Calculus Area Between Curves Practice SOLUTIONS

Find the area of the region between the curves.

1) $f(x)=x^2+2x+1, \quad g(x)=3x+3$

$x^2+2x+1=3x+3 \Rightarrow x^2-x-2=0 \Rightarrow (x-2)(x+1)=0 \Rightarrow x=-1,2$

$$\int_{-1}^{2}\left(3x+3-\left(x^2+2x+1\right)\right)dx=\int_{-1}^{2}\left(x+2-x^2\right)dx=\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}$$

$$=\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)=\frac{9}{2}$$

2) $f(x)=-x^2+4x+2, \quad g(x)=x+2$

$-x^2+4x+2=x+2 \Rightarrow x^2-3x=0 \Rightarrow x(x-3)=0 \Rightarrow x=0,3$

$$\int_{0}^{3}\left(-x^2+4x+2-(x+2)\right)dx=\int_{0}^{3}\left(3x-x^2\right)dx=\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_{0}^{3}$$

$$=\left(\frac{27}{2}-9\right)-(0-0)=\frac{9}{2}$$

3) $f(x)=\frac{1}{x^2}, \quad g(x)=0, \quad \text{on the interval } [1,5]$

$$\int_{1}^{5}\frac{1}{x^2}dx=-\frac{1}{x}\bigg|_{1}^{5}=-\frac{1}{5}-(-1)=\frac{4}{5}$$

4) $f(x)=\sqrt{3x}+1, \quad g(x)=x+1$

$\sqrt{3x}+1=x+1 \Rightarrow \sqrt{3x}=x \Rightarrow 3x=x^2 \Rightarrow x^2-3x=0 \Rightarrow x=0,3$

$$\int_{0}^{3}\left(\sqrt{3x}+1-(x+1)\right)dx=\int_{0}^{3}\left(\sqrt{3x}-x\right)dx=\int_{0}^{3}\sqrt{3x}\,dx-\int_{0}^{3}x\,dx$$

$$=\frac{1}{3}\int_{0}^{9}u^{1/2}\,du-\frac{x^2}{2}\bigg|_{0}^{3}=\frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]_{0}^{9}-\left(\frac{9}{2}-0\right)=\frac{1}{3}[18-0]-\frac{9}{2}=6-\frac{9}{2}=\frac{3}{2}$$

5) $f(x)=\sqrt[3]{x}, \quad g(x)=x$

$\sqrt[3]{x}=x \Rightarrow x=x^3 \Rightarrow x^3-x=0 \Rightarrow x=0,\pm 1$

$$\int_{-1}^{0}\left(x-\sqrt[3]{x}\right)dx+\int_{0}^{1}\left(\sqrt[3]{x}-x\right)dx=\left[\frac{x^2}{2}-\frac{3}{4}x^{4/3}\right]_{-1}^{0}+\left[\frac{3}{4}x^{4/3}-\frac{x^2}{2}\right]_{0}^{1}$$

$$=(0-0)-\left(\frac{1}{2}-\frac{3}{4}\right)+\left(\frac{3}{4}-\frac{1}{2}\right)-(0-0)=\frac{1}{2}$$

6) $x=y^2, \quad x=y+2$

$y^2=y+2 \Rightarrow y^2-y-2=0 \Rightarrow (y-2)(y+1)=0 \Rightarrow y=-1,2$

$$\int_{-1}^{2}\left(y+2-y^2\right)dy=\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_{-1}^{2}=2+4-\frac{8}{3}-\left(\frac{1}{2}-2+\frac{1}{3}\right)=\frac{9}{2}$$

7) $x=2y-y^2, \quad x=-y$

$2y-y^2=-y \Rightarrow y^2-3y=0 \Rightarrow y=0,3$

$$\int_{0}^{3}\left(2y-y^2-(-y)\right)dy=\int_{0}^{3}\left(3y-y^2\right)dy=\left[\frac{3y^2}{2}-\frac{y^3}{3}\right]_{0}^{3}$$

$$=\frac{27}{2}-\frac{27}{3}-(0-0)=\frac{27}{6}=\frac{9}{2}$$

8) $x=y^2+1, \quad x=0,$ on the interval $[-1,2]$

$$\int_{-1}^{2}\left(y^2+1\right)dy=\left[\frac{y^3}{3}+y\right]_{-1}^{2}=\frac{8}{3}+2-\left(-\frac{1}{3}-1\right)=6$$

9) $y=\dfrac{4}{2-x}, \quad y=4, \quad x=0$

$\dfrac{4}{2-x}=4 \Rightarrow 4=8-4x \Rightarrow x=1$

$$\int_{0}^{1}\left(4-\frac{4}{2-x}\right)dx=\int_{0}^{1}4\,dx-\int_{0}^{1}\frac{4}{2-x}dx \qquad \begin{matrix} u=2-x \\ du=-dx \end{matrix} \qquad =4x\Big|_{0}^{1}+4\int_{2}^{1}\frac{du}{u}=4-0+4\left[\ln u\right]_{2}^{1}$$

$$=4+4\left[\ln 1-\ln 2\right]=4-4\ln 2$$

10) $y=\dfrac{4}{x}, \quad x=0, \quad y=1, \quad y=4$

$$\int_{1}^{4}\frac{4}{x}dx=\left[4\ln x\right]_{1}^{4}=4\ln 4-4\ln 1=4\ln 4$$