Calculus Antiderivatives NOT u-substitution/Arctan SOLUTIONS

1) $\int \frac{x^3 - 4x^2 + 3x - 2}{x^4}\,dx = \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{3}{x^3} - \frac{2}{x^4} \right) dx = \ln|x| + \frac{4}{x} - \frac{3}{2x^2} + \frac{2}{3x^3} + C$

2) $\int \frac{x^3}{x^2 - 1}\,dx = \int \left(x + \frac{x}{x^2 - 1} \right) dx = \frac{x^2}{2} + \frac{1}{2} \ln|x^2 - 1| + C$

3) $\int \frac{x^2 - 1}{x^3}\,dx = \int \left(\frac{1}{x} - \frac{1}{x^3} \right) dx = \ln|x| + \frac{1}{2x^2} + C$

4) $\int \frac{x^3 - 2x^2}{x^2 + 1}\,dx = \int \left(x - 2 + \frac{-x + 2}{x^2 + 1} \right) dx = \frac{x^2}{2} - 2x - \frac{1}{2} \ln|x^2 + 1| + 2\tan^{-1} x + C$

5) $\int \frac{x^3 - 2x}{x^2 - 3}\,dx = \int \left(x + \frac{x}{x^2 - 3} \right) dx = \frac{x^2}{2} + \frac{1}{2} \ln|x^2 - 3| + C$

6) $\int \frac{x^5 - 3x^2}{x^3 + 2}\,dx = \int \left(x^2 - \frac{5x^2}{x^3 + 2} \right) dx = \frac{x^3}{3} - \frac{5}{3} \ln|x^3 + 2| + C$

7) $\int \frac{x^3 - 3x^2 + 2}{x + 3}\,dx = \int \left(x^2 - 6x + 18 - \frac{52}{x + 3} \right) dx = \frac{x^3}{3} - 3x^2 + 18x - 52\ln|x + 3| + C$

8) $\int \frac{dx}{x^2 + 16} = \int \frac{dx}{16\left(\left(\frac{x}{4}\right)^2 + 1\right)}$ $\quad u = \frac{x}{4}$, $du = \frac{1}{4}dx$ $\quad \frac{1}{4}\int \frac{du}{u^2 + 1} = \frac{1}{4}\tan^{-1} u + C = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$

9) $\int \frac{dx}{x^2 + 9} = \int \frac{dx}{9\left(\left(\frac{x}{3}\right)^2 + 1\right)}$ $\quad u = \frac{x}{3}$, $du = \frac{1}{3}dx$ $\quad \frac{1}{3}\int \frac{du}{u^2 + 1} = \frac{1}{3}\tan^{-1} u + C = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

10) $\int \frac{dx}{2x^2 + 8} = \int \frac{dx}{8\left(\left(\frac{x}{2}\right)^2 + 1\right)}$ $\quad u = \frac{x}{2}$, $du = \frac{1}{2}dx$ $\quad \frac{1}{4}\int \frac{du}{u^2 + 1} = \frac{1}{4}\tan^{-1} u + C = \frac{1}{4}\tan^{-1}\left(\frac{x}{2}\right) + C$

11) $\int \frac{dx}{x^2 + 5} = \int \frac{dx}{5\left(\left(\frac{x}{\sqrt{5}}\right)^2 + 1\right)}$ $\quad u = \frac{x}{\sqrt{5}}$, $du = \frac{1}{\sqrt{5}}dx$ $\quad \frac{\sqrt{5}}{5}\int \frac{du}{u^2 + 1} = \frac{\sqrt{5}}{5}\tan^{-1} u + C = \frac{\sqrt{5}}{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

12) $\int \frac{8dx}{x^2+2} = \int \frac{8dx}{2\left(\left(\frac{x}{\sqrt{2}}\right)^2+1\right)}$ $\quad u = \frac{x}{\sqrt{2}}$ $\quad du = \frac{1}{\sqrt{2}}dx$ $\quad 4\sqrt{2}\int \frac{du}{u^2+1} = 4\sqrt{2}\tan^{-1}u + C = 4\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

13) $\int \frac{7dx}{x^2+3} = \int \frac{7dx}{3\left(\left(\frac{x}{\sqrt{3}}\right)^2+1\right)}$ $\quad u = \frac{x}{\sqrt{3}}$ $\quad du = \frac{1}{\sqrt{3}}dx$ $\quad \frac{7\sqrt{3}}{3}\int \frac{du}{u^2+1} = \frac{7\sqrt{3}}{3}\tan^{-1}u + C = \frac{7\sqrt{3}}{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

14) $\int \frac{4dx}{\frac{x^2}{3}+3} = \int \frac{4dx}{3\left(\left(\frac{x}{3}\right)^2+1\right)}$ $\quad u = \frac{x}{3}$ $\quad du = \frac{1}{3}dx$ $\quad 4\int \frac{du}{u^2+1} = 4\tan^{-1}u + C = 4\tan^{-1}\left(\frac{x}{3}\right) + C$

15) $\int \frac{3dx}{5x^2+7} = \int \frac{3dx}{7\left(\left(x\sqrt{\frac{5}{7}}\right)^2+1\right)}$ $\quad u = x\sqrt{\frac{5}{7}}$ $\quad du = \sqrt{\frac{5}{7}}dx$ $\quad \frac{\frac{3}{7}}{\sqrt{\frac{5}{7}}}\int \frac{du}{u^2+1} = \frac{3\sqrt{7}}{7\sqrt{5}}\tan^{-1}u + C = \frac{3\sqrt{7}}{7\sqrt{5}}\tan^{-1}\left(x\sqrt{\frac{5}{7}}\right) + C$