sqrt{x+1}} \right)^2 dx = \pi \left[\ln(x+1) \right]_0^3 = \pi \ln 4$$

7) $y = \frac{1}{x}, y = 0, x = 1, x = 4$

$$V = \pi \int_1^4 \left(\frac{1}{x} \right)^2 dx = \pi \left[-\frac{1}{x} \right]_1^4 = \frac{3}{4}\pi$$

8) $y = e^{-x}, y = 0, x = 0, x = 1$

$$V = \pi \int_0^1 \left(e^{-x} \right)^2 dx = -\frac{1}{2}\pi \left[e^{-2x} \right]_0^1 = -\frac{1}{2}\pi \left[e^{-2} - 1 \right]$$

9) Find the length of the graph of $x = y^2$ from (0,0) to (1,1).

$$\frac{dx}{dy} = 2y \qquad L = \int_0^1 \sqrt{1 + \left(2y\right)^2}\, dy$$

10) Find the length of the graph of $y = \frac{1}{2}\left(e^x + e^{-x}\right)$ on the interval [0,2]

$$\frac{dy}{dx} = \frac{1}{2}\left(e^x - e^{-x}\right) \qquad L = \int_0^2 \sqrt{1 + \left(\frac{1}{2}\left(e^x - e^{-x}\right) \right)^2}\, dx$$

10) It takes 3 N of force to stretch a spring 3 m from its natural state. How much work is done to compress the spring 2 m?

$3 = 3k \Rightarrow k = 1 \quad W = \int_0^2 x\,dx = \left.\frac{x^2}{2}\right|_0^2 = 2 \text{ N - m}$

11) How much work is done lifting a 3-lb block from the ground to a platform 20 ft above the ground?

$W = 3(20) = 60 \text{ ft - lb}$

12) A 20-ft rope that weighs .75 lb/ft is hanging over the edge of a cliff. How much work is done lifting it to the top of the cliff?

$W = \int_0^{20} .75x\,dx = \left.\frac{.75x^2}{2}\right|_0^{20} = 150 \text{ ft - lb}$

13) A bucket sits on the ground and holds 50 lb of sand. It leaks so that it is empty by the time it reaches the platform 40 ft above the ground. Find the work done lifting the sand. Ignore the work done to lift the bucket and the cable attached to the bucket.

$W = \int_0^{40} \left(50 - \frac{5}{4}x\right) dx = \left[50x - \frac{5}{8}x^2\right]_0^{40} = 2000 - 1000 = 1000 \text{ ft - lb}$