Calculus p178

1) $2e^x$

2) $e^{2x}(2) = 2e^{2x}$

3) $e^{-x}(-1) = -e^{-x}$

4) $e^{-5x}(-5) = -5e^{-5x}$

5) $e^{2x/3}\left(\frac{2}{3}\right) = \frac{2}{3}e^{2x/3}$

6) $e^{-x/4}\left(-\frac{1}{4}\right) = -\frac{1}{4}e^{-x/4}$

7) $e^2 - e^x$

8) $x^2e^x + 2xe^x - \left(xe^x + e^x\right) = x^2e^x + xe^x - e^x$

9) $e^{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

10) $e^{x^2}(2x) = 2xe^{x^2}$

11) $8^x \ln 8$

12) $9^{-x} \ln 9(-1) = -9^{-x} \ln 9$

13) $3^{\csc x} \ln 3(-\csc x \cot x)$

14) $3^{\cot x} \ln 3(-\csc^2 x)$

15) $\frac{1}{x^2}(2x) = \frac{2}{x}$

16) $2(\ln x)\left(\frac{1}{x}\right) = \frac{2\ln x}{x}$

17) $\frac{1}{1/x}\left(\frac{-1}{x^2}\right) = x\left(\frac{-1}{x^2}\right) = \frac{-1}{x}$

18) $\frac{1}{10/x}\left(\frac{-10}{x^2}\right) = \frac{x}{10}\left(\frac{-10}{x^2}\right) = \frac{-1}{x}$

19) $\frac{1}{\ln x}\left(\frac{1}{x}\right) = \frac{1}{x\ln x}$

20) $x\left(\frac{1}{x}\right) + (1)\ln x - 1 = \ln x$

21) $\left(\frac{1}{x^2 \ln 4}\right)(2x) = \frac{2}{x\ln 4}(\text{my answer}) = \frac{2}{x\ln 2^2} = \frac{2}{x(2)\ln 2} = \frac{1}{x\ln 2}(\text{book answer})$

22) $\frac{1}{\sqrt{x}\ln 5}\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{2x\ln 5}$

23) $\frac{1}{(1/x)\ln 2}\left(\frac{-1}{x^2}\right) = \frac{-1}{x\ln 2}$

24) $-1(\log_2 x)^{-2}\left(\frac{1}{x\ln 2}\right)(1) = \frac{-1}{x(\log_2 x)^2 \ln x}$

25) $\ln 2\left(\frac{1}{x\ln 2}\right)(1) = \frac{1}{x}$

26) $\frac{1}{(1+x\ln 3)\ln 3}(\ln 3) = \frac{1}{1+x\ln 3}$

27) $\frac{1}{e^x \ln 10}(e^x) = \frac{1}{\ln 10}$

28) $\frac{1}{10^x}(10^x \ln 10) = \ln 10$

29)

$$\frac{dy}{dx}=3^x\ln 3 \quad 3^x\ln 3=5\Rightarrow 3^x=\frac{5}{\ln 3}\Rightarrow x\ln 3=\ln\left(\frac{5}{\ln 3}\right)\Rightarrow x=\frac{\ln\left(\frac{5}{\ln 3}\right)}{\ln 3}\Rightarrow x\approx 1.379$$

$$y=3^{1.379}+1\approx 5.549 \qquad (1.379, 5.549)$$

30)

$$\frac{dy}{dx}=2e^x \qquad 2e^x=\frac{1}{3}\Rightarrow e^x=\frac{1}{6}\Rightarrow \ln e^x=\ln\left(\frac{1}{6}\right)\Rightarrow x=-1.792$$

$$y=2e^{-1.792}-1\approx -0.667 \qquad (-1.792, -0.667)$$

31)

$$\frac{dy}{dx}=\frac{1}{2x}(2)=\frac{1}{x}$$

equation of tangent line: $y=mx$

slope of curve will equal slope of tangent at point of tangency, so

$$y=\left(\frac{1}{x}\right)x\Rightarrow y=1 \qquad 1=\ln(2x)\Rightarrow e^1=e^{\ln(2x)}\Rightarrow 2x=e\Rightarrow x=\frac{e}{2}$$

point of tangency $\left(\frac{e}{2},1\right)$ slope between pt. of tan. and origin:

$$\frac{1-0}{\frac{e}{2}-0}=\frac{2}{e}$$

32)

$$\frac{dy}{dx}=\frac{1}{x/3}\left(\frac{1}{3}\right)=\frac{1}{x}$$

equation of tangent line: $y=mx$

slope of curve will equal slope of tangent at point of tangency, so

$$y=\left(\frac{1}{x}\right)x\Rightarrow y=1 \qquad 1=\ln\left(\frac{x}{3}\right)\Rightarrow e^1=e^{\ln(x/3)}\Rightarrow \frac{x}{3}=e\Rightarrow x=3e$$

point of tangency $(3e,1)$ slope between pt. of tan. and origin:

$$\frac{1-0}{3e-0}=\frac{1}{3e}$$

33) $\frac{dy}{dx}=\pi x^{\pi-1}$

34) $\frac{dy}{dx}=\left(1+\sqrt{2}\right)x^{1+\sqrt{2}-1}=\left(1+\sqrt{2}\right)x^{\sqrt{2}}$

35) $\frac{dy}{dx}=-\sqrt{2}x^{-\sqrt{2}-1}$

36) $\frac{dy}{dx} = (1-e)x^{1-e-1} = (1-e)x^{e}$

37) $f'(x) = \frac{1}{x+2}(1) = \frac{1}{x+2}$ Domain: (-2,∞)

38) $f'(x) = \frac{1}{2x+2}(2) = \frac{1}{x+1}$ Domain: (-1, ∞)

39) $f'(x) = \frac{1}{2-\cos x}(\sin x) = \frac{\sin x}{2-\cos x}$ Domain: (-∞, ∞)

40) $f'(x) = \frac{1}{x^2+1}(2x) = \frac{2x}{x^2+1}$ Domain: (-∞, ∞)

41) $f'(x) = \frac{1}{(3x+1)\ln 2}(3) = \frac{3}{(3x+1)\ln 2}$ Domain: (1/3, ∞)

42) $f'(x) = \frac{1}{\sqrt{x+1}\ln 10}\left(\frac{1}{2\sqrt{x+1}}\right) = \frac{1}{2(x+1)\ln 10}$ Domain: (-1, ∞)

49)

$y' = e^x$, goes through the points $(0,0)$ and (x, e^x) $\quad e^x - 0 = e^x(x-0) \Rightarrow x = 1$

Point of tangency is $(1, e)$. Equation of tangent is $y - e = e(x-1)$, or $y = ex$

50) Trick question. The curve $y = xe^x$ passes through (0,0), so the equation of the tangent that passes through (0,0) must be $y = x$, as the slope of the curve is given by $e^x + xe^x$, which equals 1 at (0,0).

51) a) Initial number occurs when $t = 0$: $P(0) = \frac{300}{1+2^4} = \frac{300}{17} \approx 18$ students.

b) $\frac{dP}{dt} = \frac{(1+2^{4-t})(0) - 300(-2^{4-t}\ln 2)}{(1+2^{4-t})^2}$ $\quad \frac{dP}{dt}(4) = \frac{-300(-2^0 \ln 2)}{(1+2^0)^2} = \frac{300\ln 2}{4} \approx 52$ students per day.

c) $150 = \frac{300}{1+2^{4-t}} \Rightarrow 150 + 150(2^{4-t}) = 300 \Rightarrow 2^{4-t} = 1 \Rightarrow (4-t)\ln 2 = 0 \Rightarrow t = 4$. Rate is 52 students/day.

52) a) Initial number occurs when $t = 0$: $P(0) = \frac{200}{1+e^{5-0}} = \frac{200}{1+e^5} \approx 1$ student.

b) $\frac{dP}{dt} = \frac{(1+e^{5-t})(0) - 200(-e^{5-t})}{(1+e^{5-t})^2}$ $\quad \frac{dP}{dt}(4) = \frac{-200(-e^1)}{(1+e^1)^2} = \frac{300\ln 2}{4} \approx 39$ students per day.

c) $100 = \frac{200}{1+e^{5-t}} \Rightarrow 100 + 100(e^{5-t}) = 200 \Rightarrow e^{5-t} = 1 \Rightarrow 5 - t = 0 \Rightarrow t = 5$. Max. rate is after 5 days.

Rate is $\frac{-200\left(-e^{0}\right)}{\left(1+e^{0}\right)^{2}} = \frac{200}{4} = 50$ students/day.

53) $\frac{dA}{dt} = \frac{20\left(\frac{1}{2}\right)^{t/140}\left[\ln\left(\frac{1}{2}\right)\right]}{140}$ $\quad \frac{dA}{dt}(2) \approx 0.098$ grams/day.

54) a) $\frac{d}{dx}\left(\ln(kx)\right) = \frac{1}{kx}(k) = \frac{1}{x}$

b) $\ln(kx) = \ln k + \ln x \quad \frac{d}{dx}\left(\ln k + \ln x\right) = 0 + \frac{1}{x} = \frac{1}{x}$

55) a) $f'(x) = 2^{x}\ln 2 \quad f'(0) = 2^{0}\ln 2 = \ln 2$

b) $f'(x) = \lim_{h\to 0}\frac{2^{h}-2^{0}}{h} = \lim_{h\to 0}\frac{2^{h}-1}{h}$

c) ln2

d) ln7