Calculus 8.4/8.5 "Quest" REVIEW Name_____

1) The spring of a spring balance is 3.2 in. long when there is no weight on the balance, and it is 6.4 in. long with 1.5 lb hung from the balance. How much work is done in stretching it from 6.0 in. to a length of 13.9 in.?

$1.5 = 3.2k \Rightarrow k = \frac{15}{32}$

$W = \int_{2.8}^{10.7} \frac{15}{32} x\,dx \approx 24.996$ in-lb

2) A leaky bucket holds 350 N of sand. The bucket is lifted from the ground at a constant rate to a point 50 m above the ground. The bucket leaks at a constant rate so that only half of the original sand is still in it as it reaches the top. How much work is done lifting the water to a point 50 m above the ground? Do not include the rope and bucket.

$W = \int_0^{50} (350 - 3.5y)\,dy = 13{,}125$ N-m

3) A rescue cable attached to a helicopter weighs 2 lb/ft. A 60-lb child grabs the end of the cable and is pulled from the ocean into the helicopter. How much work is done in lifting the child if the helicopter is 18 ft above the water? (Cable AND Child)

cable: $\int_0^{18} 2y\,dy = 324$ ft-lb child: $60(18) = 1080$ ft-lb

Total: 1404 ft-lb

4) A vertical right circular cylindrical tank measures 18 ft. high and 10 ft in diameter. It is full of oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the level of the top of the tank? Give your answer to the nearest ft · lb.

$\int_0^{18} 57(25\pi)(y)\,dy \approx 725{,}236.664$ ft-lb

5) A conical tank is resting on its apex. It has a radius at the top of 10 ft and a height of 16 ft. It is filled to within 5 ft of the top with oil weighing 53 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?

$\frac{r}{h} = \frac{10}{16} \Rightarrow r = \frac{5}{8}h$

$W = \int_0^{11} 53\left(\pi\left(\frac{5}{8}y\right)^2\right)(16 - y)\,dy \approx 223{,}637.321$ ft-lb

6) Find the length of $f(x) = x^3 - 2x^2 + 3x + \cos x$ from x = -1 to x = 1.

$\int_{-1}^{1} \sqrt{1 + \left(3x^2 - 4x + 3 - \sin x\right)^2}\,dx \approx 8.408$

7) The gate of a dam is in the shape of a trapezoid that is 6 feet wide on the upper base, 8 feet wide on the lower base, and 5 feet high. The upper base is 10 feet below the surface of the water. Find the fluid pressure on the gate.

$y + 15 = -5(x - 4) \Rightarrow x = \frac{y - 5}{-5}$

$\int_{-15}^{-10} 62.4\left(2\left(\frac{y-5}{-5}\right)\right)(0 - y)\,dy = 27{,}560$ ft-lb

Calculus 7.5 "Quest" REVIEW page 2

Apply the normal probability density function $\left(f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)} \right)$ **for # 8, 9, 10.**

8) The diameters of bolts produced by a certain machine are normally distributed with a mean of 0.45 inches and a standard deviation of 0.03 inches. What percentage of bolts will have a diameter that is between 0.40 inches and 0.50 inches?

$$\int_{0.40}^{0.50} \frac{1}{(0.03)\sqrt{2\pi}} e^{-(x-0.45)^2/\left(2(0.03)^2\right)} \approx 90.442\%$$

9) A bank's loan officer rates applicants for credit. The ratings are normally distributed with a mean of 300 and a standard deviation of 25. If an applicant is randomly selected, find the probability of a rating that is between 250 and 350.

$$\int_{250}^{350} \frac{1}{(25)\sqrt{2\pi}} e^{-(x-300)^2/\left(2(25)^2\right)} \approx 95.450\%$$

10) The systolic blood pressure of 48-year-old math teachers is normally distributed with a mean of 140 mmHg and a standard deviation of 16 mmHg. What percentage of 48-year-old math teachers have a systolic blood pressure between 100 mmHg and 120 mmHg?

$$\int_{100}^{120} \frac{1}{(16)\sqrt{2\pi}} e^{-(x-140)^2/\left(2(16)^2\right)} \approx 9.944\%$$

11) A rectangular sea aquarium observation window is 20 ft wide and 15 ft high. What is the force on this window if the upper edge is 12 ft below the surface of the water. The density of seawater is 64.0 lb / ft^3.

$$\int_{-27}^{-12} 64(20)(-y)\,dy = 374{,}400 \text{ ft-lb}$$