Calculus Antiderivatives – Substitution Method Source: Calculus AP Edition (Briggs et al) ©2014

Use a change of variables to find the following indefinite integrals. Check your work by differentiation.

1) $\int 2x\left(x^2-1\right)^{99}dx$ $\quad u=x^2-1,\ du=2x\,dx$ $\quad \int u^{99}\,du=\frac{u^{100}}{100}+C=\frac{\left(x^2-1\right)^{100}}{100}+C$

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2) $\int xe^{x^2}dx$ $\quad u=x^2,\ du=2xdx\Rightarrow xdx=\frac{1}{2}du$ $\quad \frac{1}{2}\int e^u\,du=\frac{1}{2}e^u+C=\frac{1}{2}e^{x^2}+C$

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3) $\int\frac{2x^2}{\sqrt{1-4x^3}}dx$ $\quad u=1-4x^3,\ du=-12x^2dx\Rightarrow 2x^2dx=-\frac{1}{6}du$ $\quad -\frac{1}{6}\int u^{-1/2}\,du=-\frac{1}{6}\left(\frac{u^{1/2}}{1/2}\right)+C=-\frac{1}{3}\sqrt{1-4x^3}+C$

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4) $\int\frac{\left(\sqrt{x}+1\right)^4}{2\sqrt{x}}dx$ $\quad u=\sqrt{x}+1,\ du=\frac{1}{2\sqrt{x}}dx$ $\quad \int u^4\,du=\frac{u^5}{5}+C=\frac{\left(\sqrt{x}+1\right)^5}{5}+C$

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5) $\int\left(x^2+x\right)^{10}\left(2x+1\right)dx$ $\quad u=x^2+x,\ du=\left(2x+1\right)dx$ $\quad \int u^{10}\,du=\frac{u^{11}}{11}+C=\frac{\left(x^2+x\right)^{11}}{11}+C$

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6) $\int\frac{1}{10x-3}dx$ $\quad u=10x-3,\ du=10dx\Rightarrow dx=\frac{1}{10}du$ $\quad \frac{1}{10}\int\frac{1}{u}du=\frac{1}{10}\ln|u|+C=\frac{1}{10}\ln|10x-3|+C$

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7) $\int x^3\left(x^4+16\right)^6dx$ $\quad u=x^4+16,\ du=4x^3dx\Rightarrow x^3dx=\frac{1}{4}du$ $\quad \frac{1}{4}\int u^6\,du=\frac{1}{4}\left(\frac{u^7}{7}\right)+C=\frac{\left(x^4+16\right)^7}{28}+C$

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8) $\int\sin^{10}\theta\cos\theta\,d\theta$ $\quad u=\sin\theta,\ du=\cos\theta d\theta$ $\quad \int u^{10}\,du=\frac{u^{11}}{11}+C=\frac{\sin^{11}\theta}{11}+C$

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9) $\int x^9\sin x^{10}\,dx$ $\quad u=x^{10},\ du=10x^9dx\Rightarrow x^9dx=\frac{1}{10}du$ $\quad \frac{1}{10}\int\sin u\,du=\frac{1}{10}\left(-\cos u\right)+C=-\frac{1}{10}\cos x^{10}+C$

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10) $\int\left(x^6-3x^2\right)\left(x^5-x\right)dx$ $\quad u=x^6-3x^2,\ du=\left(6x^5-6x^2\right)dx\Rightarrow\left(x^5-x\right)dx=\frac{1}{6}du$ $\quad \frac{1}{6}\int u\,du=\frac{1}{6}\left(\frac{u^2}{2}\right)+C=\frac{\left(x^6-3x^2\right)^2}{12}+C$

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11) $\int\frac{8x+6}{2x^2+3x}dx$ $\quad u=2x^2+3x,\ du=\left(4x+3\right)dx\Rightarrow\left(8x+6\right)dx=2du$ $\quad 2\int\frac{1}{u}du=2\ln|u|+C=2\ln\left|2x^2+3x\right|+C$

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12)

$\int\frac{x}{x-2}dx$ Hint: let $u=x-2$ $\quad u=x-2\Rightarrow x=u+2,\ du=dx$ $\quad \int\frac{u+2}{u}du=\int\left(1+\frac{2}{u}\right)du=u+2\ln|u|+C=x-2+2\ln|x-2|+C$

Evaluate the definite integral by making a *u*-substitution and integrating from *u*(*a*) to *u*(*b*).

1) $\int_0^1 2x\left(4-x^2\right)dx$ $\quad u=4-x^2$, $du=-2xdx \Rightarrow 2xdx=-du$ $\quad -\int_4^3 u\,du = -\left[\frac{u^2}{2}\right]_4^3 = -\left[\frac{9}{2}-\frac{16}{2}\right] = \frac{7}{2}$

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2) $\int_0^2 \frac{2x}{\left(x^2+1\right)^2}dx$ $\quad u=x^2+1$, $du=2xdx$ $\quad \int_1^5 u^{-2}\,du = \left[\frac{u^{-1}}{-1}\right]_1^5 = \left[-\frac{1}{5}-\left(-\frac{1}{1}\right)\right] = \frac{4}{5}$

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3) $\int_0^{\pi/2} \sin^2\theta\cos\theta\,d\theta$ $\quad u=\sin\theta$, $du=\cos\theta d\theta$ $\quad \int_0^1 u^2\,du = \left[\frac{u^3}{3}\right]_0^1 = \left[\frac{1}{3}-0\right] = \frac{1}{3}$

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4) $\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}dx$ $\quad u=\cos x$, $du=-\sin x dx$ $\quad -\int_1^{\sqrt{2}/2} u^{-2}\,du = -\left[\frac{u^{-1}}{-1}\right]_1^{\sqrt{2}/2} = -\left[-\frac{2}{\sqrt{2}}-(-1)\right] = \frac{2}{\sqrt{2}}-1$

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5) $\int_{-1}^2 x^2 e^{x^3+1}\,dx$ $\quad u=x^3+1$, $du=3x^2dx \Rightarrow x^2dx=\frac{1}{3}du$ $\quad \frac{1}{3}\int_0^9 e^u\,du = \frac{1}{3}\left[e^u\right]_0^9 = \frac{1}{3}\left[e^9-e^0\right] = \frac{1}{3}e^9-\frac{1}{3}$

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6) $\int_0^4 \frac{p}{\sqrt{9+p^2}}dp$ $\quad u=9+p^2$, $du=2p\,dp \Rightarrow p\,dp=\frac{1}{2}du$ $\quad \frac{1}{2}\int_9^{25} u^{-1/2}\,du = \frac{1}{2}\left[\frac{u^{1/2}}{1/2}\right]_9^{25} = \frac{1}{2}\left[2\sqrt{25}-2\sqrt{9}\right] = 2$

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7) $\int_0^3 \frac{v^2+1}{\sqrt{v^3+3v+4}}dv$ $\quad u=v^3+3v+4$, $du=\left(3v^2+3\right)dv \Rightarrow \left(v^2+1\right)dv=\frac{1}{3}du$ $\quad \frac{1}{3}\int_4^{40} u^{-1/2}\,du = \frac{1}{3}\left[\frac{u^{1/2}}{1/2}\right]_4^{40} = \frac{1}{3}\left[2\sqrt{40}-2\sqrt{4}\right]$

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8) $\int_0^{1/8} \frac{x}{\sqrt{1-16x^2}}dx$ $\quad u=1-16x^2$, $du=-32xdx \Rightarrow xdx=-\frac{1}{32}du$ $\quad \int_1^{3/4} u^{-1/2}\,du = -\frac{1}{32}\left[\frac{u^{1/2}}{1/2}\right]_1^{3/4} = -\frac{1}{32}\left[2\sqrt{\frac{3}{4}}-2\right]$

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9) $\int_0^4 \frac{x}{x^2+1}dx$ $\quad u=x^2+1$, $du=2xdx \Rightarrow xdx=\frac{1}{2}du$ $\quad \frac{1}{2}\int_1^{17} \frac{1}{u}du = \frac{1}{2}\left[\ln|u|\right]_1^{17} = \frac{1}{2}\left[\ln 17-\ln 1\right]$
