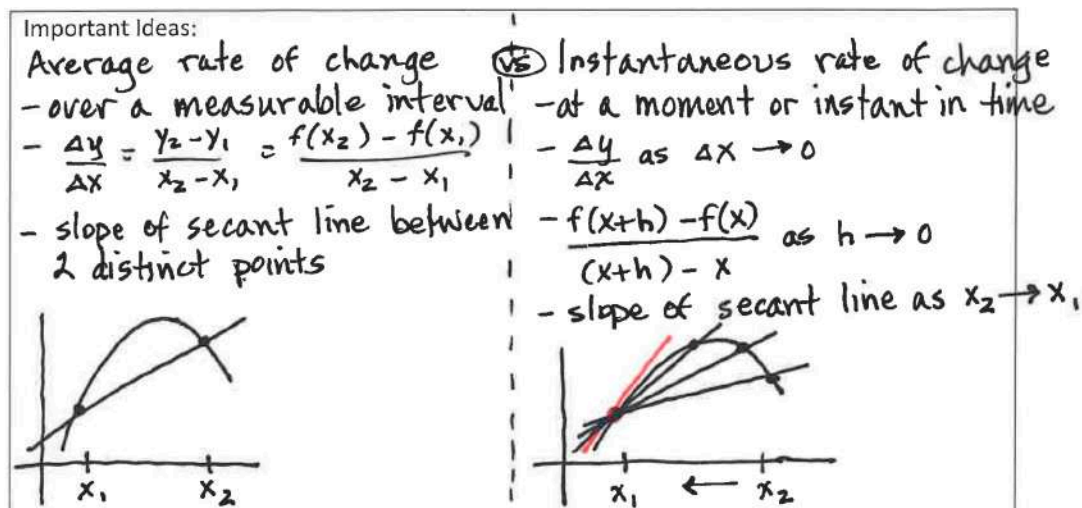


Calc Medic Important Ideas for Unit 1: Intro to Calculus

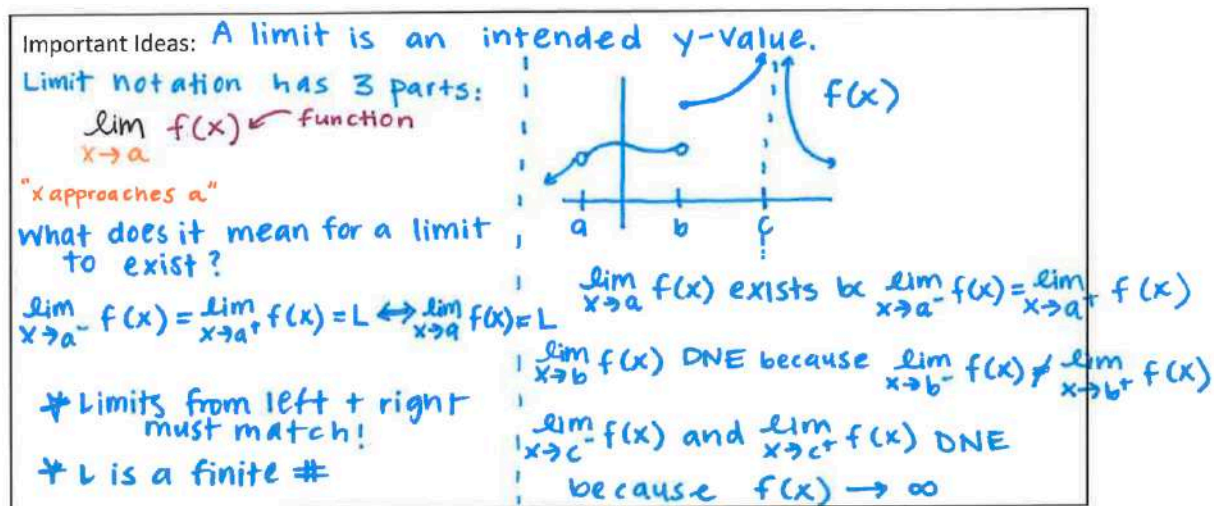
Introducing Calculus: Can Change Occur at an Instant? (Activity: A Wonder-fuel Intro to Calculus)

- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.



Defining Limits and Using Limit Notation (Activity: Can You Shoot Free Throws Like Nash?)

- Represent and interpret limits analytically using correct notation, including one-sided limits
- Estimate limits of functions using graphs or tables



Using Algebraic Approaches to Evaluate Limits (Activity: Contestants, Can You Solve This Limit?)

- Use limit properties to determine the limits of functions
- Use algebraic manipulations to determine the limits of functions

Important Ideas:

strategies to evaluate limits:

- direct substitution (try first)
- memorized forms ($\frac{\sin x}{x}$ or $\frac{|x|}{x}$...)
- factor, simplify, substitute
- algebraic manipulation (mult by a form of 1)
- graphs or tables
- limit properties to simplify functions

$\frac{k}{0} = +\infty, k \neq 0, k > 0$
 $\frac{k}{0} = -\infty, k \neq 0, k < 0$
 $\frac{0}{k} = 0, k \neq 0$
 $\frac{0}{0}, \frac{\infty}{\infty}$ are indeterminate!
 $\frac{k}{\infty} = 0$

① $\lim_{x \rightarrow a} c = c$ ② $\lim_{x \rightarrow a} c f(x) = c \lim_{x \rightarrow a} f(x)$ ③ $\lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$

④ $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$ ⑤ $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ only if this $\lim \neq 0$

Introduction to Squeeze Theorem (Activity: How Many Coffee Beans Are In The Jar?)

- Develop an understanding of bounding values and bounding functions
- Confirm the hypotheses of the Squeeze Theorem (Sandwich Theorem, Pinching Theorem, etc.) and use the theorem to justify a limit result

Important Ideas:

Conditions { If $f(x), g(x)$ and $h(x)$ are continuous functions on some interval containing a , and $g(x) \leq f(x) \leq h(x)$ on that interval,

Conclusion { then if $\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L$, then by the Squeeze Theorem

$\lim_{x \rightarrow a} f(x) = L$

How to justify w/ Squeeze Thm

- ① Verify both conditions
- ② Identify upper bound & lower bound functions
- ③ Evaluate limits of upper & lower bound functions
- ④ Make conclusion about original function's limit using the Squeeze Theorem

Continuity and Discontinuity (Activity: Soul Mates at Starbucks)

- Justify conclusions about continuity at a point using the definition.
- Determine intervals over which a function is continuous.

Important Ideas:

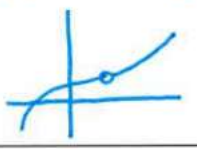
A function $f(x)$ is continuous at $x=a$ if

① $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = L$ (finite) "left and right limits match"

② $\lim_{x \rightarrow a} f(x) = f(a)$ "limit matches y-value"

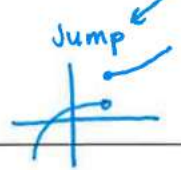
Types of Discontinuity

Removable

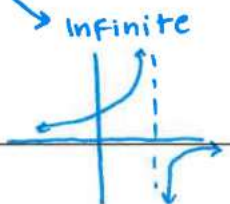


Non-removable

Jump



Infinite



Removing Discontinuities (Activity: Can This Date Be Salvaged?)

- Determine locations of removable discontinuities by graphical, numeric, or analytic methods
- Determine when and how discontinuous functions can be made continuous

Important Ideas:

Discontinuities that occur where a limit exists can be removed by defining or redefining a point on the graph. ("patch the hole")

1) Evaluate $\lim_{x \rightarrow a} f(x)$

2) If the limit exists, define $f(a)$ to be $\lim_{x \rightarrow a} f(x)$

* This creates an extended function.

Limits Involving Infinity (Activity: How Much Do We Remember From School?)

- Interpret the behavior of functions using limits involving infinity.

Important Ideas: $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$ are asking about end behavior (horizontal asymptotes)

For rational functions:

- If degree of numerator > degree of denominator, $\lim_{x \rightarrow \infty} f(x)$ DNE because $y \rightarrow \infty$
- If degree of num < degree of denom, $\lim_{x \rightarrow \infty} f(x) = 0$
- If degree of num = degree of denominator, $\lim_{x \rightarrow \infty} f(x) = \frac{\text{leading coefficient of num}}{\text{leading coefficient of denom}}$

For non-rational functions, compare dominant behavior using TFE PLC.

Tower > Factorial > Exponential > Polynomial > Logarithmic > Constant

horizontal asymptotes

Intermediate Value Theorem (Activity: Are You A 5-Star Uber Driver)

- Explain the behavior of a function on an interval using the Intermediate Value Theorem.

Important Ideas: *IVT is used to prove that a certain y-value MUST exist.

Condition: If a function $f(x)$ is continuous on $[a, b]$

Conclusion: then $f(x)$ attains or "hits" every y-value between $f(a)$ and $f(b)$.

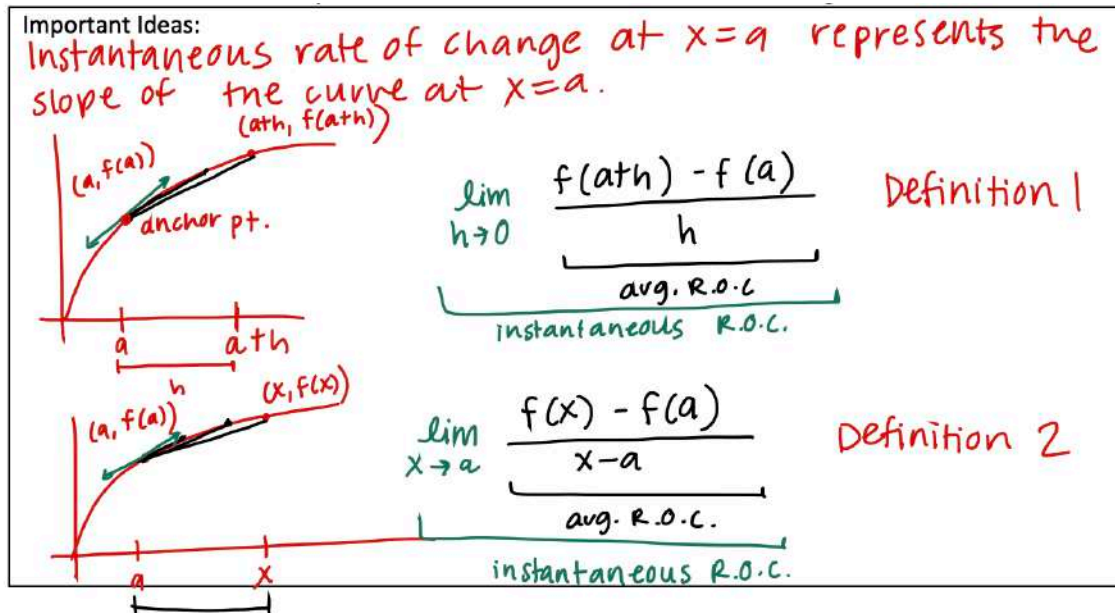
- Verify condition of continuity
- Identify $f(a)$ and $f(b)$
- check that desired value is between $f(a)$ and $f(b)$
- Make conclusion using the IVT, incorporating the question stem

Since function is continuous on $[a, b]$ and $f(a) = \underline{\hspace{2cm}}$ and $f(b) = \underline{\hspace{2cm}}$, the IVT guarantees that...

Calc Medic Important Ideas for Unit 2: Differentiation

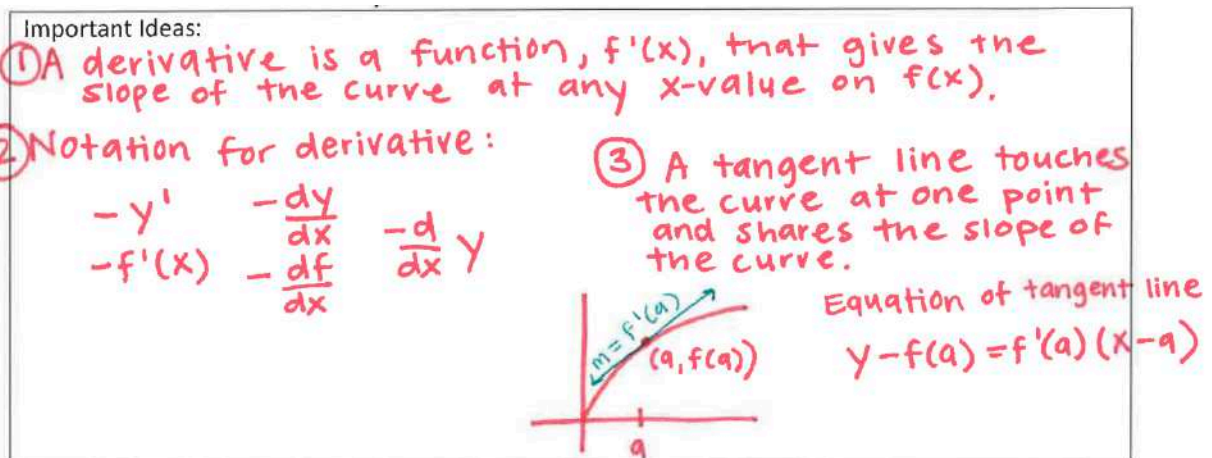
Instantaneous Rate of Change (Activity: Can a Human Break the Sound Barrier?)

- Determine average rates of change using difference quotients
- Represent the derivative of a function as the limit of a difference quotient



Defining the Derivative (Activity: The Making of a Slopes Graph)

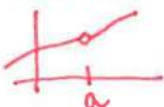

- Understand that the derivative is itself a function that outputs the slope of the curve at any point on the original function.



Continuity and Differentiability (Activity: Is This Rollercoaster Safe to Ride?)

- Estimate the derivative at a point using graphs or tables
- Explain the relationship between differentiability and continuity
- Justify how a continuous function may fail to be differentiable at a point in its domain

Important Ideas:

Differentiable functions @ $x=a$ must satisfy both conditions:	Non-differentiable functions at $x=a$:
I. $f(x)$ is continuous at $x=a$ AND II. $\lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$ * slopes must match from both sides!	* $f(x)$ not continuous  OR * slopes from both sides do <u>not</u> match 

Continuity does not imply differentiability

Derivative Shortcuts (Activity: Is There a Shortcut?)

- Calculate derivatives of familiar functions

Important Ideas:

Deriv of a constant: $\frac{d}{dx} c = 0$

Deriv of a const. multiplier: $\frac{d}{dx} c f(x) = c \frac{d}{dx} f(x) = c \cdot f'(x)$

Power Rule: $\frac{d}{dx} x^n = n x^{n-1}$ for $n \neq 0$

Deriv of Sum or Difference: $\frac{d}{dx} (f(x) \pm g(x)) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x) = f'(x) \pm g'(x)$

Derivatives of Sin x and Cos x (Activity: Toothpick Tangents)

- Calculate the derivatives of sin x and cos

Important Ideas:

If $f(x) = \sin x$, then $f'(x) = \cos x$	If $g(x) = \cos x$, then $g'(x) = -\sin x$
---	--

Derivatives of e^x and $\ln x$ (Activity: Toothpick Tangents (Part 2))

- Calculate derivatives of familiar functions

Important Ideas:

$$\text{If } f(x) = e^x \\ \text{then } f'(x) = e^x$$

$$\text{If } g(x) = \ln x \\ \text{then } g'(x) = 1/x \\ \text{"the reciprocal of } x \text{"}$$

The Product Rule (Activity: How Fast is Snapchat?)

- Calculate derivatives of products of differentiable functions
- Use the product rule in association with other derivative rules

Important Ideas:

$$\text{Let } h(x) = f(x)g(x) \\ \text{then } h'(x) = f(x)g'(x) + g(x)f'(x)$$

* Some products can be simplified to avoid using the product rule

* Some functions need to be rewritten so they appear as a product of 2 functions.

Using the Quotient Rule (Activity: Divide and Conquer)

- Use the quotient rule to find derivatives of quotients of differentiable functions
- Simplify the differentiation process by choosing the correct derivative rules

Important Ideas:

$$\text{For differentiable functions } f(x) \text{ and } g(x), \text{ and } g(x) \neq 0, \\ \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

Check results with Math 8

Check results by comparison to Product Rule
Simplify original rational function, if possible

Derivatives of Trig Functions (Activity: Tangents for Trig Functions)

- Calculate derivatives of products of differentiable functions
- Use identities to rewrite tangent, cotangent, secant, and cosecant functions and then apply derivative rules to find formulas for their derivatives
- Use the rules for derivatives of trigonometric functions in association with other derivative rules

Important Ideas:

$$\frac{d}{dx} \sin x = \cos x \quad \frac{d}{dx} \tan x = \sec^2 x \quad \frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \cos x = -\sin x \quad \frac{d}{dx} \cot x = -\csc^2 x \quad \frac{d}{dx} \csc x = -\csc x \cot x$$

Memorize these formulas!

Simplify or rewrite original functions, if possible

Calc Medic Important Ideas for Unit 3

Differentiating Composite, Implicit, and Inverse Functions

The Chain Rule (Activity: How is Lindt Chocolate Made?)

- Calculate derivatives of compositions of differentiable functions

Important Ideas:

Composite Functions

$y = f(g(x))$

outside function
inside function
" " "

Chain Rule is used for finding derivatives of composite functions.

$$y' = g'(x) \cdot f'(g(x))$$
$$\frac{dy}{dx} = \frac{d}{dx} \cdot \frac{dy}{d}$$

Implicit Differentiation (Activity: The Tangent Line Problem (Revisited))

- Find the derivative of implicitly defined functions

Important Ideas:

① Implicit functions are those where the dependent variable (y) is not isolated on one side of the equation. Ex: $x^2 + xy - y^2 = 1$

② Steps for differentiating an inverse function:

- 1) Differentiate both sides w/ respect to x .
- 2) Apply chain rule to all terms with y in them
- 3) Collect all terms w/ $\frac{dy}{dx}$ on one side of the equation
- 4) Factor out $\frac{dy}{dx}$
- 5) Solve for $\frac{dy}{dx}$ by dividing

③ To find $\frac{d^2y}{dx^2}$, the 2nd derivative, repeat the process + substitute the function for $\frac{dy}{dx}$

Derivatives of Inverse Functions (Activity: What's Your Slope?)

- Calculate derivatives of inverse functions

Important Ideas:
An inverse function is a function that reverses or "undoes" another function. If $f(a) = b$, then $f^{-1}(b) = a$.
An inverse function exists if the original function is one-to-one. (passes Horizontal Line Test)
* Slopes at inverse points are reciprocals!
If (a, b) is on the graph of f , and g is the inverse of f , then
$$g'(b) = \frac{1}{f'(a)}$$

In general:
$$\frac{d}{dx}(g(x)) = \frac{1}{f'(g(x))}$$

Annotations:
- $\frac{d}{dx}(g(x))$: the derivative of g
- $f'(g(x))$: the derivative of f
- $f'(g(x))$: evaluated at the inverse pt.
- $\frac{1}{f'(g(x))}$: the reciprocal of

Derivatives of Inverse Trigonometric Functions (Activity: Getting Triggy With It)

- Calculate derivatives of inverse trig functions

Important Ideas:

$$\frac{d}{dx}(\arcsin x) = \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arccos x) = \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\arctan x) = \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Calc Medic Important Ideas for Unit 4: Contextual Applications of Differentiation

Interpreting the Meaning of the Derivative in Context (Activity: A Summer Day of Calculus)

- Interpret the meaning of a derivative in context.

Important Ideas:

- The derivative represents the rate of change of the dependent variable with respect to the independent variable.

$$f'(x) = \frac{df}{dx}$$
- Units of $f'(x) = \frac{\text{units of } f(x)}{\text{units of } x}$
- Units of $f''(x) = \frac{\text{units of } f'(x)}{\text{units of } x}$
- Interpreting $y'(x)$
 "At $x = \underline{\hspace{2cm}}$, the y-context is increasing/decreasing at a rate of $\underline{\hspace{2cm}}$ (correct units)"

Connecting Position, Velocity and Acceleration (Activity: The Lovely Ladybug)

- Calculate rates of change in the context of straight-line motion.

Important Ideas:

- Position = $s(t)$
 Velocity = speed w/ direction
 $v(t) = s'(t)$
 Acceleration
 $a(t) = v'(t) = s''(t)$
- $v(t) = 0 \Rightarrow$ object is at rest
 $v(t) > 0 \Rightarrow$ object is moving right or up
 $v(t) < 0 \Rightarrow$ object is moving left or down
- | | |
|---------------------------|----------------------------|
| $a(t) > 0$
speeding up | $a(t) < 0$
slowing down |
| $v(t) > 0$ | $v(t) < 0$ |
| slowing down | speeding up |

Compare sign of velocity & acceleration

Rates of Change in Applied Contexts Other than Motion (Activity: How Many Shoppers on Black Friday?)

- Interpret rates of change in applied contexts.

Important Ideas:

- $y'(t)$ is the rate of change of $y(t)$.
- $y'(t) = 0 \Rightarrow$ y-context is not changing.
- $y'(t) > 0 \Rightarrow$ y-context is increasing.
- $y'(t) < 0 \Rightarrow$ y-context is decreasing.

Calculator Tips

- For rate in/ rate out problems:
 $Y_1 =$ rate in
 $Y_2 =$ rate out
 $Y_3 = Y_1 - Y_2$
- Always round to at least 3 decimal places!

Intro to Related Rates (Activity: Birthday Balloons)

- Calculate and interpret related rates in applied contexts

Important Ideas:

Related Rate Problems

- ① Draw a picture.
 - ② Write an equation that relates all the variables in the problem (usually a volume formula or Pythagorean Thm)
 - ③ Take derivative of both sides
Don't forget chain rule!
 - ④ Plug in known values + solve for the quantity you are after.
- * Determine based on context if the given rates are positive or negative.

Related Rates (Activity: "Coney" Island)

- Calculate and interpret related rates in applied contexts.

Important Ideas:

Cone Problems

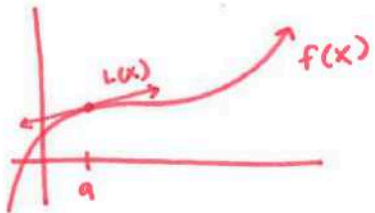
- 1) Relate the radius to the height using similar Δ s
- 2) Write volume equation only in terms of r , or only in terms of h , depending on what info is given/needed
- 3) Take derivative of both sides and solve for desired quantity
- 4) Use equation from step 1 to find rate of change of eliminated variable.

Approximating Values of a Function Using Local Linearity and Linearization (Activity: Close Enough is Good Enough!)

- Approximate the value on a curve using the equation of a tangent line

Important Ideas:

Tangent lines can be used to approximate function values near the point of tangency.



For x-values near $x=a$,
 $L(x) \approx f(x)$

- If $f(x)$ is concave up, $L(x)$ gives an underestimate. (tangent line is below curve)
- If $f(x)$ is concave down, $L(x)$ gives an overestimate. (tangent line is above curve)

L'Hospital's Rule (Activity: Mixed Messages)

- Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.

Important Ideas:

Limits resulting in $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ are considered indeterminate.

Consider the ratio of growth rates instead!

L'Hospital's Rule:

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is indeterminate, evaluate $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

If the result is still indeterminate, repeat the process by finding another derivative.

Calc Medic Important Ideas for Unit 5: Analytical Applications of Derivatives

The Mean Value Theorem (Activity: Can Calculus Get You Fined?)

- Justify conclusions about functions by applying the MVT over an interval.

Important Ideas:

Conditions: If a function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) ,

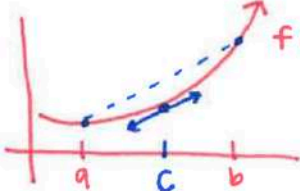
Conclusion: then there exists a value c for $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

instantaneous R.O.C. average R.O.C.

How to prove w/ MVT:

- 1) check conditions
- 2) Find average R.O.C. on $[a, b]$
- 3) Make conclusion using MVT, incorporating the question stem



Extreme Value Theorem, Absolute vs. Relative Extrema, and Critical Points (Activity: What's the Value of Apple Stock?)

- Justify conclusions about functions by applying the Extreme Value Theorem.
- Distinguish between absolute and relative extrema and critical points.

Important Ideas:

- ① **Extreme value Theorem:**
If a function $f(x)$ is continuous on $[a, b]$ then $f(x)$ must attain a maximum and minimum value on $[a, b]$.
- ② $f(x)$ has an absolute maximum at $x=c$ if $f(c) \geq f(x)$ for ALL x .
 $f(x)$ has a relative maximum at $x=c$ if $f(c) \geq f(x)$ for x near c .
 $f(x)$ has an absolute minimum at $x=c$ if $f(c) \leq f(x)$ for ALL x .
 $f(x)$ has a relative minimum at $x=c$ if $f(c) \leq f(x)$ for x near c .
- ③ **Critical points** are points where the derivative is 0 or undefined.

Determining Function Behavior from the First Derivative (Activity: Playing the Stock Market)

- Determine behaviors of a function based on the derivative of that function.

Important Ideas:

- $f(x)$ is increasing if $f'(x) > 0$
 $f(x)$ is decreasing if $f'(x) < 0$
- First Derivative Test for Relative (Local) Extrema**
 - Identify critical points ($f'(x) = 0$ or undefined)
 - Make a labeled sign chart by testing values between the critical values to determine if f' is pos or neg
 - Make conclusion
 - " $f(x)$ has a relative max of ____ at $x = \underline{\hspace{1cm}}$ because f' changes from positive to negative."
 - " $f(x)$ has a relative min of ____ at $x = \underline{\hspace{1cm}}$ because f' changes from negative to positive"

Using the Candidates Test to Determine Absolute Extrema (Activity: Are You a Stock Market Master?)

- Justify conclusions about the behavior of function based on its derivative.

Important Ideas:

Finding absolute (global) maxima + minima

- Find and list all the critical values and endpoints
- Compare the values of the function at all these locations (Make a table!)
- Write conclusion

x	$f(x)$

" $f(x)$ has an absolute max of ____ at $x = \underline{\hspace{1cm}}$."

" $f(x)$ has an absolute min of ____ at $x = \underline{\hspace{1cm}}$."

* Must refer to Candidates Test in justification

Analyzing Function Behavior with the Second Derivative (Activity: How Fast Does the Flu Spread?)

- Justify conclusions about the behavior of function based on its second derivative

Important Ideas:

$f''(x)$ tells us how $f'(x)$ (the slopes of f) are changing.

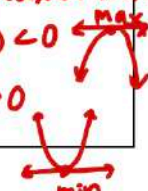
$f''(x) > 0 \Rightarrow f'(x)$ is increasing $\Rightarrow f(x)$ is concave up

$f''(x) < 0 \Rightarrow f'(x)$ is decreasing $\Rightarrow f(x)$ is concave down

$f''(x) = 0$ AND changes sign $\Rightarrow f'(x)$ has a rel max or min $\Rightarrow f(x)$ has a pt. of inflection

2nd derivative test (using concavity to determine max or min)

- $f(x)$ has a rel. max at $x = c$ if $f'(c) = 0$ and $f''(x) < 0$
- $f(x)$ has a rel. min at $x = c$ if $f'(c) = 0$ and $f''(x) > 0$



Optimization (Activity: Canalysis)

- Use derivatives to solve optimization problems.
- Interpret maximums and minimums in applied contexts.

Important Ideas:

Optimization is about finding a max or min in applied contexts

- ① Write an equation for the quantity that is to be maximized or minimized (volume, area, cost, distance, etc.)
- ② Use the constraints to find relationships between the variables
- ③ Rewrite your equation with only 1 variable
- ④ Use the 1st and 2nd derivative tests to find critical values and extrema

Exploring Behaviors of Implicit Relations (Activity: What About Us?)

- Determine critical points of implicit relations.
- Justify conclusions about the behavior of an implicitly defined relation based on evidence from its derivatives.

Important Ideas:

Implicit differentiation can be used to find 1st and 2nd derivatives of relations.

Critical points are the x and y values where $\frac{dy}{dx} = 0$ or is undefined.

*Make sure they satisfy the original equation!

A curve is increasing when $\frac{dy}{dx} > 0$ and decreasing when $\frac{dy}{dx} < 0$. (Specify x and y values!)

A curve is concave up when $\frac{d^2y}{dx^2} > 0$ and concave down when $\frac{d^2y}{dx^2} < 0$ (Specify x and y values!)

Calc Medic Important Ideas for Unit 6: Integration and Accumulation of Change

Exploring Accumulation of Change (Activity: How Much Snow Is On Janet's Driveway?)

- Interpret the meaning of area under a rate of change function in context.

Important Ideas:

- ① Accumulated quantity = Rate 1 $\cdot \Delta t_1$ + Rate 2 $\cdot \Delta t_2$ + ... + Rate $n \cdot \Delta t_n$
- ② The accumulation of a quantity is represented by the area underneath its derivative curve.
- ③ Units for area underneath a rate of change curve:
Units of rate of change \cdot Units of independent variable
- ④ Area above x-axis is positive area \Rightarrow positive accumulation \Rightarrow quantity is increasing
Area below x-axis is negative area \Rightarrow negative accumulation \Rightarrow quantity is decreasing

Approximating Areas with Riemann Sums (Activity: Fast and Curious)

- Approximate area under a curve using geometric and numerical methods

Important Ideas:

To approximate the area on $[a, b]$ with n equal subdivisions use $\frac{b-a}{n}$ as the width of each rectangle.

To find the height of each rectangle:

- LRAM - Evaluate function at left endpoint of each interval
- RRAM - Evaluate function at right endpoint of each interval
- MRAM - Evaluate function at midpoint of each interval

Total area/accumulation = $\Delta x_1 \cdot f(x_1) + \Delta x_2 \cdot f(x_2) + \dots + \Delta x_n \cdot f(x_n)$

If a function is increasing,

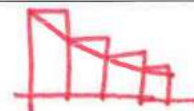
LRAM gives an underestimate

RRAM gives an overestimate

If a function is decreasing,

LRAM gives an overestimate

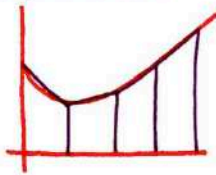
RRAM gives an underestimate



Approximating Areas with Trapezoids (Activity: Fast and Curious (Part 2))

- Approximate area under a curve using geometric and numerical methods

Important Ideas:



Area of a trapezoid = $\frac{f(b_1) + f(b_2)}{2} \cdot h$

Evaluate function at left endpoint
Evaluate function at right endpoint
 Δx

Area under $f(x) \approx$ sum of trapezoids

A trapezoidal sum gives the average of LRAM and RRAM values.

Riemann Sums, Summation Notation, and Definite Integrals (Activity: How Confident Are You?)

- Interpret and represent an infinite Riemann sum as a definite integral

Important Ideas:

To find exact area under a curve $f(x)$ over the interval $[a, b]$
Use infinitely many rectangles of infinitesimally small widths.

Summation Notation:

$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$

of rectangles
height of rectangle
width of rectangle

Integral Notation:

$\int_a^b f(x) dx$

upper limit of integration
lower limit of integration
integrand
variable of integration

The Fundamental Theorem of Calculus and Accumulation Functions (Activity: Under Cover)

- Represent accumulation functions using definite integrals.
- Find derivatives of accumulation functions.

Important Ideas:

An accumulation function outputs the area under a curve from some starting value to x (the input).

Independent variable x
rate of accumulation
starting value c

$F(x) = \int_c^x f(t) dt$

FTC (Part I): Rate of change of an accumulation function

$\frac{d}{dx} \int_c^x f(t) dt = \frac{d}{dx} F(x) = f(x)$

Applying Properties of Definite Integrals (Activity: #2020 Goals)

- Calculate a definite integral using areas and properties of definite integrals.

Important Ideas:
For $a < b < c$

$$\textcircled{1} \int_a^c f(t) dt = \int_a^b f(t) dt + \int_b^c f(t) dt$$

$$\textcircled{2} \int_b^a f(t) dt = -\int_a^b f(t) dt$$

$$\textcircled{3} \int_a^b [f(t) \pm g(t)] dt = \int_a^b f(t) dt \pm \int_a^b g(t) dt$$

$$\textcircled{4} \int_a^b k f(t) dt = k \int_a^b f(t) dt$$

$$\textcircled{5} \int_a^a f(t) dt = 0$$

The FTC and Definite Integrals (Activity: Go Figure)

- Evaluate definite integrals analytically using the Fundamental Theorem of Calculus.

Important Ideas:

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a)$$

where $F(x)$ is the antiderivative of $f(x)$ ($F'(x) = f(x)$)

$$\underbrace{F(b)}_{\text{Final value}} = \underbrace{F(a)}_{\text{Initial value}} + \underbrace{\int_a^b f(x) dx}_{\text{Accumulated change}}$$

Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation (Activity: A Match Made in Heaven)

- Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.

Important Ideas:

$\textcircled{1}$ Given a function $f(x)$, the most general antiderivative of $f(x)$ is given by

$$\int f(x) dx = F(x) + C \quad \text{where } F'(x) = f(x)$$

\int indefinite integral (no upper/lower bounds) C constant of integration

$\textcircled{2}$ Antiderivative Rules

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

Integration using Substitution (Activity: Which One Doesn't Belong?)

- Use u-substitution to find antiderivatives of composite functions

Important Ideas:

$$\frac{d}{dx}(f(u(x))) = u'(x) \cdot f'(u(x))$$

$$\int u'(x) \cdot f'(u(x)) dx = f(u(x)) + C$$

Integration with substitution reverses the chain rule!

derivative of inner function: $u'(x)$
inner function: $u(x)$
antiderivative of f' : $f(u(x))$

- Set u = inner function
- Find $\frac{du}{dx}$. Solve for du .
- "Correct" the integrand so it matches du exactly.
(can only multiply by a scalar form of 1)
- Rewrite the entire integral in terms of u .
- Find the antiderivative.
- For indefinite integrals, plug in the expression for u

Riemann Sums, Summation Notation, and Definite Integral Notation (Activity: Returning to Riemann)

- Interpret and represent an infinite Riemann sum as a definite integral

Important Ideas:

A definite integral can be represented by an infinite Riemann sum.

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \underbrace{f(x_i)}_{\text{height of } i^{\text{th}} \text{ rectangle}} \underbrace{\Delta x}_{\text{width of rectangle}}$$

where n is the # of Partitions/rectangles

Assuming equal partitions:

$$\Delta x = \frac{b-a}{n}$$

$$x_i = a + i \Delta x$$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \underbrace{f\left(a + i \left(\frac{b-a}{n}\right)\right)}_{\text{height}} \cdot \underbrace{\frac{b-a}{n}}_{\text{width}}$$

Calc Medic Important Ideas for Unit 7: Differential Equations

Modeling Differential Equations and Verifying Solutions (Activity: How Long Does Coffee Stay Hot?)

- Interpret differential equations given in context
- Verify solutions to differential equations

Important Ideas:

A differential equation is any equation that contains a derivative expression.

$\frac{dy}{dx}$ is a first order derivative, $\frac{d^2y}{dx^2}$ is a second order derivative, ...

Diff eqs. may contain the original function or be written in terms of the independent variable only.

If a rate is proportional to the current quantity then $\frac{dy}{dx} = Ky$ where "K" is the constant of proportionality.

Slope Fields (Activity: Seeing is Believing)

- Create slope fields
- Estimate solutions to differential equations

Important Ideas:

Slope fields are a graphical representation of a differential equation that allow us to visualize the family of solution curves.

Making a slope field:

- calculate slope at various ordered pairs
- plot slopes using short line segments

A solution curve will follow the trend of the slopes and must pass through the initial condition if given.

If $\frac{dy}{dx}$ is undefined, do NOT draw a slope there.

Finding General Solutions using Separation of Variables (Activity: Are you a Solution Seeker?)

- Determine general solutions to differential equations

Important Ideas:

To solve a first-order differential equation of the form $\frac{dy}{dx} = f(x)g(y)$, use separation of variables.

① Separate (Move all terms w/ y to one side and all terms w/ x to the other)

② Integrate (Don't forget tc!)

③ Isolate (Get y by itself, watch out for \pm solutions)

Finding Particular Solutions using Initial Conditions and Separation of Variables (Activity: How many Sea Lions are on Elliott Bay?)

- Determine particular solutions to differential equations

Important Ideas:

① Some particular solutions can't be written explicitly and must be defined by integrals.

If $\frac{dy}{dx} = f(x)$ and $F(a) = y_0$ then

$F(x) = y_0 + \int_a^x f(t)dt$ is a solution
antiderivative of $f(x)$

② Finding particular solutions requires 2 extra steps

Separate Integrate Solve for C Isolate select

Remember SISIS!

Exponential Models with Differential Equations (Activity: How Fast is the Coronavirus Spreading?)

- Interpret the meaning of a differential equation and its variables in context

Important Ideas:

$\frac{dy}{dt} = ky \Rightarrow$ "rate of change of a quantity is proportional to the quantity" \Rightarrow exponential growth/decay model

$k > 0 \Rightarrow$ growth

$k < 0 \Rightarrow$ decay

$\frac{dy}{dt} = ky$ has solutions of the form $y = y_0 e^{kt}$
initial condition when $t=0$

Calc Medic Important Ideas for Unit 8: Applications of Integration

Average Value of a Function (Activity: Finding the Perfect Rectangle)

- Determine the average value of a function using definite integrals

Important Ideas:

The average value of a continuous function $f(x)$ on the interval $[a, b]$ is the height of the rectangle that encompasses the same area as the area under the curve.

$$c^*(b-a) = \int_a^b f(x) dx$$
$$c^* = \frac{1}{b-a} \int_a^b f(x) dx$$

Connecting Position, Velocity, and Acceleration using Integrals (Activity: Whitney's Bike Ride)

- Determine values for positions and rates of change using definite integrals in problems involving rectilinear motion

Important Ideas:

Given velocity $v(t)$ and position $s(t)$

$$\int_a^b v(t) dt = \underbrace{s(b) - s(a)}_{\substack{\text{change in position} \\ \text{net distance} \\ \text{displacement}}}$$

or $s(b) = \underbrace{s(a)}_{\substack{\text{Final} \\ \text{position}}} + \underbrace{\int_a^b v(t) dt}_{\substack{\text{Initial} \\ \text{position} \\ \text{change in} \\ \text{position}}}$

$$\text{Total distance traveled on } [a, b] = \int_a^b |v(t)| dt$$

Using Accumulation and Definite Integrals in Applied Contexts (Activity: How many People are at the Met?)

- Interpret the meaning of a definite integral in accumulation problems
- Determine net change using definite integrals in applied contexts

Important Ideas:

Given a quantity $y(t)$ and its rate of change $y'(t)$

① $\int_a^b y'(t) dt = y(b) - y(a)$
 integral of a rate of change = net change in quantity

② The accumulation equation:

$$y(t) = y(a) + \int_a^t y'(x) dx$$

③ Sometimes $y'(t)$ consists of a rate in and a rate out.
 Calc tips:
 $y_1 = \text{rate in}$
 $y_2 = \text{rate out}$
 $y_3 = y_1 - y_2$

Finding Areas between Curves Expressed as Functions of x (Activity: How Rich are the Top 1%?)

- Calculate areas in the plane using the definite integral

Important Ideas:

• Area between $f(x)$ and $g(x)$ on $[a, b]$ when $f(x) \geq g(x)$ is given by $A = \int_a^b (f(x) - g(x)) dx$
 (upper curve - lower curve)

• The region must be bounded.

• Area is ALWAYS positive.

• Sometimes the upper + lower functions switch!

$$A = \int_a^b (f(x) - g(x)) dx + \int_b^c (g(x) - f(x)) dx$$

Finding Areas between Curves Expressed as Functions of y (Activity: How Do You Build a Deck?)

- Calculate areas in the plane using the definite integral

Important Ideas:

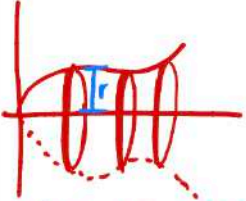
If the upper curve requires 2 or more definitions, consider using a right curve and a left curve (horizontal rectangles)

Area of Region $R = \int_c^d (f(y) - g(y)) dy$
 (right curve - left curve) width

Volume using the Disc Method (Activity: What is the Volume of a Pear?)

- Calculate volumes of solids of revolution using definite integrals

Important Ideas:
Some solids are generated by revolving 2-d regions around an axis of revolution.



If the axis of revolution is a boundary of the region, then each slice is a disk.

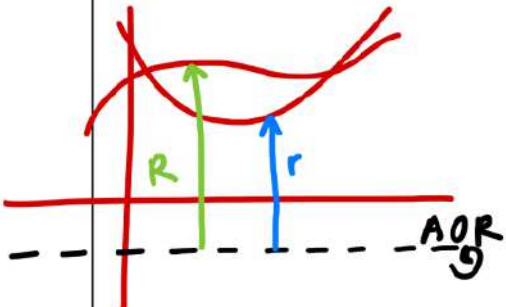
$$V = \int_a^b \pi r^2 dx = \pi \int_a^b r^2 dx \text{ or } V = \pi \int_c^d r^2 dy$$

How to find the radius:
Draw line from AOR to the curve, then "upper-lower" or "further right-closer"

Volume using the Washer Method (Activity: What's the Volume of a Bagel?)

- Calculate volumes of solids of revolution using definite integrals

Important Ideas:
use washers method when there is a space/gap between the region and axis of revolution.



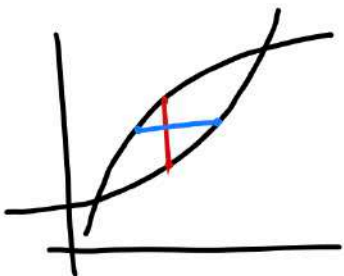
- 1) Draw sample radii from the axis of revolution to boundaries of region.
- 2) Write an expression for R and r . Think "upper-lower" or "further right-closer"
- 3) Set-up integral

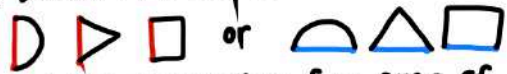
$$\pi \int_a^b [(R(x))^2 - (r(x))^2] dx \text{ or } \pi \int_c^d [(R(y))^2 - (r(y))^2] dy$$

Volumes With Cross Sections (Activity: The Best Thing Since Sliced Bread)

- Calculate volumes of solids with known cross sections using definite integrals

Important Ideas:



- 1) Draw a sample cross-section

- 2) Write expression for area of one cross-section
 $A(x)$ or $A(y)$
- 3) Set-up integral

Perpendicular to x-axis or Perpendicular to y-axis

$$\int_a^b A(x) dx \text{ or } \int_c^d A(y) dy$$