Calculus 5.4 Fundamental Theorem of Calculus (problems inspired by Stewart Calculus © 1997, 1998)

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the given function.

1) $f(x)=\int_0^x \sqrt{t}\sin t\,dt \qquad f'(x)=\sqrt{x}\sin x$

2) $g(x)=\int_6^x \left(t^3-4\right)^2 dt \qquad g'(x)=\left(x^3-4\right)^2$

3) $h(u)=\int_\pi^u \frac{1}{3+t^5}\,dt \qquad h'(u)=\frac{1}{3+u^5}$

4) $f(x)=\int_x^{-3} \sin t^4\,dt \qquad f'(x)=-\sin x^4$

5) $f(x)=\int_0^{x^3} \sqrt{t}\sin t\,dt \qquad f'(x)=3x^2\left(\sqrt{x^3}\sin x^3\right)$

6) $g(x)=\int_1^{\sqrt{x}} \frac{k^3}{k^2-3}\,dk \qquad g'(x)=\left(\frac{\left(\sqrt{x}\right)^3}{x-3}\right)\left(\frac{1}{2\sqrt{x}}\right)=\frac{x}{2x-6}$

7) $h(t)=\int_0^{1/t} \sqrt{\cos x^2}\,dx \qquad h'(t)=\sqrt{\cos\left(\frac{1}{t}\right)^2}\left(-\frac{1}{t^2}\right)$

8) $f(x)=\int_\pi^{\sin x} t\cos\left(t^2\right)dt \qquad f'(x)=\sin x\left(\cos\left(\sin x\right)^2\right)\left(\cos x\right)$

9) $f(x)=\int_{3x}^{x} \frac{y-2}{y+3}\,dy=\int_{3x}^{0} \frac{y-2}{y+3}+\int_0^x \frac{y-2}{y+3} \qquad f'(x)=-3\left(\frac{3x-2}{3x+3}\right)+\frac{x-2}{x+3}$

10) $g(x)=\int_{\cos x}^{x^3} \frac{1}{\sqrt{3-t^2}}\,dt=\int_{\cos x}^{0} \frac{1}{\sqrt{3-t^2}}+\int_0^{x^3} \frac{1}{\sqrt{3-t^2}} \qquad g'(x)=\frac{\sin x}{\sqrt{3-(\cos x)^2}}+\frac{3x^2}{\sqrt{3-\left(x^3\right)^2}}$