Calculus page 390 ff

1a) $5\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}, \frac{3\pi}{2}$ on $0 \le t \le 2\pi$

$0 \le t < \frac{\pi}{2}$: $5\cos t > 0$, so the particle is moving to the right

$\frac{\pi}{2} < t < \frac{3\pi}{2}$: $5\cos t < 0$, so the particle is moving to the left

$\frac{3\pi}{2} < t \le 2\pi$: $5\cos t > 0$, so the particle is moving to the right

The particle is stopped at $t = \frac{\pi}{2}, \frac{3\pi}{2}$

1b) displacement $= \int_0^{2\pi} 5\cos t\, dt = \left[5\sin t\right]_0^{2\pi} = 5\sin(2\pi) - 5\sin(0) = 0 - 0 = 0$

$s(2\pi) = 3$

1c) Total distance $= \int_0^{\pi/2} 5\cos t\, dt - \int_{\pi/2}^{3\pi/2} 5\cos t\, dt + \int_{3\pi/2}^{2\pi} 5\cos t\, dt = \left[5\sin t\right]_0^{\pi/2} - \left[5\sin t\right]_{\pi/2}^{3\pi/2} + \left[5\sin t\right]_{3\pi/2}^{2\pi}$

$= \left[5\sin\left(\frac{\pi}{2}\right) - 5\sin(0)\right] - \left[5\sin\left(\frac{3\pi}{2}\right) - 5\sin\left(\frac{\pi}{2}\right)\right] + \left[5\sin(2\pi) - 5\sin\left(\frac{3\pi}{2}\right)\right]$

$= \left[5 - 0\right] - \left[-5 - 5\right] + \left[0 + 5\right] = 20$

2a) $6\sin 3t = 0 \Rightarrow \sin 3t = 0 \Rightarrow t = 0, \frac{\pi}{3}$ on $0 \le t \le \frac{\pi}{2}$

$0 < t < \frac{\pi}{3}$: $6\sin 3t > 0$, so the particle is moving to the right

$\frac{\pi}{3} < t \le \frac{\pi}{2}$: $6\sin 3t < 0$, so the particle is moving to the left

The particle is stopped at $t = 0, \frac{\pi}{3}$

2b) displacement $= \int_0^{\pi/2} 6\sin 3t\, dt = \left[-2\cos t\right]_0^{\pi/2} = -2\cos\left(3\left(\frac{\pi}{2}\right)\right) - \left(-2\cos(3(0))\right) = -2(-1) + 2(1) = 2$

$s\left(\frac{\pi}{2}\right) = 5$

2c) Total distance $= \int_0^{\pi/3} 6\sin 3t\, dt - \int_{\pi/3}^{\pi/2} 6\sin 3t\, dt = \left[-2\cos 3t\right]_0^{\pi/3} - \left[-2\cos t\right]_{\pi/3}^{\pi/2}$

$= \left[-2\cos\left(3\left(\frac{\pi}{3}\right)\right) - (-2)\cos(3(0))\right] - \left[-2\cos\left(3\left(\frac{\pi}{2}\right)\right) - (-2)\cos\left(3\left(\frac{\pi}{3}\right)\right)\right]$

$= \left[2 - (-2)\right] - \left[0 + 2(-1)\right] = 4 - (-2) = 6$

3a) $49-9.8t=0 \Rightarrow t=5$ on $0 \le t \le 10$

$0 \le t < 5$: $49-9.8t>0$, so the particle is moving to the right

$0 < t \le 10$: $49-9.8t<0$, so the particle is moving to the left

The particle is stopped at $t=5$

3b) displacement $= \int_0^{10} (49-9.8t)\,dt = \left[49t-4.9t^2\right]_0^{10} = (490-490)-(0-0)=0$

$s(10)=3$

3c) Total distance $= \int_0^{5} (49-9.8t)\,dt - \int_5^{10} (49-9.8t)\,dt = \left[49t-4.9t^2\right]_0^{5} - \left[49t-4.9t^2\right]_5^{10}$

$= \left[(245-122.5)-(0-0)\right]-\left[(490-490)-(245-122.5)\right]$

$=122.5-(-122.5)=245$

4a) $6t^2-18t+12=0 \Rightarrow 6(t-2)(t-1)=0 \Rightarrow t=1,2$ on $0 \le t \le 2$

$0 \le t < 1$: $6t^2-18t+12>0$, so the particle is moving to the right

$1 < t < 2$: $6t^2-18t+12<0$, so the particle is moving to the left

The particle is stopped at $t=1,2$

4b) displacement $= \int_0^{2} (6t^2-18t+12)\,dt = \left[2t^3-9t^2+12t\right]_0^{2} = (16-36+24)-(0-0+0)=4$

$s(10)=7$

4c) Total distance $= \int_0^{1} (6t^2-18t+12)\,dt - \int_1^{2} (6t^2-18t+12)\,dt = \left[2t^3-9t^2+12t\right]_0^{1} - \left[2t^3-9t^2+12t\right]_1^{2}$

$= \left[(2-9+12)-(0-0+0)\right]-\left[(16-36+24)-(2-9+12)\right]$

$=5-(-1)=6$

5a) $5\sin^2 t\cos t=0 \Rightarrow \sin^2 t=0$ or $\cos t=0 \Rightarrow t=0,\pi,2\pi$ or $\frac{\pi}{2},\frac{3\pi}{2}$ on $0 \le t \le 2\pi$

$0 < t < \frac{\pi}{2}$: $5\sin^2 t\cos t>0$, so the particle is moving to the right

$\frac{\pi}{2} < t < \pi$: $5\sin^2 t\cos t<0$, so the particle is moving to the left

$\pi < t < \frac{3\pi}{2}$: $5\sin^2 t\cos t<0$, so the particle is moving to the left

$\frac{3\pi}{2} < t < 2\pi$: $5\sin^2 t\cos t>0$, so the particle is moving to the right

The particle is stopped at $t=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$

5b) displacement $= \int_0^{2\pi} 5\sin^2 t\cos t\,dt = \int_0^{0} 5u^2\,du = 0$

$s(2\pi)=3$

5c) Total distance $= \int_0^{\pi/2} 5\sin^2 t\cos t\,dt - \int_{\pi/2}^{\pi} 5\sin^2 t\cos t\,dt - \int_{\pi}^{3\pi/2} 5\sin^2 t\cos t\,dt + \int_{3\pi/2}^{2\pi} 5\sin^2 t\cos t\,dt$

$$= \int_0^1 5u^2\,du - \int_1^0 5u^2\,du - \int_0^{-1} 5u^2\,du + \int_{-1}^0 5u^2\,du$$

$$= \left[\frac{5u^3}{3}\right]_0^1 - \left[\frac{5u^3}{3}\right]_1^0 - \left[\frac{5u^3}{3}\right]_0^{-1} + \left[\frac{5u^3}{3}\right]_{-1}^0$$

$$= \left[\frac{5}{3} - 0\right] - \left[0 - \frac{5}{3}\right] - \left[-\frac{5}{3} - 0\right] + \left[0 - \left(-\frac{5}{3}\right)\right] = \frac{20}{3}$$

6a) $\sqrt{4-t} = 0 \Rightarrow 4 - t = 0 \Rightarrow t = 4$ on $0 \le t \le 4$

$0 \le t < 4$: $\sqrt{4-t} > 0$, so the particle is moving to the right

The particle is stopped at $t = 4$

6b) displacement $= \int_0^4 \sqrt{4-t}\,dt = -\left[\frac{2}{3}u^{3/2}\right]_4^0 = -\left[0 - \frac{16}{3}\right] = \frac{16}{3}$

$s(4) = 3 + \frac{16}{3} = \frac{25}{3}$

6c) Total distance $= \frac{16}{3}$

7a)

$e^{\sin t}\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}, \frac{3\pi}{2}$ on $0 \le t \le 2\pi$

$0 \le t < \frac{\pi}{2}$: $e^{\sin t}\cos t > 0$, so the particle is moving to the right

$\frac{\pi}{2} < t < \frac{3\pi}{2}$: $e^{\sin t}\cos t < 0$, so the particle is moving to the left

$\frac{3\pi}{2} < t \le 2\pi$: $e^{\sin t}\cos t > 0$, so the particle is moving to the right

The particle is stopped at $t = \frac{\pi}{2}, \frac{3\pi}{2}$

7b) displacement $= \int_0^{2\pi} e^{\sin t}\cos t\,dt = \int_0^0 e^u\,du = 0$

$s(4) = 3$

7c) Total distance $= \int_0^{\pi/2} e^{\sin t}\cos t\,dt - \int_{\pi/2}^{3\pi/2} e^{\sin t}\cos t\,dt + \int_{3\pi/2}^{2\pi} e^{\sin t}\cos t\,dt$

$$\int_0^1 e^u\,du - \int_1^{-1} e^u\,du + \int_{-1}^0 e^u\,du = \left[e - 1\right] - \left[e^{-1} - e\right] + \left[1 - e^{-1}\right] = 2e - \frac{2}{e} \approx 4.701$$

8a) $\frac{t}{1+t^2} = 0 \Rightarrow t = 0$ on $0 \le t \le 3$

$0 < t \le 3$: $\frac{t}{1+t^2} > 0$, so the particle is moving to the right

The particle is stopped at $t = 0$

8b) $$\text{displacement} = \int_0^3 \frac{t}{1+t^2}\,dt = \frac{1}{2}\int_1^{10} \frac{1}{u}\,du = 0 = \frac{1}{2}\Big[\ln|u|\Big]_1^{10} = \frac{1}{2}\left(\ln 10 - \ln 1\right) = \frac{\ln 10}{2}$$

$$s(4) = 3 + \frac{\ln 10}{2} \approx 4.151$$

8c) $$\text{Total distance} = \frac{\ln 10}{2} \approx 1.151$$

9a) $$v(9) = \int_0^9 \left(1 + 3\sqrt{t}\right)dt = \left[t + \frac{3t^{3/2}}{3/2}\right]_0^9 = \left(9 + 2(9)^{3/2}\right) - (0+0) = 9 + 2(27) = 63 \text{ mph}$$

9b) $$v(t) = \int \left(1 + 3\sqrt{t}\right)dt = t + 2t^{3/2} + C \qquad v(0) = 0 \Rightarrow C = 0$$

velocity is in mph, but the time is in seconds, so we must convert:

$$v(t) = \frac{t}{3600} + \frac{t^{3/2}}{1800} \text{ miles per second}$$

$$s(9) = \int_0^9 \left(\frac{t}{3600} + \frac{t^{3/2}}{1800}\right)dt = \left[\frac{t^2}{7200} + \frac{t^{5/2}}{4500}\right]_0^9 = \left(\frac{81}{7200} + \frac{243}{4500}\right) - (0+0) = \frac{261}{4000} = 0.06525 \text{ miles} = 344.52 \text{ ft}$$

10a) $$\text{displacement} = \int_0^4 (t-2)\sin t\,dt = \int_0^4 t\sin t\,dt - 2\int_0^4 \sin t\,dt$$

$\int t\sin t\,dt$ must be solved by the integration by parts method

$$\int t\sin t\,dt \qquad u = t \quad dv = \sin t\,dt \qquad du = dt \quad v = -\cos t$$

$$\int t\sin t\,dt = -t\cos t + \int \cos t\,dt + C = -t\cos t + \sin t + C$$

$$\int_0^4 t\sin t\,dt - 2\int_0^4 \sin t\,dt = \Big[-t\cos t + \sin t\Big]_0^4 - 2\Big[-\cos t\Big]_0^4$$

$$= \left(-4\cos 4 + \sin 4\right) - \left(-0\cos 0 + \sin 0\right) - 2\Big[-\cos 4 - \left(-\cos 0\right)\Big]$$

$$= -4\cos 4 + \sin 4 + 2\cos 4 - 2 \approx -1.450 \text{ meters}$$

10b)

$(t-2)\sin t = 0 \Rightarrow t = 0, 2, \pi$ on $0 \le t \le 4$

$0 < t < 2$: $(t-2)\sin t < 0$, so the particle is moving to the left

$2 < t < \pi$: $(t-2)\sin t > 0$, so the particle is moving to the right

$\pi < t \le 4$: $(t-2)\sin t < 0$, so the particle is moving to the left

distance traveled $= -\int_0^2 (t-2)\sin t\,dt + \int_2^\pi (t-2)\sin t\,dt - \int_\pi^4 (t-2)\sin t\,dt$

borrowing from part (a):

$$-\int_0^2 (t-2)\sin t\,dt = -\left(\left[-t\cos t + \sin t\right]_0^2 - 2\left[-\cos t\right]_0^2\right) = -\left(\left(\left[-2\cos 2 + \sin 2\right] - \left[-0\cos 0 + \sin 0\right]\right) - 2\left(\left[-\cos 2 - (-\cos 0)\right]\right)\right)$$

$$= -(-2\cos 2 + \sin 2 + 4\cos 2 - 2) = -2\cos 2 - \sin 2 + 2$$

$$\int_2^\pi (t-2)\sin t\,dt = -\left(\left[-t\cos t + \sin t\right]_2^\pi - 2\left[-\cos t\right]_2^\pi\right) = \left(\left(\left[-\pi\cos\pi + \sin\pi\right] - \left[-2\cos 2 + \sin 2\right]\right) - 2\left(\left[-\cos\pi - (-\cos 2)\right]\right)\right)$$

$$= (\pi + 4\cos 2 - \sin 2 - 2 - 2\cos 2) = \pi + 2\cos 2 - \sin 2 - 2$$

$$-\int_\pi^4 (t-2)\sin t\,dt = -\left(\left[-t\cos t + \sin t\right]_\pi^4 - 2\left[-\cos t\right]_\pi^4\right) = -\left(\left(\left[-4\cos 4 + \sin 4\right] - \left[-\pi\cos\pi + \sin\pi\right]\right) - 2\left(\left[-\cos 4 - (-\cos\pi)\right]\right)\right)$$

$$= -(-4\cos 4 + \sin 4 - \pi + 2\cos 4 + 2) = 2\cos 4 - \sin 4 + \pi - 2$$

Total distance: $(-2\cos 2 - \sin 2 + 2) + (\pi + 2\cos 2 - \sin 2 - 2) + (2\cos 4 - \sin 4 + \pi - 2) = 2\cos 4 - 2\sin 2 - \sin 4 + 2\pi - 2 \approx 1.91$

$v(t) = \int -32\,dt = -32t + C \quad v(0) = 90 \qquad v(t) = -32t + 90$

$v(3) = -32(3) + 90 = -6$ ft/sec

$s(t) = \int (-32t + 90)\,dt = -16t^2 + 90t + C \quad s(0) = 0 \qquad s(t) = -16t^2 + 90t$

$-16t^2 + 90t = 0 \Rightarrow t = 0, 5.625$

11a)

11b) It hits the ground when $s(t) = 0$.

The projectile hits the ground at $t = 5.625$ seconds.

11c) net distance = displacement = 0

11d) total distance = $2\int_0^{2.8125} (-32t + 90)\, dt = 253.125$ feet (graphing calculator used)

12) -4 + 5 – 24 = -23 cm

13) 4 + 5 + 24 = 33 cm

14) a: 15 – 4 = 11 b: 11 + 5 = 16 c: 16 – 24 = -8

15) acceleration is the slope of velocity, and the slope is greatest between 0 and b at $t = a$.

16) at $t = c$. (same logic as #15)

21) Total consumption = $\int_0^{10} 27.08e^{t/25}\, dt \approx 332.965$ barrels. (graphing calculator used)

22) Total consumption = $\int_0^{24} (3.9 - 2.4\sin(\pi t/12))\, dt = 93.6$ kilowatt-hours. (graphing calculator used)

23d) Total population = $\int_0^{2} 10{,}000(2 - r)(2\pi r)\, dr \approx 83{,}776$. (graphing calculator used)

24c) Rate of flow = $\int_0^{3} 8(10 - r^2)(2\pi r)\, dr = 396\pi \approx 1244.071$ in^3/sec. (graphing calculator used)

25a) Add 'em up!

27) $T = \frac{1}{2}(120 + 220 + 230 + 230 + 238 + 240 + 240 + 230 + 224 + 220 + 121) = 1156.5$

29)

$$F = kx \quad 6 = k(3) \Rightarrow k = 2 \quad F = 2x$$

$$a)\ F = 2(9) = 18 \text{ N}$$

$$b)\ W = \int_0^9 2x\, dx = x^2\Big|_0^9 = 81 - 0 = 81 \text{ N}\cdot\text{cm}$$

30)

$$F = kx \quad 10{,}0000 = k(1) \Rightarrow k = 1 \quad F = 10{,}000x$$

$$a)\ W = \int_0^{0.5} 10{,}000x\, dx = 5000x^2\Big|_0^{0.5} = 1250 - 0 = 1250 \text{ in - lb}$$

$$b)\ W = \int_{0.5}^{1} 10{,}000x\, dx = 5000x^2\Big|_{0.5}^{1} = 5000 - 1250 = 3750 \text{ in - lb}$$