Calculus Antidifferentiation by Substitution SOLUTIONS

Find the following antiderivatives by hand:

1) $\int 6x^2\cos\left(2x^3\right)dx$ $\quad u = 2x^3$, $du = 6x^2$ $\quad \int \cos u\,du = \sin u + C = \sin\left(2x^3\right) + C$

2) $2xe^{2x^2}\,dx$ $\quad u = 2x^2$, $du = 4x\,dx$ $\quad \frac{1}{2}\ e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{2x^2} + C$

3) $\int \frac{\cos 3x}{\sqrt{\sin 3x}}dx$ $\quad u = \sin 3x$, $du = 3\cos 3x\,dx$ $\quad \frac{1}{3}\int u^{-1/2}du = \frac{1}{3}\left(2u^{1/2}\right) + C = \frac{2}{3}\sqrt{\sin 3x} + C$

4) $\int \frac{3x+4}{3x^2+8x}dx$ $\quad u = 3x^2 + 8x$, $du = \left(6x+8\right)dx$ $\quad \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln\left|u\right| + C = \frac{1}{2}\ln\left|3x^2+8x\right| + C$

5) $\int x^2\sin\left(3x^3\right)dx$ $\quad u = 3x^3$, $du = 9x^2dx$ $\quad \frac{1}{9}\int \sin u\,du = \frac{1}{9}\left(-\cos u\right) + C = -\frac{1}{9}\cos\left(3x^3\right) + C$

6) $\int 3\sin^2 x\cos x\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$ $\quad 3\int u^2du = 3\left(\frac{u^3}{3}\right) + C = \sin^3 x + C$

7) $\int \cos\left(4x\right)dx$ $\quad u = 4x$, $du = 4dx$ $\quad \frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u + C = \frac{1}{4}\sin\left(4x\right) + C$

8) $\int \sec^2 x\tan^3 x\,dx$ $\quad u = \tan x$, $du = \sec^2 x\,dx$ $\quad \int u^3du = \frac{u^4}{4} + C = \frac{1}{4}\tan^4 x + C$

9) $\int x^2\sec^2 x^3\left(\tan x^3\right)dx$ $\quad u = \tan x^3$, $du = 3x^2\sec^2 x^3\,dx$ $\quad \frac{1}{3}\int u\,du = \frac{1}{3}\left(\frac{u^2}{2}\right) + C = \frac{1}{6}\tan^2 x^3 + C$

Alternate Solution: $\quad u = \sec x^3$, $du = 3x^2\sec x^3\tan x^3dx$ $\quad \frac{1}{3}\int udu = \frac{1}{3}\left(\frac{u^2}{2}\right) + C = \frac{1}{6}\sec^2 x^3 + C$

10) $\int 3x^2\sqrt{2x^3+5}\,dx$ $\quad u = 2x^3 + 5$, $du = 6x^2dx$ $\quad \frac{1}{2}\int u^{1/2}du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) = \frac{1}{3}\left(2x^3+5\right)^{3/2} + C$

11) $\int \frac{e^{1/\sqrt{x}}}{\sqrt{x^3}}dx$ $\quad u = \frac{1}{\sqrt{x}}$, $du = -\frac{1}{2}x^{-3/2}$ $\quad -2\int e^u du = -2e^u + C = -2e^{1/\sqrt{x}} + C$

12) $\int \sqrt{x}e^{x^{3/2}}\,dx$ $\quad u = x^{3/2}$, $du = \frac{3}{2}x^{1/2}dx$ $\quad \frac{2}{3}\int e^u du = \frac{2}{3}e^u + C = \frac{2}{3}e^{x^{3/2}} + C$

13) $\int xe^{x^2}\sqrt{e^{x^2}}\,dx$ $\quad$ $u = e^{x^2}$, $du = 2xe^{x^2}\,dx$ $\quad$ $\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C = \frac{1}{3}\left(e^{x^2}\right)^{3/2} + C$

14)

$$\int \sin^2 x\cos^3 x\,dx = \int \sin^2 x\cos x\left(1-\sin^2 x\right)dx = \int \sin^2 \cos x\,dx - \int \sin^4 x\cos x\,dx$$

$\int \sin^2 \cos x\,dx$ $\quad$ $u = \sin x$, $du = \cos x\,dx$ $\quad$ $\int u^2 du = \frac{u^3}{3} + C = \frac{1}{3}\sin^3 x + C$

$\int \sin^4 \cos x\,dx$ $\quad$ $u = \sin x$, $du = \cos x\,dx$ $\quad$ $\int u^4 du = \frac{u^5}{5} + C = \frac{1}{5}\sin^5 x + C$

$$\int \sin^2 \cos x\,dx - \int \sin^4 x\cos x\,dx = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C$$