Calculus Chapter 10 Review Problem Solutions

1a) $\langle -17, 32 \rangle$

1b) $\left|\langle -17, 32 \rangle\right| = \sqrt{(-17)^2 + 32^2} = \sqrt{1313}$

2a) $\langle -1, -1 \rangle$

2b) $\left|\langle -1, -1 \rangle\right| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

3a) $\langle 6, -8 \rangle$

3b) $\left|\langle 6, -8 \rangle\right| = \sqrt{6^2 + (-8)^2} = 10$

4a) $\langle 10, -25 \rangle$

4b) $\left|\langle 10, -25 \rangle\right| = \sqrt{10^2 + (-25)^2} = \sqrt{725} = 5\sqrt{29}$

5) $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle$

6) $\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

7) $\left|\langle 4, -1 \rangle\right| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$ $\quad \mathbf{u} = \left\langle \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}} \right\rangle$ $\quad \mathbf{v} = 2\left\langle \frac{4}{\sqrt{17}}, -\frac{1}{\sqrt{17}} \right\rangle = \left\langle \frac{8}{\sqrt{17}}, -\frac{2}{\sqrt{17}} \right\rangle$

8) $5(-1)\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle = \langle -3, -4 \rangle$

9a) $\frac{dy}{dx} = \frac{\frac{1}{2}\sec t(\tan t)}{\frac{1}{2}\sec^2 t} = \sin t \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \left(\frac{1}{2}\tan\left(\frac{\pi}{3}\right), \frac{1}{2}\sec\left(\frac{\pi}{3}\right)\right) = \left(\frac{\sqrt{3}}{2}, 1\right)$

Tangent equation : $y - 1 = \frac{\sqrt{3}}{2}\left(x - \frac{\sqrt{3}}{2}\right) \Rightarrow y = \frac{\sqrt{3}}{2}x + \frac{1}{4}$

9b) $\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{3}} = \left.\frac{\cos t}{\frac{1}{2}\sec^2 t}\right|_{t=\frac{\pi}{3}} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$

10a) $\frac{dy}{dx} = \frac{\frac{3}{t^2}}{\frac{-2}{t^3}} = -\frac{3}{2}t \quad -\frac{3}{2}(2) = -3 \quad \left(\frac{5}{4}, -\frac{1}{2}\right)$

Tangent equation : $y + \frac{1}{2} = -3\left(x - \frac{5}{4}\right) \Rightarrow y = -3x + \frac{13}{4}$

11) $\dfrac{dy}{dx} = \dfrac{\frac{1}{2}\sec t \tan t}{\frac{1}{2}\sec^2 t} = \sin t$ Horizontal : $\sin t = 0 \Rightarrow t = 0, \pi$ $\left(0, \frac{1}{2}\right); \left(0, -\frac{1}{2}\right)$

Vertical tangent : $\sin t$ is undefined, which never occurs.

12) $\dfrac{dy}{dx} = \dfrac{2\cos t}{2\sin t} = \cot t$ Horizontal : $\cot t = 0 \Rightarrow t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $(0,2); (0,-2)$

Vertical tangent : $\cot t$ is undefined $\Rightarrow t = 0, \pi$ $(-2,0); (2,0)$

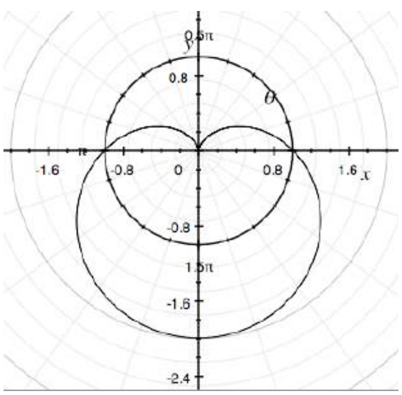
13) $\dfrac{dy}{dx} = \dfrac{-2\cos t \sin t}{\sin t} = -2\cos t$ Horizontal : $-2\cos t = 0 \Rightarrow t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $(0,0)$

Vertical tangent : $-2\cos t$ is undefined, which never occurs

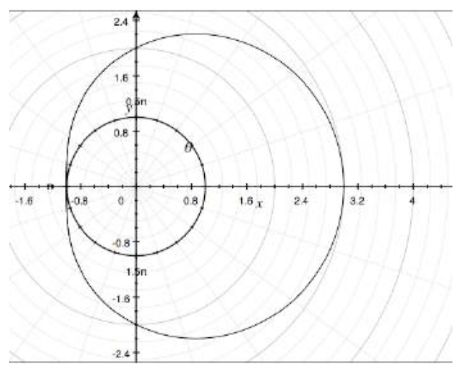
14) $\dfrac{dy}{dx} = \dfrac{9\cos t}{-4\sin t}$ Horizontal : $9\cos t = 0 \Rightarrow t = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $(0,9); (0,-9)$

Vertical tangent : $-4\sin t = 0 \Rightarrow t = 0, \pi$ $(4,0); (-4,0)$

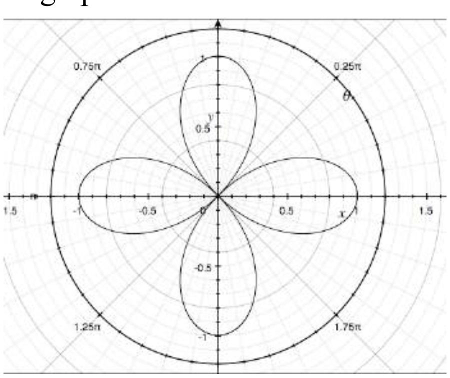
15) The graph is a cardioid



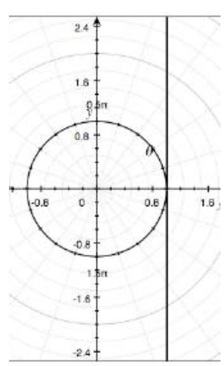
16) The graph is a convex limacon



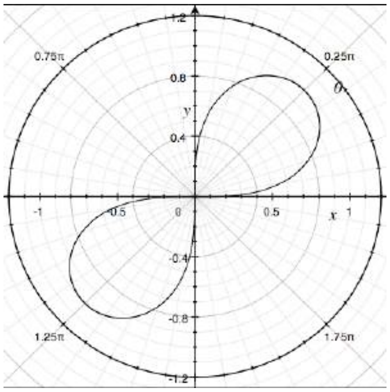
17) The graph is a rose



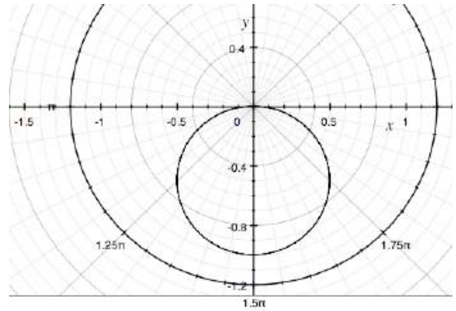
18) The graph is a line



19) The graph is a lemniscate



20) The graph is a circle



21)

$$x = \cos 2\theta(\cos\theta) \quad y = \cos 2\theta(\sin\theta)$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{3}} = \left.\frac{\cos 2\theta(\cos\theta) - 2\sin 2\theta(\sin\theta)}{\cos 2\theta(-\sin\theta) - 2\sin 2\theta(\cos\theta)}\right|_{\theta=\frac{\pi}{3}}$$

$$= \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)} = \frac{-\frac{1}{4} - \frac{3}{2}}{\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}} = \frac{-\frac{7}{4}}{-\frac{\sqrt{3}}{4}} = \frac{7}{\sqrt{3}} \approx 4.041$$

22)

$$x = (2 + \cos 2\theta)(\cos\theta) \quad y = (2 + \cos 2\theta)(\sin\theta)$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{3}} = \left.\frac{2\cos\theta + \cos 2\theta(\cos\theta) - 2\sin 2\theta(\sin\theta)}{-2\sin\theta + \cos 2\theta(-\sin\theta) - 2\sin 2\theta(\cos\theta)}\right|_{\theta=\frac{\pi}{3}}$$

$$= \frac{2\left(\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}{-2\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)} = \frac{1 - \frac{1}{4} - \frac{3}{2}}{-\sqrt{3} + \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2}} = \frac{-\frac{3}{4}}{-\frac{5\sqrt{3}}{4}} = \frac{3}{5\sqrt{3}} \approx 0.346$$

23) NOT

$$x = \left(1 - \cos\left(\frac{\theta}{2}\right)\right)(\cos\theta) = \cos\theta - \cos\theta\left(\cos\left(\frac{\theta}{2}\right)\right) \quad y = \left(1 - \cos\left(\frac{\theta}{2}\right)\right)(\sin\theta) = \sin\theta - \sin\theta\left(\cos\left(\frac{\theta}{2}\right)\right)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - \sin\theta\left(-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\right) + (-\cos\theta)\left(\cos\left(\frac{\theta}{2}\right)\right)}{-\sin\theta - \cos\theta\left(-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\right) + (\sin\theta)\left(\cos\left(\frac{\theta}{2}\right)\right)}$$

Horizontal tangent : $\cos\theta - \sin\theta\left(-\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\right) + (-\cos\theta)\left(\cos\left(\frac{\theta}{2}\right)\right) = 0 \Rightarrow \theta = 0, 2.243, 4.892, 7.675, 12.566$

$\theta = 0 \quad y = 0$ $\qquad \theta = 2.243 \quad y = 0.443$ $\qquad \theta = 4.892 \quad y = -1.739$

$\theta = 7.675 \quad y = 1.739$ $\qquad \theta = 12.566 \quad y = 0$

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27) $x = y$. the graph is a line.

28) $x = r\cos\theta \Rightarrow \cos\theta = \dfrac{x}{r} \quad r = 3\left(\dfrac{x}{r}\right) \Rightarrow r^2 = 3x \Rightarrow x^2 + y^2 = 3x$.

The graph is a circle with center $\left(\dfrac{3}{2}, 0\right)$ and radius $\dfrac{3}{2}$.

29) $r = 4\left(\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1}{\cos\theta}\right) \Rightarrow r\cos\theta = 4\left(\dfrac{y}{x}\right) \Rightarrow x = 4\left(\dfrac{y}{x}\right) \Rightarrow y = \dfrac{x^2}{4}$. The graph is a parabola.

30) Trig Identity : $\cos(u+v) = \cos u \cos v + \sin u \sin v$

$$r\cos\left(\theta + \frac{\pi}{3}\right) = 2\sqrt{3} \Rightarrow r\left[\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right] = 2\sqrt{3}$$

$$\Rightarrow r\cos\theta\left(\frac{1}{2}\right) - r\sin\theta\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3} \Rightarrow x - \sqrt{3}y = 4\sqrt{3}$$

The graph is a line.

31) $(x^2 + y^2) + 5y = 0 \Rightarrow r^2 + 5r\cos\theta = 0 \Rightarrow r = -5\cos\theta$

32) $(x^2 + y^2) - 2y = 0 \Rightarrow r^2 - 2r\sin\theta = 0 \Rightarrow r = 2\sin\theta$

33) $r^2\cos^2\theta + 4r^2\sin^2\theta = 16 \Rightarrow r^2 = \dfrac{16}{\cos^2\theta + 4\sin^2\theta}$

34) $(r\cos\theta + 2)^2 + (r\sin\theta - 5)^2 = 16$

35) $$\int_0^{2\pi} \frac{1}{2}(2 - \cos\theta)^2\, d\theta = \int_0^{2\pi} \frac{1}{2}(4 - 4\cos\theta + \cos^2\theta)\, d\theta = \int_0^{2\pi}\left(2 - 2\cos\theta + \frac{1}{2}\cos^2\theta\right) d\theta$$

$$= \left[2\theta - 2\sin\theta\right]_0^{2\pi} + \int_0^{2\pi} \frac{1 + \cos 2\theta}{4}\, d\theta = \left[2\theta - 2\sin\theta\right]_0^{2\pi} + \left[\frac{\theta}{4} + \frac{\sin 2\theta}{8}\right]_0^{2\pi}$$

$$= \left[4\pi - 0 - (0 - 0)\right] + \left[\frac{\pi}{2} + 0 - (0 + 0)\right] = \frac{9\pi}{2}$$

36) $\int_0^{\pi/3} \frac{1}{2}(\sin 3\theta)^2 d\theta = \int_0^{\pi/3} \frac{1}{2}(\sin^2 3\theta)d\theta = \int_0^{\pi/3} \frac{1-\cos 6\theta}{4} d\theta$

$= \left[\frac{\theta}{4} - \frac{\sin 6\theta}{24}\right]_0^{\pi/3} = \left[\frac{\pi}{12} - 0 - (0-0)\right] = \frac{\pi}{12}$

37)

$1 + \cos 2\theta = 1 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, etc$

$2\int_{-\pi/4}^{\pi/4} \frac{1}{2}\left((1+\cos 2\theta)^2 - 1^2\right)d\theta = 2\int_{-\pi/4}^{\pi/4} \frac{1}{2}(2\cos 2\theta + \cos^2 2\theta)d\theta = 2\int_{-\pi/4}^{\pi/4}\left(\cos 2\theta + \frac{1+\cos 4\theta}{4}\right)d\theta$

$= 2\left[\frac{\sin 2\theta}{2} + \frac{\theta}{4} + \frac{\sin 4\theta}{16}\right]_{-\pi/4}^{\pi/4} = 2\left[\frac{1}{2} + \frac{\pi}{16} + 0 - \left(-\frac{1}{2} - \frac{\pi}{16} + 0\right)\right] = 2\left(1 + \frac{\pi}{8}\right) = 2 + \frac{\pi}{4}$

38) $\int_0^{2\pi} \frac{1}{2}(2 - 2\sin\theta)^2 d\theta - \int_0^{\pi} \frac{1}{2}(2\sin\theta)^2 d\theta = \int_0^{2\pi} \frac{1}{2}(4 - 8\sin\theta + 4\sin^2\theta)d\theta - \int_0^{\pi} 2\sin^2\theta\, d\theta$

$= \int_0^{2\pi}\left(2 - 4\sin\theta + (1 - \cos 2\theta)\right)d\theta - \int_0^{\pi}(1 - \cos 2\theta)\, d\theta$

$= \left[3\theta + 4\cos\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} - \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$

$= \left[6\pi + 4 - 0 - (0 + 4 - 0)\right] - \left[\pi - 0 - (0-0)\right] = 5\pi$

39a) $\mathbf{v(t)} = \left\langle -4\sin t, \sqrt{2}\cos t \right\rangle$ $\quad$ $\mathbf{a(t)} = \left\langle -4\cos t, -\sqrt{2}\sin t \right\rangle$

39b) Speed $= \sqrt{\left(-4\sin\left(\frac{\pi}{4}\right)\right)^2 + \left(\sqrt{2}\cos\left(\frac{\pi}{4}\right)\right)^2} = \sqrt{\left(-2\sqrt{2}\right)^2 + (1)^2} = \sqrt{9} = 3$

40a) $\mathbf{v(t)} = \left\langle \sqrt{3}\sec t\tan t, \sqrt{3}\sec^2 t \right\rangle$ $\quad$ $\mathbf{a(t)} = \left\langle \sqrt{3}\sec^3 t + \sqrt{3}\sec t\tan^2 t, 2\sqrt{3}\sec^2 t\tan t \right\rangle$

40b) Speed $= \sqrt{\left(\sqrt{3}\sec(0)\tan(0)\right)^2 + \left(\sqrt{3}\sec^2(0)\right)^2} = \sqrt{(0)^2 + \left(\sqrt{3}(1)\right)^2} = \sqrt{3}$

41) $\mathbf{v(t)} = \left\langle \frac{-t}{\left(1+t^2\right)^{3/2}}, \frac{1}{\left(1+t^2\right)^{3/2}} \right\rangle$ $\quad$ Speed $= \sqrt{\left(\frac{-t}{\left(1+t^2\right)^{3/2}}\right)^2 + \left(\frac{1}{\left(1+t^2\right)^{3/2}}\right)^2} = \frac{1}{1+t^2}$

Maximum speed will occur when $t = 0$, which means that the speed = 1.

42) Angle of **r** = $\tan^{-1}\left(\dfrac{e^t \sin t}{e^t \cos t}\right) = \tan^{-1}(\tan t) = t$

$\mathbf{v} = \langle -e^t \sin t + e^t \cos t, e^t \cos t + e^t \sin t \rangle \quad \mathbf{a} = \langle -2e^t \sin t, 2e^t \cos t \rangle$

Angle of **a** = $\tan^{-1}\left(\dfrac{2e^t \cos t}{-2e^t \sin t}\right) = \tan^{-1}(-\cot t) = 90° - t$

The angle between the vectors is $t - (90° - t) = 90°$.

43) $\mathbf{r(t)} = \left\langle \int (-\sin t)\,dt, \int \cos t\,dt \right\rangle = \langle \cos t + C_1, \sin t + C_2 \rangle$

$\cos(0) + C_1 = 0 \Rightarrow C_1 = -1 \quad \sin(0) + C_2 = 1 \Rightarrow C_2 = 1$

$\mathbf{r(t)} = \langle \cos t - 1, \sin t + 1 \rangle$

44) $\mathbf{r(t)} = \left\langle \int \dfrac{1}{t^2+1}\,dt, \int \dfrac{t}{\sqrt{t^2+1}}\,dt \right\rangle = \left\langle \tan^{-1} t + C_1, \sqrt{t^2+1} + C_2 \right\rangle$

$\tan^{-1}(0) + C_1 = 1 \Rightarrow C_1 = 1 \quad \sqrt{0^2+1} + C_2 = 1 \Rightarrow C_2 = 0$

$\mathbf{r(t)} = \left\langle \tan^{-1} t + 1, \sqrt{t^2+1} \right\rangle$

45) $v\mathbf{(t)} = \left\langle \int 0\,dt, \int 2dt \right\rangle = \langle 0 + C_1, 2t + C_2 \rangle$

$0 + C_1 = 0 \Rightarrow C_1 = 0 \quad 2(0) + C_2 = 0 \Rightarrow C_2 = 0 \Rightarrow v\mathbf{(t)} = \langle 0, 2t \rangle$

$r\mathbf{(t)} = \left\langle \int 0\,dt, \int 2tdt \right\rangle = \langle 0 + C_3, t^2 + C_4 \rangle$

$0 + C_3 = 1 \Rightarrow C_3 = 1 \quad (0)^2 + C_4 = 0 \Rightarrow C_4 = 0 \Rightarrow r\mathbf{(t)} = \langle 1, t^2 \rangle$

46) $v\mathbf{(t)} = \left\langle \int -2\,dt, \int -2dt \right\rangle = \langle -2t + C_1, -2t + C_2 \rangle$

$-2(1) + C_1 = 4 \Rightarrow C_1 = 6 \quad -2(1) + C_2 = 0 \Rightarrow C_2 = 2 \Rightarrow v\mathbf{(t)} = \langle -2t + 6, -2t + 2 \rangle$

$r\mathbf{(t)} = \left\langle \int (-2t+6)\,dt, \int (-2t+2)dt \right\rangle = \langle -t^2 + 6t + C_3, -t^2 + 2t + C_4 \rangle$

$-(1)^2 + 6(1) + C_3 = 3 \Rightarrow C_3 = -2 \quad -(1)^2 + 2(1) + C_4 = 3 \Rightarrow C_4 = 2 \Rightarrow r\mathbf{(t)} = \langle -t^2 + 6t - 2, -t^2 + 2t + 2 \rangle$

47a)

$$r(t) = \left\langle 3\cos\left(\frac{\pi}{4}t\right), 5\sin\left(\frac{\pi}{4}t\right) \right\rangle \Rightarrow v(t) = \left\langle -\frac{3\pi}{4}\sin\left(\frac{\pi}{4}t\right), \frac{5\pi}{4}\cos\left(\frac{\pi}{4}t\right) \right\rangle$$

$$|v(3)| = \sqrt{\left(-\frac{3\sqrt{2}\pi}{8}\right)^2 + \left(-\frac{5\sqrt{2}\pi}{8}\right)^2} = \sqrt{\frac{18\pi^2 + 50\pi^2}{64}} = \frac{\pi\sqrt{17}}{4}$$

47)b)

$$v(t)=\left\langle -\frac{3\pi}{4}\sin\left(\frac{\pi}{4}t\right),\frac{5\pi}{4}\cos\left(\frac{\pi}{4}t\right)\right\rangle \Rightarrow a(t)=\left\langle -\frac{3\pi^2}{16}\cos\left(\frac{\pi}{4}t\right),-\frac{5\pi^2}{16}\sin\left(\frac{\pi}{4}t\right)\right\rangle$$

$$a(3)=\left\langle \frac{3\sqrt{2}\pi^2}{32},-\frac{5\sqrt{2}\pi^2}{32}\right\rangle$$

47)c)

$$\left(\frac{x}{3}\right)^2+\left(\frac{y}{5}\right)^2=1$$

48)a)

$$\frac{dy}{dx}=\frac{e^t\sin t+e^t\cos t}{e^t\cos t-e^t\sin t}$$

when $t=\pi$, $\dfrac{dy}{dx}=\dfrac{e^\pi(0)+e^\pi(1)}{e^\pi(1)-e^\pi(0)}=1$

48)b)

$$r(t)=\langle e^t\cos t, e^t\sin t\rangle \Rightarrow v(t)=\langle e^t\cos t-e^t\sin t, e^t\sin t+e^t\cos t\rangle$$

$$|v(3)|=\sqrt{\left(e^3\cos 3-e^3\sin 3\right)^2+\left(e^3\sin 3+e^3\cos 3\right)^2}=e^3\sqrt{2}$$

48)c)

$$\int_0^3\sqrt{\left(e^3\cos 3-e^3\sin 3\right)^2+\left(e^3\sin 3+e^3\cos 3\right)^2}\,dt=\left(e^3-1\right)\sqrt{2}$$

49)a)

$$r(t)=\left\langle t^2-2,\frac{2}{5}t^3\right\rangle \Rightarrow v(t)=\left\langle 2t,\frac{6}{5}t^2\right\rangle$$

$$|v(4)|=\sqrt{(8)^2+\left(\frac{96}{5}\right)^2}=\frac{104}{5}$$

49)b)

$$\int_0^4\sqrt{(2t)^2+\left(\frac{6}{5}t^2\right)^2}\,dt=\frac{4144}{135}$$

49)c)

$$\frac{dy}{dx}=\frac{\frac{6}{5}t^2}{2t}=\frac{3}{5}t \quad x=t^2-2\Rightarrow t=\sqrt{x+2}$$

$$\frac{dy}{dx}=\frac{3}{5}\sqrt{x+2}$$

50)

Endpoint of airplane vector: $(540\cos 10°, 540\sin 10°)=(531.796, 93.770)$

Endpoint of wind vector: $(540\cos(-10°), 540\sin(-10°))=(54.164, -9.55)$

Resultant vector's endpoint: $(585.96, 84.22)$

Magnitude of resultant vector: $\sqrt{585.96^2+84.22^2}\approx 591.982$ mph

Resultant direction: $\tan^{-1}\left(\frac{84.22}{585.96}\right)\approx 8.179°$ North of East